Transmit-Diversity for Spatial Modulation (SM): Towards the Design of High-Rate Spatially-Modulated Space-Time Block Codes

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Abstract— **Spatial Modulation (SM) is a recently proposed low–complexity modulation scheme for multiple–antenna wireless systems. Recent works have shown that state–of–the–art SM provides a multiplexing gain with respect to single–antenna systems by avoiding inter–channel interference, but it is inherently unable to achieve transmit–diversity. The aim of this paper is to shed light on the design of multiple–antenna wireless systems that exploit the SM concept and can achieve transmit–diversity gains. More specifically, we propose a novel modulation concept, which in a unique fashion combines the high multiplexing gain offered by SM and the transmit–diversity gain provided by Space–Time Block Codes (STBCs) technology. This modulation scheme is introduced here for the first time and referred to as "Spatially–Modulated STBCs" (SM–STBC). By using analysis and simulation, we show that SM–STBC achieves multiplexing and diversity gains, as well as still retains a single–stream receiver implementation. By using numerical simulations, we show that the proposed code can provide better performance than state–of–the–art Alamouti and rate–$3/4$ STBCs.**

I. Introduction

Multiple–antenna technology is a rich area of research [1]. In the realm of the many solutions proposed to date, Spatial Modulation (SM) is a recently proposed modulation scheme that promises a low–complexity transmitter and receiver design, along with improved system performance with respect to many state–of–the–art multiple–antenna solutions [2]–[7].

The breakthrough idea of SM is to exploit the spatial domain as an additional dimension to convey part of the information bits [6]. As a result, SM turns out to be a high spectrally–efficient transmission technology with an equivalent code rate greater than one [8]. Recent studies have shown that SM can achieve a higher capacity than multiple–antenna schemes with similar decoding complexity, such as Space–Time Block Codes (STBCs) [2], along with a smaller bit error probability than Vertical Bell Laboratories Layered Space–Time (V–BLAST) and Alamouti schemes [3].

Although originally conceived as an interference–free spatial–multiplexing scheme [3], many researchers have recently started investigating the potential of this technology for receive– and transmit–diversity. In particular, transmit–diversity is receiving a growing interest due to its suitability for downlink applications with single–antenna and low–complexity mobile units. More specifically, many researchers have focused their attention on a low complexity implementation of SM, which is known as Space Shift Keying (SSK) modulation [5]. Unlike SM, in SSK modulation only the spatial–constellation diagram is used to modulate the information bits, thus trading–off transmitter and receiver complexity for the achievable data rate [6]. As far receive– and transmit–diversity for SSK modulation is concerned, the following contributions are available in the literature. In [5] and [9], it is shown that SSK modulation can achieve a receive–diversity gain that increases linearly with the number of antennas at the receiver. In [10], it is shown that, regardless of the number of simultaneously–active antennas at the transmitter, SSK modulation is unable to provide transmit–diversity gains. In [11], a simple method is introduced to overcome that limitation. The solution is applicable to a transceiver with two transmit–antenna and one receive–antenna, and neither incurs in any spectral efficiency loss nor requires multiple simultaneously–active antennas at the transmitter. In [12], transmit–diversity is achieved by sending redundant information in non–overlapping time–slots, thus incurring in a spectral efficiency loss. In [13], it is proved that the method in [11] is unable to provide full–diversity for an arbitrary number of antennas at the transmitter and, in general, it allows us to achieve transmit–diversity only equal to two. In [14], the idea in [13] is extended and new schemes to achieve transmit–diversity greater than two are proposed. However, all these techniques are mainly concerned with SSK modulation, while, to the best of the authors knowledge, the design of transmit–diversity schemes for the more general SM concept have never been considered so far[1]. Only in [16], the authors have studied the achievable transmit–diversity of SM and have pointed out that SM cannot achieve transmit–diversity gains. However, no solutions are provided to cope with this issue and it is shown that the absence of transmit–diversity may result, especially for high correlated fading channels, in a substantial performance loss of SM with respect to the Alamouti scheme.

Motivated by all the above, this paper aims at shedding light on the issues related to the design of SM schemes with transmit–diversity. Since SM includes as special cases both SSK modulation and conventional modulation schemes [6], we show that SM needs to combine conventional transmit–diversity [17] methods (*e.g.*, Alamouti [18] and STBCs [19]) and SSK transmit–diversity solutions to be able to get full transmit–diversity gains. Thus, the design of SM schemes with transmit–diversity is a more challenging task than the design of SSK modulation and STBCs alone. More specifically, when adding the spatial–constellation diagram on top of the signal–constellation diagram of conventional modulation we have to

[1]After the submission of this paper, [15] was published. In that paper, a different approach to design STBCs for SM is proposed. We emphasize that single–stream decoding is sub–optimum for the solution in [15].

make sure that the SSK modulation component of SM does not destroy the orthogonality property of STBCs, thus still allowing the system to i) get transmit–diversity gains, and ii) enable the adoption of a single–stream receiver design. This paper provides some guidelines to construct such modulation/coding schemes. More specifically, we are interested in developing SM–STBCs with diversity order equal to two, which add the SSK modulation principle on top of the Alamouti code. In other words, this paper reports a new code with transmit–diversity equal to two, and, as opposed to classical Alamouti code, rate greater than one. Moreover, it is shown that a simple single–stream decoder can be used at the receiver without loss in optimal decoding performance. Numerical results are provided to complement our analytical investigation and to show that the proposed scheme outperforms state–of–the–art SM, Alamouti, and STBCs for the same spectral efficiency and receiver complexity. Very interestingly, it is shown that the proposed solution can outperform the more complicated rate–$3/4$ STBCs [19] with diversity gain three and four.

The reminder of this paper is organized as follows. In Section II, the system model and the main assumptions are briefly summarized. In Section III, we introduce the design of SM–STBCs for multiple–antenna systems, highlight the issues related to the efficient design of such codes, and provide some guidelines for the design of SM–STBCs with transmit–diversity equal to two and low decoding complexity. In Section IV, our claims are substantiated through Monte Carlo simulations. Finally, Section V concludes the paper.

II. System Model

We consider a Multiple–Input–Single–Output (MISO) communication system with N_t antennas at the transmitter and $N_r = 1$ antennas at the receiver. The assumption $N_r = 1$ does not limit the generality of the results derived in this paper since we are mainly interested in studying transmit–diversity schemes. By following the analysis in [9], it can be proved that the results obtained in this paper can be extended to multiple receive–antenna, and that the overall diversity achieved by the resulting system is simply multiplied by N_r. At the receiver, we assume Maximum–Likelihood (ML) decoding with Full Channel State Information (F–CSI) [4], [7], which can be obtained through the transmission of pilot symbols before data transmission. To simplify the analytical derivation, we consider a Multilevel Phase Shift Keying (M–PSK) modulation scheme (*i.e.*, the signal–constellation diagram [6]), and frequency–flat, unit–power, and independent and identically distributed (i.i.d.) Rayleigh fading channels over all the wireless links. The complex fading gain over the wireless link from the t–th transmit–antenna to the receiver is denoted by $\{h_t\}_{t=1}^{N_t}$. Finally, perfect time synchronization at the receiver and negligible differences in the propagation delays from the transmit– to the receive–antenna are assumed [7].

Additional notation used throughout this paper is as follows: i) we adopt a complex–envelope signal representation; ii) $(\cdot)^*$ is the complex–conjugate operator; iii) $|\cdot|^2$ denotes square absolute value; iv) $\mathrm{E}\{\cdot\}$ is the expectation operator; v) $\mathrm{Re}\{\cdot\}$ denotes the real part operator; vi) $Q(x) = \left(1/\sqrt{2\pi}\right)\int_x^{+\infty}\exp\left(-t^2/2\right)dt$ is the Q–function; vii) N_a is the number of simultaneously active antennas at the transmitter, with $1 \le N_a \le N_t$; viii) E_m is the average total energy transmitted by the N_a active antennas that emit a non–zero signal. It is assumed that E_m is equally distributed among the active antennas, *i.e.* each active antenna emits a signal with energy E_m/N_a; ix) T_m denotes the signaling interval for each information message; x) N_0 is the power spectral density per dimension of the Additive White Gaussian Noise (AWGN) at the receiver input; xi) for ease of notation, we set $\bar{\gamma}{=}E_m/(4N_aN_0)$; xii) S_M and S_{N_h} denote the signal– and spatial–constellation diagrams with size M and N_h, respectively. The generic element of S_M is $s_l = \tilde{s}_l\sqrt{E_m/N_a}$ for $l = 1, 2, \ldots, M$, where $\tilde{s}_l$ is a unit–energy complex number, and the generic element of S_{N_h} is the N_a–tuple of channel gains $\left\{h_{t_1}, h_{t_2}, \ldots, h_{t_{N_a}}\right\}_{t_1 \ne t_2 \ne \ldots \ne t_{N_a}=1}^{N_t}$; xiii) $\{w_t(\cdot)\}_{t=1}^{N_t}$ is the pulse shape used at the t–th transmit–antenna; xiv) $\lfloor\cdot\rfloor$ is the floor function; xv) $\binom{\cdot}{\cdot}$ is the binomial coefficient; and xvi) $\Pr\{\cdot\}$ denotes probability.

A. On the Need of STBCs to Achieve Transmit–Diversity

The distinguishable feature of SM with respect to other spatial multiplexing methods is the capability of achieving a multiplexing gain without the need to multiplex many data streams. In other words, a single transmit–antenna is active at any time instance [3]. The fundamental question is to understand whether, by still keeping a single active antenna at the transmitter, we can achieve transmit–diversity gains. As far as SSK modulation is concerned, we have shown in [11], [13] that this is actually possible via suitable pulse shaping design at the transmitter. This operation can be regarded as a form of precoding that operates on the waveform of the transmitted signal. As far as SM is concerned, in [16] the authors have shown that, regardless of the number of simultaneously–active antennas at the transmitter, no transmit–diversity gains can be achieved if pulse shaping is not adopted at the transmitter. This latter result shows that state–of–the–art SM [15] cannot achieve transmit–diversity even though multiple antennas are simultaneously transmitting. An intuitive explanation of this outcome can be obtained from the analytical model we have recently developed for SM in [6]. In [6, Eq. (11)], we have shown that the Average Bit Error Probability (ABEP) of SM is approximately given by the weighted summation of the individual ABEPs of the SSK modulation and the conventional modulation schemes composing SM. In other words, this framework highlights that a necessary condition for SM to achieve transmit–diversity, $\mathrm{Div}_{\mathrm{tx}}$, is that *both* the SSK modulation and the conventional modulation schemes inherently embedded into SM have transmit–diversity $\mathrm{Div}_{\mathrm{tx}}$. If, for example, the conventional modulation scheme embedded into SM has transmit–diversity equal to one, then the resulting SM system will always have diversity equal to one regardless of the transmit–diversity of the SSK modulation scheme. Thus, the result in [16] can be readily understood: even though multiple antennas are simultaneously active, none of the transmit–diversity methods in [17] is adopted in [16] and, so, the resulting system cannot achieve any transmit–diversity gains. Similar arguments can be used for the SSK sub–system embedded into SM.

From the arguments above, it follows that multiple transmit–antenna need to be active to get transmit–diversity gains in SM, as well as that SM should be generalized and integrated into one of the methods in [17] to increase the transmit–diversity gain. In this paper, we limit ourselves to designing SM schemes with transmit–diversity equal to two, by using the Alamouti code as the building block to get transmit–diversity gains for the conventional modulation sub–system

[18]. Generalization to other STBCs [19] is possible, but it is not pursued in this paper due to space constraints.

B. Signal Structure

According to Section II-A, we assume two simultaneously–active antennas at the transmitter, *i.e.*, $N_a = 2$. Thus, the spatial–constellation diagram might have a maximum number of points equal to $N_H = 2^{\left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor}$ [10], [14]. The goal of this paper is to find the sub–set of points[2] $N_h \le N_H$ such that the proposed system has transmit–diversity equal to two.

The working principle of the proposed scheme is as follows. At the transmitter, the bitstream emitted by a binary source is divided into blocks of $\log_2(N_h) + 2\log_2(M)$ bits each, with $\log_2(N_h)$ and $2\log_2(M)$ being the number of bits needed to identify a point (*i.e.*, a pair of antennas) in the antenna–array and the pair of symbols used by the Alamouti code in the signal–constellation diagram, respectively. Each block is split into two sub–blocks of $\log_2(N_h)$ and $2\log_2(M)$ bits. The bits in the first sub–block are used to select the $N_a = 2$ antennas that are simultaneously switched on for data transmission, while all the other antennas are kept silent in the current signaling time interval. The bits in the second sub–block are used to choose a pair of symbols in the signal–constellation diagram, which are modulated by using the Alamouti code and transmitted over two consecutive and orthogonal time slots. The receiver solves a hypothesis detection problem to jointly estimate the antennas at the transmitter that are not idle, and the two signal waveforms that are transmitted from it, which results in the estimation of the unique sequence of $\log_2(N_h) + 2\log_2(M)$ bits emitted by the encoder.

Accordingly, let (h_x, h_y) for $x \ne y = 1, 2, \ldots, N_h$ and (s_p, s_q) for $p, q = 1, 2, \ldots, M$ be the actual transmitted points of the spatial– and signal–constellation diagrams, respectively, the signals received in the first, $r_1(\cdot)$, and second, $r_2(\cdot)$, time slots are as follows, respectively ($t \in [0, T_m)$) [18]:

$$\begin{cases} r_1(t) = h_x s_p w_x(t) + h_y s_q w_y(t) + \eta_1(t) \\ r_2(t) = h_x\left(-s_q^*\right) w_x(t) + h_y s_p^* w_y(t) + \eta_2(t) \end{cases} \tag{1}$$

where $\eta_1(\cdot)$ and $\eta_2(\cdot)$ are the AWGNs in the first and second time slot, respectively.

In (1), we can recognize the structure of the Alamouti code [18]. The important difference with the Alamouti scheme is that in SM–STBC the pair of channels (h_x, h_y) is not known *a priori*, but need to be estimated among the sets of points in S_{N_h}. We will see in Section III that this might destroy the inherent orthogonality of the Alamouti code. Furthermore, let us emphasize that, for the sake of generality, we have assumed that each transmit–antenna has a different shaping filter[3] $\{w_t(\cdot)\}_{t=1}^{N_t}$. The analysis of system setups with different shaping filter is motivated by [11], [13], [14], where it has been shown that orthogonal filters can be used to improve the transmit–diversity gain of SSK modulation. Since, as explained in Section II-A, SSK modulation is embedded in SM, we will show in Section III that the shaping filters play an important role in the design of our new modulation scheme.

C. ML Receiver Design

At the destination, the receiver needs to estimate the pair of spatial– and signal–constellation points (h_x, h_y) and (s_p, s_q), respectively. By assuming a ML–optimum detector, the receiver decides in favor of the pair of points $(h_{\hat{x}}, h_{\hat{y}})$ and $(s_{\hat{p}}, s_{\hat{q}})$ that minimize the decision metric [4], [7], [19]:

$$\begin{aligned} D_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})} &= \int_{T_m} \left| r_1(t) - \left[h_{\hat{x}} s_{\hat{p}} w_{\hat{x}}(t) + h_{\hat{y}} s_{\hat{q}} w_{\hat{y}}(t) \right] \right|^2 dt \\ &+ \int_{T_m} \left| r_2(t) - \left[-h_{\hat{x}} s_{\hat{q}}^* w_{\hat{x}}(t) + h_{\hat{y}} s_{\hat{p}}^* w_{\hat{y}}(t) \right] \right|^2 dt \end{aligned} \tag{2}$$

where the $\hat{}$ symbol is used to denote the generic transmitted signal under test at the destination.

The performance of this receiver can be analyzed in terms of Pairwise Error Probability (PEP), which is the probability that having transmitted (h_x, h_y) and (s_p, s_q) the detector decides in favor of $(h_{\hat{x}}, h_{\hat{y}})$ and $(s_{\hat{p}}, s_{\hat{q}})$ with $(h_{\hat{x}}, h_{\hat{y}}) \ne (h_x, h_y)$ *or* $(s_{\hat{p}}, s_{\hat{q}}) \ne (s_p, s_q)$. In formulas, we have:

$$\begin{aligned} \mathrm{PEP} &= \mathrm{PEP}\left((h_x, h_y; s_p, s_q) \to (h_{\hat{x}}, h_{\hat{y}}; s_{\hat{p}}, s_{\hat{q}}) \right) \\ &= \Pr\left\{ D_{(h_x,h_y),(s_p,s_q)} > D_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})} \right\} \end{aligned} \tag{3}$$

The complexity of this receiver depends on how the spatial–constellation diagram is chosen. This is studied in Section III.

III. Towards the Design of Efficient SM–STBCs

To understand the main objectives of this section, let us summarize the two distinguishable features of the Alamouti code, and STBCs in general. The structure of these codes allow them to [18], [19]: i) achieve a transmit–diversity gain equal to the number of active antennas at the transmitter (*e.g.*, $\mathrm{Div_{tx}} = 2$ for the Alamouti code), and ii) use a detector whose complexity increases linearly with the size, M, of the signal–constellation diagram. In this section, we seek to design SM–STBCs that simultaneously achieve these two properties. We prove that there are fundamental limitations when adding the SSK component on top of conventional modulation schemes. Similar to [19], we use PEP analysis in our analytical derivation. Two scenarios are studied: i) a simple system setup where all the antennas at the transmitter use the same shaping filter, *i.e.*, $w_0(t) = w_t(t)$ for $t = 1, 2, \ldots, N_t$, and ii) a setup where the shaping filters can be different.

A. Same Shaping Filters

When $w_0(t) = w_t(t)$ for $t = 1, 2, \ldots, N_t$, (2) reduces to:

$$\begin{aligned} D_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})} &= \int_{T_m} \left| r_1(t) - \left[h_{\hat{x}} s_{\hat{p}} w_0(t) + h_{\hat{y}} s_{\hat{q}} w_0(t) \right] \right|^2 dt \\ &+ \int_{T_m} \left| r_2(t) - \left[-h_{\hat{x}} s_{\hat{q}}^* w_0(t) + h_{\hat{y}} s_{\hat{p}}^* w_0(t) \right] \right|^2 dt \end{aligned} \tag{4}$$

with $r_1(t) = h_x s_p w_0(t) + h_y s_q w_0(t) + \eta_1(t)$ and $r_2(t) = -h_x s_q^* w_0(t) + h_y s_p^* w_0(t) + \eta_2(t)$.

1) Analysis of Receiver Complexity: Let us first analyze if a single–stream detector can be used in this case. By substituting $r_1(\cdot)$ and $r_2(\cdot)$ into (4), and after some algebra, we get:

$$D_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})} = D_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})}^{(s_{\hat{p}})} + D_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})}^{(s_{\hat{q}})} \tag{5}$$

where:

$$\begin{aligned} D_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})}^{(s_{\hat{p}})} &= \left|h_{\hat{x}}\right|^2 \left|s_{\hat{p}}\right|^2 + \left|h_{\hat{y}}\right|^2 \left|s_{\hat{p}}\right|^2 - 2\mathrm{Re}\left\{ h_x^* h_{\hat{x}} s_p^* s_{\hat{p}} \right\} \\ &- 2\mathrm{Re}\left\{ h_y^* h_{\hat{y}} s_p s_{\hat{p}}^* \right\} - 2\mathrm{Re}\left\{ h_y^* h_{\hat{x}} s_q^* s_{\hat{p}} \right\} + 2\mathrm{Re}\left\{ h_x^* h_{\hat{y}} s_q s_{\hat{p}}^* \right\} \\ &- 2\mathrm{Re}\left\{ \bar{\eta}_1^* h_x s_{\hat{p}} \right\} - 2\mathrm{Re}\left\{ \bar{\eta}_2^* h_y s_{\hat{p}}^* \right\} \end{aligned} \tag{6}$$

$$\begin{aligned} D_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})}^{(s_{\hat{q}})} &= \left|h_{\hat{x}}\right|^2 \left|s_{\hat{q}}\right|^2 + \left|h_{\hat{y}}\right|^2 \left|s_{\hat{q}}\right|^2 - 2\mathrm{Re}\left\{ h_y^* h_{\hat{y}} s_q^* s_{\hat{q}} \right\} \\ &- 2\mathrm{Re}\left\{ h_x^* h_{\hat{x}} s_q s_{\hat{q}}^* \right\} - 2\mathrm{Re}\left\{ h_x^* h_{\hat{y}} s_p^* s_{\hat{q}} \right\} + 2\mathrm{Re}\left\{ h_y^* h_{\hat{x}} s_p s_{\hat{q}}^* \right\} \\ &- 2\mathrm{Re}\left\{ \bar{\eta}_1^* h_y s_{\hat{q}} \right\} + 2\mathrm{Re}\left\{ \bar{\eta}_2^* h_x s_{\hat{q}}^* \right\} \end{aligned} \tag{7}$$

[2] In this paper, N_h is assumed to be a power of two.

[3] To avoid any misunderstandings with our notation, we note that $w_x(t) = w_y(t)$ does *not* imply $x = y$, but $x = y$ does imply $w_x(t) = w_y(t)$.

with $\bar{\eta}_1 = \int_{T_m} \eta_1(t) w_0(t) dt$, $\bar{\eta}_2 = \int_{T_m} \eta_2(t) w_0(t) dt$.

From (5)–(7), we observe that, unlike the Alamouti code, $D^{(s_{\hat{p}})}_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})}$ and $D^{(s_{\hat{q}})}_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})}$ depend on *both* s_p and s_q. Thus, the multi–stream ML–optimum detector in (4) cannot be decomposed into two single–stream detectors without loss in performance. Only if $h_{\hat{x}} = h_x$ and $h_{\hat{y}} = h_y$ this is possible. In fact, this scenario reduces to the pure Alamouti code where $-2\mathrm{Re}\left\{h_y^* h_{\hat{x}} s_q^* s_{\hat{p}}\right\} + 2\mathrm{Re}\left\{h_x^* h_{\hat{y}} s_q s_{\hat{p}}^*\right\} = 0$ in (6) and $-2\mathrm{Re}\left\{h_x^* h_{\hat{y}} s_p^* s_{\hat{q}}\right\} + 2\mathrm{Re}\left\{h_y^* h_{\hat{x}} s_p s_{\hat{q}}^*\right\} = 0$ in (7).

In conclusion, the first result of this paper is as follows: whatever the spatial–constellation diagram is, if the shaping filters at the transmitter are all the same, adding the SSK component on top of the Alamouti code destroys its inherent orthogonally. So, no single–stream decoder can be used and the receiver complexity is of the order of $N_h M^{N_a}$ correlations.

2) Analysis of Transmit–Diversity: Although the receiver in (4) loses an important property of STBCs, it is instructive to analyze its transmit–diversity gain to understand the complexity/performance trade–off with respect to the SM scheme.

Transmit–diversity can be studied by computing the PEP in (3). In particular, by substituting (4) in (3), and after lengthly analytical computations, the following result can be obtained:

$$\mathrm{PEP} = Q\left(\sqrt{\bar{\gamma}\left[|H_1|^2 + |H_2|^2\right]}\right) \tag{8}$$

where $H_1 = h_x s_p + h_y s_q - h_{\hat{x}} s_{\hat{p}} - h_{\hat{y}} s_{\hat{q}}$ and $H_2 = -h_x s_q^* + h_y s_p^* + h_{\hat{x}} s_{\hat{q}}^* - h_{\hat{y}} s_{\hat{p}}^*$.

By carefully looking at (8), we can readily conclude that the system under analysis provides a transmit–diversity equal to two if $\rho_{12} = \mathrm{E}\{H_1 H_2^*\} = 0$, *i.e.*, if the generalized fading gains H_1 and H_2 are uncorrelated [20]. This conclusion follows from the fact that, by conditioning upon the transmitted signal–constellation point, H_1 and H_2 are complex Gaussian random variables, for which uncorrelatedness implies independence. By taking into account that we assume i.i.d. fading with unit–power, and that, by definition, $x \neq y$ and $\hat{x} \neq \hat{y}$ (and, thus, $\mathrm{E}\{h_x h_y^*\} = \mathrm{E}\{h_{\hat{x}} h_{\hat{y}}^*\} = 0$), we have:

$$\begin{aligned}\rho_{12} &= s_p s_{\hat{q}} \mathrm{E}\{h_x h_{\hat{x}}^*\} - s_p s_{\hat{p}} \mathrm{E}\{h_x h_{\hat{y}}^*\} + s_q s_{\hat{q}} \mathrm{E}\{h_y h_{\hat{x}}^*\} \\ &- s_{\hat{p}} s_q \mathrm{E}\{h_y h_{\hat{y}}^*\} + s_{\hat{p}} s_q \mathrm{E}\{h_{\hat{x}} h_x^*\} - s_{\hat{p}} s_p \mathrm{E}\{h_{\hat{x}} h_y^*\} \\ &+ s_{\hat{q}} s_q \mathrm{E}\{h_{\hat{y}} h_x^*\} - s_{\hat{q}} s_p \mathrm{E}\{h_{\hat{y}} h_y^*\}\end{aligned} \tag{9}$$

Let us analyze ρ_{12} in (9) when $h_{\hat{x}} = h_x$ and $h_{\hat{y}} = h_y$, which corresponds to the computation of the PEP of the conventional (M–PSK) modulation scheme embedded into SM. In this case, it is easy to see that $\rho_{12} = 0$. Thus, adding SM on top of the Alamouti code does not reduce the transmit–diversity. However, in order to make sure that the transmit–diversity of the system is equal to two we need to prove that *all* the PEPs have transmit–diversity equal to two.

Let us study two possibilities for the spatial–constellation diagram: i) the constellation points belong to non–overlapping sets, *i.e.*, the pairs (h_x, h_y) and $(h_{\hat{x}}, h_{\hat{y}})$ have no elements in common, and ii) the constellation points belong to overlapping sets, *i.e.*, the pairs (h_x, h_y) and $(h_{\hat{x}}, h_{\hat{y}})$ have one element in common. As an example, consider a system setup with $N_t = 4$ and $N_h = 2$. A system with spatial–constellation diagram having points (h_1, h_2) and (h_3, h_4) belongs to the first case study, while a system having points (h_1, h_2) and (h_1, h_3) belongs to the second case study. From (9), we conclude that: i) if the spatial–constellation diagram is composed by non–overlapping sets of points, then $\rho_{12} = 0$ and, thus, the transmit–diversity of the system is always $\mathrm{Div}_{\mathrm{tx}} = 2$; on the other hand ii) if the spatial–constellation diagram is composed by overlapping sets, then $\rho_{12} \neq 0$, and it is not possible to guarantee that the system has transmit–diversity equal to two.

In conclusion, the second result of this paper is as follows: if the shaping filters at the transmitter are all the same, transmit–diversity equal to two can be guaranteed by partitioning the spatial–constellation diagram into non–overlapping sets of antennas. However, a multi–stream receiver is needed at the destination for optimum ML–decoding.

B. Different Shaping Filters

In Section III-A, we have shown that, when the same shaping filters are used at the transmitter, receiver complexity is the price to be paid to get $\mathrm{Div}_{\mathrm{tx}} = 2$. In this section, we prove that the adoption of different shaping filters allows us to achieve the desired transmit–diversity gain without the need of a multi–stream decoder. The properties that the filters must satisfy to this end are derived by resorting to PEP analysis.

In this case, the decision metric in (2) with $r_1(\cdot)$ and $r_2(\cdot)$ in (1) can be still re–written as shown in (5), but $D^{(s_{\hat{p}})}_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})}$ and $D^{(s_{\hat{q}})}_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})}$ are as follows:

$$\begin{aligned}D^{(s_{\hat{p}})}_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})} &= |h_{\hat{x}}|^2 |s_{\hat{p}}|^2 + |h_{\hat{y}}|^2 |s_{\hat{p}}|^2 - 2\mathrm{Re}\left\{h_x^* h_{\hat{x}} s_p^* s_{\hat{p}} \bar{w}_{x,\hat{x}}\right\} \\ &- 2\mathrm{Re}\left\{h_y^* h_{\hat{y}} s_p s_{\hat{p}}^* \bar{w}_{y,\hat{y}}\right\} - 2\mathrm{Re}\left\{h_y^* h_{\hat{x}} s_q^* s_{\hat{p}} \bar{w}_{y,\hat{x}}\right\} \\ &+ 2\mathrm{Re}\left\{h_x^* h_{\hat{y}} s_q s_{\hat{p}}^* \bar{w}_{x,\hat{y}}\right\} - 2\mathrm{Re}\left\{\bar{\eta}_{1,\hat{x}}^* h_x s_{\hat{p}}\right\} - 2\mathrm{Re}\left\{\bar{\eta}_{2,\hat{y}}^* h_y s_{\hat{p}}^*\right\}\end{aligned} \tag{10}$$

$$\begin{aligned}D^{(s_{\hat{q}})}_{(h_{\hat{x}},h_{\hat{y}}),(s_{\hat{p}},s_{\hat{q}})} &= |h_{\hat{x}}|^2 |s_{\hat{q}}|^2 + |h_{\hat{y}}|^2 |s_{\hat{q}}|^2 - 2\mathrm{Re}\left\{h_y^* h_{\hat{y}} s_q^* s_{\hat{q}} \bar{w}_{y,\hat{y}}\right\} \\ &- 2\mathrm{Re}\left\{h_x^* h_{\hat{x}} s_q s_{\hat{q}}^* \bar{w}_{x,\hat{x}}\right\} - 2\mathrm{Re}\left\{h_x^* h_{\hat{y}} s_p^* s_{\hat{q}} \bar{w}_{x,\hat{y}}\right\} \\ &+ 2\mathrm{Re}\left\{h_y^* h_{\hat{x}} s_p s_{\hat{q}}^* \bar{w}_{y,\hat{x}}\right\} - 2\mathrm{Re}\left\{\bar{\eta}_{1,\hat{y}}^* h_y s_{\hat{q}}\right\} + 2\mathrm{Re}\left\{\bar{\eta}_{2,\hat{x}}^* h_x s_{\hat{q}}^*\right\}\end{aligned} \tag{11}$$

where $\bar{w}_{a,b} = \int_{T_m} w_a(t) w_b(t) dt$ and $\bar{\eta}_{\xi,b} = \int_{T_m} \eta_\xi(t) w_b(t) dt$ for $a = \{x, y\}$, $b = \{\hat{x}, \hat{y}\}$, $\xi = \{1, 2\}$.

1) Analysis of Receiver Complexity: From (10) and (11), we conclude that a single–stream receiver can be obtained if the following two conditions are verified simultaneously:

$$\begin{cases} \bar{w}_{x,\hat{y}} = \int_{T_m} w_x(t) w_{\hat{y}}(t) dt = 0 \\ \bar{w}_{y,\hat{x}} = \int_{T_m} w_y(t) w_{\hat{x}}(t) dt = 0 \end{cases} \tag{12}$$

which reduce to designing some shaping filters to be time–orthogonal to some others. In particular, $w_x(\cdot)$ must be orthogonal to $w_{\hat{y}}(\cdot)$, and $w_y(\cdot)$ must be orthogonal to $w_{\hat{x}}(\cdot)$.

By assuming that the orthogonality conditions in (12) are both satisfied, the ML–optimum decoder in (2) can be decomposed into two independent ML–optimum single–stream decoders (10) and (11), which depend only on $(s_p, s_{\hat{p}})$ and $(s_q, s_{\hat{q}})$, respectively. In other words, the low–complexity feature of Alamouti/STBCs is preserved. Each decoder has a computational complexity of the order of $N_h M$ correlations, and, thus, the overall complexity only increases linearly with M. We note that the decoders in (10) and (11) provide an estimate, $(h_{\hat{x}}, h_{\hat{y}})$, of (h_x, h_y), and these estimates might be different. In our receiver implementation, we randomly choose one of them with equal probability.

In conclusion, the third result of this paper is as follows: ML–optimum low–complexity single–stream decoding can be guaranteed via an adequate choice of the precoding shaping *filters at the transmitter. In particular, some pairs of filters (see (12)) should have zero cross–correlation function.*

2) Analysis of Transmit–Diversity: Finally, let us analyze if the low–complexity decoders in (10) and (11) provide the desired transmit–diversity gain. The study is here performed only for detecting symbol s_p, but a similar analytical development can be used to prove the same result for symbol s_q. The PEP is as follows:

$$\begin{aligned} \mathrm{PEP} &= \mathrm{PEP}\left(\left(h_x, h_y; s_p\right) \to \left(h_{\hat{x}}, h_{\hat{y}}; s_{\hat{p}}\right)\right) \\ &= \Pr\left\{ D_{(h_x,h_y)}^{(s_{\hat{p}})}\left(s_p\right) > D_{(h_{\hat{x}},h_{\hat{y}})}^{(s_{\hat{p}})}\left(s_p\right) \right\} \end{aligned} \tag{13}$$

where $D_{(h_{\hat{x}},h_{\hat{y}})}^{(s_{\hat{p}})}\left(s_p\right)$ is (10) by taking into account (12), *i.e.*:

$$\begin{aligned} D_{(h_{\hat{x}},h_{\hat{y}})}^{(s_{\hat{p}})}\left(s_p\right) &= \left|h_{\hat{x}}\right|^2 \left|s_{\hat{p}}\right|^2 + \left|h_{\hat{y}}\right|^2 \left|s_{\hat{p}}\right|^2 - 2\mathrm{Re}\left\{h_x^* h_{\hat{x}} s_p^* s_{\hat{p}} \bar{w}_{x,\hat{x}}\right\} \\ &- 2\mathrm{Re}\left\{h_y^* h_{\hat{y}} s_p s_{\hat{p}}^* \bar{w}_{y,\hat{y}}\right\} - 2\mathrm{Re}\left\{\eta_{1,\hat{x}}^* h_x s_{\hat{p}}\right\} - 2\mathrm{Re}\left\{\eta_{2,\hat{y}}^* h_y s_{\hat{p}}^*\right\} \end{aligned} \tag{14}$$

and $D_{(h_x,h_y)}^{(s_{\hat{p}})}\left(s_p\right)$ can be obtained from (14) by replacing $(\hat{x}, \hat{y}; \hat{p})$ with $(x, y; p)$.

We start by analyzing the transmit–diversity gain when (13) reduces to the computation of the PEP of the conventional modulation scheme embedded into SM. In this case, we have $\hat{x} = x$ and $\hat{y} = y$, and the PEP in (13) is, after some algebra:

$$\mathrm{PEP} = Q\left(\sqrt{\bar{\gamma}\left[\left(\left|h_x\right|^2 + \left|h_y\right|^2\right)\left|s_{\hat{p}} - s_p\right|^2\right]}\right) \tag{15}$$

from which, we conclude, since h_x and h_y are i.i.d., that the proposed system offers transmit–diversity equal to $\mathrm{Div}_{\mathrm{tx}} = 2$. Thus, with the adoption of the filter design in (12), adding SSK modulation on top of the Alamouti code does not introduce any reduction in the transmit–diversity gain.

Let us now study the setup when the PEP in (13) does not reduce to the computation of the probability of error of the conventional modulation scheme. The analysis of all the possible combinations of shaping filters at the transmitter is quite complicated and, due to space constraints, cannot be entirely included in this paper. So, we present here only a simple case study. Among the possible choices of the shaping filters, we consider, inspired by the analysis in Section III-A.2, the setup where the points in the spatial–constellation diagram belong to non–overlapping sets, *i.e.*, the pairs (h_x, h_y) and $(h_{\hat{x}}, h_{\hat{y}})$ have no elements in common. Furthermore, we assume $w_x = w_y$ and $w_{\hat{x}} = w_{\hat{y}}$ with $\bar{w}_{x,\hat{x}} = \int_{T_m} w_x(t)\, w_{\hat{x}}(t)\, dt = 0$. In other words, the Alamouti code emitted by each pair of antennas is shaped by the same filter, and the filters used by the antennas constituting each point in the spatial–constellation diagram are time–orthogonal. With these assumptions, the PEP reduces to:

$$\mathrm{PEP} = Q\left(\sqrt{\bar{\gamma}\left[\left|h_x\right|^2 + \left|h_y\right|^2 + \left|h_{\hat{x}}\right|^2 + \left|h_{\hat{y}}\right|^2\right]}\right) \tag{16}$$

Since all the fading gains in (16) are different and i.i.d., we conclude that (16) has a transmit–diversity gain equal to $\mathrm{Div}_{\mathrm{tx}} = 4$. We emphasize that the transmit–diversity gain of the system proposed in this paper is not four but only two since we have proved in (15) that when the PEP reduces to the analysis of the error performance of the embedded conventional modulation scheme the diversity gain is $\mathrm{Div}_{\mathrm{tx}} = 2$. However, the high diversity gain in (16) is expected to provide an additional and non–negligible coding gain, which can significantly increase the overall performance of the system.

In conclusion, the fourth result of this paper is as follows: ML–optimum low–complexity single–stream decoding with transmit–diversity of two can be guaranteed via an adequate choice of both the precoding shaping filters and the spatial–constellation diagram at the transmitter. In particular, some pairs of filters (see (12)) must have zero cross–correlation function, and the spatial–constellation diagram should be a partition[4] *of the transmit–antenna array.*

IV. Numerical Results

In this section, we show some numerical results to substantiate the claims in the sections above, and to compare the proposed solutions with state–of–the–art transmit–diversity methods. Two case studies are considered: i) a worst–case setup, which achieves $\mathrm{Div}_{\mathrm{tx}} = 1$ and needs a multi–stream decoder at the destination. This is obtained by using the same shaping filters in all the antennas at the transmitter along with a spatial–constellation diagram composed by overlapping sets of points (see Section III-A.2). This system is denoted by SM–STBC; and ii) a best–case setup, which achieves $\mathrm{Div}_{\mathrm{tx}} = 2$ and needs a single–stream decoder at the destination. This is obtained by using different and time–orthogonal shaping filters at the transmitter along with a spatial–constellation diagram composed by non–overlapping sets of points (see Section III-B.2). This system is denoted by Time–Orthogonal–Signal–Design (TOSD–) assisted SM–STBC (TOSD–SM–STBC). Hermite pulse waveforms are used to guarantee the orthogonality in the time domain [14]. The rest of the simulation setup can be found in Section II.

In Fig. 1, a performance comparison of the proposed SM–STBC and TOSD–SM–STBC modulation schemes with SM and the Alamouti code for a rate of 3 and 5 bits/s/Hz is shown. Numerical results confirm the claims in Section III: SM–STBC achieves $\mathrm{Div}_{\mathrm{tx}} = 1$, while TOSD–SM–STBC achieves $\mathrm{Div}_{\mathrm{tx}} = 2$. In particular, TOSD–SM–STBC provides a substantial performance improvement with respect to the Alamouti code with almost the same receiver complexity. The performance improvement is due to the multiplexing gain introduced by using the spatial–constellation diagram on top of the Alamouti code, and to adequately choosing it to maintain the desired transmit–diversity gain. The price to be paid for this performance improvement is the need to use $N_h = 4$ time–orthogonal shaping filters at the transmitter. Also, we notice that even SM–STBC can outperform the Alamouti code for a rate of 5 bits/s/Hz. The price to be paid in this case is the need of a multi–stream decoder at the receiver. Surprisingly, we notice that SM–STBC provides better performance when the size of the spatial–constellation diagram is smaller than the size of the signal–constellation diagram. This is an opposite behavior with respect to conventional SM [6]. The reason is that, in this case, adding the SSK modulation part on top of the Alamouti code seems to destroy most of the advantages of this latter modulation scheme. By reducing the size of the spatial–constellation diagram we minimize this effect.

In Fig. 2, the performance of SM–STBC and TOSD–SM–STBC is compared to the more powerful STBCs introduced in [19]. Let us emphasize here that STBC–H3 and STBC–H4 codes with rate–$3/4$ require three and four simultaneously–transmitting antennas to achieve transmit–diversity three and four, respectively. On the contrary, SM–STBC and TOSD–SM–STBC only require two active antennas at the transmit-

[4] A partition of a set X is a set P of non–empty subsets of X such that every element x in X is in exactly one of these subsets, *i.e.*, the union of the elements of P is equal to X, and the intersection of any two distinct subsets of P is empty.

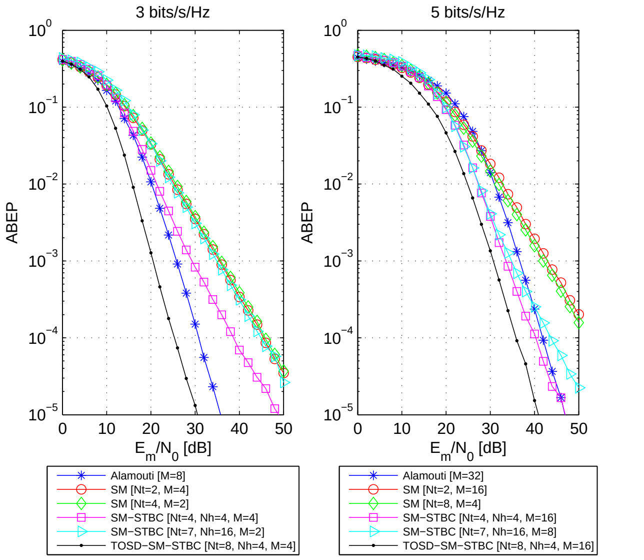

Fig. 1. ABEP against E_m/N_0. Performance comparison with SM [3] and Alamouti [18] schemes for a rate of 3 bits/s/Hz (left) and 5 bits/s/Hz (right).

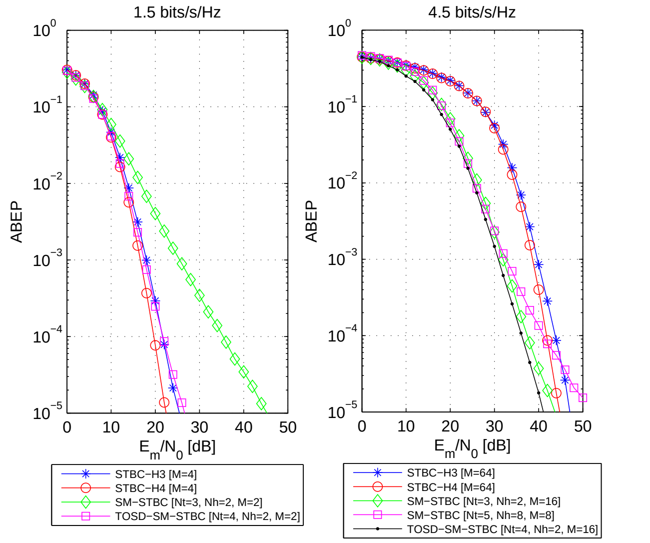

Fig. 2. ABEP against E_m/N_0. Performance comparison with STBC–H3 and STBC–H4 [19] codes for a rate of 1.5 bits/s/Hz (left) and 4.5 bits/s/Hz (right).

ter. We notice that for a low rate SM–STBC incurs in a substantial performance loss with respect to STBC–H3 and STBC–H4 codes: this is due to the impossibility of achieving transmit–diversity. On the contrary, TOSD–SM–STBC offers performance similar to the very powerful STBC–H3 and STBC–H4 codes. The ABEP of TOSD–SM–STBC degrades only for very high SNRs, where the higher diversity gain offered by STBC–H3 and STBC–H4 starts becoming more effective. Furthermore, we notice that the performance gain offered by TOSD–SM–STBC considerably increases when the data rate increases. We observe that the proposed low–complexity and full–transmit–diversity–achieving TOSD–SM–STBC modulation scheme provides much better performance than STBC–H3 and STBC–H4 codes for a very large range of SNRs. In the low–SNR regime, SM–STBC can outperform STBC–H3 and STBC–H4 codes as well, but this performance gain is achieved with a substantial increase in the receiver complexity, as SM–STBC requires a multi–stream detector.

In conclusion, the numerical examples shown in this section confirm the claims in Section III, and, more important, show that the proposed SM–STBC with optimized spatial–constellation diagram and orthogonal filter design can yield better performance than state–of–the–art multiple–antenna technology with comparable receiver complexity.

V. Conclusion

In this paper, we have introduced a new modulation/coding concept, which we have called “Spatially–Modulated STBCs”. The new proposal takes advantage of SM and STBC technologies to design transmit–diversity and high–rate modulation schemes, which exploit the location–specific property of the wireless channel as an additional dimension for data transmission. By using PEP analysis, we have proposed some general methods to design low–complexity SM–STBCs. Furthermore, numerical results have shown that SM–STBCs outperform many state–of–the–art multiple–antenna schemes specifically designed to achieve transmit–diversity.

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