Transformer Winding Losses Evaluation when Supplying Non Linear Load

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***Abstract*- Nonlinear loads produce harmonic currents, which induce additional losses in transformers and cause temperature rise, especially in the windings. The estimation of winding eddy-current loss, in the presence of harmonics, is based on the knowledge of winding eddy-current loss at power frequency. This paper deals with the transformer winding loss estimation and gives a simplified winding resistance expression that takes winding eddy-current into account and allows to estimate associated losses. The impact of transformer winding characteristics, as conductor size and layer number, on the resistance variation with frequency is shown.**

***Index Terms*-- Eddy-current, harmonics, Transformer, winding losses.**

I. Introduction

The growing use of power electronics devices in industry, office and home equipment has led to study the impact of harmonic distortion on power networks materials. Current and voltage harmonics may cause a large number of problems for electrical equipment:

- Malfunction of circuit breakers and electronic equipment [1].
- Additional losses in motors, transformers and conductors [2], [3].
- Additional heating due to these additional losses.

IEEE Std C57.110-1998 [4] was created to establish uniform methods for determining the capability of transformers to supply non linear loads. Transformer losses are separated into no load loss (core loss) and load loss [4]. Load loss (P_{ll}) is subdivided into I^2R loss and stray loss due to stray electromagnetic flux in the windings, core, core clamps, magnetic shields, enclosure or tank walls, etc. The stray loss is subdivided into winding eddy-current loss (P_{ec}) and stray loss in components other than the windings (P_{osl}) as explained in (1).

$$P_{ll} = P_j + P_{ec} + P_{osl} = R_{ac} \cdot I^2 + P_{osl} \qquad (1)$$

Where
P_{ll} is load loss.
$P_j = R_{dc} \cdot I^2$ is loss due to load current and dc resistance of the windings.
P_{ec} is winding eddy-current loss.
P_{osl} is stray loss in components other than the windings.
R_{ac} is AC winding resistance.
R_{dc} is DC winding resistance.
I is the RMS current

When a transformer supplies non linear loads, I^2R increases with the RMS current variation and winding eddy-current loss increases with the square of the load current and the square of the frequency. Equation (2) allows calculating winding eddy-current loss through a proportionality factor applied to the winding eddy-current loss at fundamental frequency [4]. The reliance on the square of the frequency is recognized in this expression with the h² term. This expression shows the necessity to know fundamental eddy-current loss to determine the harmonic impact of eddy-current loss on the transformer.

$$P_{EC} = P_{EC-o} \cdot F_{hl} \quad \text{where} \quad F_{hl} = \frac{\sum_{h=1}^{h_{\max}} \left(\frac{I_h}{I_1} \right)^2 \cdot h^2}{\sum_{h=1}^{h_{\max}} \left(\frac{I_h}{I_1} \right)^2} \qquad (2)$$

Where
P_{ec} is winding eddy-current loss.
P_{ec-o} is winding eddy-current loss at the measured current and the power frequency.
F_{hl} is harmonic loss factor for winding eddy-current.
h is harmonic order.
I_h is the RMS current at harmonic frequency of order h.
I_1 is the RMS current at fundamental frequency.

The aim of this paper is to give a winding resistance expression, which takes winding eddy-current into account and allows estimating P_{EC-o} and then winding loss in presence of harmonics. This analytical expression shows the impact of transformer winding characteristics, as conductor size and transformer winding geometry, on the resistance frequency dependence. After theoretical results on a simple conducting plate, the method to obtain the analytical expression of winding resistance is developed. Finally, some numerical results on two different transformers are presented to show the impact of winding characteristics on the AC winding resistance.

II. THEORETICAL BACKGROUND

Fig. 1 shows a simple conducting plate, assumed to be infinite in the y and z directions and carrying a sinusoidal current density *J* in the y direction, with sinusoidal magnetic fields of constant amplitudes h_1 and h_2 outside the plate in the *z* direction.

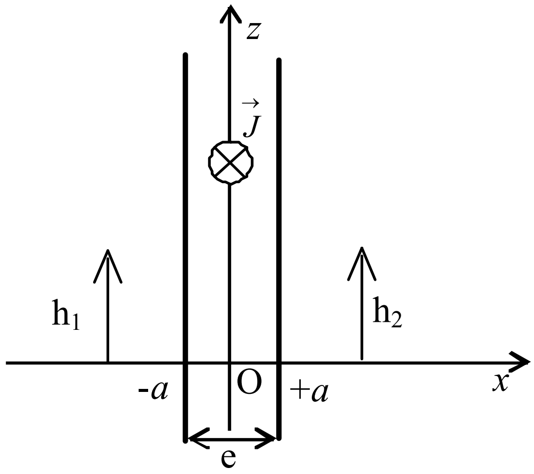

Fig. 1. Studied conducting plate.

The distribution of magnetic field *H* in metals at relatively low frequencies is described by the diffusion equation (3), which is calculated from Maxwell's equations.

$$\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} \tag{3}$$

Where μ *is* the magnetic permeability and σ *is* the electrical conductivity.

Considering that the magnetic field is only with *z* component and as there are no variations in the *y* and *z* directions, the magnetic field equation can be written:

$$\frac{\partial^2 H_z}{\partial x^2} = \mu\sigma \frac{\partial H_z}{\partial t} \tag{4}$$

A solution of this equation is of the form $H_z = Re(h(x)e^{j\omega t})$, thus equation (4) becomes:

$$\frac{d^2 h(x)}{dx^2} + k^2 h(x) = 0 \tag{5}$$

With

$$k^2 = -j\omega\mu\sigma$$

Where ω is the pulsation and j the complex number defined by $j^2 = -1$.

The equation (5) solution is

$$h(x) = a_1 e^{jkx} + a_2 e^{-jkx} \tag{6}$$

The integration constants a_1, a_2 are determined from the known boundary conditions.

$$\begin{cases} h(a) = h_2 = a_1 e^{jka} + a_2 e^{-jka} \\ h(-a) = h_1 = a_1 e^{-jka} + a_2 e^{jka} \end{cases}$$

Hence, the magnetic field expression is

$$h(x) = \frac{-h_1.\sinh[jk(x-a)] + h_2.\sinh[jk(x+a)]}{\sinh(j2ka)} \tag{7}$$

The distribution of current density J is calculated from Maxwell's equations.

$$J_y = -\frac{\partial H_z}{\partial x} \tag{8}$$

A solution of this equation is of the form $J_y = Re(j(x)e^{j\omega t})$, thus equation (8) becomes :

$$j(x) = -\frac{dh(x)}{dx} \tag{9}$$

Hence, the current density expression is

$$j(x) = jk\left\{\frac{h_1.\cosh[jk(x-a)] - h_2.\cosh[jk(x+a)]}{\sinh(j2ka)}\right\} \tag{10}$$

The loss per square meter is obtained by integrating $\rho.J^2$.

$$P_{ds} = \frac{1}{2}\rho \int_{-a}^{+a} |j(x)|^2 dx \tag{11}$$

Consequently, the Joule losses in the plate may be determined by the integral (11) and the current density expression (10) as follows

$$P_{ds} = \frac{\sqrt{\omega\mu\sigma}}{\sigma}\left\{\frac{h_1^2 + h_2^2}{2\sqrt{2}} f_1(\varphi) - \frac{2h_1h_2}{\sqrt{2}} f_2(\varphi)\right\} \tag{12}$$

With

$$\varphi = e/\delta \quad \text{and} \quad \delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$f_1(\varphi) = \frac{\sinh(2\varphi) + \sin(2\varphi)}{\cosh(2\varphi) - \cos(2\varphi)} \tag{13}$$

$$f_2(\varphi) = \frac{\cosh(\varphi).\sin(\varphi) + \sinh(\varphi).\cos(\varphi)}{\cosh(2\varphi) - \cos(2\varphi)} \tag{14}$$

Where *e* is the thickness of the plate and δ is the depth of penetration.

This expression can be rewritten as shown in equation (15).

$$P_{ds} = \frac{h_2^{\ 2}}{2\sigma\delta}\left\{(1+\frac{h_1^{\ 2}}{h_2^{\ 2}}).f_1(\varphi) - 4\frac{h_1}{h_2}.f_2(\varphi)\right\} \quad (15)$$

For conductors with small thickness compared to the depth of penetration (φ<<1), functions $f_1(\varphi)$ and $f_2(\varphi)$ can be approximated by $g_1(\varphi)$ and $g_2(\varphi)$ given in (16) and (17) respectively.

$$f_1(\varphi) \approx g_1(\varphi) = \frac{1}{\varphi}\left[1 + \frac{4}{45}\cdot\varphi^4 + \varepsilon(\varphi^8)\right] \quad (16)$$

$$f_2(\varphi) \approx g_2(\varphi) = \frac{1}{2\varphi}\left[1 - \frac{7}{90}\cdot\varphi^4 + \varepsilon(\varphi^8)\right] \quad (17)$$

Fig. 2 shows comparison, between functions $f_1(\varphi)$, $f_2(\varphi)$ and $g_1(\varphi)$, $g_2(\varphi)$ for different values of the ratio e/δ. Fig. 3 shows the relative error in per cent between the functions and their approximations.

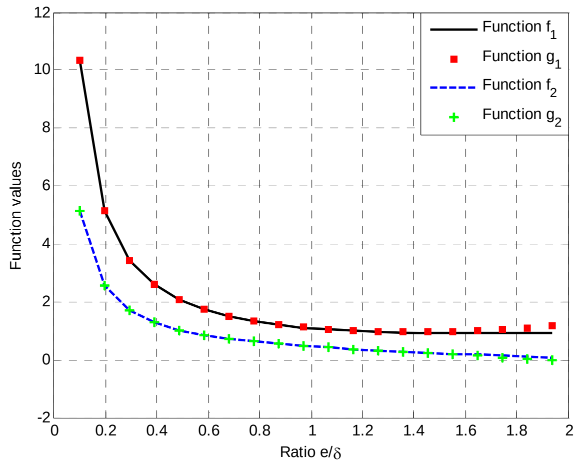

Fig. 2. Functions $f_1(\varphi)$, $f_2(\varphi)$, $g_1(\varphi)$ and $g_2(\varphi)$ versus ratio e/δ.

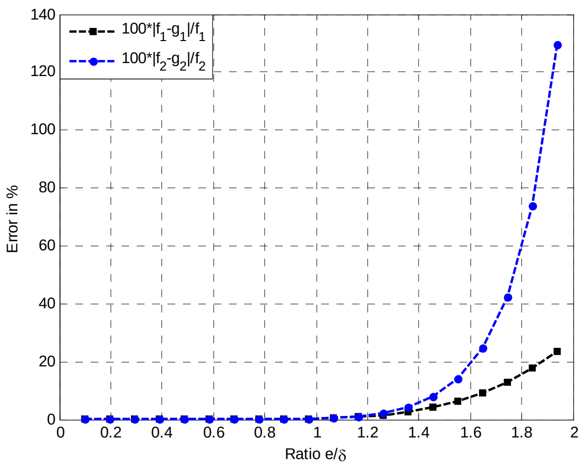

Fig. 3. Error in per cent between functions $f_1(\varphi)$& $g_1(\varphi)$ and $f_2(\varphi)$& $g_2(\varphi)$ versus ratio e/δ.

It can be seen that the two functions $g_1(\varphi)$ and $g_2(\varphi)$ give good results when the ratio between thickness and depth of penetration is less than 1.4 (errors less than 5%).

Thus, the Joule losses in the plate can be expressed according to (18) substituting $f_1(\varphi)$ and $f_2(\varphi)$ in equation (15) by $g_1(\varphi)$ and $g_2(\varphi)$.

$$P_{ds} = \frac{h_2^{\ 2}}{2e\sigma}\left\{(1-\alpha)^2 + \left(\frac{e}{\delta}\right)^4\left(\frac{4(1+\alpha^2)+7\alpha}{45}\right)\right\} \quad (18)$$

With $\alpha = \frac{h_1}{h_2}$

III. APPLICATION TO TRANSFORMER WINDINGS

For analysis, the structure of transformer windings can be approximated by a superposition of several conducting plates [5]. Fig. 4 shows the geometry of MV/LV transformer with the LV and MV windings respectively in the inner and in the outer. The curvature of the conductors, edge and end effects are neglected.

Using ampere's law, magnetic fields on both the left-hand side h_1 and right-hand side h_2 are respectively given by (19) and (20) for layer m.

$$h_1 = (m-1)\cdot\left(\frac{N}{n}\right)\frac{\hat{i}}{d} \quad (19)$$

$$h_2 = m\cdot\left(\frac{N}{n}\right)\frac{\hat{i}}{d} \quad (20)$$

With
m the layer number from the inner to the outer.
N the total number of turns in the windings.
n the total number of layers in the windings.
$\hat{i}$ the peak current in a turn.
d the height of windings.

Thus, the power per unit length, according to the y direction, dissipated in layer m (21) can be calculated using Joule losses expression in a plate (18) multiplied by the height of windings and magnetic fields expressions given by (19) and (20).

$$P_m = \left(\frac{N}{n}\right)^2\cdot\frac{I^2}{\sigma\cdot e\cdot d}\left[1+\left(\frac{e}{\delta}\right)^4\left(\frac{m^2}{3} - \frac{m}{3} + \frac{4}{45}\right)\right] \quad (21)$$

Total loss per unit length in transformer windings is then given by the sum of power dissipated in each layer (22).

$$P = \sum_{m=1}^{n} P_m = \left(\frac{N}{n}\right)^2\frac{I^2}{\sigma\cdot e\cdot d}\left[n + \left(\frac{e}{\delta}\right)^4\frac{5n^3 - n}{45}\right] \quad (22)$$

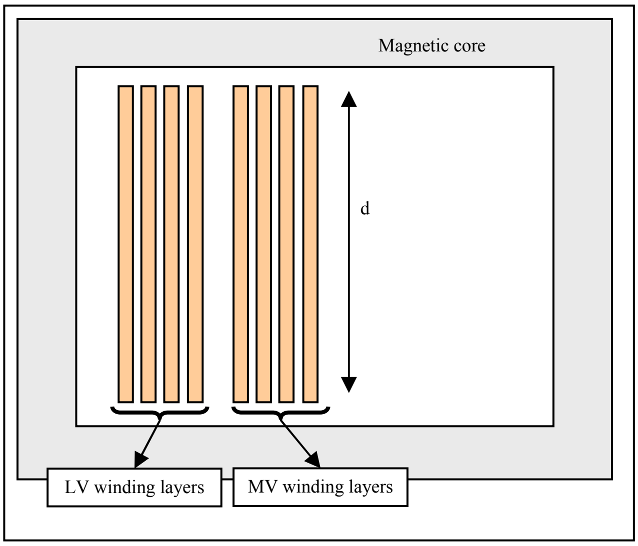

Fig. 4. Geometry of MV/LV transformer.

Introducing the DC resistance per unit length of the conductor (23), the AC resistance per unit length can be identified in the expression of total losses per unit length in transformer winding (22). Eddy-current losses are represented by the second term into brackets. Transformer geometry impact can be seen by the presence of layer number – n – and conductor thickness – e – in the expression.

$$R_{dc} = \frac{n}{de\sigma} \cdot \left(\frac{N}{n}\right)^2 \qquad (23)$$

$$P = R_{ac} \cdot I^2$$
$$with \quad R_{ac} = R_{dc}\left[1 + \left(\frac{e}{\delta}\right)^4 \left(\frac{n^2}{9} - \frac{1}{45}\right)\right] \qquad (24)$$

IV. Numerical Application

Generally, LV windings are made with rectangular or sheet conductors, while MV windings are made with small round conductors. Table I gives the winding characteristics of two typical 100 kVA transformers which are used for numerical application presented next. The ratio e/δ, given in the last column, is calculated for secondary conductors at 2.5kHz.

As primary winding characteristics are almost the same, winding ratio evolution is similar. The main difference between the transformers is the secondary winding geometry (Table I).

Then we present in Fig. 5 and Fig. 6 the R_{ac}/R_{dc} ratio evolution as a function of frequency for the secondary windings of transformer n°1 and n°2 respectively. The ratio is either calculated with (24) using g_1 and g_2, or with f_1 and f_2. Theses figures show that winding resistance with sheet conductors is less sensitive to frequency than winding resistance with rectangular conductors. Thus when a transformer is supplying non linear loads, winding eddy-current loss increase is much higher for rectangular conductors. Moreover (24) calculated with the simplified functions g_1 and g_2 gives a conservative result for transformer n°1 as soon as e/δ is greater than 1.4 (i.e. frequency greater than 800 Hz in the present case).

In presence of harmonics, eddy-current loss P_{EC} can be calculated using resistance expression (24).

$$P_{EC} = \sum_{h=1}^{h_{max}} R_{dc}\left(\frac{e}{\delta_h}\right)^4 \left(\frac{n^2}{9} - \frac{1}{45}\right) I_h^{\,2} \qquad (25)$$

Eddy-current loss at the measured current I and power frequency, $P_{EC\text{-}O}$, is

$$P_{EC-o} = R_{dc}\left(\frac{e}{\delta_R}\right)^4 \left(\frac{n^2}{9} - \frac{1}{45}\right) I^2 \qquad (26)$$

Where
δ_h is the depth of penetration for harmonic h.
δ_R is the depth of penetration at the power frequency.

Substituting (26) in (25), eddy-current loss expression can be rewritten and gives the same result that the IEEE Std C57.110-1998 (2).

TABLE I
TRANSFORMER CHARACTERISTICS

	Primary (MV)	Secondary (LV)	e/δ
Transformer n°1	Copper, ∅0.8mm 21 layers 5400 turns	Copper, rectangular Conductor 3.75*10mm² 64 turns, 2 layers	2.5 at 2.5kHz
Transformer n°2	Copper ∅ 0.75mm 19 layers 4417 turns	Copper sheet 0.2*210mm² 51 turns, 51 layers	0.13 at 2.5kHz

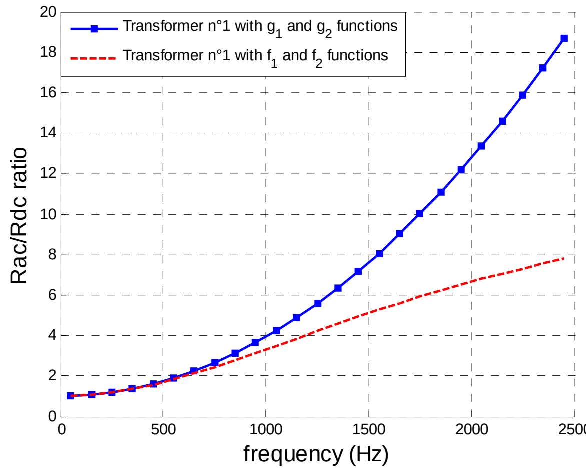

Fig. 5. Secondary winding R_{ac}/R_{dc} ratio evolution as a function of frequency for transformers n°1 presented in table I, and using functions {g_1, g_2} - equation (24) - or {f_1, f_2}.

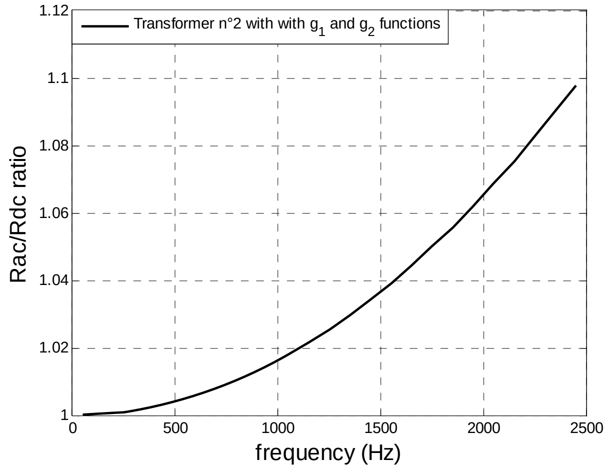

Fig. 6. Secondary winding R_{ac}/R_{dc} ratio evolution as a function of frequency for transformers n° 2 presented in table I, and using functions {g_1, g_2} - equation (24) .

The assumption that eddy-current losses in transformer windings are proportional to the square of the frequency (25) gives accurate results for small conductors or at low frequency (e/δ < 1.4). For large conductors and at high frequency (e/δ > 1.4), it leads to conservative results [7-8] as shown in Fig. 5.

V. CONCLUSIONS

The F_{hl} coefficient [4] is generally used to determine additional losses and capability of transformer when non linear loads are present, but it is necessary to know fundamental eddy-current loss to determine the harmonic impact on loss increase in transformer.

This paper gives a winding resistance expression which takes winding eddy-current into account and permits to estimate winding losses at power frequency, P_{EC-0}, and also in presence of harmonics.

This expression shows the impact of the following winding parameters: conductor thickness and layer number. This expression also underline that transformers composed of thin foil windings have winding eddy-current losses smaller than transformer with rectangular conductors. Thus, transformers with thin foil windings are less sensitive to harmonics. This work also shows that the F_{hl} coefficient gives conservative results for large conductors.

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