# How Compatible is Perfect Competition with Transmission Loss Allocation Methods?

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*Index Terms*—**Transmission loss allocation, agent-based simulation, market efficiency, electricity market**

***Abstract*—This paper addresses the problem of transmission loss allocation in a power system where the generators, the demands and the system operator are independent. We suppose that the transmission losses are exclusively charged to the generators, which are willing to adopt a perfectly competitive behavior. In this context, their offers must reflect their production costs and their transmission loss costs, the latter being unknown beforehand and having to be predicted. We assume in this paper that the generators predict their loss costs from the past observations by using a weighted average of their past allocated costs. Under those assumptions, we simulate the market dynamics for different types of transmission loss allocation methods. The results show that the transmission loss allocation scheme can lead to a poorly efficient market in terms of social welfare.**

## I. Introduction

In power systems where generation and transmission are unbundled, the transmission operational costs (e.g. transmission losses or congestion costs) are generally supported by the System Operator (SO), which charges the generators and the demands for those costs. Consequently, the transmission costs are allocated *ex post*, when the generation and demand dispatch is known [1]. In this context, allocating transmission losses has become necessary in order to define locational economic incentives towards a more rational use of ressources [2]. A possible solution could be to allocate active power losses to the market participants depending on the amount of transmission losses resulting from their injection. However, since active power losses are a non-separable nonlinear function of the bus power injections, there is no unique solution to assess each market participant's contribution in the transmission losses [3]. As emphasized in [1] and [4], alternative strategies have therefore been proposed to design Transmission Loss Allocation (TLA) methods leading to appropriate economic signals.

Generally speaking, the efficiency of those economic signals depends on the competition strategies of the market participants [5], [6]. The most common types of strategies considered to analyze electricity markets are those related to the competition models derived from game theory [7] such as, for example, the Cournot, Bertrand or Stackelberg models. Besides game theory, agent-based approaches are also used to analyze electricity markets. Those approaches model the market as a dynamic system of interacting agents. An agent refers in this context to a bundle of data and behaviorial methods representing an entity constituting part of the simulated market [8]. With respect to the models cited above (Cournot, Stackelberg), such approaches can provide a detailed modeling of the market, able to highlight phenomena that Nash-equilibria types of techniques can not. As way of example, they are advocated to study the dynamics of the market before the participants eventually settle to a Nash-equilibrium.

The goal of this paper is mainly to study the influence that TLA methods may have on electricity markets where the generators are assumed to have a perfectly competitive behavior. An agent-based approach is chosen to carry out the study. Each generator is actually modeled as an agent which formulates offers that reflect its generation cost and its transmission loss cost. Since individual lost costs are not known beforehand, each generator must predict those costs. The prediction of the loss costs may be a difficult task as the final allocation depends on the predictions of the others. While utilities could rely on sophisticated approaches to predict the loss costs, the predictions are assumed, in this paper, to be carried out by computing a weighted average of the past allocated loss costs.

The simulation process is summarized in Figure 1. First, the generators predict their loss costs based on the past allocation results. Then, their offers are computed by summing their production costs and their predicted transmission loss costs. After computing the merit order using those offers, the different transmission loss costs are sent to the generators and the overall process repeats. The results of the simulations are analyzed to determine the efficiency of three TLA methods (namely the pro-rata, proportional sharing and equivalent bilateral exchange methods), which is associated here to the social welfare of the system. This study is carried out on a

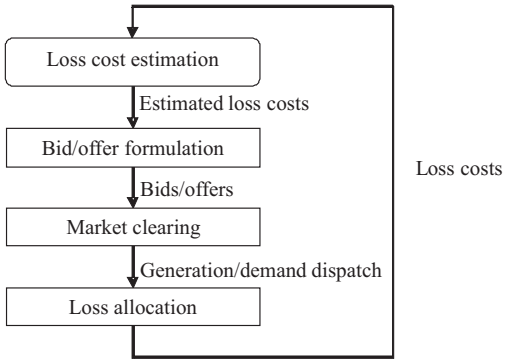


Fig. 1. Simulation scheme for the analysis of the interaction between the TLA mechanism and the generators' decisions.

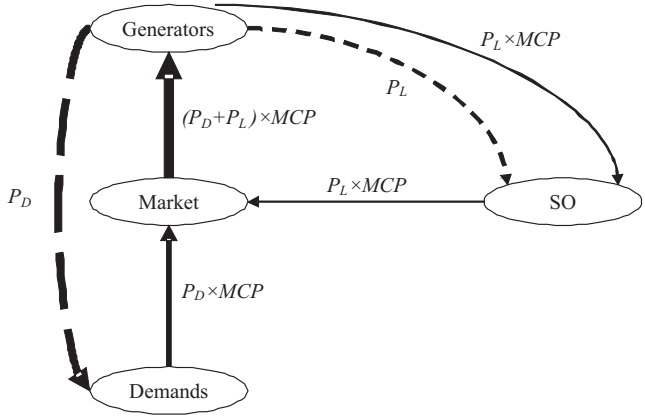


Fig. 2. Power (dashed arrows) and economic flows (continuous arrows) in the power system.

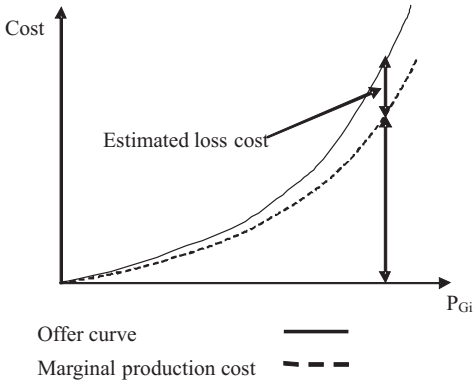


Fig. 3. Offer curve and marginal production costs of a generator $i$ as a function of its active power generation $P_{G_i}$.

2-bus system where, in order to simplify the approach, the demand is fixed and the losses are exclusively allocated to the generators.

The paper is organized as follows. In Section II, the experimental market design is detailed and a review of some common TLA approaches is provided. Then, Section III describes the prediction algorithm. Some criteria to evaluate the efficiency of TLA methods are presented in Section IV. Section V presents the results for different TLA methods and a discussion of their relative performance. And, finally, Section V concludes.

## II. Market design with TLA

In this paper, we focus on a specific market design, which is appropriate to implement different types of TLA methods. While losses could be considered in the merit order using locational marginal prices [9], this market design is based on a lossless economic dispatch. That means that the offers are selected on their price regardless of their location in the transmission grid.

In order to ease the approach, we have chosen to allocate transmission losses exclusively to the generators, which are usually considered as being the market participants with the highest price responsiveness. In addition, we consider that the demands are non-responsive to price changes, i.e. the amount of power demand is fixed.

In this context, we use a pool-based market, with no bilateral exchange contract. The inherent physical and economic flows are represented in Figure 2. Active demands ($P_D$) and losses ($P_L$) are paid at the Market Clearing Price (MCP) and the SO charges the generators for the transmission loss costs. We suppose that other transmission costs (such as investment costs, for example) are supported by the demands only, and that they are considered in the demand curve.

### A. Offer formulation

We suppose that the $N_G$ generators are willing to adopt a perfectly competitive strategy. This assumption means that the generators bid at their expected marginal cost. In the context of transmission loss allocation, the expected marginal cost encompasses the production cost and the expected loss cost. Every generator $i$ is supposed to know perfectly well its production cost $C^P_{G_i}(P_{G_i})$ and, in addition, it estimates its loss cost $\hat{C}^L_{G_i}(P_{G_i})$, which is to be integrated in its offer $\hat{C}^O_{G_i}(P_{G_i}) = C^P_{G_i}(P_{G_i}) + \hat{C}^L_{G_i}(P_{G_i})$. Figure 3 plots a typical offer curve of a generator $i$.

### B. Merit order

The process of merit order determines the generation dispatch $P_{G_i}, \ldots, P_{G_{N_G}}$ and the market clearing price ($MCP$) based on the offers $\hat{C}^O_{G_1}(P_{G_1}), \ldots, \hat{C}^O_{G_{N_G}}(P_{G_{N_G}})$. It is computed using an optimal power flow formulation as detailed below.

$$\min_{P_{G_1},\ldots,P_{G_{N_G}}} MCP(\hat{C}^O_{G_1}(P_{G_1}), \ldots, \hat{C}^O_{G_{N_G}}(P_{G_{N_G}})) \quad (1)$$

subject to:

$$\sum_{i=1}^{N_G} P_{G_i} = P_L + \sum_{j=1}^{N_D} P_{D_j} \quad (2)$$

and the other classical load-flow equations [10].

Equation (2) and the load-flow equations introduce the amount of losses in the generation dispatch. This definition of the merit order replaces the balancing mechanism, which is indeed used to compensate for the mismatch of losses in

a lossless merit order. This particular type of optimal power flow problem is solved using AMPL [11].

### C. Transmission loss allocation methods

Based on the generation dispatch $P_{G_1}, \ldots, P_{G_{N_G}}$, the amount of losses $P^L_{G_i}$ allocated to generator $i$ is assessed using a TLA method. We study three methods, namely the "Pro-Rata", the "Proportional Sharing" and the "Equivalent Bilateral Exchange" methods, which are detailed hereafter.

*1) Pro-Rata (PR):* The PR method has been used for decades in many power systems. Losses are allocated proportionally to the active power injection of every generator regardless of its location [12]. This TLA method is used for instance in France, England and Wales [13].

*2) Proportional Sharing (PS):* The PS method has been introduced by J. Bialek in [14]. This method is based on power flow tracing and relies on the assumption that a network node is a perfect mixer of incoming flows. For each node, every outcoming active power flow is proportionally composed of the incoming flows. For each line, the losses are proportionally allocated to the incoming flows into this line. This method has been vastly commented and has influenced the design of many closely related TLA schemes (see, e.g., [15]). No real-life application has yet been reported.

*3) Equivalent Bilateral Exchange (EBE):* The EBE method has been proposed by Galiana *et al.* in [16]. The equivalent bilateral exchanges are deduced from the application of the PS principle to the whole network reduced to one node. Losses are then allocated to those equivalent bilateral exchanges using incremental transmission loss factors. It is actually independent of choice of the slack bus [4]. This method has not yet been implemented.

## III. Transmission loss cost prediction

In the heart of our agent-based approach to evaluate the performances of TLA methods when assuming that the generators are willing to adopt a perfectly competitive behavior, there is a module that determines how each agent is going to predict its loss cost. The design adopted for this module is described on Figure 4. In a few words, with such a choice, a generator estimates its transmission loss cost (per generated $MWh$) by computing a weighted average of its past loss allocations (per generated $MWh$). It is clear that, in real-life, power system utilities may rely on other approaches to estimate accurately those costs. In particular, they may rely on more sophisticated algorithms or use some specific expertise to predict those costs. In this respect and by assuming that there are persons or a group of persons in charge of those prediction tasks for the utilities, one strategy to design better agents could have been to analyze their predictions using various types of data-mining techniques. Agents would then better reflect the real-life behavior of the utilities.

Since it is very likely that obtaining such prediction data is going to be difficult (and even sometimes impossible, since they can not exist for TLA schemes that have not yet been implemented), another strategy would have been to rely on experimental economics [17] to generate those data. It consists of laboratory experiments, where the humans (here playing the role of people in charge of predicting the loss costs for the utilities) are repeatedly asked to make decisions in face of a feedback signal (here the past allocations) related to their decisions. However, experimental economics is also limited since there is an obvious bias between the decisions taken in the laboratory and in real-life due to, for example, the prior-knowledge a market participant interacting with the real environment may have on the relevance of its decisions.

## IV. Performance evaluation of TLA methods

By simulating the influence of TLA methods on an electricity market, we aim to measure the efficiency of the economic signal they intend to provide.

The *market efficiency* index adopted in this paper is the social welfare of the system which is the sum of the generators' surplus and the demands' surplus [18]. The generator surplus $SP_G$ is defined by:

$$SP_G = MCP \times \sum_{j=1}^{N_D} P_{D_j} - \sum_{i=1}^{N_G} GC_{G_i}(P_{G_i}) \qquad (3)$$

where $GC_{G_i}(P_{G_i})$ is generator $i$'s total generation costs. As the demand is fixed, the demand surplus $SP_D$ is defined as follows:

$$SP_D = -MCP \times \sum_{j=1}^{N_D} P_{D_j} \qquad (4)$$

Consequently, the social welfare of the system can be written:

$$SP = -\sum_{i=1}^{N_G} GC_{G_i}(P_{G_i}) \qquad (5)$$

The system social welfare will be compared with its optimal value $SP^*$, which is computed by minimizing the total production costs of the system.

## V. Simulation results

After describing the simulation benchmark, this section reports and analyzes the simulation results.

### A. Benchmark

Simulations are run on the 2-bus system that is depicted in Figure 5. The numerical values of the different parameters of the system are given in Table I.

When the generators submit an offer equal to their marginal production cost, one can observe that a large amount of power is transmitted from bus 1 to bus 2 with a loss rate close to $15\%$.

The transmission line is modeled by a resistance $R_{1-2}$ and an inductance $X_{1-2}$. The voltage at each bus is regulated by its associated generator at $1.0$ per unit.

The generation cost supported by generator $i$ can be written as:

$$GC_{G_i}(P_{G_i}) = a_{G_i} \times {P_{G_i}}^2 + b_{G_i} \times P_{G_i} + c_{G_i} \qquad (6)$$

**Input:** The loss cost prediction for time $t$: $\hat{C}_{G_i}^{L,t}$, the quantity of losses allocated to generator $i$ at time $t$: $TLA_{G_i}^t$, its active power injection at time $t$: $P_{G_i}^t$ and the market price at time $t$: $MCP^t$.
**Parameter:** A memory factor $\beta$ ($\beta \in [0,1]$) that weights the actual loss cost at time $t-k$ by a factor $\beta^{t-k}$ in the average-based predictions.
**Output:** The loss cost prediction for time $t+1$: $\hat{C}_{G_i}^{L,t+1}$.
**Algorithm:**

- **Step 1:** Generator $i$ computes the average transmission loss cost per $MWh$ at time $t$: $C_{G_i}^{L,t} = \frac{TLA_{G_i} \times MCP}{P_{G_i}}$.
- **Step 2:** Generator $i$ computes the predicted loss costs for time $t+1$ as follows: $\hat{C}_{G_i}^{L,t+1} = \frac{t \times \beta \times \hat{C}_{G_i}^{L,t} + (t \times (1-\beta)+1) C_{G_i}^{L,t}}{t+1}$.

Fig. 4. The procedure a generator $i$ uses at time $t$ to predict its transmission loss cost per generated $MW$ at time $t+1$ ($\hat{C}_{G_i}^{L,t+1}$). At time $t=0$, no data for the past loss cost allocations are available and, $\hat{C}_{G_i}^{L,0}$ and $C_{G_i}^{L,0}$ are chosen equal to 0.

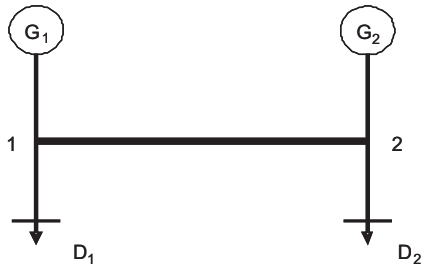


Fig. 5. 2-bus system.

| Symbol | Unit | Value |
|---|---|---|
| $P_{D_1}$ | $MW$ | 300 |
| $P_{D_2}$ | $MW$ | 1200 |
| $Q_{D_1}$ | $MVar$ | 100 |
| $Q_{D_A}$ | $MVar$ | 100 |
| $R_{1-2}$ | $p.u.$ | 0.0002 |
| $X_{1-2}$ | $p.u.$ | 0.0005 |
| $a_{G_1}$ | €$/MWh^2$ | 0.0147 |
| $b_{G_1}$ | €$/MWh$ | 20 |
| $c_{G_1}$ | € | 0 |
| $a_{G_2}$ | €$/MWh^2$ | 0.0588 |
| $b_{G_2}$ | €$/MWh$ | 20 |
| $c_{G_2}$ | € | 0 |

TABLE I
NUMERICAL DATA FOR THE 2-BUS SYSTEM.

This leads to an optimal system welfare $SP^*$ equal to $-66959$€.

At time $t=0$, generators' estimated loss costs are set to $0$€$/MW$. In our simulations, a memory factor $\beta$ has been chosen equal to $0.5$. We have observed that this parameter does not significantly impact the results, except for the speed of convergence.

### B. Numerical results

*1) Market efficiency:* Results in terms of system welfare are represented in Figure 6. One can observe that the pro-rata allocation induces no change in the generation dispatch. The system welfare remains equal to $-70425$€ in this case. As emphasized in [19], the pro-rata method is poorly efficient in the context of an asymmetrical system with high losses, as it is the case here.

On the other hand, EBE and PS methods provide an economic signal which motivates a change in the generation dispatch, leading to a greater welfare. After a few iterations (four to six), the system welfare reaches $-68192$€ and $-68685$€ for the PS and EBE methods, respectively.

When compared to $SP^*$, the TLA methods under consideration may however appear to be inefficient. The lack of performance might be caused by structural defaults of the methods themselves (as for the pro-rata method, for example). It also shows that further research may be needed to design more efficient allocation schemes. While allocating an amount of losses larger than the physical losses $P_L$ is controversial [4], one could also think of defining negative allocations for generators that help decreasing the amount of transmission losses when they inject more power into the grid. However, this may potentially lead to volatile markets where predictions could be less accurate and, therefore, discourage generators to adopt a perfectly competitive behavior since they may prefer to cover themselves for the risk of underestimating their loss costs.

*2) Market price:* Results in terms of $MCP$ are represented in Figure 7. Four to six iterations are required before the market price and the system welfare can be assumed to have converged. Moreover, simulations run with an initial value of the predicted loss costs $C_{G_i}^{L,0}$ set to $20.0$€$/MW$ rather than $0.0$€$/MW$ have shown convergence to the same equilibrium point.

As one can observe, with the agents chosen to model the generators, the market clearing price grows with the loss cost predictions. Except for the first iteration, the $MCP$ is always larger than the marginal production costs of the generators. This was expected since, even if the generators adopt a perfectly competitive behavior, they submit an higher offer than their marginal production cost to cover the transmission loss cost they are expecting to be charged for.

*3) Discrepancy between the predicted and the actual loss costs:* We have observed that the transmission loss costs tended to be underestimated by the different generators. This is mainly due to the fact that the initial estimation of those costs is set equal to $0$. We note however that this under estimation

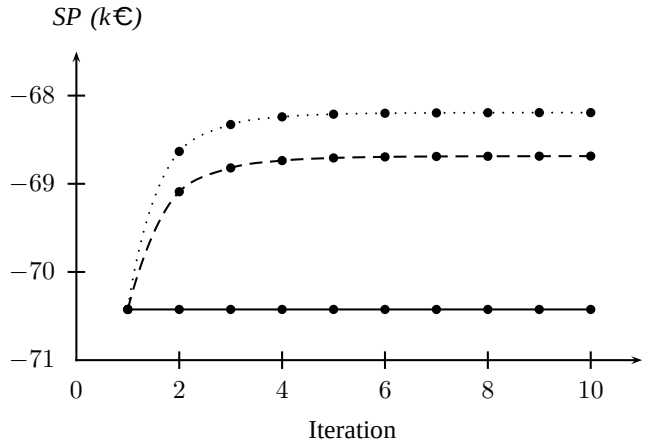


Fig. 6. Social welfare at each iteration for the 2-bus system with an average-based loss cost prediction. The solid, dotted and dashed lines correspond to the pro-rata, PS and EBE methods, respectively.

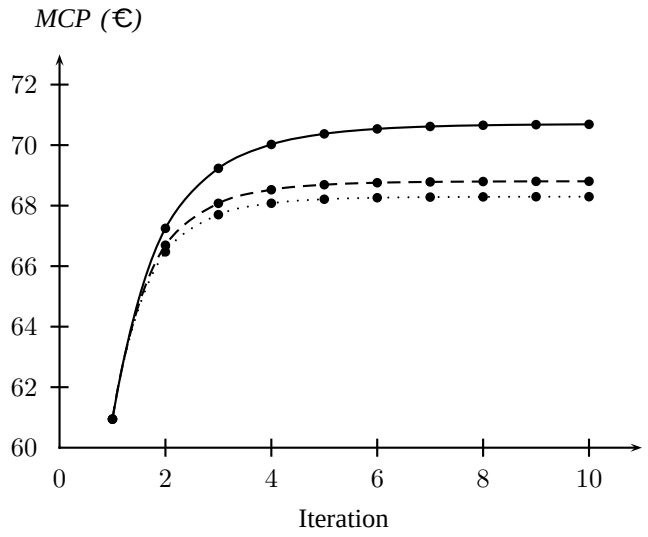


Fig. 7. Market price $MCP$ at each iteration for the 2-bus system with an average-based loss cost prediction. The solid, dotted and dashed lines correspond to the pro-rata, PS and EBE methods, respectively.

vanishes with the iterations.

## VI. Conclusion

We have analyzed in this paper the performance of three transmission loss allocation methods, namely the pro-rata, proportional sharing and equivalent bilateral exchange methods, for a market in which the power generation companies are willing to adopt a perfectly competitive behavior. To perform this analysis, we have relied on an agent-based approach.

As main finding of our analysis, we noted that the three allocation methods under consideration were leading to a social welfare which was smaller that the one that could be obtained by considering an optimized vertically integrated system. The pro-rata method offered the poorest performance while the proportional Sharing and the equivalent bilateral exchange were giving similar performance. While many simplifying assumptions have been adopted in our simulations (time-invariant system, simple loss cost prediction algorithms, 2-bus power system, etc), those results suggest that there is still room for designing more efficient TLA methods. In particular, we believe that new TLA methods should at least intrinsically lead to a nearly optimal social welfare when the generators are willing to adopt a perfectly competitive behavior.

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## References

[1] A. Conejo, J. Arroyo, N. Alguacil, and A. Guijarro, "Transmission loss allocation: A comparison of different practical algorithms," *IEEE Transactions on Power Systems*, vol. 17, no. 3, pp. 571–576, August 2002.

[2] F. Galiana, A. Conejo, and I. Kockar, "Incremental transmission loss allocation under pool dispatch," *IEEE Transactions on Power Systems*, vol. 17, no. 1, pp. 26–32, February 2002.

[3] A. Conejo, F. Galiana, and I. Kockar, "Z-bus loss allocation," *IEEE Transactions on Power Systems*, vol. 16, no. 1, pp. 105–110, February 2001.

[4] J. Mateus and P. Franco, "Transmission loss allocation through equivalent bilateral exchanges and economical analysis," *IEEE Transactions on Power Systems*, vol. 40, no. 4, pp. 1799–1807, November 2005.

[5] T. Peng and K. Tomsovic, "Congestion influence on bidding strategies in an electricity market," *IEEE Transactions on Power Systems*, vol. 18, no. 3, pp. 1054–1061, August 2003.

[6] E. Bompard, W. Lu, and R. Napoli, "Network constraint impacts on the competitive electricity markets under supply-side strategic bidding," *IEEE Transactions on Power Systems*, vol. 21, no. 1, pp. 160–170, February 2006.

[7] A. Rahimi and A. Sheffrin, "Effective market monitoring in deregulated electricity markets," *IEEE Transactions on Power Systems*, vol. 18, no. 2, pp. 486–493, May 2003.

[8] L. Tesfatsion and K. Judd, *Handbook of Computational Economics: Agent-Based Computational Economics*. Elsevier Science and Technology Books, 2006.

[9] S. Stoft, *Power System Economics: Designing Markets for Electricity*. IEEE Computer Society Press, 2002.

[10] J. Carpentier, "Contribution à l'étude du dispatching économique," *Bulletin de la Société Française des Electriciens*, vol. 3, pp. 431–447, 1962.

[11] R. Fourer, D. Gay, and B. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*. Duxbury Press / Brooks/Cole Publishing Company, 2002.

[12] F. Galiana and M. Phelan, "Allocation of transmission loss to bilateral contracts in a competitive environment," *IEEE Transactions on Power Systems*, vol. 15, no. 1, pp. 143–150, February 2000.

[13] J. Bialek, S. Ziemianek, and N. Abi-Samra, "Tracking-based loss allocation and economic dispatch," in *Proceedings of the 13th Power Systems Computation Conference*, Trondheim, Norway, June 1999, pp. 375–381.

[14] J. Bialek, "Tracing the flow of electricity," in *IEE Production, Generation, Transmission and Distribution No. 4*, July 1996, pp. 313–320.

[15] J. Bialek and P. Kattuman, "Proportional sharing assumption in tracing methodology," *IEE Proceedings Genereration, Transmission and Distribution*, vol. 151, no. 4, pp. 526–532, July 2004.

[16] F. Galiana, A. Conejo, and H. Gil, "Transmission network cost allocation based on equivalent bilateral exchanges," *IEEE Transactions on Power Systems*, vol. 18, no. 4, pp. 1425–1431, November 2003.

[17] V. Smith, "Economics in the laboratory," *The Journal of Economic Perspectives*, vol. 8, no. 1, pp. 113–131, May 1994.

[18] A. Mas-Collel, *Microeconomic Theory*. Whinston and Green, 1995.

[19] Y. Phulpin, M. Hennebel, and S. Plumel, "A market based approach to evaluate the efficiency of transmission loss allocation," in *Proceedings of the Modern Electric Power System conference*, Wroclaw, Poland, September 2006, pp. 57–61.

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