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Optimal Matching Between a Permanent Magnet Synchronous Machine and a Wind Turbine – Statistical Approach

Darío Morales, Miguel López, Jean-Claude Vannier

***Abstract*—This paper proposes a methodology to decide the optimal matching between the size of the rotor of a wind turbine and the rated power of a permanent magnet synchronous machine. This is made taking into account the average electrical energy produced over a period of time. The analytical model of the wind energy conversion system is used to calculate the output power curve. The influence of the number of pairs of poles over the average power is also studied.**

***Index Terms*— average wind power, battery charging, permanent magnet synchronous machine.**

I. INTRODUCTION

The average electric power produced by a small wind energy conversion system (WECS) depends on the generator's rated power, the total swept area of the wind turbine, the gearbox's transformation ratio, the battery voltage and the wind speed probability function. This article proposes a methodology to determine the optimal matching between the generator's rated power and the wind turbine's rotor size.

The system studied in this paper consists of 220 (V)/50 (Hz) Permanent Magnet Synchronous Machine (PMSM) connected to a 50 (V) DC Bus through a rectifier and a DC/DC buck power converter as shown in Fig. 1.

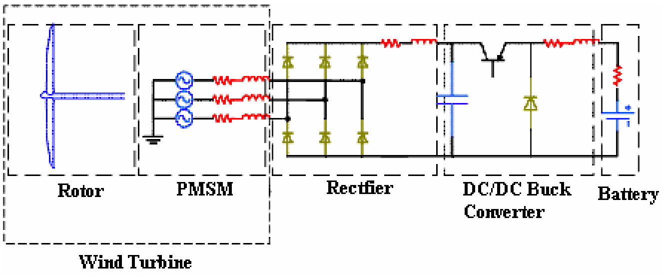

Fig.1 Wind energy conversion system for battery charging purposes

The blades of the wind turbine convert the energy of the wind into rotational forces that drive the PMSM. For a given rotor size, the amount of power that can be transferred to the shaft depends on the wind speed and the rotational speed of the system. The portion of this power that can be converted into electric energy will be limited by the rated power of the generator.

The proposed methodology is based on the mathematical model of the steady-state stator current expressed as function of the generator's rated power and the mechanical angular velocity. Through numerical simulations, this model is used to determine the WECS's output power as function of the wind speed.

The average power produced by the WECS is expressed as function of the its output power and the wind speed probability function. In this paper, the curves of average power are determined for generators up to 5 kW. The study also shows the influence of the number of pairs of poles of the generator on the system's average output power.

II. PROBLEM STATEMENT

The problem consists in calculating the average power produced by the WECS as function of a set of relevant parameters. In this article, the influence of the alternator's rated power, the number of pairs of poles and the wind turbine rotor size over the average output power are studied.

The average power produced by the WECS depends on the following:

- the output power curve which represents the electrical power produced by the WECS at each wind speed
- the wind speed probability function.

The average power may be calculated form the following equation:

$$\langle P_w \rangle = \int_0^{\infty} P(v_w) \cdot p(v_w) \cdot dv_w \qquad (1)$$

Where P(v_w) represents the curve of output power versus the wind speed of the WECS and $p(v_w)$ represents the probability of each wind speed. In general, wind resource may be represented by the Weibull probability function as follows:

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$$p(v_w)=\left(\frac{k}{c}\right)\cdot\left(\frac{v_w}{c}\right)^{k-1}\cdot exp\left[-\left(\frac{v_w}{c}\right)^{k}\right] \qquad (2)$$

where,

$$c=\frac{\langle v_w\rangle}{\Gamma\left(1+\frac{1}{k}\right)}\ ;\ \Gamma(x)=\int_0^\infty e^{-t}\cdot t^{x-1}\cdot dt$$

The coefficient **k** is the shape factor and $<v_w>$ is the mean wind speed.

The integral equation may be approximated by a summation over N_B bins. Thus, the average output power can be calculated by the following expression [1]:

$$\langle P_w\rangle=\sum_{j=1}^{N_B}P_w\left(\frac{v_{w_{j-1}}+v_{w_j}}{2}\right)\left\{exp\left[-\left(\frac{v_{w_{j-1}}}{c}\right)^{k}\right]-exp\left[-\left(\frac{v_{w_j}}{c}\right)^{k}\right]\right\} \qquad (3)$$

$P_w((v_{j-1}+v_{w\,j})/2)$ represents the output power of the WECS calculated at the midpoint between v_{wj-1} and v_{wj}.

From the above equations it can be noticed that the calculation of the output power of the wind turbine as function of the alternator's rated power is the key point to solve the problem.

III. ELECTRICAL OUTPUT POWER CALCULATION: PROPOSED METHOD

Assuming that the battery imposes a constant voltage to the generator, the stator current and the electrical output power depend on the rotor angular velocity. In order to determine the output power curve, it is necessary to compute, for each wind speed, the corresponding rotational speed of the system. In the following section, the stator current of the generator will be developed as function of the angular velocity. Then, the wind turbine characteristics will be described. The curve of the alternator's power and wind turbine characteristics will be used to compute the optimal operating point of the system. Finally, the equations used to express the average output power as function of the alternator's rated power are developed.

A. Mathematical model of the stator current

The DC/DC buck converter adapts the 50(V) battery voltage to the rectifier output voltage. For purposes of simplifying the analysis, the input voltage of the DC/DC converter is represented as a controlled voltage source whose voltage depends on the value of the duty cycle α. The simplified circuit is shown in Fig. 2.

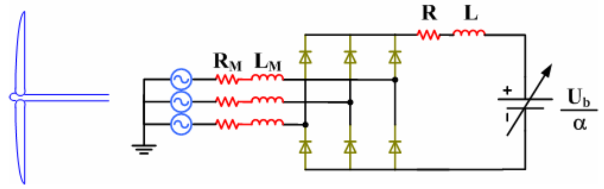

Figure 2. WECS using the DC/DC equivalent circuit

where,

Rm = stator resistance
Lm = stator inductance
R = rectifier output resistance
L = rectifier output inductance
U_b = battery voltage
α = buck converter duty cycle

Assuming that the rectifier imposes the fundamental voltage and current to be in-phase, the phasor diagram is represented by the following figure [2]:

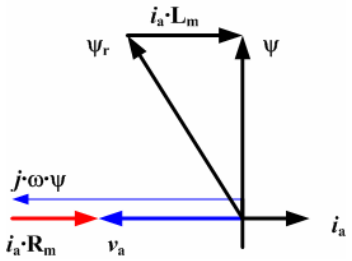

Figure 3. Phasor diagram at the operating point

Where i_a is the peak value of the stator current, v_a is the peak value of the stator voltage, ω is the electrical frequency, ψr is the rotor magnetic flux and ψ is the total magnetic flux.

From the above figure, the total magnetic flux, the stator current and the stator voltage may be related as follows:

$$(\psi_r)^2=(\psi)^2+(L_m\cdot i_a)^2 \qquad (4)$$

$$\psi=\frac{v_a}{\omega_e}+\frac{R_m\cdot i_a}{\omega_e} \qquad (5)$$

Combining (4) and (5), the expression for the stator current as function of the electrical frequency and the stator voltage may be written as [2]:

$$i_a=\frac{-\frac{v_a\cdot R_m}{\omega_e^2}+\sqrt{\frac{v_a^2\cdot R_m^2}{\omega_e^4}-\left(\frac{R_m^2}{\omega_e^2}+L_m^2\right)\cdot\left(\frac{v_a^2}{\omega_e^2}-\psi_r^2\right)}}{\left[\frac{R_m^2}{\omega_e^2}+L_m^2\right]} \qquad (6)$$

where,

$$v_a=\frac{2\cdot U_b}{\pi\cdot\alpha}$$

The electrical frequency and the mechanical angular velocity may be related as:

$$\Omega_m = \frac{\omega_e}{\mathrm{p_n}} \tag{7}$$

p_n represents the number of pairs of poles.

The curve of the peak value of the steady-state stator current as function of the mechanical angular velocity is shown in Fig. 3.

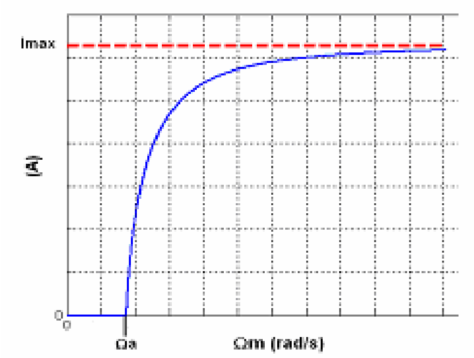

Figure 3. Curve of the peak value of the stator current i_a versus mechanical angular velocity Ωm

The maximum value of the stator current I_{max} and the cut-in mechanical angular velocity may be calculated as follows:

$$\mathrm{I_{max}} = \frac{\psi_r}{\mathrm{L_m}} \tag{8}$$

$$\Omega_a = \frac{2}{p_n} \cdot \frac{U_b}{\alpha \cdot \pi \cdot \psi_r} \tag{9}$$

The equation (9) shows that the PMSM will generate power only when the rotational speed is fast enough as to produce an e.m.f. higher than the voltage imposed by the battery in the rectifier input terminals.

B. Electrical power as function of the rotational velocity

The power delivered by the wind turbine to the PMSM may be expressed as function of the terminal voltage and the stator current as follows:

$$P_a = \frac{3}{2} \cdot v_a \cdot i_a + P_{cupper-losses} \tag{10}$$

Substituting (6) in (10), the equation for the alternator's input power (P_a) may be written:

$$P_a = \frac{3 \cdot U_b}{\alpha \cdot \pi} \cdot i_a + \frac{3}{2} \cdot i_a^2 \cdot \mathrm{R_m} \tag{11}$$

From the above expression, it can be noticed that P_a depends on the PMSM electrical parameters (ψ_r, R_m, L_m) and the duty cycle of the DC/DC converter (α).

As the generator terminal voltage is imposed by the battery bank and the value of α[3], the power delivered by the generator to the load depends on the value of the stator current which depends on the alternator frequency. Let us assume that, when operating in steady state conditions, the PMSM terminal voltage can not be higher than its rated voltage, and that the stator current can not be higher than its rated current. Thus, there is an $\alpha=\alpha_{min}$ that imposes the rated voltage to the generator and a maximal amount of power delivered which is imposed by the rated stator current (I_{rated}). When the buck DC/DC converter operates at $\alpha=\alpha_{min}$, the alternator delivers its rated power to the load (rated voltage and rated stator current). However, if the DC/DC power converter is controlled to operate with α greater than α_{min}, the power delivered by the alternator decreases because of the voltage reduction. In this case, the PMSM operates only at rated stator current. Also it can be noticed that, according to (9), if α increases the cut-in rotational velocity decreases. Thus, the value of the duty cycle allows controlling the power and the rotational velocity of the alternator. In Fig. 4, the curves of P_a versus the alternator frequency for different values of α are shown.

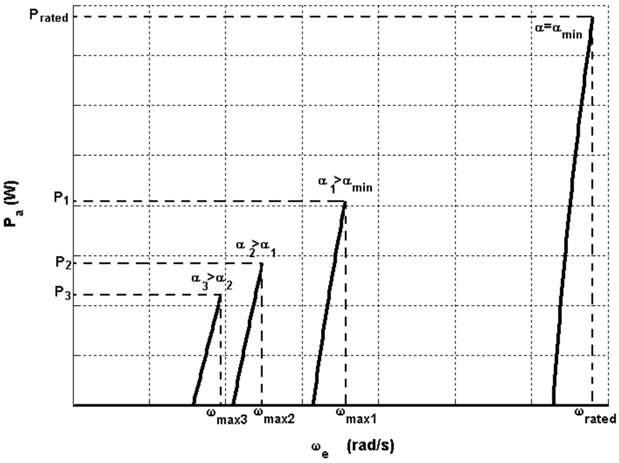

Figure 4. Alternator power versus alternator frequency for various duty cycle α.

C. R_m and L_m as function of the alternator's rated power

As shown in (6), (10) and (11), the power delivered to the load depends on the resistance (R_m) and the inductance (L_m).

The following equations allow expressing the alternators parameters as function of its rated apparent power.

$$R_m = R_{pu} \cdot \frac{3}{S_{3\phi}} \cdot \left[\frac{2 \cdot U_b}{\sqrt{2} \cdot \pi \cdot \alpha_{\min}} \right]^2 \tag{12}$$

$$L_m = \frac{X_{pu}}{\omega_{nom}} \cdot \frac{3}{S_{3\phi}} \cdot \left[\frac{2 \cdot U_b}{\sqrt{2} \cdot \pi \cdot \alpha_{\min}} \right]^2 \tag{13}$$

D. Mathematical model of the wind turbine

The mechanical power developed by the wind turbine depends on the power coefficient (C_p), the radius of the turbine (R), the air density (ρ) and the wind speed as follows:

$$P_w=\frac{1}{2}\cdot\rho\cdot C_p(\lambda)\cdot\pi\cdot R^2\cdot v_w^3 \tag{14}$$

In the above relation, C_p is a dimensionless coefficient which represents the efficiency between the power available in the wind and the mechanical power transferred to the shaft. This coefficient may be expressed as function of the ratio between the linear speed of the tip of the blade and the wind speed (v_w). The tip-speed ratio (λ) is:

$$\lambda=\frac{\Omega_m\cdot R}{v_w} \tag{15}$$

The power coefficient of a small three-blade wind turbine may be expressed by the following equation:

$$C_p(\lambda)=0.19\cdot\frac{\lambda\cdot(8.08-\lambda)}{1.56^2+(8.08-\lambda)^2} \tag{16}$$

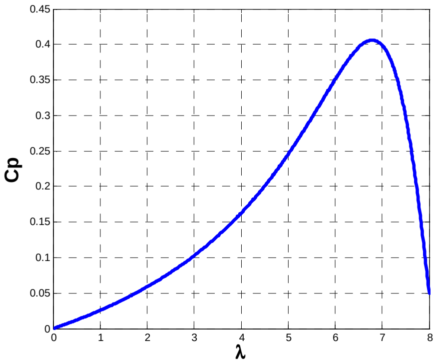

Figure 5. Power coefficient (Cp) versus tip-speed ratio (λ)

If we consider a wind turbine of radius R, according to (16), the power available may be plotted as function of the rotational velocity. The Fig. 6 shows the power developed by a turbine whose rotor radius is 1.25m at various wind speeds.

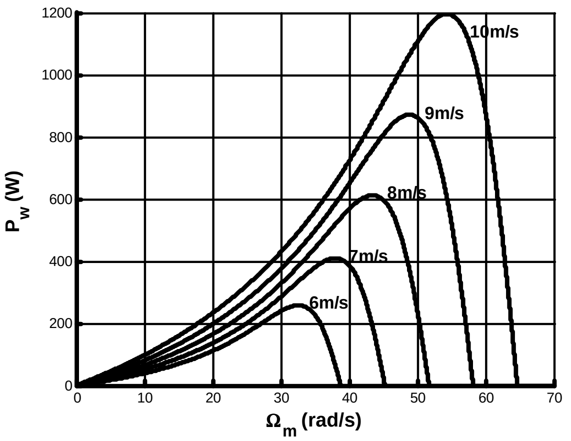

Figure 6. Power available in the wind versus the wind turbine's rotational velocity. R=1.25m

It can be noticed that for each wind speed, there is an optimal rotational velocity which maximizes the amount of power extracted from the wind.

The power available in the wind depends also on the size of the rotor as shown in Fig. 7.

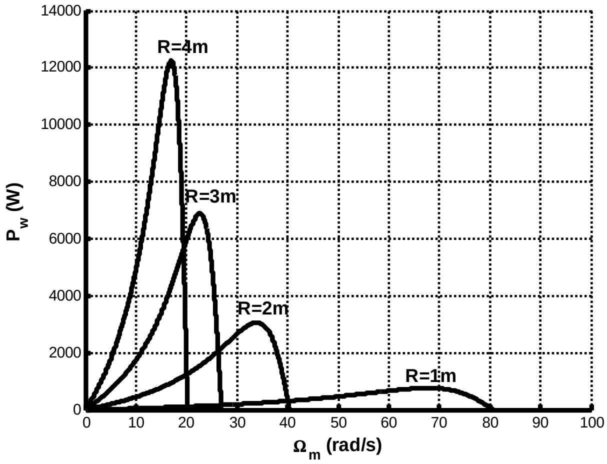

Figure 7. Power available in the wind as function of the wind turbine's radius (R). vw=10m/s

For a given wind speed, the maximal power available in the wind increases together with the size of the rotor, meanwhile, the angular velocity at which this power is available decreases.

E. *Determination of the output power curve*

The steady-state operation is achieved by the WECS when the power available in the wind (input power) equals the alternator's power (resistant power). Graphically, the steady-state operating speed is given by the intersection point between the curves of wind power and the alternator's power.

The next figure compares the determination of the operating point using the analytical model proposed in this paper with the curve obtained using a specially developed SIMULINK© model (used as reference).

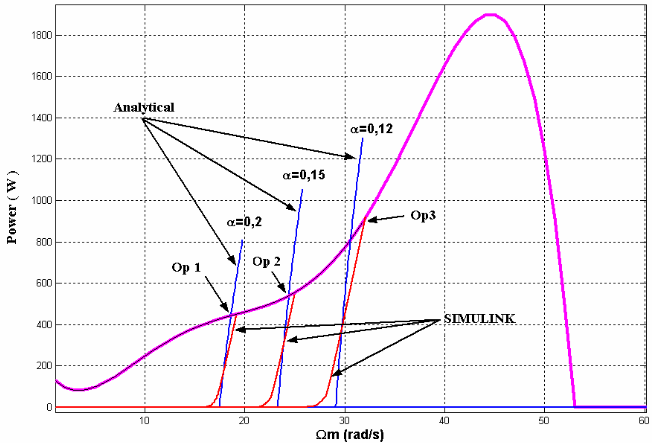

Figure 8. Determination of the steady-state operating point for different values of α. Comparison between the proposed analytical model and a specially developed SIMULINK© model. Wind speed 10 (m/s).

The previous figure shows that the error in the determination of the output power varies between 1-10% whereas the error associated to the rotational velocity varies between 1-5%. These errors do not have a big influence over the calculation of the average output power.

IV. RESULTS

All the results are obtained using the Weibull probability function with a 6m/s average wind speed and form coefficient of k=2.

For each alternator's rated power, the value of its resistance and inductance are calculated using (12) and (13). Then, the output power curve is calculated. Finally, the average output power is determined using (3).

A. Influence of the alternator's rated power over the average output power

The curves of the average output power versus the alternator's rated power for different rotor radius are shown in Fig. 9. It can be noticed that there are two stages in the evolution of the average output power. In the first one, the average power increases almost proportionally to the rated power whereas in the second one, the average output power increases very slowly.

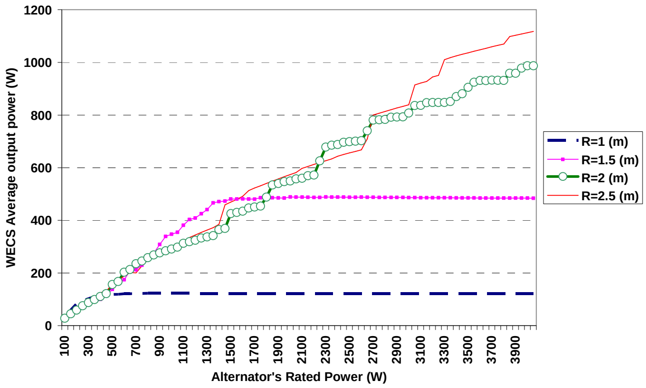

Fig. 9. WECS average output power versus alternator's rated power for various rotor sizes (R).

The phenomenon of saturation of the average output power can be explained with the help of Fig. 10. The probability function of wind speed is plotted together with the output power curves of three machines (500W, 1500W and 1800W). The rotor's size considered in this case is 1.5m. The difference between the output power curves of the alternators of 1500W and 1800W appears only for high wind speeds with a low probability of occurrence.

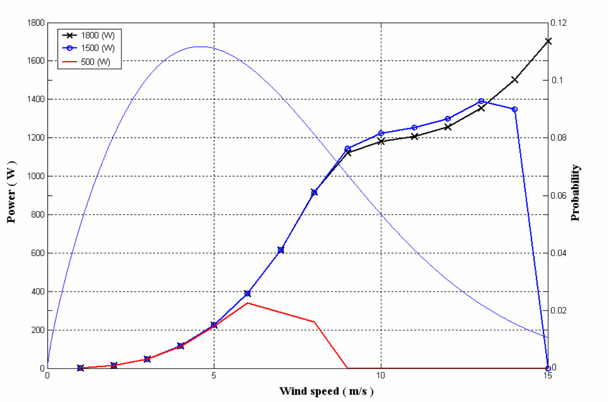

Fig. 10 . Power and wind speed probability versus wind speed for alternators of 500W, 1500W and 1800W of rated power.

Fig. 9 also reveals that the increase of the rotor's size does not necessarily increases the value of the average power. For instance, for the 1kW PMSM, using a rotor of radius 2.5m will not increase the average output power when compared with a rotor of 2m radius.

Fig. 11, shows the available wind power at 6m/s versus the rotational velocity for two rotors of radius 2m and 2.5m respectively. The power curve of a 1kW alternator is shown for the two steady-state operating points. In the case of the rotor of 2m, the system operates at α=0.17 producing about 520W whereas in the case of the 2.5m, wind turbine the system operates at α=0.21 producing about 500W. When the size of the rotor is increased, the available wind power is also increased. The additional power is now available at lower rotational velocities. The only way to produce power is through the reduction of the duty cycle which causes a reduction of the output power.

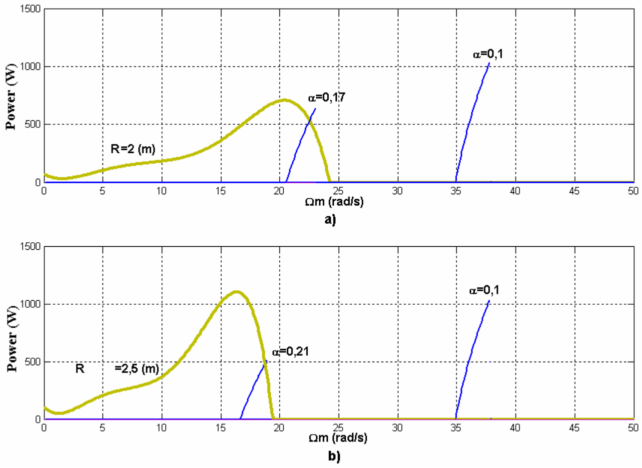

Fig. 11. Wind power versus rotational velocity for two rotor's size and alternators power versus rotational velocity for different values of duty cycle a) Rotor's size R=2m b) Rotor's size R=2.5m

B. Influence of the number of pairs of poles over the average output power

The influence of the number of pairs of poles is analyzed in this article because this parameter influences the cut-in rotational speed.

The curves of the average output power versus the alternator's rated power considering two machines of 4 and 9 pairs of poles are shown in Fig. 12.

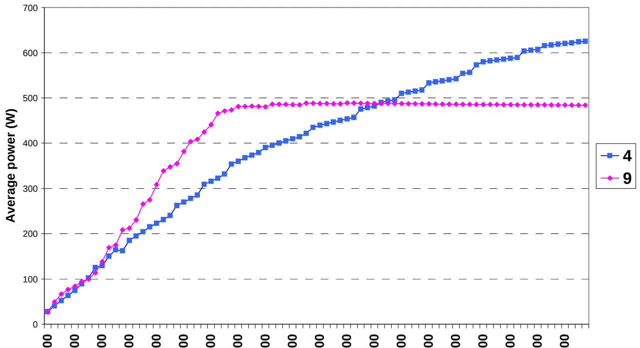

Fig. 12. Average wind power versus alternator's rated power. Comparison between 4 and 9 pairs of poles.

In this particular case it can be noticed that, when the alternator's rated power is inferior to 2.7kW, the machine with 9 pairs of poles produces a higher value of average output power than the alternator with 4 pairs of poles. However, if the alternator rated power is superior to 2.7kW, the opposite phenomenon occurs. Figures 13 and 14 show the available wind power for different wind speeds and the alternator's power curve for two machines of 1.5kW and 3.5kW respectively.

For an average wind speed of 6m/s, Fig. 13 shows that for a rated power of 1kW, the PMSM with 9 pairs of poles can track the optimal power point until 8m/s. The machine with 4 pairs of poles is forced to work at a higher value of duty cycle to reduce its rotational velocity. Thus, if we consider an average wind speed of 6m/s, it will be very difficult to this system to track the optimal power point for the most frequent wind speeds. This effect reduces the average output power.

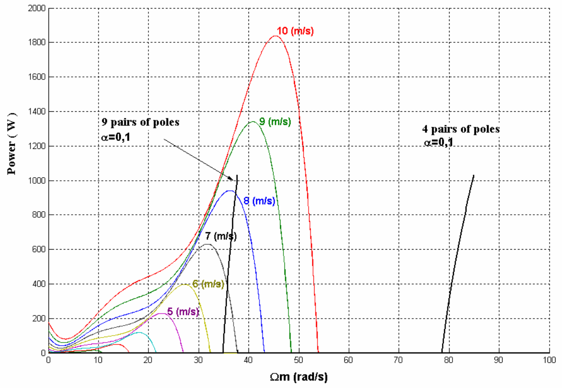

Fig. 13. Average wind power versus alternator's rated power. Comparison between 4 and 9 pairs of poles.

By the other hand, even increasing the value of α, the 3.5kW alternator can track the optimal power point for a much larger range of wind speeds, as can be seen in Fig. 14.

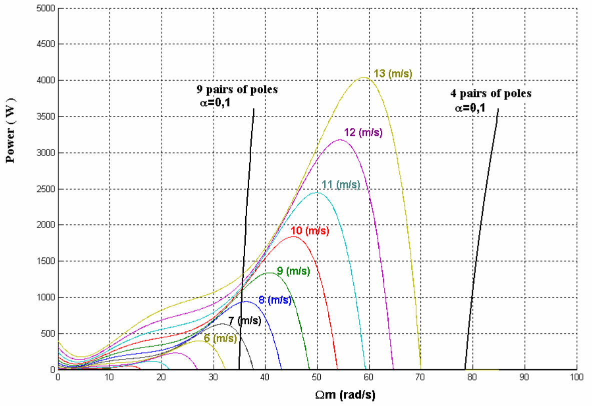

Fig. 14. Average wind power versus alternator's rated power. Comparison between 4 and 9 pairs of poles.

V. CONCLUSION

This paper studies the influence of three parameters of a WECS over the average power produced. These parameters are: the rotor size of the wind turbine, the rated power and the number of pairs of poles of the alternator. This paper demonstrates that that the analytical model of the stator current can be used to correctly estimate the steady-state operating point of the system.

The results obtained in this paper show that increasing the size of the rotor or the rated power of the alternator, not necessarily will increase the value of the average output power, thus when designing a wind energy conversion system, the statistical analysis appears to be fundamental to optimize the energy production. This conclusion is confirmed by the fact that the number of pairs of poles influences the cut-in rotational speed of the system which has a direct influence over the average output power.

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