Delay-Tolerant Decode-and-Forward Based Cooperative Communication over Ricean Channels

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Abstract

In this paper, we propose a TDMA based simple transmission scheme, which overcomes the effect of the delays caused by the poor synchronization of the relaying nodes over Ricean channels. The proposed scheme is able to provide an optimized coding gain in unsynchronized cooperative networks as compared to the existing delay tolerant distributed space-time block codes.

I. Introduction

One of the recently discussed problems of the cooperative communication is the asynchronization of the relaying nodes [1]. Due to the asynchronous transmissions a traditionally designed structure of distributed space-time code is destroyed at the reception and it looses the diversity and coding gain. This point is throughly explained in [2]. In a delay constrained cooperative system, the data from different relays reach the destination after different delays. It is shown in [1] that the received delayed distributed space-time block code loses diversity for all well-known codes. The first reported delay tolerant codes for asynchronous cooperative network were proposed in [2]. The work of [2] is generalized and refined in [3] to include full-diversity delay tolerant space-time trellis codes (STTC) of minimum constrained length. In [1], delay tolerant distributed space-time block codes based on threaded algebraic space-time (TAST) codes [4] are designed for unsynchronized cooperative network. The distributed TAST codes of [1] preserve the rank of the space-time codewords under arbitrary delays at the reception of different rows of the codeword matrix. A lattice based decoder is used for decoding of the delayed codewords, which is computationally more complex than the decoupled decoding. One important observation is that the TAST codes provide optimized coding gains for the *synchronized* MIMO system, where the codeword is received without any shift between the rows. In the asynchronous cooperative network, note that it is

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not possible to obtain the optimal coding gain of the TAST codes because of the relative shifts between the rows of the received codeword.

In this letter, we propose a simple time-division multiple access (TDMA) based distributed transmission scheme for delay perturbed decode-and-forward based cooperative network which achieves full diversity under arbitrary delays over uncorrelated Ricean channels. Since the Rayleigh channel is a special case of the Ricean channel, the proposed scheme is also applicable over uncorrelated Rayleigh channels. The proposed scheme also provides optimized coding gain in an unsynchronized cooperative network. In addition, the proposed cooperative scheme performs better than the same rate existing delay tolerant distributed space-time code based cooperative scheme.

Notation: Upper (lower) bold face letters are used for matrices (row or column vector); $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ are the transpose, conjugate, and Hermitian of a matrix or vector; $\otimes$ denotes the Kronecker product; $K \times K$ identity matrix is shown as $\boldsymbol{I}_K$. Let $\boldsymbol{X}$ be a $v \times w$ matrix, then $\boldsymbol{X}(m:n, p:q)$ represents a $(n-m+1) \times (q-p+1)$ matrix formed by m to n, $1 \leq m \leq n \leq v$ sequential rows and p to q, $1 \leq p \leq q \leq w$ sequential columns of $\boldsymbol{X}$, $\boldsymbol{X}(:, p:q)$ stands a $v \times (q-p+1)$ matrix formed by p to q, $1 \leq p \leq q \leq w$ sequential columns of $\boldsymbol{X}$, and $\boldsymbol{X}(m:n, :)$ denotes a $(n-m+1) \times w$ matrix formed by m to n, $1 \leq m \leq n, n \leq v$ sequential rows of $\boldsymbol{X}$, $\boldsymbol{X}(i, :)$ represents i-th row of $\boldsymbol{X}$; $\boldsymbol{X}(:, i)$ represents i-th column of $\boldsymbol{X}$; $\mathbf{0}_{a \times b}$ is an all zero matrix of size $a \times b$; $\boldsymbol{e}_c^T$ is row vector consisting 1 at c position and rest of all elements as 0; diag $\{\boldsymbol{w}\}$ is a diagonal matrix with the elements of the vector $\boldsymbol{w}$ on its diagonal.

II. System Model

We consider a cooperative communication system, which consists of one source (S), N relays ($\mathrm{R}_1, \mathrm{R}_2, ..,$ R_N), and one destination (D) terminal as shown in Fig. 1. Each of them can either transmit or receive a signal at a time. It is assumed that the relays decode the received data without any error. As the transmitters within the relays are distributed in different locations and there is no central local oscillator in contrast to a co-located antenna array, there are relative timing errors between the different relays. The timing errors can be arbitrary values. The data transmitted by the relays is received by D with delay profile $\boldsymbol{\Delta} = (\delta_1, \delta_2, .., \delta_N)$, where δ_i denotes the relative delay of the signal received from the i-th relay as reference to the earliest received relay signal. The maximum relative delay is assumed to be $\delta_{\max}$. The channel of links are assumed to be uncorrelated Ricean distributed. Let us state the major assumptions as follows:

A1. $0 \leq \delta_1, \delta_2, .., \delta_N \leq \delta_{\max}$.

A2. The source and relays do not know the delay profile $\boldsymbol{\Delta}$ but they know $\delta_{\max}$ perfectly. However, the destination knows both the delay profile $\boldsymbol{\Delta}$ and maximum delay $\delta_{\max}$ perfectly.

A3. All channels are fast fading and can vary from one time interval to another.

A4. The destination knows the channel between the relays and itself perfectly. Similarly, each relay knows the channel between the source and itself perfectly.

A5. No errors occur in the channels between the source and the relays.

Since it is difficult to acquire perfect knowledge of the channel which does not remain constant over several time intervals, Assumptions **A3** and **A4** seem to be contradictory to each other. However, we made Assumptions **A3** and **A4** in order to obtain a generalized delay tolerant scheme which can work under an *idealized* set-up. Nevertheless, the proposed scheme is applicable for practical block fading channels which remain constant over multiple time intervals and can be perfectly estimated at the receiver. Though Assumption **A5** seem to be very strong, however, it is shown in Subsection VI-C, that **A5** is approximately valid when the relaying nodes are close to the source.

III. Distributed Space-Time Coding for Asynchronous Cooperative Network

It is clear from the assumptions **A1-A3** that even if all relays start transmitting all rows of a distributed space-time block code (STBC) simultaneously, different rows will reach D with different delays $\delta_i \leq \delta_{\text{max}}, i \in \{1, 2, .., N\}$. If all relays continuously (without any pause between the transmission of two consecutive codewords) transmit the rows of different distributed STBC at different blocks, then the data of two consecutively transmitted STBC can be overlapped due to the timing errors. Hence, in order to avoid this problem, the transmission of a STBC in a distributed manner from N asynchronous relays is performed by using simultaneous transmission and pause (STP) strategy as follows [1]: All relays start transmitting the assigned rows of the codeword simultaneously and as the values of the relatives delays are unknown, therefore, each of them waits for δ_{max} time intervals after the transmission of the codeword is finished. Due to the delays in the reception, an $N \times T$ transmitted STBC $\boldsymbol{S}$ is transformed into an $N \times (T + \delta_{\text{max}})$ codeword at the receiver as follows:

$$\boldsymbol{S}^{\boldsymbol{\Delta}} = \begin{bmatrix} \mathbf{0}_{1\times\delta_1} & \boldsymbol{S}(1,:) & \mathbf{0}_{1\times(\delta_{\text{max}}-\delta_1)} \\ \mathbf{0}_{1\times\delta_2} & \boldsymbol{S}(2,:) & \mathbf{0}_{1\times(\delta_{\text{max}}-\delta_2)} \\ \vdots & \vdots & \vdots \\ \mathbf{0}_{1\times\delta_N} & \boldsymbol{S}(N,:) & \mathbf{0}_{1\times(\delta_{\text{max}}-\delta_N)} \end{bmatrix}, \tag{1}$$

where a $\mathbf{0}$ represents no transmission. Let W symbols be encoded into the original STBC $\boldsymbol{S} \in \mathbb{C}^{N\times T}$, then it can be seen from (1) that by following the STP strategy it takes $T + \delta_{\text{max}}$ time intervals for transmitting $\boldsymbol{S}$. Hence, the effective data rate in the asynchronous cooperative network is $W/(T + \delta_{\text{max}})$, which is less than the data rate in a synchronized system W/T for which the STBC is traditionally designed. In [1], distributed TAST codes were designed for delay constrained asynchronous cooperative network to provide full diversity. However, optimized coding gain is not guaranteed in asynchronous cooperative network for these codes.

IV. Delay Independent Transmission Scheme

The block diagram of the proposed transmission scheme is shown in Fig. 2. Let $\boldsymbol{s}_k = \left[s_1^k, s_2^k, .., s_L^k\right]^T$, where s_i^k belongs to an arbitrary constellation $\mathcal{A}$ with J signal points, represent a data vector to be transmitted in the k-th block/frame. It is assumed that $L = MN$, where M is a positive integer. The data vector $\boldsymbol{s}_k$ is passed through a grouping block $\boldsymbol{\Pi}$ at each relay as shown in Fig. 2. The operation of $\boldsymbol{\Pi}$ can be compactly written as

$$\boldsymbol{G}_k = \left[\boldsymbol{\Pi}_1 \boldsymbol{s}_k, \boldsymbol{\Pi}_2 \boldsymbol{s}_k, .., \boldsymbol{\Pi}_M \boldsymbol{s}_k\right], \tag{2}$$

where $\boldsymbol{\Pi}_n$ is a $N \times L$ matrix defined as $\boldsymbol{\Pi}_n = \boldsymbol{I}_L\left((n-1)N + 1 : nN, :\right)$ and $n \in \{1, 2, .., M\}$. The grouped data matrix is applied to the precoder vector $\boldsymbol{\theta}_i$, where $\boldsymbol{\theta}_i$ has size $1 \times N$. The design of $\boldsymbol{\theta}_i$ will be discussed in Section V. The transposed precoded data vector $\boldsymbol{z}_i^k = (\boldsymbol{\theta}_i \boldsymbol{G}_k)^T$ of size $M \times 1$ is parsed through a $(L + N\delta_{\text{max}}) \times M$ multiplexing matrix $\boldsymbol{\Gamma}_i$ given as

$$\boldsymbol{\Gamma}_i = \left[\mathbf{0}_{M \times (i-1)(M+\delta_{\text{max}})}, \boldsymbol{I}_M, \mathbf{0}_{M \times (N-i)M + (N-i+1)\delta_{\text{max}}}\right]^T. \tag{3}$$

The parsed data vector $\boldsymbol{x}_i^k = \boldsymbol{\Gamma}_i \boldsymbol{z}_i^k$ of size $(L + N\delta_{\text{max}}) \times 1$ is transmitted through the i-th relay (elements of a column vector are transmitted sequentially). The data transmitted from the relays will undergo the delay profile and the destination receives delayed versions of them. The multiplexing matrix $\boldsymbol{\Gamma}_i$ introduces ordering in the transmissions from the relaying nodes. It ensures that the data transmitted from two consecutive relays is separated by δ_{max} time intervals and each relay transmits for M non-overlapping time intervals and remain silent (transmitting a 0 signal) at other time intervals such that the received data at each time interval consists of data transmitted by most one relay (only). We call this strategy as orthogonal transmission and pause (OTP). By using OTP, we are able to transmit L symbols in $L + N\delta_{\text{max}}$ time intervals. Hence, the effective data rate in symbol per channel use (SPCU) and bit per channel use (BPCU) is

$$\frac{L}{L + N\delta_{\text{max}}} \text{ SPCU} = \frac{M}{M + \delta_{\text{max}}} \text{ SPCU} = \frac{M}{M + \delta_{\text{max}}} \log_2 J \text{ BPCU}, \tag{4}$$

where $L = MN$ and J is the total number of points in the signal constellation $\mathcal{A}$. It can be seen from (4) that the proposed scheme provides data rate less than 1 SPCU. However, for a fixed data rate in BPCU the delay in decoding $L + N\delta_{\text{max}}$ symbols can be reduced by increasing J, i.e., using higher order signal constellations at the cost of performance degradation. In the case of BPSK, i.e., $J = 2$, the data rate of 1 BPCU can be obtained as follows:

$$\lim_{L \to \infty} \frac{L}{L + N\delta_{\text{max}}} \text{ SPCU} = \lim_{M \to \infty} \frac{M}{M + \delta_{\text{max}}} \text{ BPCU} = 1 \text{ BPCU}, \tag{5}$$

meaning that in OTP using BPSK constellation, we can obtain the data rate of 1 BPCU if infinite delay in the decision is allowed. Nonetheless, if $N\delta_{\text{max}} << L$, approximately full rate can be achieved for finite values of L in the case of BPSK. It will be shown in the simulations that the proposed scheme achieves better coding gain than the same rate existing best delay tolerant distributed STBC [1].

Let us represent all channel gains from the N relays to the destination, while each relay actively transmits, into an $N \times M$ matrix $\boldsymbol{H}_k$ as follows:

$$\boldsymbol{H}_k = \begin{bmatrix} h^k_{1,1} & h^k_{1,2} & \dots & h^k_{1,M} \\ h^k_{2,1} & h^k_{2,2} & \dots & h^k_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ h^k_{N,1} & h^k_{N,2} & \dots & h^k_{N,M} \end{bmatrix}, \tag{6}$$

where $h^k_{i,n}$ denotes the non-zero mean and Gaussian distributed (Ricean) complex channel coefficient utilized during the transmission of the non-zero data from the i-th relay in n-th time interval (out of M consecutive time intervals when i-th relay remains active) in the k-th block. The received signal $\boldsymbol{y}^{\boldsymbol{\Delta}}_k \in \mathbb{C}^{(L+N\delta_{\text{max}})\times 1}$ can be written as

$$\boldsymbol{y}^{\boldsymbol{\Delta}}_k = \sum_{i=1}^{N} \text{diag}\left\{\boldsymbol{h}^k_i\right\} \boldsymbol{x}^k_i + \boldsymbol{q}_k, \tag{7}$$

where $\boldsymbol{h}^k_i = \left[\boldsymbol{0}_{1\times((i-1)M+\delta_i+(i-1)\delta_{\text{max}})}, \boldsymbol{H}_k\left(i,:\right), \boldsymbol{0}_{1\times(L-iM+(N-i+1)\delta_{\text{max}}-\delta_i)}\right]$ is an $1 \times (L + N\delta_{\text{max}})$ row vector and $\boldsymbol{q}_k$ is an $(L + N\delta_{\text{max}}) \times 1$ column vector consisting of additive white Gaussian noise (AWGN). At destination, $\boldsymbol{y}^{\boldsymbol{\Delta}}_k$ is passed through the grouping block $\boldsymbol{\Xi}$, which performs the following operation:

$$\boldsymbol{Y}_k = \left[\boldsymbol{\Xi}_1\boldsymbol{y}^{\boldsymbol{\Delta}}_k, \boldsymbol{\Xi}_2\boldsymbol{y}^{\boldsymbol{\Delta}}_k, .., \boldsymbol{\Xi}_M\boldsymbol{y}^{\boldsymbol{\Delta}}_k\right], \tag{8}$$

where $\boldsymbol{\Xi}_n$ is an $N \times (L + N\delta_{\text{max}})$ matrix with i-th row given by $\boldsymbol{e}^T_{\delta_i+(i-1)(M+\delta_{\text{max}})+n}$ and $\boldsymbol{Y}_k \in \mathbb{C}^{N\times M}$ contains the data received only during the active periods of all relays. Next, the group of symbols represented by $\boldsymbol{s}^k_n = \boldsymbol{\Pi}_n\boldsymbol{s}_k$ can be decoded from $\boldsymbol{Y}_k\left(:,n\right)$ as follows:

$$\hat{\boldsymbol{s}}^k_n = \arg \min_{\boldsymbol{s}^k_n\in\mathcal{A}^N} \left\|\boldsymbol{Y}_k\left(:,n\right) - \text{diag}\left\{\boldsymbol{H}_k\left(:,n\right)\right\}\boldsymbol{\Theta}\boldsymbol{s}^k_n\right\|^2, \tag{9}$$

where $\boldsymbol{\Theta} = \left[\boldsymbol{\theta}^T_1, \boldsymbol{\theta}^T_2, .., \boldsymbol{\theta}^T_N\right]^T$. Since $\text{diag}\left\{\boldsymbol{H}_k\left(:,n\right)\right\}$ is a diagonal matrix and $\boldsymbol{\Theta}\boldsymbol{s}^k_n$ is a column vector, the ML metric (9) can be alternately written as

$$\hat{\boldsymbol{s}}^k_n = \arg \min_{\boldsymbol{s}^k_n\in\mathcal{A}^N} \left\|\boldsymbol{Y}_k\left(:,n\right) - \text{diag}\left\{\boldsymbol{\Theta}\boldsymbol{s}^k_n\right\}\boldsymbol{H}_k\left(:,n\right)\right\|^2. \tag{10}$$

It can be seen from (9) and (10) that the ML decoding of the transmitted data does not depend upon the delay profile, contrary to the existing delay tolerant space-time block code [1]. Hence, the decoding complexity of the proposed scheme is independent of the delay profile, whereas the decoding complexity of the existing delay tolerant scheme [1] varies with delay.

V. Performance Analysis

A Chernoff bound over the pair-wise error probability (PEP) in the case when $\boldsymbol{\Theta}\left(\boldsymbol{s}_n^k\right)^0$ is transmitted and $\boldsymbol{\Theta}\boldsymbol{s}_n^k$ is decoded, $\left(\boldsymbol{s}_n^k\right)^0 \neq \boldsymbol{s}_n^k$, can be obtained as [5, Theorem 4.2]

$$\Pr\left(\left(\boldsymbol{s}_n^k\right)^0 \rightarrow \boldsymbol{s}_n^k \,|\boldsymbol{H}_k(:,n)\right) \leq \exp\left(-\frac{\left\|\left(\mathrm{diag}\left\{\boldsymbol{\Theta}\left(\boldsymbol{s}_n^k\right)^0\right\} - \mathrm{diag}\left\{\boldsymbol{\Theta}\boldsymbol{s}_n^k\right\}\right)\boldsymbol{H}_k(:,n)\right\|^2}{4\sigma^2}\right), \tag{11}$$

where σ^2 is the variance of the AWGN noise $\boldsymbol{q}_k$. As $\boldsymbol{H}_k(:,n) \sim \mathcal{CN}\left(\sqrt{\frac{1}{1+K}}\rho\bar{\boldsymbol{H}}_k(:,n), \frac{K}{1+K}\rho^2\boldsymbol{I}_N\right)$, where ρ^2 is the transmit power, $\bar{\boldsymbol{H}}_k(:,n)$ is a column vector consisting of the mean values of $\boldsymbol{H}_k(:,n)$, therefore, by using the probability density function (p.d.f.) [6, Eq. (2.16)] and the moment generating function (M.G.F.) [6, Eq. (2.16)] of a non-central chi-square distributed random variable, (11) can be averaged over $\boldsymbol{H}_k(:,n)$ with the help of the procedure given in [5, Section 4.4] in order to obtain the following upper bound of PEP (UBPEP):

$$\mathbb{E}_{\boldsymbol{H}}\left[\Pr\left\{\left(\boldsymbol{s}_n^k\right)^0 \rightarrow \boldsymbol{s}_n^k\right\}\right] \leq \frac{e^{\left(K(1+K)\bar{\boldsymbol{H}}_k^H(:,n)\left(\boldsymbol{\Phi}^{-1}-\frac{1}{1+K}\boldsymbol{I}_N\right)\bar{\boldsymbol{H}}_k(:,n)\right)}}{(1+K)^{-n_t n_r}\left|\boldsymbol{\Phi}\right|}, \tag{12}$$

where $\boldsymbol{\Phi} = (1+K)\boldsymbol{I}_N + \frac{\rho^2}{4\sigma^2}\left(\mathrm{diag}\left\{\boldsymbol{\Theta}\left(\boldsymbol{s}_n^k\right)^0\right\} - \mathrm{diag}\left\{\boldsymbol{\Theta}\boldsymbol{s}_n^k\right\}\right)\left(\mathrm{diag}\left\{\boldsymbol{\Theta}\left(\boldsymbol{s}_n^k\right)^0\right\} - \mathrm{diag}\left\{\boldsymbol{\Theta}\boldsymbol{s}_n^k\right\}\right)^H$. The UBPEP for Rayleigh channels can be obtained by substituting $K = 0$ in (12).

A. Precoder Design

From (12), the following conditions can be pointed out, which must be satisfied by $\boldsymbol{\Theta}$:

- In order to obtain full diversity, $\mathrm{diag}\left\{\boldsymbol{\Theta}\left(\boldsymbol{s}_n^k\right)^0\right\} - \mathrm{diag}\left\{\boldsymbol{\Theta}\boldsymbol{s}_n^k\right\} = \mathrm{diag}\left\{\boldsymbol{\Theta}\left(\left(\boldsymbol{s}_n^k\right)^0 - \boldsymbol{s}_n^k\right)\right\}$ must be full rank matrix, i.e., $\boldsymbol{\Theta}\left(\boldsymbol{s}_n^k\right)^0$ should be different for all N elements from all possible $\boldsymbol{\Theta}\boldsymbol{s}_n^k$ provided that $\left(\boldsymbol{s}_n^k\right)^0 \neq \boldsymbol{s}_n^k$.
- If $\boldsymbol{\Theta}$ is a unitary matrix, then it ensures that the translation $\boldsymbol{\Theta}\left(\left(\boldsymbol{s}_n^k\right)^0 - \boldsymbol{s}_n^k\right)$ will not alter the distance $\left(\boldsymbol{s}_n^k\right)^0 - \boldsymbol{s}_n^k$.
- In order to satisfy $\mathbb{E}\left[\mathrm{Tr}\left\{\mathrm{diag}\left\{\boldsymbol{\Theta}\boldsymbol{s}_n^k\right\}\mathrm{diag}^H\left\{\boldsymbol{\Theta}\boldsymbol{s}_n^k\right\}\right\}\right] = N$, i.e., the average power constraint over $\mathrm{diag}\left\{\boldsymbol{\Theta}\boldsymbol{s}_n^k\right\}$, it must be ensured that $\mathbb{E}\left[\mathrm{Tr}\left\{\boldsymbol{s}_n^k\left(\boldsymbol{s}_n^k\right)^H\right\}\right] = N$ and $\mathrm{Tr}\left\{\boldsymbol{\Theta}\boldsymbol{\Theta}^H\right\} = N$.
- $\boldsymbol{\Theta}$ must be chosen to minimize UBPEP.

The optimization problem can be expressed as follows:

$$\min_{\left\{\boldsymbol{\Theta}|\boldsymbol{\Theta}\in\mathbb{U}, \mathrm{Tr}\left\{\boldsymbol{\Theta}\boldsymbol{\Theta}^H\right\}=N, \left(\boldsymbol{s}_n^k\right)^0 \neq \boldsymbol{s}_n^k\right\}} \mathrm{UBPEP}, \tag{13}$$

where $\mathbb{U}$ is the group of all unitary matrices. As a closed-form solution of optimized $\boldsymbol{\Theta}$ is difficult to find, we will provide an iterative numerical method for finding the optimized values of $\boldsymbol{\Theta}$. As $\boldsymbol{\Theta}$ has a unitary structure, first we need to parametrize $\boldsymbol{\Theta}$ [7]–[9].

Lemma 1: ***Parametrization of Skew Hermitian Matrix:*** *Let $\boldsymbol{S} \in \mathbb{C}^{N\times N}$ be skew Hermitian such that $\boldsymbol{S} = -\boldsymbol{S}^H$, then $\boldsymbol{S}$ can be parametrize as follows:*

$$\mathrm{vec}\left(\boldsymbol{S}\right) = \jmath \boldsymbol{L}_d \boldsymbol{r} + \boldsymbol{L}_l \boldsymbol{c} - \boldsymbol{L}_u \boldsymbol{c}^*, \tag{14}$$

where $\boldsymbol{r}$ is a $N \times 1$ real entries vector containing the imaginary values of main diagonal of $\boldsymbol{S}$, $\boldsymbol{c} \in \mathbb{C}^{N(N-1)/2\times 1}$ contains strictly below main diagonal elements of $\boldsymbol{S}$ taken in a column wise vector in the same way as the vec *operator, $\boldsymbol{c}^* \in \mathbb{C}^{N(N-1)/2\times 1}$ consists strictly of above diagonal elements of $\boldsymbol{S}$ taken in row wise order starting from first row from left to right, $\boldsymbol{L}_d$ is a $N^2 \times n$ matrix that takes care of the elements of the main diagonal of $\boldsymbol{S}$, $\boldsymbol{L}_l$ is a $N^2 \times N(N-1)/2$ matrix taking care of the elements strictly below the diagonal of $\boldsymbol{S}$, and $\boldsymbol{L}_u$ is a $N^2 \times N(N-1)/2$ matrix takes care of the elements strictly above the diagonal of $\boldsymbol{S}$ [7, Eq. (27)].*

The proof of Lemma 1 can be seen in [7], [8].

Lemma 2: ***Parametrization of Unitary Matrix:*** *A complex valued $N \times N$ unitary matrix $\boldsymbol{\Theta}$ can be parametrize as follows:*

$$\boldsymbol{\Theta} = \exp\left(\boldsymbol{S}\right), \tag{15}$$

where $\boldsymbol{S} \in \mathbb{C}^{N\times N}$ skew Hermitian matrix, where $\exp\left(\boldsymbol{S}\right) \triangleq \sum_{m=0}^{\infty} \frac{1}{m!}\boldsymbol{S}^m$, and $\boldsymbol{S}^0 \triangleq \boldsymbol{I}_N$.

The proof of Lemma 2 can be seen in [9].

From (12), (14), and (15), UBPEP can be expressed as a real-valued function of $\boldsymbol{r}$ and $\boldsymbol{c}$ as

$$\mathrm{UBPEP} = f\left(\boldsymbol{r}, \boldsymbol{c}, \boldsymbol{c}^*\right). \tag{16}$$

It can be seen from (16) that the optimization problem of (13) reduces into obtaining optimized values of $\boldsymbol{r}$ and $\boldsymbol{c}$. From [7, Theorem 2], the update equation of the steepest descent method for optimization of (13) can be obtained as

$$\begin{bmatrix} \boldsymbol{r}_{k+1} \\ \boldsymbol{c}_{k+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{r}_{k} \\ \boldsymbol{c}_{k} \end{bmatrix} + \mu \begin{bmatrix} \left(\mathcal{D}_{\boldsymbol{r}} f\left(\boldsymbol{r}_k, \boldsymbol{c}_k, \boldsymbol{c}_k^*\right)\right)^T \\ 2\left(\mathcal{D}_{\boldsymbol{c}^*} f\left(\boldsymbol{r}_k, \boldsymbol{c}_k, \boldsymbol{c}_k^*\right)\right)^T \end{bmatrix}, \tag{17}$$

where μ is a real negative constant, $\boldsymbol{r}_k \in \mathbb{R}^{N\times 1}$ and $\boldsymbol{c}_k \in \mathbb{C}^{N(N-1)/2\times 1}$ are the value of $\boldsymbol{r}$ and $\boldsymbol{c}$, respectively after k iterations, and $\mathcal{D}_{\boldsymbol{r}}$ and $\mathcal{D}_{\boldsymbol{c}^*}$ are the first order derivative [10] w.r.t. $\boldsymbol{r}$ and $\boldsymbol{c}^*$, respectively. The first order derivatives can be obtained by using chain rule [10] and properties of the trace and the vec operators [11] as follows:

$$\begin{aligned} \mathcal{D}_{\boldsymbol{r}} f\left(\boldsymbol{r}_k, \boldsymbol{c}_k, \boldsymbol{c}_k^*\right) &= \jmath\left(\boldsymbol{a} - \boldsymbol{b}\right)\boldsymbol{L}_d, \\ \mathcal{D}_{\boldsymbol{c}^*} f\left(\boldsymbol{r}_k, \boldsymbol{c}_k, \boldsymbol{c}_k^*\right) &= -\boldsymbol{a}\boldsymbol{L}_u + \boldsymbol{b}\boldsymbol{L}_l, \end{aligned} \tag{18}$$

where $\boldsymbol{a} \in \mathbb{C}^{1\times N^2}$ and $\boldsymbol{b} \in \mathbb{C}^{1\times N^2}$ are defined as

$$
\begin{aligned}
\boldsymbol{a} \triangleq & -\mathrm{vec}^H\left(\frac{\rho^2}{4\sigma^2}\mathrm{diag}\left\{\boldsymbol{\Theta}\left(\left(\boldsymbol{s}_n^k\right)^0-\boldsymbol{s}_n^k\right)\right\}\boldsymbol{\Phi}^{-1}\left(K(1+K)\bar{\boldsymbol{H}}_k\left(:,n\right)\bar{\boldsymbol{H}}_k^H\left(:,n\right)\boldsymbol{\Phi}^{-1}+\boldsymbol{I}_N\right)\right)\boldsymbol{L}_d \\
& \times\left(\left(\left(\boldsymbol{s}_n^k\right)^0-\boldsymbol{s}_n^k\right)^T\otimes\boldsymbol{I}_N\right)\sum_{t=0}^{\infty}\frac{1}{(t+1)!}\sum_{p=0}^{t}\left(\boldsymbol{S}^T\right)^{t-p}\otimes\boldsymbol{S}^p, \\
\boldsymbol{b} \triangleq & -\mathrm{vec}^H\left(\frac{\rho^2}{4\sigma^2}\mathrm{diag}^*\left\{\boldsymbol{\Theta}\left(\left(\boldsymbol{s}_n^k\right)^0-\boldsymbol{s}_n^k\right)\right\}\boldsymbol{\Phi}^{-1}\left(K(1+K)\bar{\boldsymbol{H}}_k\left(:,n\right)\bar{\boldsymbol{H}}_k^H\left(:,n\right)\boldsymbol{\Phi}^{-1}+\boldsymbol{I}_N\right)\right)\boldsymbol{L}_d \\
& \times\left(\left(\left(\boldsymbol{s}_n^k\right)^0-\boldsymbol{s}_n^k\right)^H\otimes\boldsymbol{I}_N\right)\sum_{t=0}^{\infty}\frac{1}{(t+1)!}\sum_{p=0}^{t}\left(\boldsymbol{S}^H\right)^{t-p}\otimes\left(\boldsymbol{S}^*\right)^p.
\end{aligned}
\tag{19}
$$

Optimized precoder for Rayleigh channels can be obtained by setting $K=0$ in (19). The optimization problem (13) can be seen as linear constellation precoder (LCP) design [12, Eq. (7)]. In [12], [13], LCP-A and LCP-B precoders are developed based on linear algebraic constructions. It is shown in [12, Table I], that unitary LCP-A provides better coding gain than LCP-B precoder. Further, it is shown in [12, Section III-B] that unitary LCP-A precoder is a Vandermonde matrix given by

$$
\boldsymbol{\Theta}=\frac{1}{\sqrt{N}}\begin{bmatrix} 1 & \beta_0 & \cdots & \beta_0^{N-1} \\ 1 & \beta_1 & \cdots & \beta_1^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \beta_N & \ldots & \beta_N^{N-1} \end{bmatrix}, \tag{20}
$$

where $\beta_l = e^{\jmath\left(\frac{2\pi l}{N}+\frac{\pi}{2N}\right)}$, $l=0,1,..,N-1$. Therefore, we can also use Vandermonde matrix (20) for deciding $\boldsymbol{\Theta}$ and $\boldsymbol{\theta}_i, i\in\{1,2,..,N\}$ is chosen as corresponding row of (20). However, it is shown in the next section that with the help of the proposed optimization method, significant performance enhancements can be obtained for more than two relays as compared to LCP-A precoder [12].

VI. Simulation Results

A. Optimization of $\boldsymbol{\Theta}$

We used the procedure given in Subsection V-A for optimizing $\boldsymbol{\Theta}$ at each SNR value. An $N\times N$ discrete Fourier transform (DFT) matrix is used as an initialization matrix for starting the steepest descent algorithm. The optimized value of the analytical PEP is obtained by plugging the optimized value of $\boldsymbol{\Theta}$ into (12). We have also obtained the analytical PEP for the Vandermonde matrix or LCP-A precoder [12] of (20) and the best known *sub-optimal* full diversity unitary rotation matrix given as follows [14]:

$$
\boldsymbol{\Psi}=\begin{bmatrix} -0.3279852776 & -0.5910090485 & -0.7369762291 \\ -0.7369762291 & -0.3279852776 & 0.5910090485 \\ -0.5910090485 & 0.7369762291 & -0.3279852776 \end{bmatrix}. \tag{21}
$$

The sub-optimal rotation matrix of (21) provides the best coding gain along with full diversity *out of all* known *sub-optimal* full diversity rotation matrices, however, it does not achieve an upper bound of the coding gain [14]. For the case of two relays, we found that the optimized precoder coincides with the LCP-A precoder. We have shown the plots of analytical PEP over Rayleigh and Ricean channels

$K \in \{0, 2, 5, 10\}$, and $\bar{\boldsymbol{H}}_k(:, n) = [1, 1, 1]^T$ with the $\boldsymbol{\Theta}$ obtained by proposed numerical optimization method, from the Vandermonde matrix, and from 3×3 unitary rotation matrix of (21) in Fig. 3 for the cooperative system with 3 relays. It can be seen from Fig. 3 that from low to moderate SNR, the optimized precoder and Vandermonde matrix performs similarly. However, from moderate to high SNR (SNR above about 15 dB) the proposed optimized precoder significantly outperforms the Vandermonde matrix based precoder. In addition, the preposed precoder also works better than the sub-optimal precoder of (21). The effect of the Ricean coefficient K on the code design can also be observed from Fig. 3. It can be seen from Fig. 3 that for high values of the K factor the proposed optimized precoder provides more improvement as compared to the LCP-A precoder or (21) which is designed for Rayleigh fading channels.

B. Comparison of the Proposed Delay Tolerant Scheme with the Conventional DTTAST Code

In Fig. 4 we show the SER versus SNR performance of previously proposed delay tolerant TAST (DTTAST) codes [1] and the proposed OTP scheme with BPSK constellation, in a cooperative system with two asynchronous relays $N = 2$, $\delta_{\text{max}} = 3$, and over Rayleigh and Ricean channels $K \in \{0, 2, 5\}$ with $\bar{\boldsymbol{H}}_k(:, n) = [1, 1]^T$. A 2×3 DTTAST code is given as [1, Eq. (17)]

$$\boldsymbol{X}^{\text{DT}} = \begin{bmatrix} x_1 & \phi y_2 & \phi y_3 \\ \phi y_1 & x_2 & x_3 \end{bmatrix}, \tag{22}$$

where $[x_1, x_2, x_3]^T = \boldsymbol{\Phi}[s_1, s_2, s_3]^T$ and $[y_1, y_2, y_3]^T = \boldsymbol{\Phi}[s_4, s_5, s_6]^T$, $s_i \in \mathcal{A}$, $\phi = e^{\jmath 2\pi/15}$, and $\boldsymbol{\Phi}$ is given in (21). The DTTAST code of (22) is used for transmission which transmits 6 BPSK symbols in one codeword block and provides an effective data rate 1 in the unsynchronized cooperative network with $\delta_{\text{max}} = 3$. In Fig. 4, the performance of DTTAST code averaged over all possible delay profiles is shown. The performance of the proposed delay independent scheme is plotted with $L = 200$. As $N\delta_{\text{max}} << L$, the rate of the proposed scheme $R = L/(L + N\delta_{\text{max}}) = 0.97 \approx 1$. It can be seen from Fig. 4 that the proposed scheme significantly outperforms the DTTAST codes at all SNRs and for both Rayleigh and Ricean channels. For example, a gain of 8.5 dB is achieved at SER=10^{-3} over Rayleigh channels $K = 0$. All simulations are performed over 10^5 channel realizations.

C. Effects of Wrong Relaying on the Performance of the Proposed Delay Tolerant Scheme

In Fig. 5, we have shown the performance of the proposed scheme assuming that one out of three relaying nodes is in outage over Ricean fading channels with $K = 1$. Simulations are performed by assuming different SNR values over the link between the source and the relay in outage (SNR$\in \{5, 15, 25, 35\}$ dB). The higher the value of SNR, the closer the relay in outage is to the source. It can be seen from Fig. 5 that, because of wrong relaying, there is an error floor in the performance of the proposed scheme at higher SNR between the relays and the destination when the relay in outage is far from the source. However, when the SNR between the source and relay in outage is very high, i.e., the relay is very close to the source, the proposed scheme performs very close to the system where the relays have perfect knowledge of the data in the relays to the destination SNR range of $0-15$ dB as assumed in Assumption **A5**. Fig. 5 suggests that Assumption **A5** is approximately valid when the relaying nodes are very close to the source and the SNR between the relays and the destination lies in the range of low to moderate values. All simulations are performed over 10^5 channel realizations.

VII. Conclusions

We have proposed a simple TDMA based transmission scheme for decode-and-forward based cooperative systems. In addition, we have also designed a PEP based precoder for unsynchronized cooperative networks over uncorrelated Ricean channels. It is shown by simulations that by proper scheduling of transmissions and optimized precoder significant coding gain and full diversity can be achieved under arbitrary delay profile. Moreover, the proposed scheme significantly outperforms the approximate same rate existing delay tolerant distributed space-time block code.

References

[1] M. O. Damen and A. R. Hammons, “Delay-tolerant distributed-TAST codes for cooperative diversity,” *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3755–3773, Oct. 2007.

[2] Y. Li and X.-G. Xia, “Full diversity distributed space-time trellis codes for asynchronous cooperative communications,” *Proc. Int. Symp. on Information Theory*, pp. 911–915, Sep. 2005, Adelaide, Australia.

[3] Y. Shang and X.-G. Xia, “Limited-shift-full-rank matrices with applications in asynchronous cooperative communications,” *IEEE Trans. Inform. Theory*, vol. 53, no. 11, pp. 4119–4126, Nov. 2007.

[4] H. E. Gamal and M. O. Damen, “Universal space-time coding,” *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1097–1119, May 2003.

[5] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*. Cambridge, UK: Cambridge University Press, 2003.

[6] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*. New York, NY, USA: John Wiley & Son Inc., 2000.

[7] A. Hjørungnes and D. P. Palomar, “Patterned complex-valued matrix derivatives,” *In Proc. for Fifth IEEE Workshop on Sensor Array and Multi-Channel Signal Processing, (SAM 2008)*, pp. 1–5, Jul. 2008, Darmstadt, Germany.

[8] ——, “Finding patterned complex-valued matrix derivatives by using manifolds,” *In Proc. for the First Symposium on Applied Sciences in Biomedical and Communication Technologies, (ISABEL 2008)*, pp. 1–5, Oct. 2008, Aalborg, Denmark.

[9] R. F. Rinehart, “The exponential representation of unitary matrices,” *Mathematics Magazine*, vol. 37, no. 2, pp. 111–112, Mar. 1964.

[10] A. Hjørungnes and D. Gesbert, “Complex-valued matrix differentiation: Techniques and key results,” *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2740–2746, Jun. 2007.

[11] J. R. Magnus and H. Neudecker, *Matrix Differential Calculus with Application in Statistics and Econometrics*. Essex, England: John Wiley & Sons, Inc., 1988.

[12] Y. Xin, Z. Wang, and G. B. Giannakis, “Space-time diversity systems based on linear constellation precoding,” *IEEE Trans. Wireless Commun.*, vol. 2, no. 2, pp. 294–309, Mar. 2003.

[13] R. Vishwanath and M. R. Bhatnagar, “Optimum linear constellation precoding for space time wireless systems,” *Wireless Personal Communications*, vol. 40, no. 4, pp. 511–521, Mar. 2007.

[14] F. Oggier and E. Viterbo, “Table of best known full diversity algebraic rotations,” available on "http://www1.tlc.polito.it/∼viterbo/rotations/rotations.html".

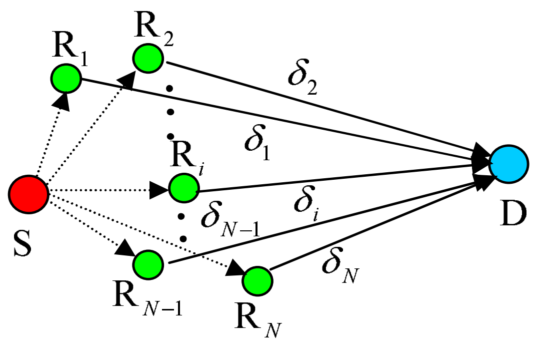

Fig. 1. Cooperative system with one source-destination pair and N relays. The relative delay in link between the i-th relay and the destination is shown by δ_i.

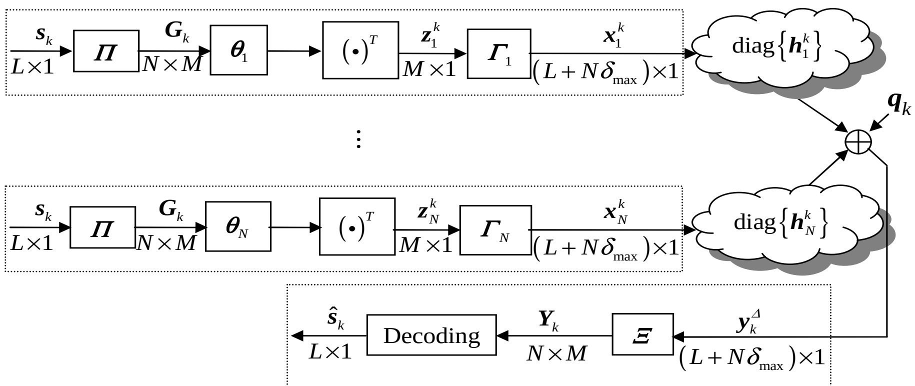

Fig. 2. Block diagram of the proposed delay independent cooperative transmission scheme.

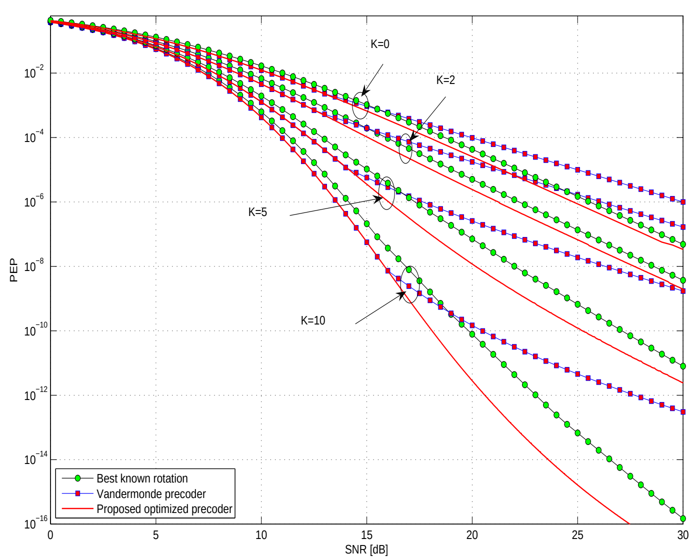

Fig. 3. Analytical PEP versus SNR plots of asynchronous cooperative system with three relays over Rayleigh and Ricean channels $K \in \{0, 2, 5, 10\}$ with Vandermonde precoder [12], known best full diversity rotation [14], and proposed optimized precoder.

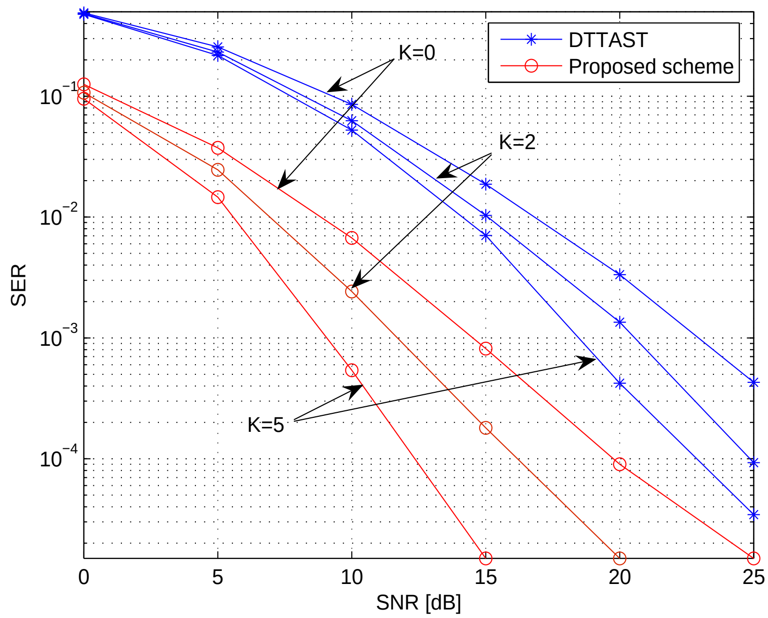

Fig. 4. Comparison of DTTAST codes and the proposed scheme with two unsynchronized relays over Rayleigh and Ricean channels $K \in \{0, 2, 5\}$.

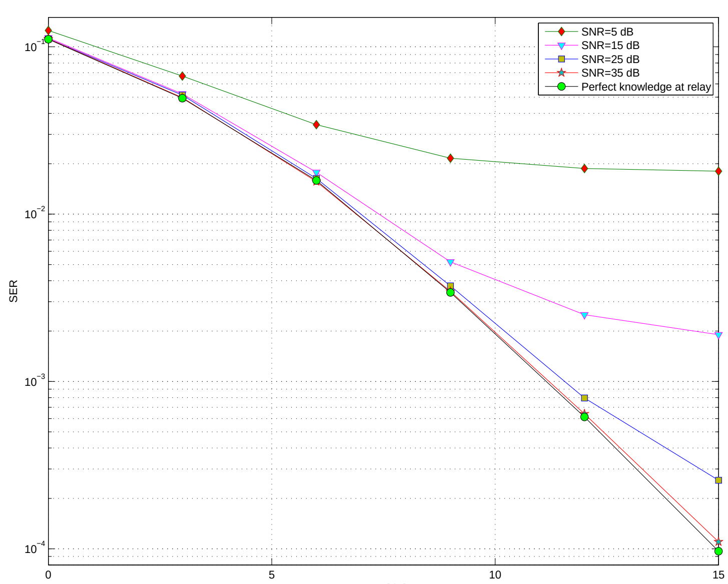

Fig. 5. Effect of wrong relaying on the performance of the proposed scheme with three unsynchronized relays over Ricean channels $K = 1$.