

# A Market Based Approach to Evaluate the Efficiency of Transmission Loss Allocation

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**Abstract - In the deregulated electrical power system, the allocation of transmission losses is becoming a key issue. The electric transmission power system is more and more constrained due to the increasing volume of power flows, while the active power losses' costs are increasing. In this context, the traditional "postage stamp" allocation is no longer sufficient to give right incentives. One solution could be to charge each actor depending on his responsibilities on the active power losses. This is why a number of transmission loss allocation (TLA) methods have recently been proposed. Their economic impact, however, has rarely been taken into account. This paper resumes the most common TLA methods and designs a frame to compare their efficiency. A simplified market design is proposed in order to evaluate the impact of the integration of transmission loss costs. These methods are applied to the IEEE 14 bus system. An over cost of 1.16% of the global production costs is then estimated for the "postage stamp" allocation while flow based methods reach 0.38%. The influence of the simulation parameters is analyzed.**

**Keywords: power system economics, load flow analysis, transmission losses, market efficiency**

## 1. INTRODUCTION

In the deregulated electrical power system, the allocation of transmission losses is becoming a key issue. The electric transmission power system is more and more constrained due to the increasing volume of power flows, while the active power losses' costs are increasing. In this context, the traditional "postage stamp" allocation (PS) [1] is no longer sufficient to give right incentives [2]. One solution could be to charge each user depending on his responsibilities on the active power losses [3]. This is why a number of transmission loss allocation (TLA) methods have recently been proposed [4]. Their economic impact, however, has

rarely been taken into account. This paper resumes the most common TLA methods and designs a frame to compare their efficiency. The efficiency of TLA methods is then estimated in case of the IEEE 14-bus system. Further, the influence of the simulation parameters is analyzed.

## 2. TRANSMISSION LOSS ALLOCATION

Allocating transmission losses to users, generators or loads, requires the knowledge of the power system state. Injections and consumptions are supposed to be known, as well as the characteristics of the network itself. Power flows and losses are deduced from a load flow program. The goal is to assess responsibilities of each user over the active losses, which are known as the difference between injected and demanded active power. A non-exhaustive set of interesting allocation methods is resumed in the following and presented in Fig. 1.

A domain approach (MD) has been developed [3] in order to define responsibility areas for each user with respect to direct active power flows. Assuming a proportional repartition at each node, a flow allocation is made, leading to a loss allocation.

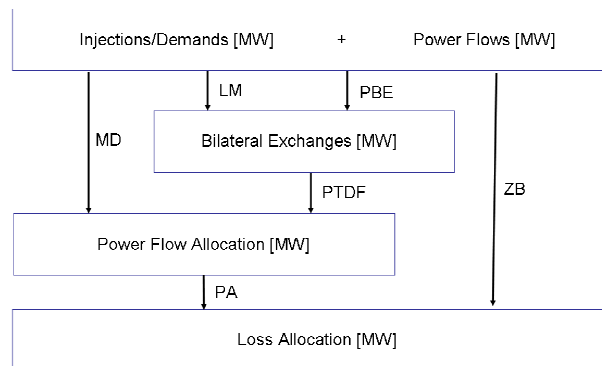


Fig. 1. Transmission Loss Allocation Methods

Further, circuit theory has lead to the Z-Bus allocation (ZB) that is dependent of the network impedance matrix [5].

Finally, Proportional Bilateral Exchange method (PBE) [6] and Loss Minimization method (LM) [7] deal with equivalent bilateral exchanges. A flow allocation is computed using power transfer distribution factors (PTDF) [8].

Losses are then allocated to bilateral exchanges proportionally to the flow allocation (PA). According to this method, negative losses may be associated with counter-flows. Other ways of allocating losses to power flows are developed in [6] and [9]. They are not analyzed in this paper. Finally, losses are allocated to each user as a partial sum of the allocation to all bilateral exchanges.

### 3. INTERNALIZATION OF TRANSMISSION LOSS COSTS INTO THE MARKET

As the knowledge of the network configuration and injections/demands is required, the transmission cost allocation is an ex post process. However, each user has to internalize the allocated costs in its offer/demand price. In this paper, it is assumed that each user has an accurate forecast of the demand/generation level and can thus predict the losses it will be charged for.

Moreover, the internalization of transmission loss costs may only be efficient if users adapt their injection/demand to the financial signal that is given. Considering that demands are independent of electricity price, or poorly flexible, the transmission loss costs are exclusively supported by generators.

Usually, losses are supported by independent/transmission system operators. Users are then charged proportionally to the transmission losses they are allocated. However, in order to simplify the market design, the allocation is done considering the generators' brut injection. This brut injection  $B_{Gi}$  is then divided into two parts: a net production  $N_{Gi}$ , which may be sold, and the transmission losses  $TLA_{Gi}$ .

$$B_{Gi} = N_{Gi} + TLA_{Gi} \quad (1)$$

Each generator production costs  $PC_{Gi}$  are supposed to be known. In this paper, they are quadratic with respect to brut injections  $B_{Gi}$ :

$$PC_{Gi}(B_{Gi}) = \alpha_{Gi} + \beta_{Gi} \times B_{Gi} + \gamma_{Gi} \times B_{Gi}^2 \quad (2)$$

The marginal price  $MP_{Gi}$  of each generator corresponds to the derivative of PC with respect to the net injection  $N_{Gi}$ . As the transmission loss allocation  $TLA_{Gi}$  may be non linear, its derivative and thus the derivative of  $PC_{Gi}$  may be hard to estimate. Therefore, the marginal price is assessed as the net marginal price plus an average loss price.

$$MP_{Gi}(N_{Gi}) = \frac{d PC_{Gi}(N_{Gi})}{dN_{Gi}} + \frac{PC_{Gi}(B_{Gi}) - PC_{Gi}(N_{Gi})}{N_{Gi}} \quad (3)$$

### 4. BASIC MARKET DESIGN WITH INTEGRATED TRANSMISSION LOSSES

All demands are assumed to buy their electricity in a market in which the rules have been simplified. Indeed, it is considerable that they all buy at the market marginal price  $MP_{eq}$ , which is by definition the marginal price of the marginal generator.

The market equilibrium may thus be defined as the injection plan  $(B_{Gi})_{eq}$  for which all demands are satisfied at a minimal marginal price.

$$MP_{eq} = \min_{(B_{Gi})} (\max_{G_i} (MP_{G_i})) \quad (4)$$

In order to compute the market equilibrium state, a minimization algorithm is required. Traditional algorithms seem difficult to apply to this problem because of the non-linearity of some transmission loss allocation methods. Thus, a progressive heuristic algorithm has been used.

The starting production plan is defined by the optimal power flow that minimizes the total production costs. The generators' marginal prices are then different. In the simplified market design, demands would then buy more electricity to the "cheapest" generator and less to the most expensive one. The production of the generator with the highest marginal cost is therefore gradually decreased while the "cheapest" generation is increased. New marginal prices are computed after each change in the production plan, leading rapidly to a minimal market price and the equilibrium production plan.

The maximal market efficiency corresponds to the minimal market deviation, which is estimated as the difference between the total production cost, obtained through the market simulation and the minimum amount it can reach in case of the optimal power flow.

$$D = \frac{(\sum_{G_i} PC(B_{G_i}))_{eq}}{(\sum_{G_i} PC(B_{G_i}))_{OPF}} - 1 \quad (5)$$

In fact, the best method maximizes the social welfare that is traditionally described as the difference between what demands could pay and what injection may be paid. In a market where demands are fixed, it consists of minimizing total production costs.

## 5. EFFICIENCY OF TRANSMISSION LOSS ALLOCATION METHODS

### 5.1. Comparison Framework

The proposed Transmission Loss Allocation methods have been applied to the IEEE 14 bus system [9] described in Fig. 2. with the following attributes.

- Demands' consumption level is fixed and given in Table II
- Generators' production costs are known. Coefficients are given in Table I.
- Generators' output voltage is set at the optimal power flow reference value. It is not affected by their injection
- Generators internalize transmission loss costs as defined in 3.
- Demands buy at the market marginal price

Demands' and compensators' configuration is defined in Table II.

In order to compare the efficiency of the presented methods in a broad range of conditions, different scenarios have been defined. The first scenario deals with the network configuration that is described in Table I and II. The other scenarios are defined using drastic modifications of demand level (s2 to s5), or important changes in the demand location (s6 to s11). Finally, production cost coefficients are modified (s12 to s15). All scenarios are presented in Table III.

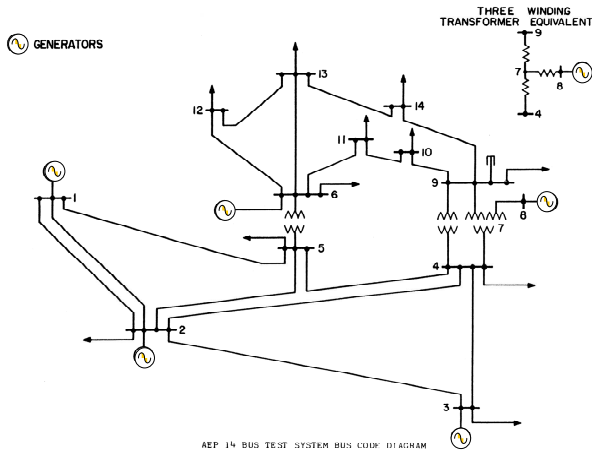


Fig. 2. IEEE 14-bus system

TABLE I  
PRODUCTION COST COEFFICIENTS IN THE IEEE 14 BUS SYSTEM

Bus #	$P_{Gmin}$	$P_{Gmax}$	$\alpha_{Gi}$	$\beta_{Gi}$	$\beta_{Gi}$
1	0	332	0	20	0.043
2	0	140	0	20	0.25
3	0	100	0	40	0.10
6	0	100	0	40	0.10
8	0	100	0	40	0.10

Injections are in MW,  $a_{Gi}$  in €,  $b_{Gi}$  in €/MW,  $c_{Gi}$  in €/MW<sup>2</sup>

TABLE II  
IEEE 14 BUS SYSTEM: BUS CONFIGURATION

Bus	$P_D$	$Q_D$	$P_G$	$Q_G$	$V$	$\theta$
1	0	0	232.4	-16.9	1.06	0
2	21.7	12.7	40	42.4	1.04	-2.02
3	94.2	19	0	23.4	1.01	-8.30
4	47.8	-3.9	0	0	1.04	-5.98
5	7.6	1.6	0	0	1.04	-4.80
6	11.2	7.5	0	12.2	1.07	-5.72
7	0	0	0	0	1.06	-7.18
8	0	0	0	17.4	1.09	-6.12
9	29.5	16.6	0	0	1.05	-8.49
10	9	5.8	0	0	1.04	-8.28
11	3.5	1.8	0	0	1.05	-7.14
12	6.1	1.6	0	0	1.05	-6.72
13	13.5	5.8	0	0	1.05	-6.94
14	14.9	5	0	0	1.03	-8.82

Injections are in MW or MVar. Voltage is in p.u., voltage angle in degrees

TABLE III  
TEST SCENARIOS

Scenario	<i>injections</i>	<i>demands</i>
s1	see Table I	see Table II
s2	see Table I	$P_{Dj} = 125\%$
s3	see Table I	$P_{Dj} = 150\%$
s4	see Table I	$P_{Dj} = 175\%$
s5	see Table I	$P_{Dj} = 200\%$
s6	see Table I	$P_{Dj} = 200\%$ $P_{D2} = 0\%$
s7	see Table I	$P_{Dj} = 150\%$ $P_{D3} = 0\%$
s8	see Table I	$P_{Dj} = 150\%$ $P_{D2} = P_{D3} = 0\%$
s9	see Table I	$P_{Dj} = 150\%$ $P_{D4} = 0\%$
s10	see Table I	$P_{Dj} = 125\%$ $P_{D9} = 0\%$
s11	see Table I	$P_{Dj} = 125\%$ $P_{D9:D14} = 50\%$
s12	$\beta_{Gi} = 20$ €/MW	$P_{Dj} = 150\%$
s13	$\beta_{Gi} = 10$ €/MW	$P_{Dj} = 150\%$
s14	$\beta_{Gi} = 20$ €/MW $\alpha_{Gi} = 0.1$ €/MW <sup>2</sup>	$P_{Dj} = 150\%$
s15	$\beta_{Gi} = 20$ €/MW $\alpha_{Gi} = 0.2$ €/MW <sup>2</sup>	$P_{Dj} = 150\%$

Reference values are listed in Table I and II

### 5.2. Results

All the methods that are described in this paper have been implemented and compared with the traditional “postage stamp” allocation. As the Z-bus allocation assesses the loss dedicated to each node, losses have been allocated proportionally to each generator's allocation.

An important parameter regarding the level of the market deviation is the loss ratio, which is defined in (6). As its value depends on the production plan, it has been estimated in case of the OPF. Its values are presented in Fig. 3.

$$LR = \frac{(\sum_{G_i} P_{G_i} - \sum_{D_j} P_{D_j})_{OPF}}{(\sum_{D_j} P_{D_j})_{OPF}} \quad (6)$$

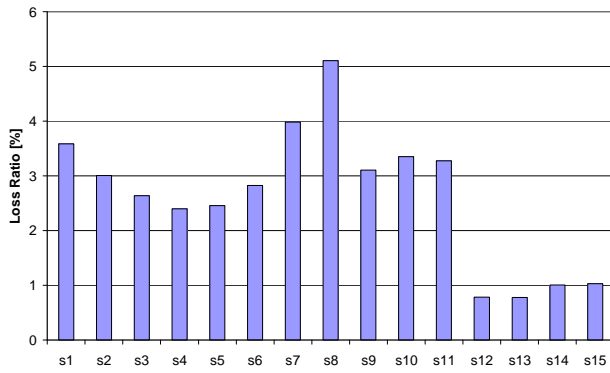


Fig. 3. Loss Ratio for scenarios s1 through s15.

Market deviations, defined in (5), are presented in Fig. 4 for scenarios s1 through s5. The loss ratio decreases smoothly from 3.6% (s1) to 2.4% (s4 and s5).

The important deviation of the market social welfare in case of the “postage stamp” method (PS) characterizes the need for an efficient method. In this particular case and for scenarios s1 to s5, the global production costs may be decreased by around 1.12%. The variance of the market deviation is relatively low. This shows that the loss ratio has a relatively poor effect on this deviation.

The Z-Bus (ZB) allocation seems quite ineffective, as its deviation is even higher than the postage stamps deviation (1.28%). However, it is important to note, that its efficiency level is quite dependent upon the network configuration, as its higher variance shows.

The other TLA methods show a higher efficiency than the postage stamp method. The domain method (MD) has a deviation of 0.95% while the equivalent bilateral exchange method (PBE) reaches 0.30% and the Loss Minimization method (LM) respectively 0.28%. The difference between these two methods is not very important due to the fact that the network is particularly meshed.

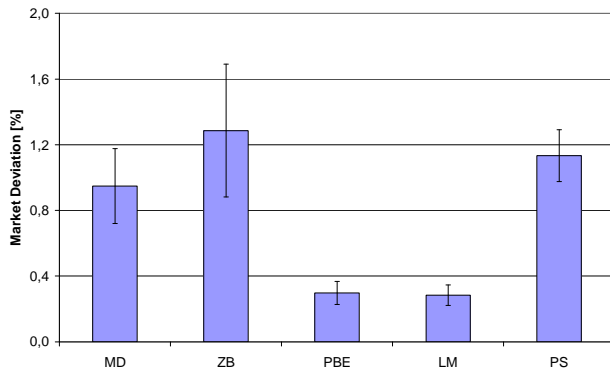


Fig. 4. Average and variance of the Market Deviation  $D$  for scenarios s1 through s5.

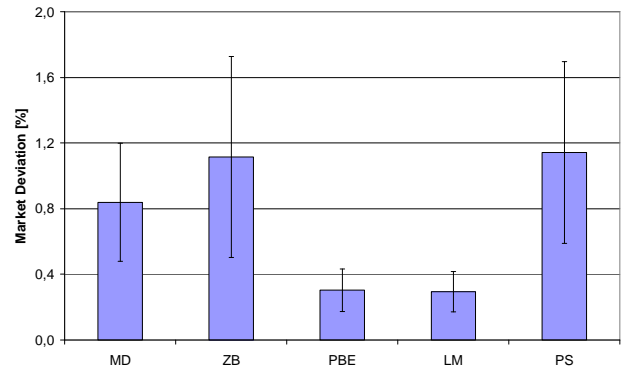


Fig. 5. Average and variance of the Market Deviation  $D$  for scenarios s5 through s11.

It is of great interest to see how this deviation behaves, when the network topology changes. The placement and the level of the demand have been changed in Scenarios s5 through s11. Market deviation average and variance for each TLA method are presented in Fig. 5. The loss ratio evolves between 2.5% and 5%. The variance is therefore higher than in Fig. 4.

It is to be noted that the average market deviation is very similar as the one obtained for scenarios s1 to s5. The variance is a bit higher than in Fig. 4. The market deviation obtained with the “postage stamp” method is of 1.14%, which is approximately the same level as in the previous scenarios. The Z bus allocation does not represent an interesting solution (1.12%), while the efficiency of the domain method is also restricted (0.84%). Flow-based methods PBE (0.30%) and LM (0.29%) are the most efficient transmission loss allocation methods.

### 5.3. Influence of simulation parameters

The influence of the production cost coefficients is analyzed in scenarios s12 through s15. In these cases, all generators are given the same production cost coefficients. Instead of being concentrated in bus 1 and 2, the production is much more distributed leading to a lower loss ratio (Fig. 3). Although these scenarios are unrealistic representations (the loss ratio in large scale power systems is usually around 2.5% or more), it allows one to analyze the impacts of different parameters on the results presented in this paper. Simulating with such a low loss ratio has a considerable impact on the level of deviation that may be observed. Because of this, these results are presented separately.

Fig. 6 represents the market deviation for scenarios s12 and s13. A very low market deviation is observable: around 0.01%. However, the relative efficiency of each method is preserved, except for the method Z-Bus method, which shows the lowest market deviation in this case. A drop of the linear cost coefficient seems to create an increase of the market deviation.

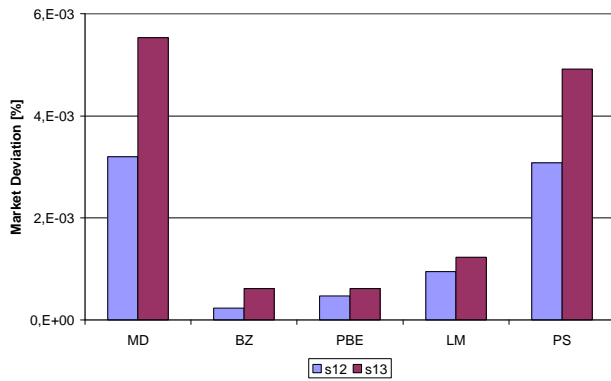


Fig. 6. Market Deviation for scenarios s12 and s13

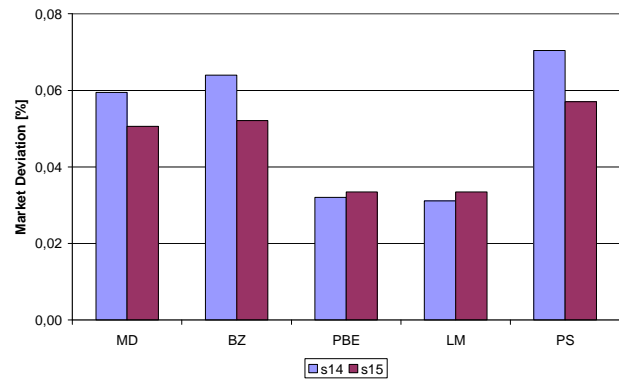


Fig. 7. Market Deviation for scenarios s14 and s15

Fig. 7 represents the market deviation for scenarios s14 and s15. The deviation is lower than the one that was observed for scenarios s1 through s11. The same hierarchy is observed. The impact of the quadratic term is small, and only slightly measurable. Furthermore the evolution is not uniform between methods. It may be assumed that this parameter is not very important regarding the efficiency of each method.

## 6. CONCLUSION

The need for an adequate transmission loss allocation method has been highlighted. An economic gain of around 1.1 % could be reached in case of the IEEE 14-Bus System. Several methods have been presented and applied to a “perfect competition” market, whose users integrate the loss allocation cost into their offer. Simulating the market equilibrium has shown that flow based methods are particularly efficient. Z-Bus and Domain methods seem to be of limited interest if the goal is to allocate losses for a greater social welfare.

The same level of market deviation has been observed after very important changes in the demand level and location. This gain could thus be expected in case of a large scale power system with a loss ratio of almost 3%. The algorithm that was presented in this paper is very fast and will be used in order to apply the comparison frame at a larger scale.

The analysis of the simulation parameters has shown that the loss ratio plays a major role in the magnitude of the expected gain. This loss ratio apparently has no observable effect upon the relative efficiency of each method. The production cost coefficients themselves do not importantly affect this gain.

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