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On the Fairness of Centralised Decision-Making Strategies in multi-TSO Power Systems

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Abstract - In this paper, we consider an interconnected power system, where the different Transmission System Operators (TSOs) have agreed to transferring some of their competences to a Centralised Control Center (CCC). In such a context, a recurrent difficulty for the CCC is to define decision-making strategies which are fair enough to every TSO of the interconnected system. We address this multi-objective problem when the objective of every TSO can be represented by a real-valued function. We propose an algorithm to elect the solution that leads to the minimisation of the distance with the utopian minimum after having normalised the different objectives. We analyse the fairness of this solution in the sense of economics. We illustrate the approach with the IEEE 118 bus system partitioned in 3 areas having as local objective the minimisation of active power losses, the maximisation of reactive power reserves, or a combination of both criteria.

Keywords - multi-area power system, centralised control, multi-objective optimisation, fairness.

1 Introduction

THE operational coordination of Transmission System Operators (TSOs), particularly with regard to security operation, is an important issue in interconnected power systems [1, 2]. To address this problem of coordination, new control strategies have been proposed [3, 4] and recent work, such as [5], has highlighted the potential benefits of a Centralised Control Center (CCC) to make decisions for multi-TSO systems. In large-scale interconnected power systems such as the UCTE system for example, a CCC could thus be created in order to coordinate the actions of the different TSOs.

It is, however, expected that, even with the creation of a CCC, every TSO will preserve some prerogatives on its own system operation. Indeed, operational objectives are likely to remain defined by the TSOs since they may be dependent on local topology, system architecture, generation capacity or continuation of traditional engineering practices [6]. Moreover, prior to agreeing to transferring some of their competences to a higher decision level, the TSOs would probably require some guarantees regarding the fulfillment of their own objectives by the CCC. This issue may be conflicting as satisfying the objective of a single TSO may adversely affect other TSOs. The CCC will thus be continuously confronted with an arbitrage, which should be "fair enough" to every TSO.

While this arbitrage could be carried out through a negotiation between different parties for long-term decisions (e.g., transmission investments), decision-making at an operational level should handle short-term operation and should thus rely on some well-established procedures. This paper proposes an optimisation scheme for the CCC when the objective of every TSO can be represented by a real-valued function, which may be of non-economic nature and will from now on be referred to as "cost function". Such a scheme could be used as a systematic procedure to address short-term operation issues. The scheme relies on the formulation of the problem as a multi-objective optimisation problem and selects a solution which could at least, in principle, bring consensus among the different TSOs. Besides the fact that the solution minimises the distance with the utopian minimum in a normalised multi-dimensional space, we also analyse its fairness in the sense of economics. Many of the discussions will be based on an illustrative example which will also serve as common thread in this paper.

The paper is organised as follows. Section 2 defines the multi-objective problem and presents the illustrative example. In Section 3, we propose a normalisation of the multi-objective problem and a procedure for identifying the element that stands closest to the origin of the normalised space. Section 4 introduces the concept of fairness in economics and we show in Section 5 that our method has indeed some properties of fairness. Finally, some opportunities for further research are outlined in Section 6.

2 Formulation and illustrative example

In the first part of this section, we introduce some notations that will be used through this paper to study the multi-objective problem which the CCC is supposed to solve. Afterwards, our illustrative example is presented.

2.1 *Formulation of the problem*

We focus on a system partitioned in $nbArea$ areas. Each area i has a TSO i, which has its own objective expressed by an objective function C_i. We also assume that its ability to influence the system has been transferred to a CCC. Let $\mathbf{u}$ denote the joint control variable[1] that appends every TSO's individual control variables and U the set of joint control variables. We assume that C_i is defined on U, that is $C_i(\mathbf{u}) : U \mapsto \Re$. Also, the set of constraints imposed by TSO i is denoted by $g_i(\mathbf{u}) \leq 0$. The joint constraint function $g(\mathbf{u})$ for the entire system is obtained by appending the $nbArea$ functions $g_i(\mathbf{u})$.

The constrained multi-objective optimisation problem faced by the CCC can thus be formulated as follows:

$$\min_{\mathbf{u}}[C_1(\mathbf{u}), C_2(\mathbf{u}), \ldots, C_{nbArea}(\mathbf{u})] \tag{1}$$

subject to:

$$g(\mathbf{u}) \leq 0 \tag{2}$$

The cost associated with a solution $\mathbf{u}$ can be represented in a $nbArea$-dimensional cost-space by a vector $[C_1(\mathbf{u}), C_2(\mathbf{u}), \ldots, C_{nbArea}(\mathbf{u})]$. It is commonly adopted in the multi-objective optimisation literature that the solution of such a problem is characterised by its Pareto-front.

A Pareto-front is defined in the $nbArea$-dimensional cost-space as the set of non-dominated solutions. A solution $\mathbf{u_p}$ is non-dominated if there exists no other solution $\mathbf{u} \in U$ such that for every $i \in [1, 2, \ldots, nbArea]$, $C_i(\mathbf{u}) \leq C_i(\mathbf{u_p})$. Should the Pareto-front be reduced to a single element, the solution of the arbitrage made by the CCC would then be this element. In such a particular context, there would indeed exist a solution minimising every single objective. However, in general, the Pareto-front is composed of many elements (possibly an infinite number of them) and the CCC must choose one of those elements. In this paper, the arbitrage problem to which the CCC is confronted will therefore be the problem of selection of the fairest solution on the Pareto-front.

Numerous papers have proposed methods for tracing the Pareto-front of multi-objective optimisation problems. Many of these techniques compute the Pareto-front by exploiting the property that by minimising a linear combination of the individual objectives, one has a solution which stands on the Pareto-front [10,11]. To compute the Pareto-front, we use in this paper the so-called normal boundary intersection approach, already used in [12] to solve a 2-objective OPF problem. As proven in [13], its output solutions are evenly distributed unlike those obtained with other techniques.

2.2 *Illustrative example*

The benchmark power system used herewith is the IEEE 118 bus system, commonly used as test system by the power system researchers (see e.g. [14, 15]). We have partitioned the system into 3 areas referred to as 1, 2 and 3. This system is shown in Figure 1.

[1] Multi-dimensional variables are represented by bold fonts in this paper.

The proposed methodology is applied to a multi-TSO reactive power scheduling problem, which is a particular type of Optimal Power Flow (OPF). In such a context, the load demand is supposed to be static, as well as the generation dispatch. A decentralised slack bus is used in our simulations, which may slightly change this dispatch. Every element $\mathbf{u} \in U$ is composed of the generators' output voltages, the capacitor banks' or FACTS' reactive power injections and the tap settings. The inequality (2) represents the limits on voltage magnitude at each bus and on reactive power injections for every generator or compensator, as well as the fact that the active power exports are maintained constant.

Every TSO i has an objective of a different nature. TSO 1 focuses only on the minimisation of its own active power losses APL_1, TSO 2 on the minimisation of a quadratic sum of its reactive power injections (QSQ_2) and TSO 3 has an objective function which is a linear combination of APL_3 and QSQ_3. Such types of objectives are commonly used in the literature to describe operational objectives of TSOs (see for example [7–9]). These 3 objectives can be represented by the generic expression:

$$C_i(\mathbf{u}) = \gamma_i APL_i(\mathbf{u}) + (1 - \gamma_i) QSQ_i(\mathbf{u}) \tag{3}$$

where $\gamma_i \in [0, 1]$ is the weight coefficient for area i, $APL_i(\mathbf{u})$ is the amount of active power losses in area i and $QSQ_i(\mathbf{u})$ is the sum of the square of the reactive power injections in area i. In our example, γ_1 is equal to 1, γ_2 to 0 and γ_3 to 0.9. The Pareto-front of our illustrative case is represented in Figure 2.

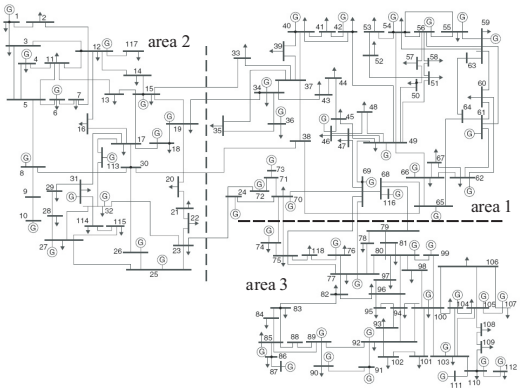


Figure 1: IEEE 118 bus system with 3 TSOs.

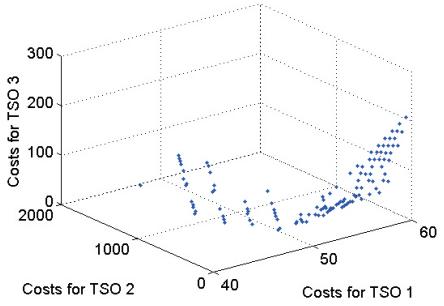


Figure 2: Representation of the Pareto-front for the IEEE 118 bus system with 3 areas.

3 An algorithm to make decision for multi-objective problems

In this section, we propose an approach for electing the point on the Pareto-front that could satisfy the different parties. The approach relies on finding a solution as close as possible to the "utopian minimum" C^{ut} defined as follows:

$$C^{ut} = [C_1(\mathbf{u}_1^*), C_2(\mathbf{u}_2^*), \ldots, C_{nbArea}(\mathbf{u}_{nbArea}^*)] \quad (4)$$

where $\mathbf{u}_i^*$ is the solution of Problem (5) which optimises the entire system with a unique objective $C_i(\mathbf{u})$ under Constraint (2), that is:

$$\mathbf{u}_i^* = \arg\min_{\mathbf{u}\in U} C_i(\mathbf{u}) \quad (5)$$

This approach is based on the following principle: should a "utopian minimum" exist, it would then be picked as solution since everyone of TSOs' objectives are optimised with that solution. However, we know that, except if the Pareto-front is reduced to a single element, the "utopian minimum" can not be reached. That is why we choose the solution that minimises the distance (related to an Euclidian norm after having normalised the cost functions) with the "utopian minimum".

The method for normalising the cost functions is presented in Section 3.1, while the algorithm for finding the point on the Pareto-front is described in Section 3.2. Finally, the approach is illustrated on the test system described in Section 2.2.

3.1 Normalisation

We explain hereafter the normalisation process that can be adopted to obtain a fair arbitrage. Its rationale is twofold. First, every local objective can have a different nature (e.g., minimisation of active power losses, maximisation of reactive power reserves, etc.). This problem should naturally be addressed by the normalisation process. Second, it also makes sense to normalise the cost functions in order to penalise the TSOs whose objective fulfillment is detrimental to other TSOs' objectives and favor those whose objectives are particularly compatible with the others.

For a cost function C_i, the normalisation factor will be the product of the two terms C_i° and χ_i. The normalised cost function $\overline{C}^i$ will thus be computed using the following equation:

$$\overline{C_i}(\mathbf{u}) = \frac{C_i(\mathbf{u})}{C_i^\circ \times \chi_i} \quad (6)$$

Before defining the two terms, since we will pick a solution on the Pareto-front that stands closest to the "utopian minimum" according to a Euclidian distance, a small normalisation factor for TSO i will have for effect to give more weight to its own objective function C_i and will then favor it.

The term C_i° is defined as follows:

$$C_i^\circ = \sum_j \frac{C_i(\mathbf{u}_j^*) - C_i(\mathbf{u}_i^*)}{nbArea} \quad (7)$$

and has been introduced for two main reasons. First, it is expressed in the same unit as C_i and will therefore make possible the comparison between objective functions having different natures. In particular, it will make our approach independent of any scaling factor that may affect the different cost functions C_i. Second, the term C_i° will also favor a TSO whose objective fulfillment is weakly penalised by the fulfillment of the other objectives. Indeed, C_i° being the average value of the overcosts supported by TSO i for the $nbArea$ control variables $\mathbf{u}_1^*$, $\mathbf{u}_2^*$, …, $\mathbf{u}_{nbArea}^*$, this term will be particularly small if the overcosts induced by other objective fulfillments $C_i(\mathbf{u}_j^*)$ are small.

The term χ_i is defined as follows:

$$\chi_i = \sum_j \frac{C_j(\mathbf{u}_i^*) - C_j(\mathbf{u}_j^*)}{C_j^\circ} \quad (8)$$

and has been introduced to penalise the detrimental impact of TSO i's objective achievement on the other TSOs' costs, represented by the term $C_j(\mathbf{u}_i^*) - C_j(\mathbf{u}_j^*)$. We note that this difference term is divided by C_j°. Thus, this division allows to sum up unitless overcosts having different natures. Also, this normalisation aims to leverage the penalisation that TSO i endures when its optimal control variables are detrimental to the objective of another TSO j, which is itself compatible with the other TSO's objective.

By anticipating the results of Section 5, we find that, by using the normalisation factor $C_i^\circ \times \chi_i$, the solution of the arbitrage has some properties of fairness in the economic sense. It is obvious that other approaches to compute the normalisation factors could also be designed.

3.2 Optimisation of the normalised problem

As mentioned earlier, our approach will elect the solution $\mathbf{u}^*$, whose cost vector $C(\mathbf{u}^*)$ minimises (in the normalised cost space) the Euclidian distance to the "utopian minimum". This problem can be formulated as follows:

$$\mathbf{u}^* = \arg\min_{\mathbf{u}\in U} \sum_{i=1}^{nbArea} (\overline{C_i}(\mathbf{u}) - \overline{C_i}(\mathbf{u}_i^*))^2 \quad (9)$$

under Constraint (2).

Solving this problem is equivalent to finding the point on the Pareto-front that minimises the distance to the utopian minimum. As proof, suppose that $\mathbf{u}^*$ is not on the Pareto-front and is solution of (9) under Constraint (2). Then, there would exist a solution $\mathbf{u}'$ such that $C_i(\mathbf{u}') \leq C_i(\mathbf{u}^*)$ for every $i \leq nbArea$. In this case, for every area i, we would have $\overline{C_i}(\mathbf{u}') \leq \overline{C_i}(\mathbf{u}^*)$ and consequently, $\sum_{i=1}^{nbArea} (\overline{C_i}(\mathbf{u}') - \overline{C_i}(\mathbf{u}_i^*))^2 \leq \sum_{i=1}^{nbArea} (\overline{C_i}(\mathbf{u}^*) - \overline{C_i}(\mathbf{u}_i^*))^2$. Therefore, $\mathbf{u}^*$ would not be the solution of (9), which is contradiction.

Table 1 summarises the procedure for computing, according to our strategy, a point on the Pareto-front which

could satisfy the different TSOs. This procedure implies solving the optimisation problem (9) under Constraint (2). This problem can be solved using a standard OPF algorithm [16–18]. In principle, its resolution should not be more difficult than a classical OPF problem, at least if we assume that solving an OPF problem whose cost function is the sum of squared individual OPF-like functions is not more difficult than solving any other OPF problem. Such an assumption makes sense since the convexity properties of the different C_i, usually assumed by OPFs, are preserved in (9). As optimisation tools, we have used MINOS [19] with a formulation of the problem in AMPL (A Mathematical Programming Language) [20].

Input: For every TSO i, a real-valued objective function $C_i(\mathbf{u})$ and a constraint vector $\mathbf{g}_i(\mathbf{u})$.
Output: A vector of control variables $\mathbf{u}^*$.
Step 1: For every TSO i, compute $\mathbf{u}_i^*$, solution of:
$\arg\min_{\mathbf{u}\in U} C_i(\mathbf{u})$ with respect to: $g(\mathbf{u}) \leq 0$.
Step 2: Compute the solution $\mathbf{u}^*$ of:
$\arg\min_{\mathbf{u}\in U} \sum_{i=1}^{nbArea} (\overline{C_i}(\mathbf{u}) - \overline{C_i}(\mathbf{u}_i^*))^2$
with respect to: $g(\mathbf{u}) \leq 0$
where $\overline{C_i}(\mathbf{u}) = \frac{C_i(\mathbf{u})}{C_i^\circ \times \chi_i}$
with $C_i^\circ = \sum_j \frac{C_i(\mathbf{u}_j^*) - C_i(\mathbf{u}_i^*)}{nbArea}$
and $\chi_i = \sum_j \frac{(C_j(\mathbf{u}_i^*) - C_j(\mathbf{u}_j^*))}{C_j^\circ}$.

Table 1: An algorithm for identifying a fair solution of the multi-objective optimisation problem.

3.3 Example

The proposed method is illustrated hereafter with the test system described in Section 2.2. Table 2 gives the different costs $C_i(\mathbf{u}_j^*)$, the normalised overcosts $\overline{C_i}(\mathbf{u}_j^*) - \overline{C_i}(\mathbf{u}_i^*)$ and the terms involved in the computation of the normalisation factors. The bottom of the table also gives the costs $C_i(\mathbf{u}^*)$ and the normalised overcosts $\overline{C_i}(\mathbf{u}^*) - \overline{C_i}(\mathbf{u}_i^*)$ supported by each TSO. As one can see, those overcosts are particularly small. Figure 3 represents the localisation of the normalised costs corresponding to $\mathbf{u}^*$ on the normalised Pareto-front.

	$i=1$	$i=2$	$i=3$
$C_i(\mathbf{u}_1^*)$	43.02	1359.8	66.65
$C_i(\mathbf{u}_2^*)$	59.40	0.0	211.70
$C_i(\mathbf{u}_3^*)$	51.79	1999.9	37.84
C_i°	8.38	1119.9	67.56
χ_i	1.64	4.53	2.83
$\overline{C_i}(\mathbf{u}_1^*) - \overline{C_i}(\mathbf{u}_i^*)$	0	0.2682	0.1506
$\overline{C_i}(\mathbf{u}_2^*) - \overline{C_i}(\mathbf{u}_i^*)$	1.1910	0	0.9088
$\overline{C_i}(\mathbf{u}_3^*) - \overline{C_i}(\mathbf{u}_i^*)$	0.6375	0.3944	0
$C_i(\mathbf{u}^*)$	43.17	60.65	38.85
$\overline{C_i}(\mathbf{u}^*) - \overline{C_i}(\mathbf{u}_i^*)$	0.0111	0.0120	0.0053

Table 2: Values of the different costs $C_i(\mathbf{u})$ and normalised overcosts $\overline{C_i}(\mathbf{u}) - \overline{C_i}(\mathbf{u}_i^*)$ for every solution $\mathbf{u}_j^*$ of the single objective optimisations and for the solution $\mathbf{u}^*$ of the centralised decision making scheme. Values of C_i° and χ_i for TSO i are also reported.

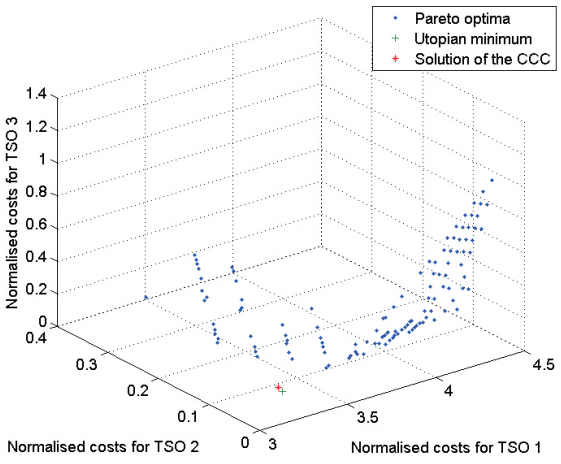


Figure 3: Localisation of the CCC's solution on the normalised Pareto-front for the IEEE 118 bus system partitioned into 3 areas.

4 Fairness criteria for an arbitrage

The notion of fairness has been widely studied in economics. A consensus is that this notion is highly subjective [21] and that different arbitrages can be simultaneously considered as "fair" for any given situation. However, the classification proposed by J. Konow in [22] provides some objective criteria for assessing the fairness of a particular allocation, namely the "efficiency", the "accountability" and the "altruism". Those criteria have been defined by analysing experimental data obtained by polling people on their opinions concerning the fairness of different types of allocations.

4.1 On efficiency

According to J. Konow, an arbitrage can not be qualified as fair if it is poorly efficient, that is, if considerable resources are not allocated. While he does not define explicitly the level of efficiency of a given arbitrage for a multi-objective problem, we will consider here that the level of performance is optimal if there exists no other arbitrage which can lead to a better outcome for all the parties. In this paper, where all objectives are expressed using real-valued functions, we propose to relate the efficiency of an arbitrage to the distance between its outcome and the Pareto-front of the problem. Since, as stated in Section 2.1, the problem of arbitrage is reduced here to the choice of the fairest solution on the Pareto-front, this efficiency criterion, or at least the interpretation we make of it, will not help in this task.

There has been a trend in the economic literature to oppose the Pareto efficiency and fairness of an arbitrage (see e.g. [23]). In our view, this trend is not conflicting with our approach to fairness, since efficiency is not sufficient to guarantee the fairness of an arbitrage.

4.2 On accountability

"Acountability" is defined by J. Konow as follows: "if the individual with more has earned his superior position". An example of an accountable arbitrage is given in [22]: consider two individuals with the same abilities

and a global earning that should be divided between them, if one chooses to work 50% less, an accountable notion of fairness would allocate him less earning than to the other individual.

4.3 On altruism

The notion of "altruism" is defined by J. Konow in [22]. He states that what parties can not influence should not affect the allocation. J. Konow gives in the following example for altruism: if two individuals having different abilities work each at 100% of their capabilities, an altruist notion of fairness would allocate them the same share of the global earning. This notion is also developed by M. Rabin, who associates in [24] the fairness with the concept of "reciprocity".

5 Evaluation of the centralised strategy

In Section 3, we have presented a method for electing one solution of the multi-objective optimisation problem described in Section 2. In Section 4, we reviewed three criteria used in economics for assessing the fairness of an arbitrage. We assess hereafter whether the arbitrage proposed in Section 3 satisfies those criteria for our example. Afterwards, we discuss the sensitivity of our allocation scheme with respect to false information a TSO may be tempted to provide in order to gain more.

5.1 Efficiency

As proven in Section 3.2, the solution of our optimisation scheme is on the Pareto-front. The solution elected has thus the property of maximum efficiency whatever the objective functions are.

5.2 Accountability

As introduced in Section 4, an allocation is "accountable" if it is profitable for the party investing more effort. In the context of MVAr scheduling, this notion is hardly quantifiable. However, we will consider here that an "effort" of TSO i could be to make the constraints $g_i(\mathbf{u}) \leq 0$ less strict. Let us define, for example, that an "effort" would be the increase of the range of possible bus voltages in an entire control area (from $[0.94, 1.06]$ to $[0.92, 1.08]$).

In order to study the accountability of our arbitrage strategy, we have optimised the base case system with no effort and with an effort from each TSO, successively. Table 3 presents the costs and normalised overcosts supported by each TSO in every case.

When comparing the results obtained with different localisations of the effort, one can notice that if a TSO makes an effort, the final allocation is generally more profitable for this TSO, at least in the original cost space. This "accountability" can also be observed in the normalised space, where the overcosts $\overline{C_i}(\mathbf{u}^*) - \overline{C_i}(\mathbf{u}_i^*)$ tend to decrease when TSO i makes an effort (except for TSO 3 in this example).

Effort	$C_1(\mathbf{u}^*)$	$C_2(\mathbf{u}^*)$	$C_3(\mathbf{u}^*)$
None	43.17	60.65	38.85
TSO 1	42.34	49.40	38.58
TSO 2	43.13	44.25	38.45
TSO 3	43.10	61.00	38.59
Effort	$\overline{C_1}(\mathbf{u}^*) - \overline{C_1}(\mathbf{u}_1^*)$	$\overline{C_2}(\mathbf{u}^*) - \overline{C_2}(\mathbf{u}_2^*)$	$\overline{C_3}(\mathbf{u}^*) - \overline{C_3}(\mathbf{u}_3^*)$
None	0.0111	0.0120	0.0053
TSO 1	0.0088	0.0107	0.0058
TSO 2	0.0083	0.0081	0.0052
TSO 3	0.0114	0.0119	0.0056

Table 3: Values of the cost functions $C_i(\mathbf{u}^*)$ and normalised overcosts $\overline{C_i}(\mathbf{u}^*) - \overline{C_i}(\mathbf{u}_i^*)$ in every area of the test system. Four cases have been studied: no extra effort, effort from TSO 1, effort from TSO 2 and effort from TSO 3.

However, those observations can not be generalised since there are some cases for which the final allocation is not accountable. Indeed, let us consider the case where a TSO i makes an "effort" from which it does not benefit directly ($C_i(\mathbf{u}_i^*)$ remains constant). In such a context, its effort could allow the other TSOs to increase their possible benefits by increasing their use of TSO i's resources. This could change the normalisation factors, especially the ones related to TSO i, and the location of the utopian minimum so that the final allocation could be less profitable for TSO i. In particular, this situation has arisen when defining as "effort" from TSO 2 an increase of the reactive power injection capabilities for every generator in its control area. For such a case, $C_2(\mathbf{u}_2^*)$ remains the same (0.0, see Table 2) but its resources can be used by the other TSOs, which leads to an increase of C_2° (from 1119.9 to 1971.0) while χ_2 does not decrease significantly (from 4.53 to 4.28) and the other normalisation factors tend to decrease. This leads to an increase of $C_2(\mathbf{u}^*)$ (from 60.65 to 86.19) and so, penalises TSO 2, which has made more effort.

Our allocation is also not accountable if applied to a system with only two parties. The normalisation factors for TSO 1 and TSO 2 would then be $C_1^\circ \times \chi_1 = C_1(\mathbf{u}_2^*) - C_1(\mathbf{u}_1^*)$ and $C_2^\circ \times \chi_2 = C_2(\mathbf{u}_1^*) - C_2(\mathbf{u}_2^*)$, respectively. Therefore, one TSO would be rewarded if its objective fulfillment is highly penalising its neighbor and the arbitrage could not be accountable. This flaw disappears, however, when considering systems with 3 TSOs or more. Indeed, the more TSOs participate in the process, the more importance is given to a local objective which affects slightly the other TSOs' objectives.

5.3 Altruism

One concept behind altruism is that a parameter which does not depend on TSOs' actions should not affect the allocations. The interpretation we make here of this concept is that a TSO i, whose control variables have little impact on the objectives of the other TSOs, should not have much influence in their allocations, whatever its objective function C^i. However, since the dynamics of the different areas of our benchmark system were highly coupled, we have been unable to check on the test problem whether this concept was indeed satisfied.

Another concept behind altruism is that the allocations should not be biased toward the TSOs with the greatest "abilities". Indeed, as written in Section 4.3, the overcosts should rather be shared according to the efforts made by the different TSOs. In the context of MVAr scheduling, we consider that the ability of a TSO is related to its influence on the dynamics of the system. Thus, the TSOs that have a strong influence on the system should not have a highly negative impact on the other TSOs. In this respect, our allocation scheme has clearly some altruism properties since the terms χ_i and C_i^0 penalise the TSO i whose objective fulfilment is not compatible with the other objective fulfilments.

5.4 Sensitivity to biased information

In the context of a CCC, the TSOs may be tempted to provide wrong information in order to gain more. We discuss here how sensitive our arbitrage scheme is with respect to biased information concerning the objective functions or the constraints (e.g., limitations on voltage or reactive power injections).

We first focus on a biased formulation of a TSO's cost functions. More precisely, we suppose that a TSO i provides the CCC with a function C_i^w rather than C^i and that any other type of information provided is correct.

If $C_i^w(\mathbf{u}) = a \times C_i(\mathbf{u}) + b$ with $a, b \in \Re$, the allocation strategy is not affected since, as emphasised in Section 3.2, our arbitrage strategy has the property of being immune to any linear transformation of the objective functions[2].

Now, let us consider the case, where $C_i^w(\mathbf{u}) = C_i(\mathbf{u}) \times C_i(\mathbf{u})$. Intuitively, with such a wrong formulation of its objective function, TSO i could obtain a better allocation, since it may give to the CCC the impression that a deviation from $\mathbf{u}_i^*$ is worse for it than it is in reality. However, such a strategy is not systematically beneficial for a TSO. For example, if TSO 1, which focuses on minimisation of its active power losses APL_1, asks the CCC to minimise the square of APL_1, the arbitrage leads to a solution were $APL_1 = 43.178$ rather than $APL_1 = 43.172$ if TSO 1 were to provide its true objective function. Therefore, such a strategy of overestimating its costs may be counter-productive.

Even if it is clear that by truncating their objective function, the TSOs could bias the allocation in their favor, such a problem could be avoided in practice by constraining the TSOs to select their cost function in a set of reasonable formulations for the objectives.

Another way for the TSOs to bias the arbitrage scheme in their favor would be to report more restrictive constraints g_i such that they avoid non-accountable efforts. We refer to Section 5.2, for an example of the potential benefits of a TSO when it provides wrong information about its voltage constraints. The lack of accountability of our scheme may indeed induce such types of gaming. However, this strategy could be avoided if TSOs have to declare only one set of constraints, which would serve for scheduling very different types of situations (e.g., peak load or low demand). A biased formulation of the constraints could then be profitable in one situation but counter-productive in others.

6 Conclusions

In this paper, we have addressed the problem of centralised decision making for a multi-TSO power system for which every TSO's individual objective can be represented by a real-valued cost function. We emphasise that the problem could be reduced to the election of the fairest point on the Pareto-front. First, we advocated using "common engineering sense" to select the point which is closest (according to a specific distance measure) to the defined utopian minimum. We also proposed an algorithm for computing this point. This approach was illustrated with the IEEE 118 bus system partitioned in 3 areas having as local objective the minimisation of active power losses, the maximisation of reactive power reserves, or a combination of both criteria. Afterwards, we introduce briefly the concept of fairness as studied in economics and we show that our approach is indeed satisfying, at least to some extent, the fairness criteria.

While the number of potential applications of our method is large (any allocation that can be formulated as a multi-objective problem could be solved through our method), its Achilles' heel is related to the way we define the "fairest allocation" and, more specifically, to the cost functions normalisation procedure. This definition is in essence subjective. It may perhaps even be naïve to assess the fairness of an allocation without consulting the different parties.

In the framework of multi-TSO power system operation, there is a multitude of tasks, such as dynamic security assessment or transmission investments for which the objective of each party can not be expressed as a real-valued cost function. In such contexts, it would also be interesting, even challenging, to attempt to define the concept of fairest allocation.

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