

Adaptive Equalization for Calibration of Subband Hybrid Filter Banks A/D Converters

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Abstract—Hybrid Filter Banks (HFB) A/D converters (ADC) are attractive to software-defined radio application, however their high sensitivity to analog imperfections is still a bottleneck for their realization. In this paper, adaptive equalization is applied for calibrating a subband HFB ADC. Thus, the synthesis filter coefficients are iteratively adjusted for compensating the mismatches between the analog part and the digital part. Simulations show that the robustness of subband HFB ADC is therefore improved.

I. INTRODUCTION

In the context of cognitive radio, Hybrid Filter Banks (HFB) A/D converters (ADC) may be an attractive solution to meet the future A/D conversion system requirements [1]. HFB are parallel architectures that consist in an analog frequency band decomposition (analysis bank), a conversion-stage (ADC bank) and a digital reconstruction (synthesis bank). From a given sampling rate on each channel, classical HFBs enlarge the conversion bandwidth [2],[3],[4] with a resolution related to the digital reconstruction complexity. Also, it is possible to improve the resolution in a narrow band by concentrating the digital part effort on the reconstruction of the band of interest. This can be achieved by using a subband HFB (Figure 1) for which only the digital part differ from the classical HFB. This architecture has been proposed in [5] and make it possible to reconstruct one or several subbands of the wide-band signal. Thus, it is possible to adapt by software the conversion characteristics (bandwidth and resolution) to the required specifications.

However, the imperfections, such as the analog realization errors, degrade the performances of the above-mentioned HFB ADCs dramatically [5], [6]. The manufacturing of high precision or tunable analog components is so costly that the adjustment of synthesis filter coefficients after the analog filter implementation may be an appropriate solution. Such a method should make the HFB recover the expected reconstruction quality. In [7], a calibration method using decorrelation technique can be applied to a multi-channel IF receiver to suppress the interference among the subband signals, whereas it is necessary to estimate the values describing the analog filters. Furthermore, a complex cross-correlation matrix needs be inverted, which is a critical point for realization. [8] proposes a method that uses a known signal at the input of HFB to estimate the analog filter frequency responses. The synthesis

filter coefficients are then calculated from the measurement of the analog filter frequency responses. This method offers good performance but such a calibration cannot be implemented in a final product because it does require a too complex calculation (a large matrix inversion).

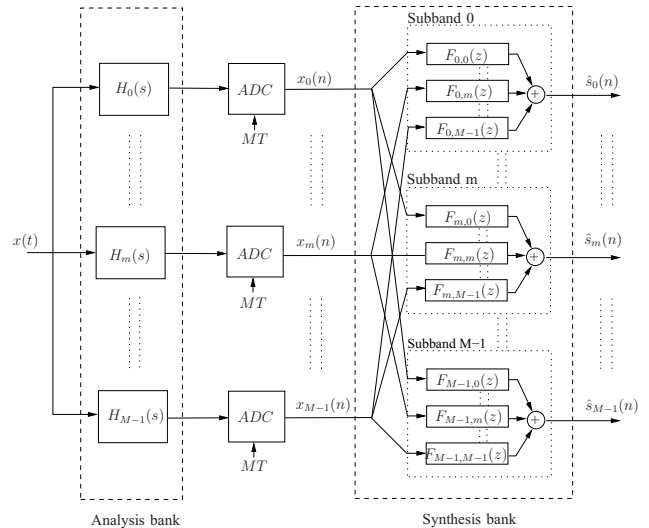


Fig. 1. Continuous-time M -channel subband HFB ADC, T denotes the Nyquist sampling period of input wide-band signal, $m \in [0, M - 1]$.

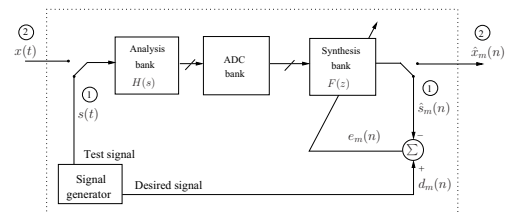


Fig. 2. Phase 1: calibration of HFB ADC using adaptive channel equalization; phase 2: conversion of the extern wideband signal $x(t)$.

In this paper, channel adaptive equalization is proposed to calibrate a subband HFB. With this method, widely used in many applications [9], the synthesis filter coefficients should be iteratively adjusted to minimize an error signal that should represent the reconstruction quality. In Figure 2, the phase 1 shows the proposed adaptive equalization principle in the HFB context. The signal generator provides two linked signals. $s(t)$

is a wide-band test signal for the HFB input. $d_m(n)$ is the desired signal at the output of the HFB. It corresponds to the selection of the subband of interest signal of the signal $s(t)$. The error signal $e(n)$ is used to adjust the synthesis filter coefficients. The HFB calibration will be achieved once the mean-square error (MSE) of $e(n)$ signal is low enough to meet the reconstruction accuracy requirements. Thus, fixing the synthesis filter coefficients achieved in the phase 1 and switching to the phase 2, the subband HFB can convert the extern wideband signal $x(t)$. The quantization errors are neglected here.

In section II, the proposed method is first theoretically justified upon a subband HFB ADC discrete-time model. In section III, the method is extended to a continuous-time subband HFB. An eight-channel HFB with initial good performance and then affected by analog errors is simulated in time domain. The adaptive equalization calibration is simulated and discussed.

II. ADAPTIVE CHANNEL EQUALIZATION FOR A DISCRETE-TIME SUBBAND HFB ADC MODEL

A. Perfect Reconstruction (PR) conditions of a subband HFB ADC

Figure 3 shows a theoretical model of a subband HFB in discrete-time domain. The input signal $x(t)$ is assumed to be band-limited that it could be sampled at $\frac{1}{T}$ without aliasing. This model, first proposed in [5], involves ideal filters $G_m(z)$ that select each subband (Figure 4). The matrix $\mathbf{H}(z)$ reconstructs the real signals $x_m(n)$ (ADC output signal) from the ideal subband signals. The synthesis filter matrix $\mathbf{F}(z)$ reconstructs an estimation of each subband signal. The matrix $\mathbf{H}(z)$ is a multiple-input multiple-output (MIMO) model consisting in M -by- M FIR filters. The outputs of the analysis bank can be presented in frequency domain by

$$X_i(e^{j\omega}) = \sum_{m=0}^{M-1} H_{i,m}(e^{j\omega}) S_m(e^{j\omega}), \quad (1)$$

where $H_{i,m}(z)$ is the element of branch i , subband m in $\mathbf{H}(z)$ matrix. Likewise, the outputs $\hat{s}_m(n)$ of the synthesis filter banks are given by

$$\hat{S}_m(e^{j\omega}) = \sum_{i=0}^{M-1} F_{m,i}(e^{j\omega}) X_i(e^{j\omega}), \quad (2)$$

where $F_{m,i}(z)$ is the element of subband m , branch i in $\mathbf{F}(z)$ matrix. Substituting the term $X_i(e^{j\omega})$ in (2) with (1), the relation between the estimated output $\hat{s}_m(n)$ and the inputs subband signals is presented in frequency domain by

$$\hat{S}_m(e^{j\omega}) = \sum_{i,j=0}^{M-1} F_{m,i}(e^{j\omega}) H_{i,j}(e^{j\omega}) S_j(e^{j\omega}) \quad (3)$$

For the subband m , let's define

$$\mathcal{T}_{m,m}^\dagger(e^{j\omega}) = \sum_{i=0}^{M-1} F_{m,i}(e^{j\omega}) H_{i,m}(e^{j\omega}) \quad (4)$$

$$\mathcal{T}_{m,j}^\dagger(e^{j\omega}) = \sum_{\substack{i,j=0 \\ j \neq m}}^{M-1} F_{m,i}(e^{j\omega}) H_{i,j}(e^{j\omega}) \quad (5)$$

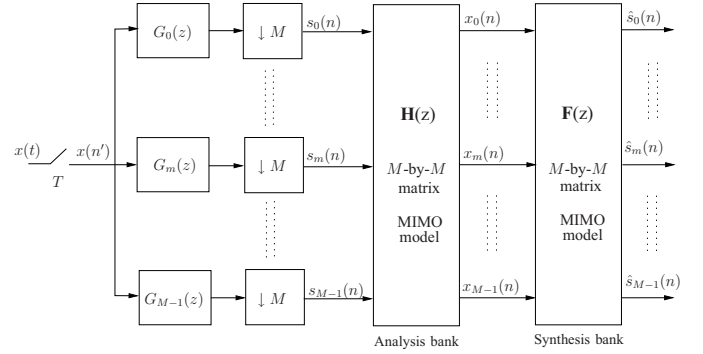


Fig. 3. Discrete-time subband M -channel HFB ADC model, n' et n denote represent respectively the discrete-time indices associated the sampling rate $\frac{1}{T}$ et $\frac{1}{MT}$.

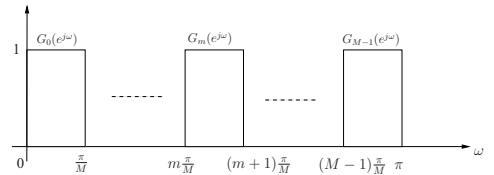


Fig. 4. Ideal filters for selecting the subbands $[m\frac{\pi}{M}, (m+1)\frac{\pi}{M}]$, $m \in [0, M-1]$.

$\mathcal{T}_{m,m}^\dagger(e^{j\omega})$ is usually called the distortion function, which is expected to be a pure delay n_d and scaled by a factor c . $\mathcal{T}_{m,j}^\dagger(e^{j\omega})$, $j \in [0, M-1] \setminus \{m\}$, are the aliasing functions, which are desired to be 0. The PR conditions for the subband HFB ADC are defined here by

$$\begin{cases} \mathcal{T}_{m,m}^\dagger(e^{j\omega}) = e^{-j\omega n_d} \\ \mathcal{T}_{m,j}^\dagger(e^{j\omega}) = 0 \quad (j \in [0, M-1] \setminus \{m\}) \end{cases} \quad (6)$$

Hence, the outputs for the subband m of HFB ADC can be presented as follows

$$\hat{S}_m(e^{j\omega}) = \underbrace{\mathcal{T}_{m,m}^\dagger(e^{j\omega}) S_m(e^{j\omega})}_{\text{Distortion term}} + \underbrace{\sum_{j=0; j \neq m}^{M-1} \mathcal{T}_{m,j}^\dagger(e^{j\omega}) S_j(e^{j\omega})}_{\text{Aliasing terms}} \quad (7)$$

Assuming that the synthesis filters are L -length FIR, their optimal coefficient values can be achieved by solving an overdetermined linear system [1], [5]. To do that, (6) is written for a set of N discrete frequencies, with N larger than L .

B. Adaptive channel equalization for subband HFB ADC calibration

Because the problem is similar for all subbands, we can just focus on the reconstruction of one subband signal $s_m(n)$. Due to analog realization errors, the theoretical original response $H_{i,m}(e^{j\omega})$ are changed to $\tilde{H}_{i,m}(e^{j\omega})$; the corresponding ADC output signals are $\tilde{x}_i(n)$. The PR conditions (6) require that $F_{m,i}(e^{j\omega})$ should be accordingly modified to keep the channels equalized. During the proposed calibration phase, the synthesis coefficient vectors noted $\hat{\mathbf{f}}_{m,i} =$

$[\tilde{f}_{m,i}(0), \tilde{f}_{m,i}(1), \dots, \tilde{f}_{m,i}(L-1)]$ are adaptively equalized in order to minimize the reconstruction error (Figure 5). Let $\tilde{\mathbf{f}}_m$ denote the synthesis filter coefficients in the subband m

$$\tilde{\mathbf{f}}_m = [\tilde{f}_{m,0}, \tilde{f}_{m,1}, \dots, \tilde{f}_{m,M-1}] \quad (8)$$

Let $\tilde{\mathbf{x}}(n)$ denote the inputs of the synthesis filters in subband m ,

$$\tilde{\mathbf{x}}(n) = [\tilde{x}_0(n), \tilde{x}_1(n), \dots, \tilde{x}_{M-1}(n)]^T, \quad (9)$$

where \mathbf{v}^T means the transpose of the vector \mathbf{v} , and $\tilde{x}_i(n)$ is denoted by

$$\tilde{x}_i(n) = [\tilde{x}_i(n), \tilde{x}_i(n-1), \dots, \tilde{x}_i(n-L+1)] \quad (10)$$

Let $e_m(n)$ denote the difference between the desired signal and the estimated subband signal $\hat{s}_m(n)$. \hat{s}_m and $e_m(n)$ are defined by

$$\hat{s}_m(n) = \tilde{\mathbf{f}}_m^T \tilde{\mathbf{x}}(n) \quad (11)$$

$$e_m(n) = d_m(n) - \hat{s}_m(n) \quad (12)$$

The chosen cost function is the MSE \mathbf{J}_m defined by

$$\begin{aligned} \mathbf{J}_m &= \mathbf{E}(|e_m(n)|^2) \\ &= \mathbf{E}(|d_m(n)|^2) - 2\tilde{\mathbf{f}}_m^T \mathbf{E}(\tilde{\mathbf{x}}(n)d_m(n)) + \\ &\quad \tilde{\mathbf{f}}_m^T \mathbf{E}(\tilde{\mathbf{x}}(n)^T \tilde{\mathbf{x}}(n)) \tilde{\mathbf{f}}_m \end{aligned} \quad (13)$$

The expectation $\mathbf{E}(\tilde{\mathbf{x}}(n)^T \tilde{\mathbf{x}}(n))$ is equal to the auto-correlation of the vector $\tilde{\mathbf{x}}(n)$, noted \mathbf{R}_{xx} ; the expectation $\mathbf{E}(\tilde{\mathbf{x}}(n)d_m(n))$ is equal to the cross-correlation between $\tilde{\mathbf{x}}(n)$ and $d_m(n)$, noted \mathbf{p}_m . So, (13) can be written

$$\mathbf{J}_m = \sigma_{d_m}^2 - 2\tilde{\mathbf{f}}_m^T \mathbf{p}_m + \tilde{\mathbf{f}}_m^T \mathbf{R}_{xx} \tilde{\mathbf{f}}_m \quad (14)$$

Hence, the optimal coefficients \mathbf{f}_m^{opt} achieved by the Wiener-Hopf method to minimize \mathbf{J}_m [9] is

$$\mathbf{f}_m^{opt} = \mathbf{R}_{xx}^{-1} \mathbf{p}_m \quad (15)$$

The calculation of \mathbf{f}_m^{opt} with (15) is obviously too costly. Here, the LMS algorithm is chosen for its simplicity. It consists of updating iteratively the synthesis filter coefficients \mathbf{f}_m^{opt} with the formula

$$\tilde{\mathbf{f}}_m(n+1) = \tilde{\mathbf{f}}_m(n) + \mu e_m(n) \tilde{\mathbf{x}}(n), \quad (16)$$

where μ is named step-size. The condition of stability of long LMS FIR filters is shown below [9]

$$0 < \mu < \frac{2}{LX_{max}}, \quad (17)$$

where X_{max} is the maximum value of the power spectral density of the inputs $\tilde{\mathbf{x}}_m(n)$. Therefore, the adaptive equalization seems theoretically suitable to the discrete-time model of subband HFB. In the next section, this method is extended to the continuous-time HFB architecture presented in Figure 1.

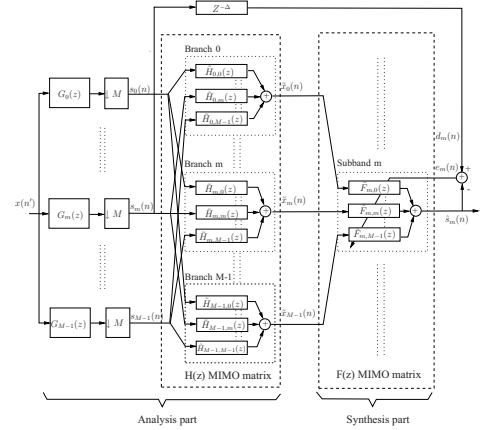


Fig. 5. Adaptive equalization of a subband HFB ADC discrete-time model.

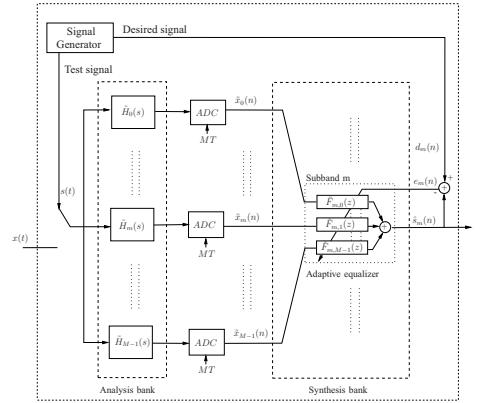


Fig. 6. Continuous-time adaptive subband M -channel HFB ADC architecture: calibration of the subband m before converting a wide-band signal $x(t)$.

III. ADAPTIVE CHANNEL EQUALIZATION FOR A CONTINUOUS-TIME HFB ADC ARCHITECTURE

Figure 6 shows the adaptive equalization for the continuous-time HFB ADC calibration. $\tilde{H}_m(s)$ denotes the analog filters transfer function $H_m(s)$ with errors. The signal generator provides two signals $s(t)$ and $d_m(n)$ for calibration purpose. $s(t)$ is a wide-band signal. To make sure that the calibration is efficient, this signal should be spectrally rich enough. The desired signal $d_m(n)$ is the signal $s(t)$ filtered in the subband of interest. The calibration is achieved once the MSE is below a predetermined threshold. Then the digital coefficients are fixed. The HFB is ready to convert other signals. In order to measure and compare the performances of HFBs in different cases in time-domain, the distortion-to-aliasing ratio (DAR_{dB}) is defined in (18). $\hat{s}_{m_dis}(n)$ denotes the distortion term, that is to say the useful signal at the HFB output, $\hat{s}_{m_alia}(n)$ is the sum of the aliasing terms of the subband m . K is the number of the considered samples.

$$DAR_{dB} = 10 * \log \frac{\sum_{k=0}^{K-1} |\hat{s}_{m_dis}(n-k)|^2}{\sum_{k=0}^{K-1} |\hat{s}_{m_alia}(n-k)|^2} \quad (18)$$

A. Simulation conditions

Considering the frequency band [800MHz 1.2GHz], an eight-channel subband HFB has been simulated in time domain with Matlab/Simulink. The adaptive equalization is shown in Figure 6 has been simulated for the calibration of the first subband [800 MHz 850MHz]. The analysis filter bank consists in eight pure Gm-LC resonators [1] whose central frequencies are $2\pi\Omega_i$ and the quality factor, Q_i . The filter central frequencies are assumed to be affected by a frequency offset of $2\pi\Delta\Omega$. The transfer function of each analysis filter with analog errors is then

$$\tilde{\mathbf{H}}_i(s) = \frac{\frac{\Omega_i + \Delta\Omega}{Q_i} s}{s^2 + \frac{\Omega_i + \Delta\Omega}{Q_i} s + (\Omega_i + \Delta\Omega)^2} \quad (19)$$

Synthesis filters are 128-length FIR. The test signal and the desired signal are obtained thanks to a pseudo-random sequence and several filters. The pseudom-random signal is filtered through a filter bank, each filter selecting one subband. The desired signal is the one corresponding to the band to reconstruct and the test signal is the sum of the signals of all subbands. Practically, the test signal and the desired signal samples could be calculated only once and stored in a memory in the device. The test signal should be converted into an analog signal by a DAC, which should not be a limitation because, often, for the same technology, DAC offers a much better speed performance than ADC.

In order to reduce the aliasing terms, it is efficient to allocate a small part of frequency borders (low and high frequencies) at each subband spectrum as guard band (GB) [5], which is equal to 7% of the subband bandwidth in this paper.

B. Simulation results

First, the ideal synthesis filters \mathbf{f}_m^{goal} corresponding to the analysis bank with errors $\tilde{\mathbf{H}}_i(s)$ ($\Delta\Omega = 1\% * \frac{\pi}{M}$) are calculated in the frequency domain with the least mean square global approximation synthesis method [1]. As expected, this subband HFB ($\tilde{\mathbf{H}}_i(s)$ with \mathbf{f}_m^{goal}) achieves a DAR of 96 dB. This performance can be considered as the goal of the calibration.

Then, the synthesis filters are set to the theoretical values \mathbf{f}_m^{th} corresponding to the analysis bank without errors. The DAR goes down to 53 dB. The adaptive equalization is started and processed with a variable step-size beginning at 0.3 and ending at $1.8 * 10^{-3}$. Figure 7 shows the convergence of two selected synthesis filter coefficients and the DAR . After 10^6 iterations which corresponds to a duration of 10ms in real-time, a stable DAR of 87 dB is achieved which is sufficient for high resolution telecommunication applications (typical resolution of 14 bit). The corresponding synthesis filter coefficient set is called \mathbf{f}_m^{opt} . The convergence time could be improved with a more advanced adaptive algorithm.

Last, to verify the performance of the subband HFB with \mathbf{f}_m^{opt} (phase 2 in Figure 2), a realistic wide-band signal is generated for the input. The results show also rather good performance of 86 dB.

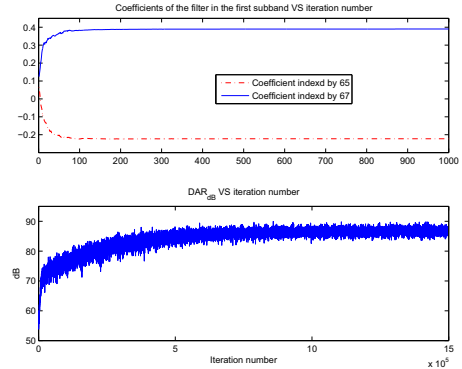


Fig. 7. Above: the convergence of synthesis filter coefficients. $\Delta\Omega = 1\% * \frac{\pi}{M}$. Below: the convergence of DAR_{dB} VS the iteration number.

TABLE I
SUBBAND HFB ADC CALIBRATION PERFORMANCES (DAR_{dB}),
($\Delta\Omega = err * \frac{\pi}{M}$).

Analog Error (err)	DAR_{dB}			
	Goal of calibration (\mathbf{f}_m^{goal})	Before calibration (\mathbf{f}_m^{th})	After calibration (\mathbf{f}_m^{opt})	Wide-band signal (\mathbf{f}_m^{opt})
1%	96	53	87	86
5%	96	39	83	79

Another simulation is performed for an analog error of 5%. All the simulation results are included in Table I.

IV. CONCLUSION

In this paper, an adaptive subband HFB ADC architecture is proposed. An optimal coefficient set can be approached iteratively to compensate the analog errors by applying the LMS algorithm. Such a calibration can be easily implementable in a digital circuit unlike other calibration techniques that require a large matrix inversion. Simulation results show that a signal-to-aliasing ratio can be achieved to 87 dB.

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