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*Ecole Centrale Paris*

*Laboratoire Génie Industriel*

## **Cahier d'Études et de Recherche / Research Report**

Parameterize a territorial risk evaluation scale using multiple experts knowledge through risk assessment examples

Olivier Cailloux and Vincent Mousseau

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# Parameterize a territorial risk evaluation scale using multiple experts knowledge through risk assessment examples

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**ABSTRACT:** Evaluating and comparing the threats and vulnerabilities associated with territorial zones according to multiple criteria (industrial activity, population, etc.) can be a time-consuming task and often requires the participation of several experts and decision makers. Rather than a direct evaluation of these zones, building a risk evaluation scale and using it in a formal procedure permits to automate the assessment and therefore to apply it in a repeated way and in large-scale contexts and, provided the chosen procedure and scale are accepted, to make it objective. One of the main difficulty of building such a formal evaluation procedure is to account for the multiple experts knowledge and decision makers preferences. The procedure used in this article, ELECTRE TRI, uses the performances of each territorial zone on multiple criteria, together with preferential parameters, to qualitatively assess their associated risk level. The preferential parameters to be determined are the category limits, i.e. the limits on each criterion of the performance range associated with a given risk level, and the weights associated with the different criteria. To obtain these parameters with no direct questioning of the stakeholders, tools based on mathematical programming have been developed to deduce these preferential parameters from assessment examples. In this article, several such tools are applied together in order to build a complete example leading to the definition of a territorial risk evaluation scale taking into account the preferences of multiple stakeholders.

## 1 RISK ASSESSMENT MODELLED AS MULTICRITERIA SORTING MODELS

Assessing the risk levels associated with geographical zones can be a difficult task. It involves multiple, and often conflicting, point of views: a zone may have a low risk according to one criterion while being exposed to a critical risk according to an other one. Associating one risk level to a zone involves aggregating these point of views. This article suggests to use the tools developed in the domain of multicriteria decision aid, which enable a formal approach to that aggregation problem when assessing risk.

The paper is structured as follows. This section describes the relation between a risk assessment problem and a multicriteria sorting problem. Section 2 describes a specific multicriteria sorting method, ELECTRE TRI (Figueira et al. 2005, Mousseau et al. 2000, Roy 1991), and discusses existing methodologies to elicit an ELECTRE TRI based risk model with experts. Section 3 suggests a strategy to build a risk evaluation model as an ELECTRE TRI model using indirect parameters elicitation. An illustrative example is given in Section 4. The algorithm used for the inference is detailed in Section 5.

### 1.1 MCDA problem statements

Real-world decision problems such as territorial risk evaluation usually involve multiple points of view on evaluation of zones they concern. These points of view are relevant for a Decision Maker (DM), or expert (who is either a single person or a collegial body). They are formally represented by real-valued functions called criteria. Multicriteria (MC) decision aiding aims at recommending a decision which is consistent with a value system of the DM. Various methodologies have been proposed to support DMs facing a MC decision problem (Keeney and Raiffa 1976, Roy 1996, Bouyssou et al. 2006). Three main problematics structure the MC decision aiding field: choice, ranking or sorting (Tsoukiàs 2007). Given a finite set  $A$  of alternatives, the choice problematic consists in selecting a subset of alternatives (as small as possible) being judged as the most satisfactory. The ranking problematic consists in establishing a preference pre-order (either partial or complete) on the set of alternatives. These two problematics are said to be comparative, as they require to compare alternatives one to another in order to set up the choice set or the preference pre-order. The sorting problem-

atic concerns ordinal classification of alternatives and consists in assigning each alternative to one of some pre-defined and ordered categories. The assignment of an alternative to an appropriate category relies on the alternative's intrinsic value (and not on the comparison to others). In this paper, we consider the multicriteria (MC) sorting problematic to represent qualitative risk assessment models.

### 1.2 Defining a qualitative risk assessment scale

Most concepts used in a risk assessment problem easily map to concepts used in the MC sorting problematic. What is called a category in such a problematic represents the set of possible risk levels that the territorial zones must be mapped into. They can be e.g.: {Critical risk, Medium risk, Low risk}. Ideally these risk levels should be defined according to some precautionary measures associated with them, to give these categories a precise meaning. The point of views involved in a risk evaluation problem correspond to the criteria in a MC sorting problem. Examples include the presence of a school or the percentage of vulnerable persons.

Building an evaluation scale thus amounts to build a MC sorting model.

### 1.3 Building a sorting model

Each zone being described by a vector of risk factors associated with the point of views involved in the problem, the task at hand consists in assigning these zones to a set of risk categories. This is done using a MC sorting preference model. This model contains a set of subjective data representing the preferences of the considered DM with regard to, e.g., the relative importance of each of the criteria. These objective and subjective data together with a sorting method allow to aggregate the different point of views to assess the risk level of each considered zone.

These preferential parameters may be elicited in a direct way, but this is often difficult as it requires the DM to understand the fine details of their use in the considered MC sorting method. That is why it has been suggested to deduce the preferential parameters in an inverse way, by asking the DM examples of alternatives, or zones, and the category, or risk level, they would consider appropriate for these.

A supplementary difficulty arises when the evaluation method to be defined involve multiple DMs, as different stakeholders may favor different subjective parameter values. Applying inverse elicitation in a multiple DMs context amounts to ask each DM for a set of examples, which may be conflicting, and deduce preferential parameter values that may be either entirely shared by the DMs, or shared for a part of the parameters, and individual for other values. This is the approach we use in this article.

## 2 ELECTRE TRI FOR RISK EVALUATION

The MC sorting method used here is a simplified version of ELECTRE TRI. It is appropriate for a risk evaluation setting as it only requires ordinally evaluated performances on the different criteria. The version considered here is very close to the version studied by Bouyssou and Marchant (2007a, 2007b).

### 2.1 Sorting procedure

ELECTRE TRI requires, as a definition of the preferences of a DM, criteria importance parameters and category limits separating the categories. The criteria importance parameters include a weight for each of the criteria and a majority threshold that defines when a set of criteria is good enough to be decisive. The category limits separate, for each criterion, two consecutive risk levels.

Consider a finite set of territorial zones  $A$ , a set of category limits  $B = \{b_0, \dots, b_k\}$ , and a finite set of criteria indices  $J$ . A criterion  $g_j$  ( $j \in J$ ) is a function from  $A \cup B$  to  $\mathbb{R}$  where  $g_j(a)$  denotes the performance of the zone  $a$  on the criterion  $g_j$ . The zones have to be sorted in  $k$  risk levels,  $c_1, \dots, c_k$ , ordered by their desirability.  $c_1$  is the worst (i.e. highest) risk level, and  $c_k$  is the best (the lowest) one. Each risk level  $c_h$  is defined by the performances of its lower frontier, or category limit,  $b_{h-1}$  and its upper frontier  $b_h$  of  $B$ . The performances are here supposed to be such that a higher value denotes a better performance (i.e. associated with less risks) and the performances on the frontiers are non-decreasing, i.e.  $\forall j \in J, 1 \leq h \leq k : g_j(b_{h-1}) \leq g_j(b_h)$ .

To sort the zones, ELECTRE TRI uses the concept of outranking relation. The assignment rule used here, known as the pessimistic rule, assigns a zone  $a$  to the highest possible risk level  $c_h$  such that the zone outranks the category's lower frontier  $b_{h-1}$ . A zone  $a$  outranks a frontier  $b_{h-1}$  if and only if there is a sufficient coalition of criteria supporting the assertion " $a$  is at least as good as  $b_{h-1}$ ", and no criterion strongly opposes (vetoes) that assertion. To compute this, preferential parameters given by a DM are used. The coalition of criteria in favor of the outranking,  $\forall a \in A, 1 \leq h \leq k$ , is defined as

$$\sum_{j \in J} w_j C_j(a, b_{h-1}), \quad (1)$$

where  $w_j$  is the weight of the criterion  $g_j$ , and  $C_j(a, b_{h-1}) \in \{0, 1\}$  measures whether  $a$  is at least as good as  $b_{h-1}$  from the point of view of the criterion  $j$ :  $C_j(a, b_{h-1}) = 1 \Leftrightarrow g_j(a) \geq g_j(b_{h-1})$ , 0 otherwise. The weights are defined so that they sum to one ( $\sum_{j \in J} w_j = 1$ ). The coalition is compared to a majority threshold  $\lambda \in [0.5, 1]$  defined by the decision maker along with the weights. If

$\sum_{j \in J} w_j C_j(a, b_{h-1}) < \lambda$ , the coalition is not a sufficient coalition and the zone does not outrank the frontier  $b_{h-1}$  and will therefore be assigned in a risk level below  $c_h$ .

Even when the coalition is strong enough, a criterion may veto the outranking situation. It happens when  $g_j(a) > v_j^{h-1}$ . The veto threshold  $v_j^{h-1}$  is a value that the DM may define and represents the performance that, if not reached by some zone  $a$ , forbids the zone to have a risk label of  $c_h$ . To summarize, the zone  $a$  outranks the frontier  $b_{h-1}$  (and therefore is assigned to at least the category  $c_h$ ) if and only if  $\sum_{j \in J} w_j C_j(a, b_{h-1}) \geq \lambda$  and  $\forall j \in J : g_j(a) > v_j^{h-1}$ .

In a case involving a single DM, the weights and majority thresholds (defining the sufficient coalitions) and the category limits may be given directly by him. However, this requires that the DM understands how these values will be used. It is moreover a difficult process to directly ask the DM for these parameters. The approach used here supposes that he provides assignment examples which are used to infer the preferential parameters.

The situation is even more complex when several DMs are involved: there is no reason to suppose that all DMs a priori agree on the importance of the criteria or on the frontiers parameters. Hence, in addition to the difficulty related to the inference of an ELECTRE TRI model, an additional level of complexity is introduced by a possible divergence in the preferences expressed by the different DMs.

## 2.2 Inference of preferential parameters in a multiple DM context

The ELECTRE TRI preferential parameters to elicit are the category limits, the weights, and the vetoes.

Previous works aiming to infer preferential parameters for the ELECTRE TRI procedure on the basis of assignment examples exist, but they mostly involve a single DM. Existing approaches suggest to find the entire ELECTRE TRI preference model parameters (Mousseau and Slowinski 1998) from assignment examples, or find the importance coefficients only (Mousseau et al. 2001), or the categories limits (Ngo The and Mousseau 2002), the other parameters being supposedly known. Robust approaches are suggested which compute for each alternative a range of possible categories to which alternatives can be assigned under incomplete determination of the parameters (Dias and Clímaco 1999, Dias and Clímaco 2000, Dias et al. 2002). Some tools deal with the problem of non existing preference model solutions which may arise because of an inconsistent set of assignment examples (i.e. assignment examples that do not match ELECTRE TRI) (Mousseau et al. 2003, Mousseau et al. 2006). While the above approaches target a unique DM, Damart et al. (2007) propose a method involving a group of DMs that iteratively build, in parallel, in-

Table 1: A summary of the main features proposed by other articles and by this one (last row). For each article, the second column indicates the expected input of the main tool proposed in the article, the last one shows its output.  $i$  designates assignment examples from a single DM,  $i^*$  designates possibly inconsistent assignment examples from a single DM,  $g$  is a group of DMs' assignment examples,  $\mathcal{P}$  is a set of category limits,  $W$  is a set of weights. The computations are based on linear (or mixed integer and linear) programming, except for the first one.

Article	input	output
MS98	$i$	$\mathcal{P}, W$ (non-linear)
MFN01	$i, \mathcal{P}$	$W$
NM00	$i, W$	$\mathcal{P}$
DMFC02	$i$	robust model ( $\mathcal{P}, W$ )
MDFGC03	$i^*$	how to restore consistency
MDF06	$i^*$	how to restore consistency
DDM07	$g, \mathcal{P}$	progressive collective model ( $W$ )
This article	$g$	collective model ( $\mathcal{P}$ )

dividual preference models and a collective preference model representing the group consensus. Table 1 presents a summary of the available tools for indirect preference elicitation related to ELECTRE TRI.

## 3 BUILDING A RISK EVALUATION MODEL WITH MULTIPLE EXPERTS

Contrary to the above approaches, the method we propose consists in eliciting category limits shared by all DMs while leaving weights of criteria possibly different for each DM. This enables a consensus to be approached by first validating a part of the preference model (the category limits) with all DMs before eliciting the weights of criteria.

The process is the following.

- Obtain from each DM typical zones that correspond to each specific risk levels. These zones are defined by their evaluations on the criteria and will be used as indirect information to elicit the risk evaluation model.
- Search for an ELECTRE TRI model without vetoes representing the zone examples. A mathematical program is thus built in order to compute category limits compatible with the typical zones provided by the DMs. The program is defined in Section 5.
- If no ELECTRE TRI model without vetoes is able to represent all zone examples, it is possible to allow vetoes to be used and define the corresponding mathematical program.
- If still no satisfying ELECTRE TRI model is found, it is possible to check which maximal subsets of zone examples can be represented (Mousseau et al. 2003, Mousseau et al. 2006). The DMs may then be asked individually if they agree to remove or change specific zone examples to restore consistency.

- When shared category limits are found, they should be presented to the DMs for validation. If they disagree, they may propose additional zone examples to further constrain the model and iteratively converge towards satisfactory category limits.
- At this stage, the DMs agree on a set of category limits but weights of criteria are still possibly distinct for each DM. The approach suggested by Damart et al. (2007) may then be used to build a consensus on the weights.

One of the important features of the proposed approach is that, as opposed to existing methods, the proposed method applies to a group DM context and does not suppose that part of the preference model is known beforehand.

#### 4 ILLUSTRATIVE EXAMPLE

Let us illustrate the method on the following hypothetical scenario. This simplistic example role is to illustrate the proposed method rather than provide an in-depth case study.

A group of experts would like to develop a scale permitting to evaluate the level of risk of each territorial zone around a given industrial installation related to a possible hazard. Each zone is to be determined as belonging to one of the three categories {High risk  $\prec$  Medium risk  $\prec$  Low risk}. Each of these categories are associated with specific precautionary measures (e.g. evacuate the population). Building a risk evaluation scale relative to hazards related to industrial installations is typically important in decision processes such as the PPRT (Plan de Prévention des Risques Technologiques) used in France (PPRT 2011).

The four members of the expert group consider that the following six criteria should be used to evaluate the risk associated to each zone.

- d The distance to the hazardous industrial installation, evaluated on a 3 points ordinal scale. 0: less than 500 meters; 1: between 500 and 2000 meters; 2: more than 2 km.
- sc Presence or absence school in the zone, a binary assessment. No school is encoded with a 1, presence of school with a 0.
- bv Building vulnerability, evaluated on a 5 points ordinal scale.
- pe Presence of public environment and technical assets, evaluated on a 5 points ordinal scale.
- i Impact on other industrial installations, which could lead to cascaded effects. Evaluated on a 5 points ordinal scale.

Table 2: Inferred category limits.  $b_1$  separate the categories “High risk” and “Medium risk” and  $b_2$  separate the categories “Medium risk” and “Low risk”.

Frontier	bv	pe	d	i	sc	pr
$b_1$	2.8	1.2	1.8	2.8	0.8	40.7
$b_2$	3.2	3.2	2.2	3.2	0.8	81.8

pr Proportion of the population who is vulnerable (children under 15 and elderly persons), evaluated in percentage.

We suppose that each expert provides 30 zone examples and their associated risk evaluation (that could correspond to real zones the experts have previously evaluated or fictitious zones defined by their evaluation vectors). The data is given in Annex A.

Using the assignment examples, the program Infer Limits (described in Section 5) is used to find category limits shared by the experts which match their individual assignment examples. The results are provided in Table 2.

The numerical values found as frontiers by the mathematical program should be interpreted in terms of the scales used for the evaluations of the related point of views. For example, the evaluation of the frontier  $b_2$  on the point of view of the building vulnerability criterion, as displayed in Table 2, is 3.2. As that criterion scale is discrete, a value of at least 3.2 corresponds to a value of at least 4. This means that an evaluation of 4 on the point of view of the building vulnerability for a given zone counts as an argument in favor of that zone to be assigned to a category better than  $b_2$ , i.e. the category “Low risk”. If the zone has an evaluation of at least 2.8, the evaluation of  $b_1$  on the building vulnerability criterion, but not better than 3.2, hence, an evaluation of 3, this point of view argues in favor of that zone being assigned to the risk level “Medium risk”. If the evaluation is less than 2.8, thus is 0 to 2, the risk level recommended by this point of view is “High risk”.

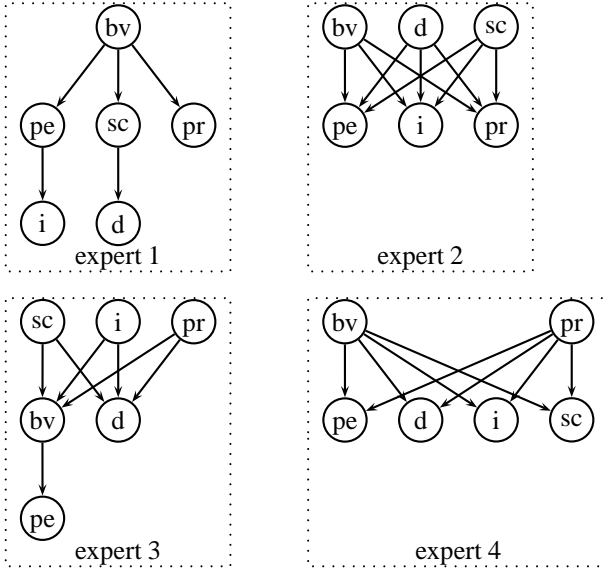
The table shows a possible set of frontiers evaluations that may be shared by all DMs such that when used in an ELECTRE TRI model it is possible to reproduce their zone examples, provided adequate weights are used. At this stage the DMs do not share the weight values yet. Table 3 shows for each DM a set of weights matching the assignment examples with the common category limits. Note that these weights are not the only ones that reproduce all assignment examples with the frontier values shown in Table 2.

Choosing these category limits impose some constraints, for each expert, on the criteria importance that may be used to match the zone examples. It is possible to compute the restrictions imposed to each expert on the relative importance of the criteria. In Figure 1, for a given expert, an arrow is drawn from a criterion  $j$  to a criterion  $j'$  if all ELECTRE TRI models matching the zone examples for that expert, considering the category limits in Table 2, are such that the

Table 3: A set of weights, as found by the Infer Limits program, matching zone examples of each expert when used together with the inferred category limits.

DM	bv	pe	d	i	sc	pr	$\lambda$
dm1	0.375	0.25	0	0	0.25	0.125	0.69
dm2	0.75	0	0.125	0	0.125	0	0.94
dm3	0.125	0	0.125	0.25	0.25	0.25	0.56
dm4	0.875	0	0	0	0	0.125	0.94

Figure 1: The restrictions on the weights imposed by the choice of the common frontiers, for each expert. An arrow from a criterion  $j$  to a criterion  $j'$  states that the choice of these frontiers implies a greater weight for  $j$  than for  $j'$  for this expert.



weight on the criterion  $j$  is more important than the weight on the criterion  $j'$ .

These importance relations are to be discussed and accepted by all experts for these shared frontiers to be accepted. If a DM disagree on some importance comparisons, the Infer Limits program should be run again with some additional constraints concerning unacceptable weights comparisons.

It may be seen that a consensus on the weights cannot be readily found using these category limits as e.g. expert 3 has sc necessarily more important than bv while experts 1 and 4 have the inverse preference. To get closer to a consensus, the third decision expert might be asked if he could consider bv to be more important than sc and the Infer Limits program be run again with that supplementary constraint included.

In our example, we suppose that when presented the category limits, one of the expert disagree with one of the values, saying that in his opinion the category limit  $b_1$  on the criterion pe must have a value of at least 2, thus, that a zone should have an evaluation of at least 2 according to the point of view of the public environment and technical assets for this point of view to argue in favor of the zone to be in the “Medium risk” or “Low risk” category. This constraint is thus included in the program and a new so-

Table 4: Inferred category limits on the second run, with the added constraint that  $b_1$  on pe must be at least 2.  $b_1$  separate the categories “High risk” and “Medium risk” and  $b_2$  separate the categories “Medium risk” and “Low risk”.

Frontier	bv	pe	d	i	sc	pr
$b_1$	2.8	2.8	1.8	2.2	0.8	40.8
$b_2$	3.2	3.2	2.2	3.8	0.8	74.2

Table 5: A set of weights, as found by the Infer Limits program, matching zone examples of each expert when used together with the inferred category limits.

DM	bv	pe	d	i	sc	pr	$\lambda$
dm1	0.375	0.25	0.125	0	0.125	0.125	0.69
dm2	0.75	0	0.125	0	0.125	0	0.94
dm3	0.125	0	0.125	0.25	0.25	0.25	0.56
dm4	0.5	0	0	0	0	0.5	0.56

lution found, displayed in Table 4.

A set of weights compatible with the zone examples using these frontiers is presented in Table 5.

Supposing that all DMs agree with the shared frontier values found, the approach suggested by Damart et al. (2007) may then be used to iteratively build consensual weights among the group of experts.

## 5 ALGORITHMIC DESCRIPTION

In this section we detail the mathematical program which allow to elicit the category limits for a set of DMs. This requires the following supplementary notation. For each DM  $l \in \mathcal{L}$ , the set of examples  $E^l$  is the set of pairs  $(a, h) \in A \times [1, k]$  specifying that the zone  $a$  is assigned to the risk level  $c_h$  by  $l$ . We write  $a \rightarrow h$  to denote such a zone example.

Infer Limits is a mixed integer program that finds, if possible, a set of category limits such that it is possible to satisfy the zone examples of each DMs using individual criteria importance parameters without using veto thresholds.

### 5.1 Stating the problem

Having a set of zones  $A$  used as examples, a set of criteria indices  $J$ , the evaluations of the zones  $g_j(a) \forall a \in A, j \in J$ , the number of categories  $k$ , a set of DMs  $\mathcal{L}$ , zone examples  $E^l$ , the goal of Infer Limits is to determine the performances of category limits  $g_j(b_h), \forall j \in J, 1 \leq h \leq k - 1$  shared among the DMs, together with individual weights  $w_j^l$  and majority thresholds  $\lambda^l$ , matching all zone examples. Note that the lowest category limit,  $b_0$ , must be outranked by every zone (because they must go into at least category  $c_1$ , the worst one) and hence the performances  $g_j(b_0)$  on every criterion necessarily consist in performances lower than every other performances on the

same criterion. The upper frontier of the best category, that would be  $b_k$ , is used only when an other assignment procedure known as the optimistic procedure is used. Therefore, the lowest and best frontiers are not considered in our inference programs. Satisfying these assignment examples amounts to satisfy,  $\forall l \in \mathcal{L}, \forall a \rightarrow h \in E^l$  :

$$\sum_{j \in J: g_j(a) \geq g_j(b_{h-1})} w_j^l \geq \lambda^l, \text{ and} \quad (2)$$

$$\sum_{j \in J: g_j(a) \geq g_j(b_h)} w_j^l < \lambda^l. \quad (3)$$

Equation (2) ensures that the example zone is assigned to a risk level at least as good as  $c_h$ , and (3) makes sure that it is assigned to a risk level not better than  $c_h$ .

## 5.2 Constraints

$\forall j \in J, a \in A, 1 \leq h \leq k-1$  :

$$\begin{cases} \frac{1}{M_j + \delta_j} ((g_j(a) - g_j(b_h)) + \delta_j) \leq C_j(a, b_h); \\ C_j(a, b_h) \leq \frac{1}{M_j} (g_j(a) - g_j(b_h)) + 1. \end{cases} \quad (4)$$

The constraints in (4) define  $C_j(a, b_h)$ , binary variables indicating the agreement of the criterion  $j$  to say that  $a$  is at least as good as  $b_h$ . See Figure 2.  $\delta_j, \forall j \in J$ , is a value smaller than the minimal difference between any two different performances on  $j$  ( $\forall j \in J : \min_j = \min_{a, b \in A} |g_j(a) - g_j(b)|$ ) divided by  $k$ . If there is no two different performances for a given criterion,  $\delta_j$  is defined arbitrarily to 1. It is defined so that adding this value to the constraints does not restrict the space of possibilities about  $g_j(b_h)$  and may thus be used to change the strict inequality to a loose inequality.  $\forall j \in J, M_j = \bar{g}_j - \underline{g}_j + \min_j$  is a scaling factor representing the maximal performance difference on the criterion  $j$  including one extreme frontier.  $\bar{g}_j$  and  $\underline{g}_j$  are the best and worst performances on the criterion  $j$ .

The sum of the weights are modelled using the following constraints,  $\forall l \in \mathcal{L}, a \rightarrow h \in E^l, j \in J$  :

$$\begin{cases} \sigma_j^l(a, b_h) \leq w_j^l \\ \sigma_j^l(a, b_h) \geq 0 \\ \sigma_j^l(a, b_h) \leq C_j(a, b_h) \\ \sigma_j^l(a, b_h) \geq C_j(a, b_h) + w_j^l - 1. \end{cases} \quad (5)$$

The variables  $\sigma_j^l(a, b_h)$  represent the sum of the support for saying that  $a$  is at least as good as  $b_h$

Figure 2: constraining  $C_j$  to the appropriate value

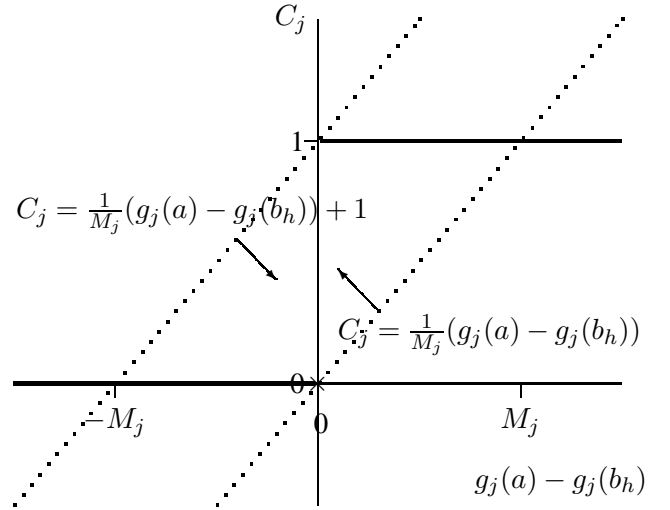
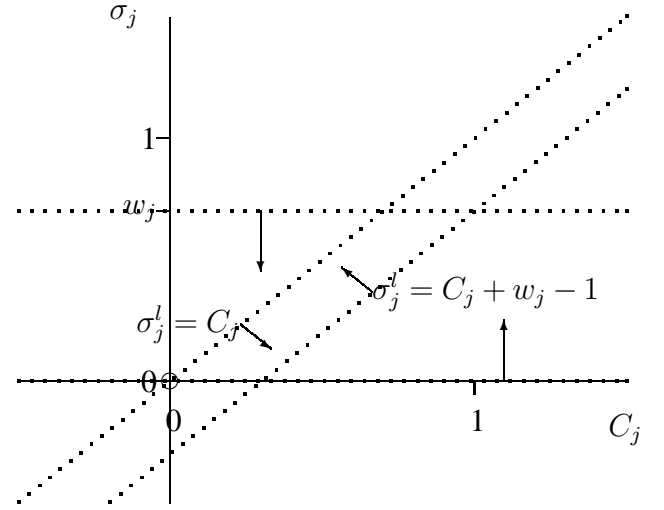


Figure 3: constraining  $\sigma_j^l$  to the appropriate value



while avoiding the non-linear expression  $\sigma_j^l(a, b_h) = w_j^l C_j(a, b_h)$  (Meyer et al. 2008). See also Figure 3.

Then we need to ensure that the sum of supports for each examples are greater than the related majority thresholds, i.e.,  $\forall l \in \mathcal{L}, \forall a \rightarrow h \in E^l, h < k$  :

$$\sum_{j \in J} \sigma_j^l(a, b_h) + s \leq \lambda^l - \delta_\lambda, \quad (6)$$

and  $\forall l \in \mathcal{L}, \forall a \rightarrow h \in E^l, h > 1$  :

$$\sum_{j \in J} \sigma_j^l(a, b_{h-1}) \geq \lambda^l + s. \quad (7)$$

$\delta_\lambda$  is an arbitrary small positive value,  $s$  is a slack variable used in the objective function.

$$\forall l \in \mathcal{L} : \sum_{j \in J} w_j^l = 1. \quad (8)$$



Constraints (8) ensure that the weights sum to one.

$$\forall j \in J, 2 \leq h \leq k - 1 : g_j(b_{h-1}) \leq g_j(b_h). \quad (9)$$

Ensuring that the category limits are ordered correctly is achieved through Constraints (9).

The last constraint ensures that the best and worst frontiers evaluations are not more than  $\min_j$  apart from the closest zone evaluation.

$$\forall j \in J : \begin{cases} g_j(b_k - 1) \leq \overline{g}_j + \min_j, \\ g_j(b_1) \geq \underline{g}_j - \min_j. \end{cases} \quad (10)$$

### 5.3 Objective function

In order to maximize the separation between the sum of support and the majority thresholds, the slack variable  $s$  used in Constraints (6), (7) must be maximized. It is also possible to simply search for a solution to the program without specifying any objective.

## 6 CONCLUSION

In this article we show how to express the elaboration of an ordinal multicriteria risk scale as a multicriteria sorting problem. This highlights the usefulness for risk analysis of some tools and approaches developed in the field of multicriteria decision analysis. Moreover, we propose a method to elicit an appropriate multicriteria sorting model with multiple experts on the basis of the ordinal evaluation of typical zones. The method emphasizes and fosters the necessary interaction between experts during the elicitation of the model.

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A ILLUSTRATIVE EXAMPLE DATA

dm	zone	bv	pe	d	i	sc	pr	category
dm1	Zone08	5	4	1	4	1	90.1	Low
dm1	Zone11	5	3	2	3	0	92.7	Medium
dm1	Zone12	1	3	3	2	1	42.7	High
dm1	Zone13	2	4	3	5	0	90.5	High
dm1	Zone15	4	1	3	2	1	97.9	Low
dm1	Zone20	1	2	3	1	0	60.1	High
dm1	Zone22	3	3	3	2	1	16.7	Medium
dm1	Zone23	1	3	1	4	0	44.8	High
dm1	Zone24	4	2	3	2	1	84.7	Low
dm1	Zone25	4	3	1	2	1	40.5	Medium
dm1	Zone29	5	4	1	1	1	40.8	Low
dm1	Zone32	3	5	3	3	1	35.2	Medium
dm1	Zone34	4	4	1	5	1	90.5	Low
dm1	Zone41	2	3	1	4	0	52.7	High
dm1	Zone45	4	1	2	4	1	75.8	Medium
dm1	Zone48	4	5	1	3	0	66.1	Medium
dm1	Zone52	4	4	1	3	1	2.2	Low
dm1	Zone54	2	1	1	2	1	41.9	High
dm1	Zone56	3	1	2	2	0	6.6	High
dm1	Zone57	3	3	2	2	0	53.1	Medium
dm1	Zone60	1	1	1	5	0	8.6	High
dm1	Zone70	3	1	1	2	0	51.4	High
dm1	Zone76	1	3	2	1	1	52.4	High
dm1	Zone82	3	5	2	1	0	85.3	Medium
dm1	Zone83	3	3	3	5	0	81.2	Medium
dm1	Zone84	4	3	2	1	1	58.2	Medium
dm1	Zone88	3	3	2	1	0	45.3	Medium
dm1	Zone89	5	4	1	3	1	84.2	Low
dm1	Zone90	3	3	1	5	1	1.3	Medium
dm1	Zone99	1	2	3	4	0	79.8	High
dm2	Zone00	3	3	2	4	1	63.6	Medium
dm2	Zone03	2	1	2	3	1	3.4	High
dm2	Zone05	5	5	3	4	1	13.2	Low
dm2	Zone10	2	4	1	3	0	35.3	High
dm2	Zone11	5	3	2	3	0	92.7	High
dm2	Zone14	4	4	1	5	1	16.7	High
dm2	Zone16	4	3	3	1	1	72.1	Low
dm2	Zone20	1	2	3	1	0	60.1	High
dm2	Zone25	4	3	1	2	1	40.5	High
dm2	Zone26	1	5	3	3	1	28.6	High
dm2	Zone27	5	2	3	4	0	29.1	High
dm2	Zone30	3	1	2	1	1	59.2	Medium
dm2	Zone32	3	5	3	3	1	35.2	Medium
dm2	Zone34	4	4	1	5	1	90.5	High
dm2	Zone36	4	2	1	2	1	90.9	High
dm2	Zone38	3	4	2	3	0	78.2	High
dm2	Zone52	4	4	1	3	1	2.2	High
dm2	Zone55	2	5	1	4	1	16.7	High
dm2	Zone58	2	1	2	3	1	74.2	High
dm2	Zone63	5	1	2	5	0	46.9	High
dm2	Zone64	2	2	3	2	1	40.7	High
dm2	Zone69	3	3	1	2	1	56.9	High
dm2	Zone84	4	3	2	1	1	58.2	Medium
dm2	Zone86	4	2	2	3	1	56.8	Medium
dm2	Zone88	3	3	2	1	0	45.3	High
dm2	Zone89	5	4	1	3	1	84.2	High
dm2	Zone92	3	5	3	1	0	2.9	High
dm2	Zone97	3	3	2	5	1	41.3	Medium
dm2	Zone98	1	3	2	2	0	82.7	High
dm2	Zone99	1	2	3	4	0	79.8	High
dm3	Zone00	3	3	2	4	1	63.6	Medium
dm3	Zone01	2	5	3	3	0	83.8	Medium
dm3	Zone05	5	5	3	4	1	13.2	Low
dm3	Zone06	2	2	1	2	1	8.3	High
dm3	Zone08	5	4	1	4	1	90.1	Low
dm3	Zone17	4	2	1	4	0	83.0	Low
dm3	Zone20	1	2	3	1	0	60.1	High

dm3	Zone21	5	5	3	1	1	50.6	Medium
dm3	Zone31	2	3	3	3	1	0.5	Medium
dm3	Zone46	1	2	3	1	1	22.2	High
dm3	Zone50	3	5	3	2	0	90.6	High
dm3	Zone51	4	4	2	1	0	32.1	High
dm3	Zone55	2	5	1	4	1	16.7	High
dm3	Zone58	2	1	2	3	1	74.2	Medium
dm3	Zone59	2	1	2	2	0	82.0	High
dm3	Zone64	2	2	3	2	1	40.7	Medium
dm3	Zone69	3	3	1	2	1	56.9	Medium
dm3	Zone70	3	1	1	2	0	51.4	High
dm3	Zone71	4	3	1	5	0	83.0	Low
dm3	Zone72	4	3	3	4	0	99.7	Low
dm3	Zone74	1	5	2	4	0	17.0	High
dm3	Zone75	1	5	2	4	0	85.7	Medium
dm3	Zone78	1	2	2	5	1	84.1	Low
dm3	Zone81	3	2	1	5	0	20.4	High
dm3	Zone82	3	5	2	1	0	85.3	High
dm3	Zone86	4	2	2	3	1	56.8	Medium
dm3	Zone87	2	1	1	5	0	91.4	High
dm3	Zone95	5	2	3	3	0	70.9	Medium
dm3	Zone96	5	5	2	4	0	7.5	High
dm3	Zone98	1	3	2	2	0	82.7	High
dm4	Zone02	3	1	2	5	1	68.8	Medium
dm4	Zone05	5	5	3	4	1	13.2	High
dm4	Zone09	5	4	3	3	1	37.4	High
dm4	Zone10	2	4	1	3	0	35.3	High
dm4	Zone18	2	3	3	4	1	77.8	High
dm4	Zone19	4	1	2	3	1	31.9	High
dm4	Zone23	1	3	1	4	0	44.8	High
dm4	Zone24	4	2	3	2	1	84.7	Low
dm4	Zone28	2	2	1	2	1	19.7	High
dm4	Zone29	5	4	1	1	1	40.8	Medium
dm4	Zone32	3	5	3	3	1	35.2	High
dm4	Zone34	4	4	1	5	1	90.5	Low
dm4	Zone35	3	1	3	4	1	81.2	Medium
dm4	Zone36	4	2	1	2	1	90.9	Low
dm4	Zone48	4	5	1	3	0	66.1	Medium
dm4	Zone49	3	4	3	5	1	25.4	High
dm4	Zone60	1	1	1	5	0	8.6	High
dm4	Zone67	1	1	3	5	1	10.1	High
dm4	Zone71	4	3	1	5	0	83.0	Low
dm4	Zone74	1	5	2	4	0	17.0	High
dm4	Zone80	4	1	2	3	1	14.1	High
dm4	Zone81	3	2	1	5	0	20.4	High
dm4	Zone83	3	3	3	5	0	81.2	Medium
dm4	Zone84	4	3	2	1	1	58.2	Medium
dm4	Zone85	4	2	3	1	1	31.1	High
dm4	Zone86	4	2	2	3	1	56.8	Medium
dm4	Zone87	2	1	1	5	0	91.4	High
dm4	Zone90	3	3	1	5	1	1.3	High
dm4	Zone98	1	3	2	2	0	82.7	High
dm4	Zone99	1	2	3	4	0	79.8	High