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1. Introduction

Wireless networked systems arise in various communication contexts, and are becoming a bigger and integral part of our everyday life. In today practical networked systems, information delivery is accomplished through routing: network nodes simply store-and-forward data, and processing is accomplished only at the end nodes. Network Coding (NC) is a recent field in electrical engineering and computer science that breaks with this assumption: instead of simply forwarding data, intermediate network nodes may recombine several input packets into one or several output packets (Ahlswede et al., 2000). NC offers the promise of improved performance over conventional network routing techniques. In particular, NC principles can significantly impact the next-generation of wireless *ad hoc*, sensor, and cellular networks, in terms of both energy efficiency and throughput (Ho et al., 2003).
However, besides the many potential advantages and applications of NC over classical routing, the NC principle is not without its drawbacks. A fundamental problem that NC needs to face over lossy (*e.g.*, wireless) networks is the so-called error propagation problem: corrupted packets injected by some intermediate nodes might propagate through the network until the destination, and might render impossible to decode the original information (Cai & Yeung, 2002). As a matter of fact, the application of NC to a wireless context needs to take into account that the wireless medium is highly unpredictable and inhospitable for adopting the existing NC algorithms, which have mostly been designed by assuming wired (*i.e.*, error-free)

networks as the blueprint. Furthermore, in contrast to routing, this problem is crucial in NC due to the algebraic operations performed by the internal nodes of the network: the mixing of packets within the network makes every packet flowing through it statistically dependent on other packets, so that even a single erroneous packet might affect the correct detection of all the other packets (Koetter & Kschischang, 2008). On the contrary, the same error in networks using just routing would affect only a single source-to-destination path. An up-to-date overview of recent results and open problems related to the application of NC to a wireless context can be found in (Di Renzo et al., 2010a), (Di Renzo et al., 2010b).

NC finds successful application in wireless cooperative networks (Pabst, 2004), (Scaglione et al., 2006), since it offers an efficient way for overcoming their limitations in terms of achievable throughput (Katti et al., 2008b). More specifically, in wireless cooperative networks multiple radios are deployed in a neighborhood. The radios connect to each other via wireless links to form a multi-hop wireless network, with a few nodes acting as gateways that connect the wireless network to, *e.g.*, the Internet. Packets traverse multiple wireless links before reaching the gateway and finally the wired network (and, thus, the destination). Multi-hop networks extend the coverage area without expensive wiring, thus offering cheap and moderately fast connectivity, which is often sufficient for accessing the Internet assuming normal browsing habits (Pabst, 2004). However, the price to be paid for this improved robustness in the transmission of data is the reduction of the achievable data rate, which is due to two main reasons: i) the transmission of redundant information to achieve the benefits of spatial diversity, and ii) the practical need to adopt the half-duplex constraint, which precludes the nodes to transmit and receive data simultaneously (Wang et al., 2009). In this context, NC offers an intelligent solution to boost the channel capacity of multi-hop wireless networks by combining information from different packets at the packet (Katti et al., 2008a), (Chachulski et al., 2007), symbol (Katti et al., 2008c), or signal (Katti et al., 2007) level: data mixing allows the system to offset the throughput limitations set by the half-duplex constraint and to reduce the amount of redundant data to be transmitted. For example, the inherent capability offered by NC to recover the throughput of cooperative networking has been clearly assessed for the so-called two-way relay channel (Zhang et al., 2006).

Moving from the considerations above, it is evident that the design of efficient and robust protocols and algorithms to exploit the properties of NC in a cooperative networking scenario will play an important role for the next generation of wireless networks that require high data transmission rates. In particular, to take full advantage of the benefits of NC in a cooperative scenario, the network codes have to be properly designed in order to: i) maximize the probability of correct decoding at the destination nodes in order to minimize the effects of the error propagation problem, ii) reduce the energy consumptions of the overall cooperative network with the aim to prolong its operational life, and iii) keep the complexity of the relay nodes performing NC at a low level. Originally, the design of network codes has mainly been concerned with methods to achieve the maximum information flow (Ahlswede et al., 2000), (Li et al., 2003), (Koetter & Medard, 2003), (Ho et al., 2006). However, in the recent period considerable effort has been devoted to the design of efficient network codes to attain the maximum diversity gain (Xiao & Skoglund, 2009a), (Xiao & Skoglund, 2009b), Rebelatto et al. (2010a), (Rebelatto et al., 2010b), (Topakkaya & Wang, 2010), which is known to determine the Bit Error Probability (BEP) for high Signal-to-Noise-Ratios (SNRs) (Wang & Giannakis, 2003). More specifically, as far as a multi-source multi-relay cooperative scenario is concerned, in (Xiao & Skoglund, 2009a) it has been shown that binary NC is sub-optimal for achieving full-diversity, and in (Rebelatto et al., 2010b) it has been pointed out that max-diversity-achieving network codes can be obtained by resorting to the theory of non-binary linear block codes. For example, as far as the canonical two-source two-relay cooperative network is concerned, the methods proposed in (Xiao & Skoglund, 2009a), (Xiao & Skoglund, 2009b), (Rebelatto et al., 2010a), (Rebelatto et al., 2010b) can achieve full-diversity, when, instead, XOR-based binary NC (Katti et al., 2008a) cannot. The solution proposed by all these papers to overcome the limitation in the achievable diversity is based on using network codes in a non-binary Galois field. However, the price to be paid for this performance improvement is the additional complexity required at the relay nodes, which must network-code the received packets by using non-binary arithmetic. Also, longer decoding delays are, in general, required to design full-diversity-achieving network codes (Rebelatto et al., 2010b). Furthermore, the solutions available in (Xiao & Skoglund, 2009a), (Xiao & Skoglund, 2009b), Rebelatto et al. (2010a), (Rebelatto et al., 2010b) aim at guaranteeing the same diversity gain for all the active sources of the network, which, in general, leads to an inflexible network code design as multiple sources might have different Quality-of-Service (QoS) requirements, and, so, might need different diversity gains. In conclusion, the solutions available so far seem to be still inflexible to accommodate the needs of the multi-source multi-relay scenario, as well as to keep the computational complexity of the relay nodes at a low level.

Motivated by these design challenges, the aim of this book chapter is to propose a new and flexible method to design network codes for cooperative wireless networks with the objectives of: i) improving the diversity gain of conventional relay-only (Scaglione et al., 2006) and XOR-based binary NC (Katti et al., 2008a), ii) keeping at a low level the complexity of the relays, and iii) having the flexibility of assigning to each source a different diversity gain according to the desired QoS. In this book chapter, we show that these design goals can be simultaneously achieved by exploiting the theory of Unequal Error Protection (UEP) linear block codes for the flexible and robust design of network codes for multi-source multi-relay networks (Masnick & Wolf, 1967), (Boyarinov & Katsman, 1981). In particular, by focusing on the canonical two-source two-relay network scenario we prove that, by adopting a simple (4,2,2) UEP code (Van Gils, 1983, Table I) as a network code, at least one source can achieve a better diversity gain than conventional relay-only or XOR-only NC protocols without the need to either use non-binary operations or require additional time-slots. The adoption of UEP coding theory for wireless relay networks (Nguyen et al., 2010) and for random NC with application to multimedia content distribution (Thomos & Frossard, 2004) is receiving a growing interest. However, to the best of the authors knowledge, none of the available works have exploited UEP coding theory for the flexible design of distributed network codes for diversity purposes.

The reminder of this book chapter is organized as follows. In Section 2 and Section 3, system model and network code design are introduced, respectively. In Section 4, a low-complexity detector based on the Maximum-Likelihood (ML) principle for demodulation and network decoding at the destination node is derived. In Section 5, an analytical framework to compute the Average BEP (ABEP) and the diversity gain of various network codes is proposed. In Section 6, numerical results are shown to substantiate our claims and compare the UEP-based network code design with conventional relay and NC methods. Finally, Section 7 concludes the book chapter.

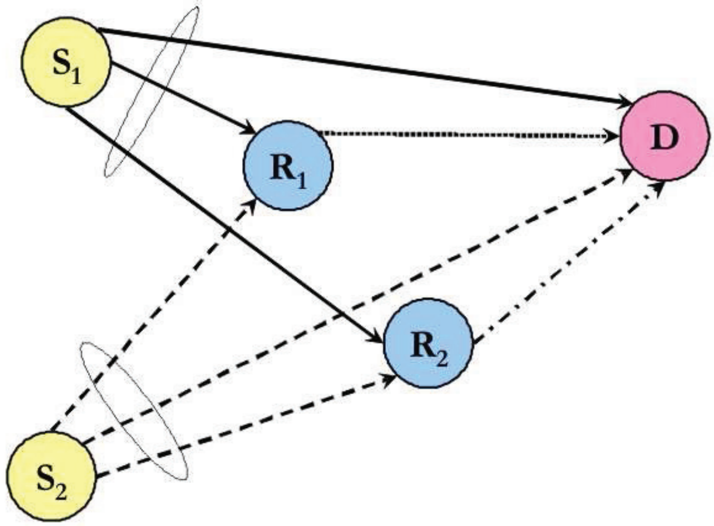

Fig. 1. Two-source two-relay network topology. Different line-styles denote transmission over orthogonal channels (*e.g.*, time-slots (Scaglione et al., 2006)) to avoid mutual interference: S_1 transmits in time-slot 1 (solid lines), S_2 in time-slot 2 (dashed lines), R_1 in time-slot 3 (dotted lines), and R_2 in time-slot 4 (dashed-dotted lines).

2. System model

Let us consider the canonical two-source two-relay cooperative network shown in Figure 1. Generalization to multi-source multi-relay networks is possible, but it is not considered in this book chapter due to space constraints. The working principle of the network in Figure 1 is as follows. In time-slots $t = 1, 2$, source node S_t broadcasts a modulated symbol, x_{S_t}, with average energy E_m. For analytical tractability, Binary Phase Shift Keying (BPSK) modulation is considered, *i.e.*:

$$x_{S_t} = \sqrt{E_m}(1 - 2b_{S_t}) \tag{1}$$

where $b_{S_t} = \{0, 1\}$ is the bit emitted by S_t.

Accordingly, the signals received at relays R_1, R_2, and destination D are:

$$\begin{cases} y_{S_tR_1} = h_{S_tR_1}x_{S_t} + n_{S_tR_1} \\ y_{S_tR_2} = h_{S_tR_2}x_{S_t} + n_{S_tR_2} \\ y_{S_tD} = h_{S_tD}x_{S_t} + n_{S_tD} \end{cases} \tag{2}$$

where h_{XY} is the fading coefficient from node X to node Y, which is a circular symmetric complex Gaussian random variable with zero mean and variance σ^2_{XY} per dimension (*i.e.*, a Rayleigh fading channel model is considered). For analytical tractability, independent and identically distributed (i.i.d.) fading over all the wireless links is considered, *i.e.*, $\sigma^2_0 = \sigma^2_{XY}$ for any X and Y. Furthermore, n_{XY} denotes the complex Additive White Gaussian Noise (AWGN) at the input of node Y and related to the transmission from node X to node Y. The

AWGNs in different time-slots are independent and identically distributed with zero mean and variance $N_0/2$ per dimension.

Upon reception of $y_{S_1R_1}$, $y_{S_1R_2}$, $y_{S_2R_1}$, and $y_{S_2R_2}$, the relays R_1 and R_2 attempt to decode the symbols transmitted by S_1 and S_2 in a similar fashion as in a Decode-and-Forward (DF) cooperative protocol (Scaglione et al., 2006). Unlike other solutions available in the literature for network code design for cooperative networks (Xiao & Skoglund, 2009a) (Xiao & Skoglund, 2009b), (Rebelatto et al., 2010a), (Rebelatto et al., 2010b), we do not rely on powerful (*i.e.*, Shannon-like) channel codes at the physical layer, which allow each relay to detect correct and wrong packets, and enable them to forward only the former ones. We consider a very simple implementation in which the relays demodulate-network-code-and-forward (D-NC-F) each received symbol without checking whether the symbol is correct or wrong. The main aim of this assumption is to keep the complexity of the relays at a very low level (Koetter & Kschischang, 2008), and to understand the robustness, in terms of coding and diversity gain, of the error propagation problem (Di Renzo et al., 2010a), (Di Renzo et al., 2010b) on various network codes.

In particular, we consider ML-optimum demodulation that exploits Channel State Information (CSI) about the source-to-relay channels at each relay node, as follows ($t = 1, 2$):

$$\begin{cases} \hat{b}_{S_tR_1} = \underset{\tilde{b}_{S_t}\epsilon\{0,1\}}{\operatorname{argmin}}\{|y_{S_tR_1} - \sqrt{E_m}h_{S_tR_1}(1-2\tilde{b}_{S_t})|^2\} \\ \hat{b}_{S_tR_2} = \underset{\tilde{b}_{S_t}\epsilon\{0,1\}}{\operatorname{argmin}}\{|y_{S_tR_2} - \sqrt{E_m}h_{S_tR_2}(1-2\tilde{b}_{S_t})|^2\} \end{cases} \tag{3}$$

where ˆ denotes the estimated symbol, and ˜ denotes the trial symbol used in the hypothesis-detection problem.

After demodulating $\hat{b}_{S_tR_q}$ for $t = 1, 2$ and $q = 1, 2$, each relay, R_q, executes the following operations: i) it performs NC on these symbols, ii) it re-modulates the network-coded symbol, and iii) it transmits the modulated symbol to the destination during the third (if $q = 1$) and fourth (if $q = 2$) time-slot. By denoting with $f_{R_q}(\cdot,\cdot)$ the NC operation performed by relay R_q, and with b_{R_q} the network-coded symbol, *i.e.*, $b_{R_q} = f_{R_q}(\hat{b}_{S_1R_q}, \hat{b}_{S_2R_q})$, the signal received at the destination, D, is:

$$y_{R_qD} = h_{R_qD}x_{R_q} + n_{R_qD} \tag{4}$$

where $x_{R_q} = \sqrt{E_m}(1 - 2b_{R_q})$.

After four time-slots, the destination has four received signals, *i.e.*, $y_{S_1D}, y_{S_2D}, y_{R_1D}, y_{R_2D}$, from which it tries to infer the pair of symbols b_{S_1} and b_{S_2} transmitted by S_1 and S_2, respectively. The derivation of the detector used by the destination, D, can be found in Section 4, and its performance analysis in Section 5.

3. UEP–based network code design

In (4), we have implicitly described the network code used by relay R_q with $b_{R_q} = f_{R_q}(\cdot,\cdot)$. In this book chapter, four network codes (or NC scenarios) are investigated:

- *Scenario 1*: $b_{R_1} = \hat{b}_{S_1R_1}$ and $b_{R_2} = \hat{b}_{S_2R_2}$. This network code corresponds to the working scenario in which relays R_1 and R_2 only decode-and-forward the signals received from sources S_1 and S_2, respectively (relay-only scenario) (Scaglione et al., 2006).

- *Scenario 2*: $b_{R_1} = \hat{b}_{S_1R_1} \oplus \hat{b}_{S_2R_1}$ and $b_{R_2} = \hat{b}_{S_1R_2} \oplus \hat{b}_{S_2R_2}$, where $\oplus$ denotes bit-wise XOR operation. This network code corresponds to the working scenario in which relays R_1 and R_2 demodulate-network-code-and-forward the signals received from sources S_1 and S_2, respectively. Furthermore, they use conventional binary NC (XOR-only scenario) (Katti et al., 2008a).
- *Scenario 3*: $b_{R_1} = \hat{b}_{S_1R_1} \oplus \hat{b}_{S_2R_1}$ and $b_{R_2} = \hat{b}_{S_2R_2}$. This scenario corresponds to using a distributed network code obtained from a $(4,2,2)$ UEP code (Van Gils, 1983, Table I), where a higher diversity gain has to be assigned to source S_2.
- *Scenario 4*: $b_{R_1} = \hat{b}_{S_1R_1}$ and $b_{R_2} = \hat{b}_{S_1R_2} \oplus \hat{b}_{S_2R_2}$. This scenario corresponds to using a distributed network code obtained from a $(4,2,2)$ UEP code (Van Gils, 1983, Table I), where a higher diversity gain has to be assigned to source S_1.

Scenario 1 and *Scenario 2* correspond to state-of-the-art distributed coding techniques (Rebelatto et al., 2010b), while *Scenario 3* and *Scenario 4* are the flexible network codes we are interested in studying in this book chapter. The reason why UEP coding theory can be a suitable tool to design distributed network codes for application scenarios in which different sources require a different diversity gain (see also Section 1) has its information-theoretic foundation in (Zhang, 2008). In fact, in (Zhang, 2008) it is shown that the minimum distance of a network code plays the same role as it plays in classical coding theory. Furthermore, from classical coding theory we know that the minimum distance of a linear block code directly determines the diversity gain over fully-interleaved fading channels (Proakis, 2000, Ch. 8), (Simon & Alouini, 2000, Ch. 12). In UEP linear codes, each systematic bit has its own minimum distance, and the set of these distances is known as *separation vector* (Masnick & Wolf, 1967), (Boyarinov & Katsman, 1981). From (Van Gils, 1983, Table I), the network code of *Scenario 3* is a UEP distributed code with separation vector $[2,3]$, which means that the minimum distance for the bits sent by S_1 and S_2 is equal to 2 and 3, respectively. Likewise, the network code of *Scenario 4* is a UEP distributed code with separation vector $[3,2]$, which means that the minimum distance for the bits sent by S_1 and S_2 is equal to 3 and 2, respectively. Thus, from (Zhang, 2008) it follows that by using a network code constructed from UEP coding theory we can individually assign different diversity gains to different sources. Also, note that, unlike (Xiao & Skoglund, 2009a), (Rebelatto et al., 2010b), this is obtained by neither using a non-binary Galois field nor introducing extra delays. The complexity and decoding latency of all the network codes studied in this book chapter are, on the other hand, the same. The downside is that only one source can achieve full-diversity. To the best of the authors knowledge, the adoption of UEP coding theory to design distributed network codes for multi-hop/cooperative networks with noisy and faded source-to-relay channels has never been addressed in the literature. Finally, we note that although, for analytical tractability and space constraints, only the two–source two-relay network topology is here investigated, UEP coding theory can be applied to generic multi-source multi-relay networks, by using, *e.g.*, the codes available in (Van Gils, 1983, Table I).

4. Receiver design

We consider that the destination, D, uses a detector based on a low–complexity implementation of the Maximum Likelihood Sequence Estimation (MLSE) criterion. In particular, according to the MLSE criterion, given y_{S_1D}, y_{S_2D}, y_{R_1D}, and y_{R_2D} introduced in Section 2, the proposed receiver estimates the distributed codeword that has most probably

been transmitted (Proakis, 2000). However, in order to keep the complexity of the detector at a low level, we introduce some simplifications in the analytical development with the aim of reducing the computational complexity and the *a priori* CSI required at the destination for optimal decoding. Of course, this leads to a sub–optimal receiver design, but allows the destination not to estimate the wireless channel over all the wireless links of the network.
Before proceeding with the development of the detector, we need to identify the codebook, *i.e.*, the set of distributed codewords that can be received by the destination in a noise-less and fading-less scenario. The codebook is denoted by $\mathcal{C} = \{\mathbf{c}^{(1)}, \mathbf{c}^{(2)}, \mathbf{c}^{(3)}, \mathbf{c}^{(4)}\}$, where $\mathbf{c}^{(j)}$ is the j-th codeword of $\mathcal{C}$, and $c_i^{(j)}$ is the i-th element of $\mathbf{c}^{(j)}$ for $i = 1, 2, 3, 4$. More specifically, in a noise-less and fading-less scenario we have: $c_1^{(j)} = b_{S_1}$, $c_2^{(j)} = b_{S_2}$, $c_3^{(j)} = b_{R_1}$, and $c_4^{(j)} = b_{R_2}$. As far as the NC scenarios described in Section 3 are concerned, the following codebooks can be obtained:

- $\mathcal{C} = \{0000, 0101, 1010, 1111\}$ for *Scenario 1*
- $\mathcal{C} = \{0000, 0111, 1011, 1100\}$ for *Scenario 2*
- $\mathcal{C} = \{0000, 0111, 1010, 1101\}$ for *Scenario 3*
- $\mathcal{C} = \{0000, 0101, 1011, 1110\}$ for *Scenario 4*

Detection at the destination encompasses two main steps, which involve physical and network layers, respectively.

1. *Step 1 (Physical Layer)*: From the received signals, $y_{S_1D}, y_{S_2D}, y_{R_1D}$, and y_{R_2D}, hard-decision estimates of $[b_{S_1}, b_{S_2}, b_{R_1}, b_{R_2}]$ are provided by using a ML-optimum receiver, which exploits channel information on the source-to-destination and relay-to-destination wireless links ($t = 1, 2$ and $q = 1, 2$):

$$\begin{cases} \hat{b}_{S_tD} = \underset{\tilde{b}_{S_t} \epsilon \{0,1\}}{\operatorname{argmin}} \{|y_{S_tD} - \sqrt{E_m} h_{S_tD}(1 - 2\tilde{b}_{S_t})|^2\} \\ \hat{b}_{R_qD} = \underset{\tilde{b}_{R_q} \epsilon \{0,1\}}{\operatorname{argmin}} \{|y_{R_qD} - \sqrt{E_m} h_{R_qD}(1 - 2\tilde{b}_{R_q})|^2\} \end{cases} \quad (5)$$

2. *Step 2 (Network Layer)*: The hard-decision estimates $\hat{\mathbf{c}} = [\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4] = [\hat{b}_{S_1D}, \hat{b}_{S_2D}, \hat{b}_{R_1D}, \hat{b}_{R_2D}]$ are input to the network layer, which uses a MLSE–optimum criterion to jointly estimate the bits emitted by the sources S_1 and S_2, as follows (Proakis, 2000), (Simon & Alouini, 2000):

$$\left[\hat{b}_{S_1}, \hat{b}_{S_2}\right] = \underset{\tilde{b}_{S_1} \in \{0,1\},\, \tilde{b}_{S_2} \in \{0,1\}}{\arg\max} \left\{ \Pr\left\{ \tilde{b}_{S_1}, \tilde{b}_{S_2} \middle| \hat{b}_{S_1D}, \hat{b}_{S_2D}, \hat{b}_{R_1D}, \hat{b}_{R_2D} \right\} \right\} \quad (6)$$

   where $\Pr\{\cdot\}$ denotes probability.

   Since the network codes studied in Section 3 can be regarded as systematic linear block codes (Proakis, 2000), (6) can be re-written as follows:

$$\left[\hat{b}_{S_1}, \hat{b}_{S_2}\right] = \left[c_1^{(\hat{j})}, c_2^{(\hat{j})}\right] = \underset{\left\{c_1^{(\tilde{j})}, c_2^{(\tilde{j})}\right\} \in \mathcal{C}}{\arg\max} \left\{ \Pr\left\{ c_1^{(\tilde{j})}, c_2^{(\tilde{j})} \middle| \hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4 \right\} \right\} \quad (7)$$

Finally, by applying the Bayes theorem, by taking into account that the symbols emitted by the sources (and, thus, the codewords) are equiprobable, and by noticing that the hard-decisions $\hat{\mathbf{c}} = [\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4] = [\hat{b}_{S_1D}, \hat{b}_{S_2D}, \hat{b}_{R_1D}, \hat{b}_{R_2D}]$ are independently computed, (7) simplifies as follows:

$$\left[\hat{b}_{S_1}, \hat{b}_{S_2}\right] = \left[c_1^{(\hat{j})}, c_2^{(\hat{j})}\right] = \underset{\left\{c_1^{(\tilde{j})}, c_2^{(\tilde{j})}\right\} \in \mathcal{C}}{\arg\max} \left\{ \prod_{i=1}^{4} \left[\Pr\left\{ \hat{c}_i \middle| c_i^{(\tilde{j})} \right\} \right] \right\} \tag{8}$$

where, according to Section 3, we have: $c_3^{(\tilde{j})} = f_{R_1}\left(c_1^{(\tilde{j})}, c_2^{(\tilde{j})}\right)$ and $c_4^{(\tilde{j})} = f_{R_2}\left(c_1^{(\tilde{j})}, c_2^{(\tilde{j})}\right)$.

By carefully looking at (8), we notice that the computation of $\Pr\left\{ \hat{c}_i \middle| c_i^{(\tilde{j})} \right\}$ for $i = 1, 2, 3, 4$ and $\tilde{j} = 1, 2, 3, 4$ requires the knowledge of the channel gains over all the wireless links of the network, including the source-to-relay links to which the destination has not direct access. The availability of this information typically requires some cross-layer interactions between physical and network layers, along with the design of a so-called channel-aware detector (Chamberland & Veeravalli, 2007), (Di Renzo et al., 2009). With the aim to simplify the complexity of the detector, we retain two main assumptions (A) in this book chapter: A1) we consider that the destination has no knowledge of the channels over the source-to-relay links, and A2) we consider that the network layer only knows the fading distribution of the source-to-destination and relay-to-destination wireless links, but it does not know the exact realization of the channel gains.

From A1), it can be readily proved that $\Pr\left\{ \hat{c}_i \middle| c_i^{(\tilde{j})} \right\}$ for $i = 1, 2, 3, 4$ and $\tilde{j} = 1, 2, 3, 4$ follows a Bernoulli distribution (Proakis, 2000) that does not depend on the source-to-relay links, as shown below:

$$\begin{cases} \Pr\left\{ \hat{c}_1 \middle| c_1^{(\tilde{j})} \right\} = \left(1 - P_{S_1D}\right)^{1 - d_1(\tilde{j})} P_{S_1D}^{d_1(\tilde{j})} \\ \Pr\left\{ \hat{c}_2 \middle| c_2^{(\tilde{j})} \right\} = \left(1 - P_{S_2D}\right)^{1 - d_2(\tilde{j})} P_{S_2D}^{d_2(\tilde{j})} \\ \Pr\left\{ \hat{c}_3 \middle| c_3^{(\tilde{j})} \right\} = \left(1 - P_{R_1D}\right)^{1 - d_3(\tilde{j})} P_{R_1D}^{d_3(\tilde{j})} \\ \Pr\left\{ \hat{c}_4 \middle| c_4^{(\tilde{j})} \right\} = \left(1 - P_{R_2D}\right)^{1 - d_4(\tilde{j})} P_{R_2D}^{d_4(\tilde{j})} \end{cases} \tag{9}$$

where: i) $P_{XD} = Q\left(\sqrt{\bar{\gamma} \left|h_{XD}\right|^2}\right)$ with $X = \{S_1, S_2, R_1, R_2\}$ is the error probability over the link from node X to node D, ii) $\bar{\gamma} = 2E_m/N_0$, iii) $Q(x) = \left(1/\sqrt{2\pi}\right) \int_0^{+\infty} \exp(-t^2/2) dt$ is the Gaussian Q-function, and iv) $d_i\left(\tilde{j}\right) = d_H\left(\hat{c}_i, c_i^{(\tilde{j})}\right) = \left|\hat{c}_i - c_i^{(\tilde{j})}\right|$ for $i = 1, 2, 3, 4$, where $d_H(a, b)$ denotes the Hamming distance between two bits a and b (Proakis, 2000). From A2), as the network layer has no access to the actual fading gains but only knows their distribution, we need to replace P_{XD} in (9) with its average value, as follows:

$$\bar{P}_{XD} = \mathrm{E}\{P_{XD}\} = \int_0^{+\infty} Q\left(\sqrt{\bar{\gamma}\xi}\right) g_{|h_{\mathrm{XD}}|^2}(\xi)\, d\xi \tag{10}$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation operator computed over fading channel statistics, and $g_{|h_{\mathrm{XD}}|^2}(\cdot)$ is the Probability Density Function (PDF) of the fading power gain $|h_{\mathrm{XD}}|^2$.

Since, according to Section 2, we consider i.i.d. Rayleigh fading channels, $\bar{P}_{XD}$ can be readily computed in closed–form as follows (Simon & Alouini, 2000) (for $X = \{S_1, S_2, R_1, R_2\}$):

$$\bar{P} = \bar{P}_{XD} = \frac{1}{2}\left(1 - \sqrt{\frac{2\sigma_0^2\bar{\gamma}}{1 + 2\sigma_0^2\bar{\gamma}}}\right) \tag{11}$$

Finally, by substituting (9) and (11) into (8), and computing the logarithm, we obtain, after some algebra, the result as follows:

$$\begin{aligned}\left[\hat{b}_{S_1}, \hat{b}_{S_2}\right] = \left[c_1^{(\hat{j})}, c_2^{(\hat{j})}\right] &= \underset{\left\{c_1^{(\tilde{j})}, c_2^{(\tilde{j})}\right\}\in\mathcal{C}}{\arg\max} \left\{\ln\left(\frac{\bar{P}}{1-\bar{P}}\right)\sum_{i=1}^{4} d_i\left(\tilde{j}\right)\right\} \\ &= \underset{\left\{c_1^{(\tilde{j})}, c_2^{(\tilde{j})}\right\}\in\mathcal{C}}{\arg\max} \left\{\ln\left(\frac{\bar{P}}{1-\bar{P}}\right)\sum_{i=1}^{4} \left|\hat{c}_i - c_i^{(\tilde{j})}\right|\right\}\end{aligned} \tag{12}$$

By taking into account that $\ln\left(\bar{P}/(1-\bar{P})\right)$ is a negative (for $\bar{P} \in [0, 1/2]$) factor that does not effect the outcome of the detector, (12) simplifies as follows:

$$\left[\hat{b}_{S_1}, \hat{b}_{S_2}\right] = \left[c_1^{(\hat{j})}, c_2^{(\hat{j})}\right] = \underset{\left\{c_1^{(\tilde{j})}, c_2^{(\tilde{j})}\right\}\in\mathcal{C}}{\arg\min} \left\{\sum_{i=1}^{4} \left|\hat{c}_i - c_i^{(\tilde{j})}\right|\right\} \tag{13}$$

which turns out to be a (distributed) Minimum Distance Decoder (MDD) receiver for network decoding.

In (13), it is important to note that multiple codewords of a codebook might have the same Hamming distance from the hard-decisions $\hat{\mathbf{c}} = [\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4] = [\hat{b}_{S_1D}, \hat{b}_{S_2D}, \hat{b}_{R_1D}, \hat{b}_{R_2D}]$ provided by the physical layer. In this case, we simply assume that the detector randomly chooses one of them with equal probability.

5. Performance analysis

The objective of this section is to develop an accurate analytical framework to compute the ABEP of the MDD receiver in (13) for the four NC scenarios described in Section 3. To this end, we compute first the BEP conditioned upon fading channel statistics, and then the average over the wireless channel.

5.1 Conditional bit error probability (BEP)

To compute the BEP, we take into account that:

1. For analytical tractability, we use union bound methods that require the estimation of the Pairwise Error Probability (PEP) for each pair of codewords of the codebook (Proakis, 2000).
2. We assume that the codewords of the distributed network code are equiprobable.
3. We separately compute the BEP of sources S_1 and S_2, since, as mentioned in Section 1 and Section 3, we are interested in showing that UEP-based distributed network codes provide different performance for different sources, according to the specified separation vector.

Accordingly, the BEP of source S_t for $t = 1,2$ can be upper-bounded as follows ($t = 1,2$)[1]:

$$\mathrm{BEP}^{(S_t)} = \frac{1}{4}\sum_{j_1=1}^{4}\sum_{j_2 \neq j_1=1}^{4} \mathrm{PEP}^{(S_t)}\left(j_1 \to j_2\right) \tag{14}$$

where $\mathrm{PEP}^{(S_t)}\left(j_1 \to j_2\right)$ is defined as:

$$\mathrm{PEP}^{(S_t)}\left(j_1 \to j_2\right) = \Pr\left\{\mathbf{c}^{(j_1)} \to \mathbf{c}^{(j_2)} \left| c_t^{(j_1)} \neq c_t^{(j_2)} \right.\right\} \tag{15}$$

where $\Pr\left\{\mathbf{c}^{(j_1)} \to \mathbf{c}^{(j_2)} \left| c_t^{(j_1)} \neq c_t^{(j_2)} \right.\right\} = \Pr\left\{\mathbf{c}^{(j_1)} \to \mathbf{c}^{(j_2)}\right\}$ if $c_t^{(j_1)} \neq c_t^{(j_2)}$ and $\Pr\left\{\mathbf{c}^{(j_1)} \to \mathbf{c}^{(j_2)} \left| c_t^{(j_1)} \neq c_t^{(j_2)} \right.\right\} = 0$ if $c_t^{(j_1)} = c_t^{(j_2)}$, and it allows us to compute the BEP of each source individually. As a matter of fact, the detector might be wrong in estimating the transmitted codeword, but this does not necessarily lead to a decoding error in the bits transmitted by *both* sources. As an example, let us consider *Scenario 3* and the transmission of $\mathbf{c}^{(2)} = 0111$. If the receiver (wrongly) decodes $\hat{\mathbf{c}}^{(2)} = \mathbf{c}^{(4)} = 1101$, then this results in an error only for the bit emitted by S_1, while there is no error for S_2.

From (15) and by grouping together common PEPs, it can be shown that the BEP of S_1 and S_2 in (14) simplifies as follows:

$$\begin{cases} \mathrm{BEP}^{(S_1)} = \mathrm{PEP}^{(S_1)}\left(1 \to 3\right) + \mathrm{PEP}^{(S_1)}\left(1 \to 4\right) \\ \mathrm{BEP}^{(S_2)} = \mathrm{PEP}^{(S_2)}\left(1 \to 2\right) + \mathrm{PEP}^{(S_2)}\left(1 \to 4\right) \end{cases} \tag{16}$$

where we conclude that only three PEPs need to be computed to estimate the error performance, as it can be shown that: $\mathrm{PEP}\left(1 \to 2\right) = \mathrm{PEP}^{(S_1)}\left(1 \to 2\right) = \mathrm{PEP}^{(S_2)}\left(1 \to 2\right)$, $\mathrm{PEP}\left(1 \to 3\right) = \mathrm{PEP}^{(S_1)}\left(1 \to 3\right) = \mathrm{PEP}^{(S_2)}\left(1 \to 3\right)$, and $\mathrm{PEP}\left(1 \to 4\right) = \mathrm{PEP}^{(S_1)}\left(1 \to 4\right) = \mathrm{PEP}^{(S_2)}\left(1 \to 4\right)$.

[1] Note that, to simplify the notation, we avoid to emphasize that BEP and PEP are conditioned upon the fading channel. Instead, we use ABEP and APEP to denote the same functions when averaged over fading channel statistics.

The PEPs in (16) can be computed from (13). In particular, by direct inspection of (13), the generic PEP can be explicitly written as follows:

$$\begin{aligned}\mathrm{PEP}^{(S_t)}(j_1 \to j_2) &= \Pr\left\{\mathbf{c}^{(j_1)} \to \mathbf{c}^{(j_2)} \left| c_t^{(j_1)} \neq c_t^{(j_2)} \right.\right\} \\ &= \Pr\left\{D_{j_1} > D_{j_2} \left| c_t^{(j_1)} \neq c_t^{(j_2)} \right.\right\} + \frac{1}{2}\Pr\left\{D_{j_1} = D_{j_2} \left| c_t^{(j_1)} \neq c_t^{(j_2)} \right.\right\}\end{aligned} \quad (17)$$

where we have defined $D_j = \sum_{i=1}^{4} \left|\hat{c}_i - c_i^{(j)}\right|$ for $j = 1, 2, 3, 4$. Note that the second addend in the second line of (17) is due to the closing comment made in Section 4, where we have remarked that the detector randomly chooses with equal probability (*i.e.*, $1/2$) one of the two decision metrics D_{j_1} and D_{j_2} in (17) if they are exactly the same.

Let us now introduce the random variable:

$$D_{j_1,j_2} = D_{j_1} - D_{j_2} = \sum_{i=1}^{4}\left[\left|\hat{c}_i - c_i^{(j_1)}\right| - \left|\hat{c}_i - c_i^{(j_2)}\right|\right] \quad (18)$$

Then, by denoting the probability density function of D_{j_1,j_2} conditioned upon $c_t^{(j_1)} \neq c_t^{(j_2)}$ in (18) by $g_{D_{j_1,j_2}}\left(\cdot \left| c_t^{(j_1)} \neq c_t^{(j_2)} \right.\right)$, the PEP in (17) can be formally re-written as follows:

$$\mathrm{PEP}^{(S_t)}(j_1 \to j_2) = \int_{0^+}^{+\infty} g_{D_{j_1,j_2}}\left(\xi \left| c_t^{(j_1)} \neq c_t^{(j_2)} \right.\right) d\xi + \frac{1}{2}\int_{0^-}^{0^+} g_{D_{j_1,j_2}}\left(\xi \left| c_t^{(j_1)} \neq c_t^{(j_2)} \right.\right) d\xi \quad (19)$$

Closed-form expressions of $\mathrm{PEP}^{(S_t)}(j_1 \to j_2)$ are computed in Section 5.3.

5.2 Average bit error probability (ABEP)

The ABEP can be readily computed from (14) by exploiting the linearity property of the expectation operator. In formulas, we have:

$$\mathrm{ABEP}^{(S_t)} = \mathrm{E}\left\{\mathrm{BEP}^{(S_t)}\right\} = \frac{1}{4}\sum_{j_1=1}^{4}\sum_{j_2 \neq j_1 = 1}^{4} \mathrm{APEP}^{(S_t)}(j_1 \to j_2) \quad (20)$$

where $\mathrm{APEP}^{(S_t)}(j_1 \to j_2) = \mathrm{E}\left\{\mathrm{PEP}^{(S_t)}(j_1 \to j_2)\right\}$.

The APEPs in (20) can be computed by taking the expectation of (19) after computing the integrals. Closed-form expressions of these APEPs are given in Section 5.3.

5.3 Average pairwise error probability (APEP)

The closed-form computation of the APEPs in (20) requires the knowledge of the probability density function $g_{D_{j_1,j_2}}\left(\cdot \left| c_t^{(j_1)} \neq c_t^{(j_2)} \right.\right)$ in (19). In Section 2, we have mentioned that, as opposed to many state-of-the-art research works, our system setup accounts for errors over the source-to-relay links. More specifically, (3) shows that the relays might incorrectly demodulate the bits transmitted by the sources. Even though the MDD receiver in (13) is unaware of these decoding errors, as explained in Section 4, they affect its performance and need to be carefully taken into account for computing the APEPs.

More specifically, in Section 4 we have shown that the relays operate in a D-NC-F mode, which means that they perform two error-prone operations: i) they use the DF protocol for relaying the received symbols, and ii) they combine the symbols received from the sources by using NC. The accurate computation of the APEPs in (20) requires that the error propagation caused by DF and NC operations at the relays are accurately quantified.

5.3.1 DF and NC operations: The effect of realistic source-to-relay channels

As far as DF is concerned, the error propagation of this relay protocol in two-hop relay networks has already been quantified in the literature. In particular, in (Hasna & Alouini, 2003) the following result is available.

Given a two-hop, source-to-relay-to-destination (S-R-D), wireless network, the end-to-end (*i.e.*, at destination D) probability of error, P_{SRD}, is given by:

$$P_{SRD} = P_{SR} + P_{RD} - 2P_{SR}P_{RD} \tag{21}$$

where P_{SR} and P_{RD} are the error probabilities over the source-to-relay and relay-to-destination links, respectively.

By taking into account the analysis in Section 4, it can be readily proved that $P_{\mathrm{SR}} = Q\left(\sqrt{\bar{\gamma}\,|h_{\mathrm{SR}}|^2}\right)$ and $P_{\mathrm{RD}} = Q\left(\sqrt{\bar{\gamma}\,|h_{\mathrm{RD}}|^2}\right)$. The average end-to-end probability of error, $\bar{P}_{SRD}$, can be computed from (10) and (11), and by taking into account that channel fading over the two links is uncorrelated. The final result from (21) is:

$$\bar{P}_{SRD} = \mathrm{E}\left\{P_{SRD}\right\} = \bar{P}_{SR} + \bar{P}_{RD} - 2\bar{P}_{SR}\bar{P}_{RD} = 2\bar{P} - 2\bar{P}^2 \tag{22}$$

Let us now consider the error propagation effect due to NC operations and caused by errors over the source-to-relay channels. In this book chapter, NC, when performed by the relays, only foresees binary XOR operations (see Section 3). Thus, we analyze the error propagation effect in this case only. The result is summarized in Proposition 1.

Proposition 1. *Let b_{S_1} and b_{S_2} be the bits emitted by two sources S_1 and S_2 (see, e.g., (1)). Furthermore, let $\hat{b}_{S_1}$ and $\hat{b}_{S_2}$ be the bits estimated at relay R (see, e.g., (3)) after propagation through the wireless links S_1-to-R and S_2-to-R, respectively. Finally, let $b_R = \hat{b}_{S_1} \oplus \hat{b}_{S_2}$ be the network-coded bit computed by the relay R. Then, the probability, P_R, that the network-coded bit, b_R, is wrong due to fading and noise over the source-to-relay channels is as follows:*

$$P_R = \Pr\left\{\left(\hat{b}_{S_1} \oplus \hat{b}_{S_2}\right) \neq \left(b_{S_1} \oplus b_{S_2}\right)\right\} = P_{S_1R} + P_{S_2R} - 2P_{S_1R}P_{S_2R} \tag{23}$$

where P_{S_1R} and P_{S_2R} are the error probabilities over the S_1-to-R and S_2-to-R wireless links, respectively. Similar to the analysis of the DF relay protocol, it can be readily proved that $P_{\mathrm{S_1R}} = Q\left(\sqrt{\bar{\gamma}\,|h_{\mathrm{S_1R}}|^2}\right)$ and $P_{\mathrm{S_2R}} = Q\left(\sqrt{\bar{\gamma}\,|h_{\mathrm{S_2R}}|^2}\right)$.

Proof: The result in (23) can be proved by analyzing all the error events related to the estimation of $\hat{b}_{S_1}$ and $\hat{b}_{S_2}$ at relay R. In particular, four events have to be analyzed: (a) no decoding errors over the S_1-to-R and S_2-to-R links, *i.e.*, $\hat{b}_{S_1} = b_{S_1}$ and $\hat{b}_{S_2} = b_{S_2}$; (b) decoding

(a) No decoding errors

b_{S_1}	b_{S_2}	$b_{S_1} \oplus b_{S_2}$	$\hat{b}_{S_1}$	$\hat{b}_{S_2}$	b_R
0	0	0	0	0	0
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	1	1	0

(b) Decoding errors over the $S_1 - R$ link

b_{S_1}	b_{S_2}	$b_{S_1} \oplus b_{S_2}$	$\hat{b}_{S_1}$	$\hat{b}_{S_2}$	b_R
0	0	0	1	0	1
0	1	1	1	1	0
1	0	1	0	0	0
1	1	0	0	1	1

(c) Decoding errors over the $S_2 - R$ link

b_{S_1}	b_{S_2}	$b_{S_1} \oplus b_{S_2}$	$\hat{b}_{S_1}$	$\hat{b}_{S_2}$	b_R
0	0	0	0	1	1
0	1	1	0	0	0
1	0	1	1	1	0
1	1	0	1	0	1

(d) Decoding errors over both links

b_{S_1}	b_{S_2}	$b_{S_1} \oplus b_{S_2}$	$\hat{b}_{S_1}$	$\hat{b}_{S_2}$	b_R
0	0	0	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

Table 1. Error propagation effect due to NC at the relays for realistic source-to-relay channels.

errors only over the S_1-to-R link, *i.e.*, $\hat{b}_{S_1} \neq b_{S_1}$ and $\hat{b}_{S_2} = b_{S_2}$; (c) decoding errors only over the S_2-to-R link, *i.e.*, $\hat{b}_{S_1} = b_{S_1}$ and $\hat{b}_{S_2} \neq b_{S_2}$; and (d) decoding error over both S_1-to-R and S_2-to-R links, *i.e.*, $\hat{b}_{S_1} \neq b_{S_1}$ and $\hat{b}_{S_2} \neq b_{S_2}$. These events are summarized in Table 1. In particular, we notice that errors occur if and only if there is a decoding error over a single wireless link. On the other hand, if errors occur in both links they cancel out and there is no error in the network-coded bit. Accordingly, P_R can be formally written as follows:

$$\begin{aligned} P_R &= \Pr\left\{\hat{b}_{S_1} \neq b_{S_1}\right\} + \Pr\left\{\hat{b}_{S_2} \neq b_{S_2}\right\} - 2\Pr\left\{\hat{b}_{S_1} \neq b_{S_1} \text{ and } \hat{b}_{S_2} \neq b_{S_2}\right\} \\ &= \Pr\left\{\hat{b}_{S_1} \neq b_{S_1}\right\} + \Pr\left\{\hat{b}_{S_2} \neq b_{S_2}\right\} - 2\Pr\left\{\hat{b}_{S_1} \neq b_{S_1}\right\} \Pr\left\{\hat{b}_{S_2} \neq b_{S_2}\right\} \end{aligned} \tag{24}$$

which leads to the final result in (23). This concludes the proof of Proposition 1. □

Finally, we note that, from (23), the average probability of error at the relay with NC, $\bar{P}_R$, can be computed from (10) and (11), and by taking into account that the fading over two links is uncorrelated. The final result from (23) is:

$$\bar{P}_R = \mathrm{E}\{P_R\} = \bar{P}_{S_1R} + \bar{P}_{S_2R} - 2\bar{P}_{S_1R}\bar{P}_{S_2R} = 2\bar{P} - 2\bar{P}^2 \tag{25}$$

Very interestingly, by comparing (22) and (25) we notice that DF and NC produce the same error propagation effect. Thus, by combining them, as the network codes in Section 3 foresee, we can expect an error concatenation problem. In particular, by combining the results in (22) and (25), the end-to-end error probability of the bits emitted by sources S_1 and S_2 and received by destination D (denoted by $P_{S_1(R_1R_2)D}$ and $P_{S_2(R_1R_2)D}$, respectively) can be computed as shown in (26)-(29) for *Scenario 1*, *Scenario 2*, *Scenario 3*, and *Scenario 4*, respectively:

$$\begin{cases} P_{S_1(R_1R_2)D} = P_{S_1R_1} + P_{R_1D} - 2P_{S_1R_1}P_{R_1D} \\ P_{S_2(R_1R_2)D} = P_{S_1R_2} + P_{R_2D} - 2P_{S_1R_2}P_{R_2D} \end{cases} \tag{26}$$

$$\begin{cases} P_{S_1(R_1R_2)D} = [P_{S_1R_1} + P_{S_2R_1} - 2P_{S_1R_1}P_{S_2R_1}] + P_{R_1D} - 2[P_{S_1R_1} + P_{S_2R_1} - 2P_{S_1R_1}P_{S_2R_1}]P_{R_1D} \\ P_{S_2(R_1R_2)D} = [P_{S_1R_2} + P_{S_2R_2} - 2P_{S_1R_2}P_{S_2R_2}] + P_{R_2D} - 2[P_{S_1R_2} + P_{S_2R_2} - 2P_{S_1R_2}P_{S_2R_2}]P_{R_2D} \end{cases} \tag{27}$$

$$\begin{cases} P_{S_1(R_1R_2)D} = [P_{S_1R_1} + P_{S_2R_1} - 2P_{S_1R_1}P_{S_2R_1}] + P_{R_1D} - 2[P_{S_1R_1} + P_{S_2R_1} - 2P_{S_1R_1}P_{S_2R_1}]P_{R_1D} \\ P_{S_2(R_1R_2)D} = P_{S_1R_2} + P_{R_2D} - 2P_{S_1R_2}P_{R_2D} \end{cases} \tag{28}$$

$$\begin{cases} P_{S_1(R_1R_2)D} = P_{S_1R_1} + P_{R_1D} - 2P_{S_1R_1}P_{R_1D} \\ P_{S_2(R_1R_2)D} = [P_{S_1R_2} + P_{S_2R_2} - 2P_{S_1R_2}P_{S_2R_2}] + P_{R_2D} - 2[P_{S_1R_2} + P_{S_2R_2} - 2P_{S_1R_2}P_{S_2R_2}]P_{R_2D} \end{cases} \tag{29}$$

The average values of $P_{S_1(R_1R_2)D}$ and $P_{S_2(R_1R_2)D}$, *i.e.*, $\bar{P}_{S_1(R_1R_2)D} = \mathrm{E}\left\{P_{S_1(R_1R_2)D}\right\}$ and $\bar{P}_{S_2(R_1R_2)D} = \mathrm{E}\left\{P_{S_2(R_1R_2)D}\right\}$ can be computed by using arguments similar to (22) and (25). The final result is here omitted due to space constraints and to avoid redundancy.

5.3.2 Closed–form expressions of APEPs

From (16) and (20), it follows that only three APEPs need to be computed, for each NC scenario in Section 3, to estimate the ABEP of both sources. Due to space constraints, we avoid to report the details of the derivation of each APEP for all the NC scenarios. However, since the derivations are very similar, we summarize in Appendix A the detailed computation of a generic APEP. All the other APEPs can be derived by following the same procedure.

In particular, by using the development in Appendix A the following results can be obtained:

Scenario 1:

$$\begin{cases} \mathrm{APEP}\,(1 \to 2) &= \bar{P}_{S_2D}\bar{P}_{S_2(R_1R_2)D} + (1/2)\,(1-\bar{P}_{S_2D})\,\bar{P}_{S_2(R_1R_2)D} \\ &+ (1/2)\left(1-\bar{P}_{S_2(R_1R_2)D}\right)\bar{P}_{S_2D} \\ \mathrm{APEP}\,(1 \to 3) &= \bar{P}_{S_1D}\bar{P}_{S_1(R_1R_2)D} + (1/2)\,(1-\bar{P}_{S_1D})\,\bar{P}_{S_1(R_1R_2)D} \\ &+ (1/2)\left(1-\bar{P}_{S_1(R_1R_2)D}\right)\bar{P}_{S_1D} \\ \mathrm{APEP}\,(1 \to 4) &= (1/2)\,(1-\bar{P}_{S_1D})\,(1-\bar{P}_{S_2D})\,\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ &+ (1/2)\,(1-\bar{P}_{S_1D})\left(1-\bar{P}_{S_1(R_1R_2)D}\right)\bar{P}_{S_2D}\bar{P}_{S_2(R_1R_2)D} \\ &+ (1/2)\,(1-\bar{P}_{S_1D})\left(1-\bar{P}_{S_2(R_1R_2)D}\right)\bar{P}_{S_2D}\bar{P}_{S_1(R_1R_2)D} \\ &+ (1/2)\,(1-\bar{P}_{S_2D})\left(1-\bar{P}_{S_1(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_2(R_1R_2)D} \\ &+ (1/2)\,(1-\bar{P}_{S_2D})\left(1-\bar{P}_{S_2(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_1(R_1R_2)D} \\ &+ (1/2)\left(1-\bar{P}_{S_1(R_1R_2)D}\right)\left(1-\bar{P}_{S_2(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_2D} \\ &+ (1-\bar{P}_{S_1D})\,\bar{P}_{S_2D}\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ &+ (1-\bar{P}_{S_2D})\,\bar{P}_{S_1D}\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ &+ \left(1-\bar{P}_{S_1(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_2D}\bar{P}_{S_2(R_1R_2)D} \\ &+ \left(1-\bar{P}_{S_2(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_2D}\bar{P}_{S_1(R_1R_2)D} \\ &+ \bar{P}_{S_1D}\bar{P}_{S_2D}\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \end{cases} \tag{30}$$

Scenario 2:

$$\begin{cases} \text{APEP}\,(1 \to 2) & = & (1-\bar{P}_{S_2D})\,\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ & + & \left(1-\bar{P}_{S_1(R_1R_2)D}\right)\bar{P}_{S_2D}\bar{P}_{S_2(R_1R_2)D} \\ & + & \left(1-\bar{P}_{S_2(R_1R_2)D}\right)\bar{P}_{S_2D}\bar{P}_{S_1(R_1R_2)D} \\ & + & \bar{P}_{S_2D}\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ \text{APEP}\,(1 \to 3) & = & (1-\bar{P}_{S_1D})\,\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ & + & \left(1-\bar{P}_{S_1(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_2(R_1R_2)D} \\ & + & \left(1-\bar{P}_{S_2(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_1(R_1R_2)D} \\ & + & \bar{P}_{S_1D}\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ \text{APEP}\,(1 \to 4) & = & (1/2)\,(1-\bar{P}_{S_1D})\,\bar{P}_{S_2D} + (1/2)\,(1-\bar{P}_{S_2D})\,\bar{P}_{S_1D} \\ & + & \bar{P}_{S_1D}\bar{P}_{S_2D} \end{cases} \tag{31}$$

Scenario 3:

$$\begin{cases} \text{APEP}\,(1 \to 2) & = & (1-\bar{P}_{S_2D})\,\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ & + & \left(1-\bar{P}_{S_1(R_1R_2)D}\right)\bar{P}_{S_2D}\bar{P}_{S_2(R_1R_2)D} \\ & + & \left(1-\bar{P}_{S_2(R_1R_2)D}\right)\bar{P}_{S_2D}\bar{P}_{S_1(R_1R_2)D} \\ & + & \bar{P}_{S_2D}\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ \text{APEP}\,(1 \to 3) & = & \bar{P}_{S_1D}\bar{P}_{S_1(R_1R_2)D} + (1/2)\,(1-\bar{P}_{S_1D})\,\bar{P}_{S_1(R_1R_2)D} \\ & + & (1/2)\left(1-\bar{P}_{S_1(R_1R_2)D}\right)\bar{P}_{S_1D} \\ \text{APEP}\,(1 \to 4) & = & (1-\bar{P}_{S_1D})\,\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ & + & \left(1-\bar{P}_{S_1(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_2(R_1R_2)D} \\ & + & \left(1-\bar{P}_{S_2(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_1(R_1R_2)D} \\ & + & \bar{P}_{S_1D}\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \end{cases} \tag{32}$$

Scenario 4:

$$\begin{cases} \text{APEP}\,(1 \to 2) &= (1/2)\left(1-\bar{P}_{S_2D}\right)\bar{P}_{S_2(R_1R_2)D} + \bar{P}_{S_2D}\bar{P}_{S_2(R_1R_2)D} \\ &+ (1/2)\left(1-\bar{P}_{S_2(R_1R_2)D}\right)\bar{P}_{S_2D} \\ \text{APEP}\,(1 \to 3) &= \left(1-\bar{P}_{S_1D}\right)\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ &+ \left(1-\bar{P}_{S_1(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_2(R_1R_2)D} \\ &+ \left(1-\bar{P}_{S_2(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_1(R_1R_2)D} \\ &+ \bar{P}_{S_1D}\bar{P}_{S_1(R_1R_2)D}\bar{P}_{S_2(R_1R_2)D} \\ \text{APEP}\,(1 \to 4) &= \left(1-\bar{P}_{S_1D}\right)\bar{P}_{S_2D}\bar{P}_{S_1(R_1R_2)D} \\ &+ \left(1-\bar{P}_{S_2D}\right)\bar{P}_{S_1D}\bar{P}_{S_1(R_1R_2)D} \\ &+ \left(1-\bar{P}_{S_1(R_1R_2)D}\right)\bar{P}_{S_1D}\bar{P}_{S_2D} \\ &+ \bar{P}_{S_1D}\bar{P}_{S_2D}\bar{P}_{S_1(R_1R_2)D} \end{cases} \tag{33}$$

5.4 Diversity analysis

Let us now study the performance (ABEP_∞) of the MDD receiver for high SNRs, which allows us to understand the diversity gain provided by the network codes described in Section 3 (Wang & Giannakis, 2003). To this end, we need to first provide a closed-form expression of the ABEP of S_1 and S_2 from the APEPs computed in Section 5.3.2. By taking into account that the wireless links are i.i.d. and that the average error probability over a single-hop link is given by $\bar{P}$ in (11), from (20), (30)-(33), and some algebra, the ABEPs for *Scenario 1*, *Scenario 2*, *Scenario 3*, and *Scenario 4* are as follows, respectively:

Scenario 1: $\text{ABEP}^{(S_1)} = \text{ABEP}^{(S_2)} = (1/2)\bar{P}_1 + (1/2)\bar{P}_3 + (1/2)\bar{P}_1\bar{P}_2 + (1/2)\bar{P}_1\bar{P}_3 + (1/2)\bar{P}_1\bar{P}_4 + (1/2)\bar{P}_2\bar{P}_3 + (1/2)\bar{P}_3\bar{P}_4 - (1/2)\bar{P}_1\bar{P}_2\bar{P}_3 - (1/2)\bar{P}_1\bar{P}_2\bar{P}_4 - (1/2)\bar{P}_1\bar{P}_3\bar{P}_4 - (1/2)\bar{P}_2\bar{P}_3\bar{P}_4 - (1/2)\bar{P}_1\bar{P}_2\bar{P}_3\bar{P}_4$, where we have defined $\bar{P}_1 = \bar{P}_2 = \bar{P}$ and $\bar{P}_3 = \bar{P}_4 = 2\bar{P} - 2\bar{P}^2$.

Scenario 2: $\text{ABEP}^{(S_1)} = \text{ABEP}^{(S_2)} = (1/2)\bar{P}_1 + (1/2)\bar{P}_2 + \bar{P}_1\bar{P}_3 + \bar{P}_1\bar{P}_4 + \bar{P}_3\bar{P}_4 - \bar{P}_1\bar{P}_2\bar{P}_4 - \bar{P}_1\bar{P}_3\bar{P}_4$, where we have defined $\bar{P}_1 = \bar{P}_2 = \bar{P}$ and $\bar{P}_3 = \bar{P}_4 = 3\bar{P} - 6\bar{P}^2 + 4\bar{P}^3$.

Scenario 3: $\text{ABEP}^{(S_1)} = (1/2)\bar{P}_1 + (1/2)\bar{P}_3 + \bar{P}_1\bar{P}_2 + \bar{P}_1\bar{P}_4 + \bar{P}_2\bar{P}_4 - \bar{P}_1\bar{P}_2\bar{P}_4$ and $\text{ABEP}^{(S_2)} = \bar{P}_1\bar{P}_2 + \bar{P}_1\bar{P}_3 + \bar{P}_1\bar{P}_4 + 2\bar{P}_2\bar{P}_4 + \bar{P}_3\bar{P}_4 - 2\bar{P}_1\bar{P}_2\bar{P}_4 - 2\bar{P}_2\bar{P}_3\bar{P}_4$, where we have defined $\bar{P}_1 = \bar{P}_2 = \bar{P}$, $\bar{P}_3 = 3\bar{P} - 6\bar{P}^2 + 4\bar{P}^3$, and $\bar{P}_4 = 2\bar{P} - 2\bar{P}^2$.

Scenario 4: $\text{ABEP}^{(S_1)} = \bar{P}_1\bar{P}_2 + 2\bar{P}_1\bar{P}_3 + \bar{P}_1\bar{P}_4 + \bar{P}_2\bar{P}_3 + \bar{P}_3\bar{P}_4 - 2\bar{P}_1\bar{P}_2\bar{P}_3 - 2\bar{P}_1\bar{P}_3\bar{P}_4$ and $\text{ABEP}^{(S_2)} = (1/2)\bar{P}_2 + (1/2)\bar{P}_4 + \bar{P}_1\bar{P}_2 + \bar{P}_1\bar{P}_3 + 2\bar{P}_2\bar{P}_3 - \bar{P}_2\bar{P}_4 - 2\bar{P}_1\bar{P}_2\bar{P}_3$, where we have defined $\bar{P}_1 = \bar{P}_2 = \bar{P}$, $\bar{P}_3 = 2\bar{P} - 2\bar{P}^2$, and $\bar{P}_4 = 3\bar{P} - 6\bar{P}^2 + 4\bar{P}^3$.

From the results above, we notice that in *Scenario 1* and *Scenario 2* both sources have the same ABEP. Furthermore, for all the NC scenarios we can easily compute ABEP_∞ and the diversity gain (Div) of S_1 and S_2, as shown in Table 2. In particular, from Table 2 we observe that, by using UEP coding theory for network code design (*i.e.*, *Scenario 3* and *Scenario 4*), at least one source can achieve a diversity gain greater than that obtained by using relay–only or XOR–only network codes (*i.e.*, *Scenario 1* and *Scenario 2*). Furthermore, this performance improvement is obtained by increasing neither the Galois field nor the number of time-slots

	$\text{ABEP}_{\infty}^{(S_1)}$	$\text{ABEP}_{\infty}^{(S_2)}$	Div_{S_1}	Div_{S_2}
Scenario 1	$(3/2)\bar{P}$	$(3/2)\bar{P}$	1	1
Scenario 2	$\bar{P}$	$\bar{P}$	1	1
Scenario 3	$2\bar{P}$	$16\bar{P}^2$	1	2
Scenario 4	$16\bar{P}^2$	$2\bar{P}$	2	1

Table 2. ABEP_{∞} of S_1 and S_2 and diversity gain.

	$\text{ABEP}_{\infty}^{(S_1)}$	$\text{ABEP}_{\infty}^{(S_2)}$	Div_{S_1}	Div_{S_2}
Scenario 1	$\bar{P}$	$\bar{P}$	1	1
Scenario 2	$\bar{P}$	$\bar{P}$	1	1
Scenario 3	$\bar{P}$	$6\bar{P}^2$	1	2
Scenario 4	$6\bar{P}^2$	$\bar{P}$	2	1

Table 3. ABEP_{∞} of S_1 and S_2 and diversity gain with ideal source-to-relay channels.

(Rebelatto et al., 2010b). Finally, by studying the diversity gain provided by the network codes obtained from UEP coding theory in terms of separation vector (SP), we observe that the achievable diversity gain is equal to $\text{Div} = \text{SP} - 1$. From the theory of linear block codes, we know that this is the best achievable diversity for a (4,2,2) UEP-based code that uses a MDD receiver design at the destination (Proakis, 2000), (Simon & Alouini, 2000). Better performance can only be achieved by using a more complicated receiver design, which, *e.g.*, exploits CSI at the network layer.

5.5 Effect of realistic source-to-relay channels

In Section 2, we have mentioned that the relays simply D-NC-F the received bits even though the source-to-relay channels are error-prone, and so the transmission is affected by the error propagation problem. Thus, it is worth being analyzed whether this error propagation effect can decrease the diversity gain achieved by the MDD receiver or whether only a worse coding gain can be expected. To understand this issue, in this section we study the performance of an idealized working scenario in which it is assumed that there are no decoding errors at the relays. In other words, we assume $\hat{b}_{S_t R_q} = b_{S_t}$ for $t = 1, 2$ and $q = 1, 2$ in (3). In this case, the expression of the ABEP for high SNRs can still be computed from (20) and (30)-(33), but by taking into account that $\bar{P} = \bar{P}_{S_1 D} = \bar{P}_{S_2 D} = \bar{P}_{S_1 (R_1 R_2) D} = \bar{P}_{S_2 (R_1 R_2) D}$. The final result of ABEP_{∞} for S_1 and S_2 is summarized in Table 3.
By carefully comparing Table 2 and Table 3, we notice that there is no loss in the diversity gain due to decoding errors at the relay. However, for realistic source-to-relay channels the ABEP is, in general, slightly worse. Interestingly, we notice that *Scenario 2* is the most robust to error propagation, and, asymptotically, there is no performance degradation.

6. Numerical examples

In this section, we show some numerical results to substantiate claims and analytical derivations. A detailed description of the simulation setup can be found in Section 2. In particular, we assume: i) BPSK modulation, ii) $\sigma_0^2 = 1$, and iii) according to Section 5.5, both scenarios with and without errors on the source-to-relay wireless links are studied.

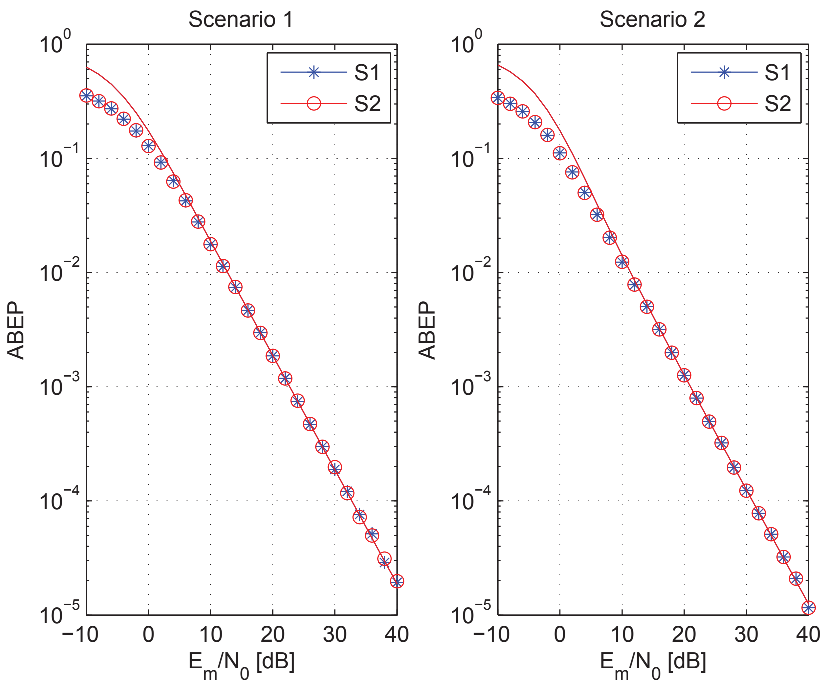

Fig. 2. ABEP against E_m/N_0. Solid lines show the analytical model and markers Monte Carlo simulations ($\sigma_0^2 = 1$).

The results are shown in Figure 2 and Figure 3 for realistic source-to-relay links, and in Figure 4 and Figure 5 for ideal source-to-relay links, respectively. By carefully analyzing these numerical examples, the following conclusions can be drawn: i) our analytical model overlaps with Monte Carlo simulations, thus confirming our findings in terms of achievable performance and diversity analysis; ii) as expected, it can be noticed that the ABEP gets slighltly worse in the presence of errors on the source-to-relay wireless links for *Scenario 1*, *Scenario 3*, and *Scenario 4*, while, as predicted in Table 3, the XOR–only network code (*Scenario 2*) is very robust to error propagation and there is no performance difference between Figure 2 and Figure 4; and iii) the network code design based on UEP coding theory allows the MDD receiver to achieve, for at least one source, a higher diversity gain than conventional relaying and NC methods, and without the need to use either additional time-slots or non-binary operations.

More specifically, the complexity of UEP–based network code design is the same as relay–only and XOR–only cooperative methods. For example, by looking at the results in Figure 3 and Figure 5, we observe that the network code in *Scenario 3* is the best choice when the data sent by S_2 needs to be delivered i) either with the same transmit power but with better QoS or ii) with the same QoS but with less transmit power if compared to S_1. The working principle of the network code in *Scenario 3* has a simple interpretation: if S_2 is the "golden user", then we should dedicate one relay to only forward its data without performing NC on the data of S_1. A similar comment can be made about *Scenario 4* if S_1 is the "golden user". This result

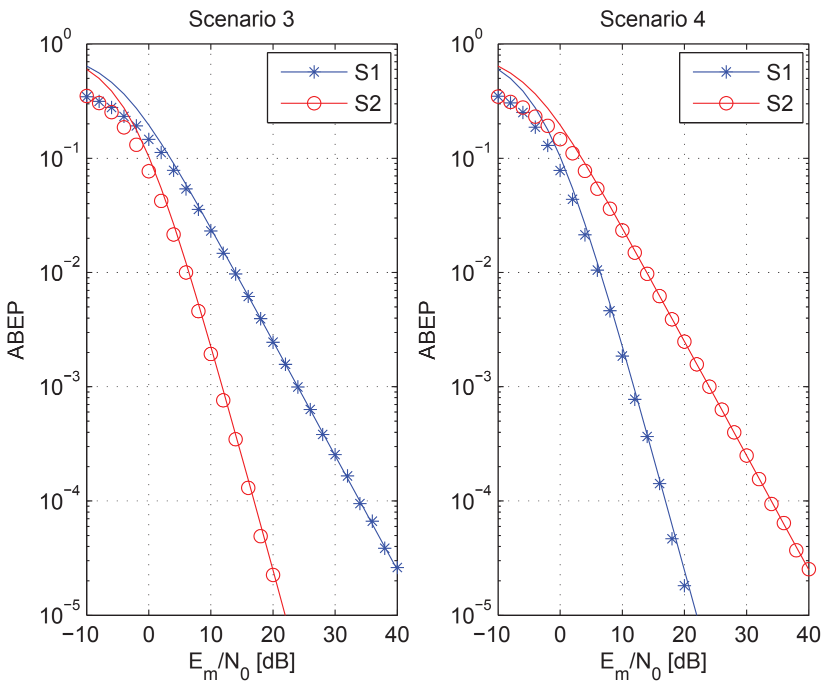

Fig. 3. ABEP against E_m/N_0. Solid lines show the analytical model and markers Monte Carlo simulations ($\sigma_0^2 = 1$).

highlights that, from the network optimization point of view, there might be an optimal choice of the relay nodes that should perform relay-only and NC coding operations. By constraining the relays to perform simple operations (*e.g.*, to work in a binary Galois field), this hybrid solution might provide better performance than scenarios where all the nodes perform NC. However, analysis and numerical results shown in this book chapter have also highlighted some important limitations of the MDD receiver. As a matter of fact, with conventional relaying and NC methods only diversity equal to one can be obtained, while with UEP-based NC at least one user can achieve diversity gain equal to two. However, the network topology studied in Figure 1 would allow each source to achieve a diversity gain equal to three, as three copies of the messages sent by both sources are available at the destination after four time-slots. This limitation is mainly due to the adopted detector, which does not exploit channel knowledge at the network layer and does not account for the error propagation caused by realistic source-to-relay wireless links. The development of more advanced channel-aware receiver designs is our ongoing research activity.

7. Conclusion

In this book chapter, we have proposed UEP coding theory for the flexible design of network codes for multi-source multi-relay cooperative networks. The main advantage of the proposed method with respect to state-of-the-art solutions is the possibility of assigning the diversity

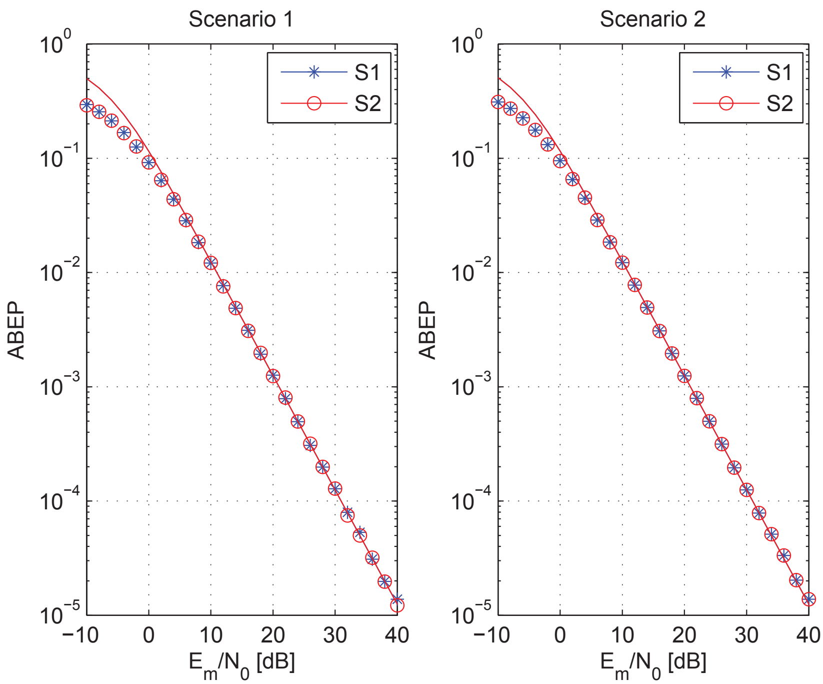

Fig. 4. ABEP against E_m / N_0. Solid lines show the analytical model and markers Monte Carlo simulations ($\sigma_0^2 = 1$). Ideal source-to-relay channels.

gain of each user individually. This offers a great flexibility for the efficient design of network codes for cooperative networks, as energy consumption, performance, number of time-slots required to achieve the desired diversity gain, and complexity at the relay nodes for performing NC can be traded-off by taking into account the specific and actual needs of each source, and without the constraint of over-engineering (*e.g.*, working in a larger Galois field or using more time-slots than actually required) the system according to the needs of the source requesting the highest diversity gain.

Ongoing research is now concerned with the development of more robust receiver schemes at the destination, with the aim of better exploiting the diversity gain provided by the UEP-based network code design.

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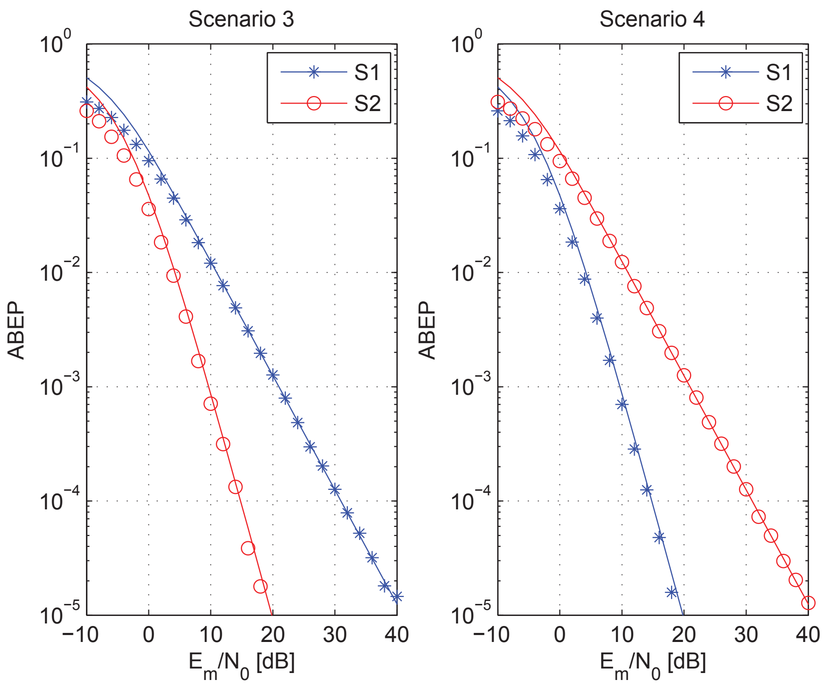

Fig. 5. ABEP against E_m / N_0. Solid lines show the analytical model and markers Monte Carlo simulations ($\sigma_0^2 = 1$). Ideal source-to-relay channels.

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A. Appendix: Proof of (30)–(33)

To understand how (30)-(33) are computed, in this section we provide a step-by-step derivation of the computation of $\text{APEP}\left(1 \to 3\right)$ for source S_1 and *Scenario 1*, *i.e.*, $\text{APEP}^{(S_1)}\left(1 \to 3\right) = \Pr\left\{0000 \to 1010\right\}$. Note that since $c_1^{(1)} = 0 \neq c_1^{(3)} = 1$, we avoid to emphasize, for the sake of simplicity, this conditioning in what follows. Other APEPs, for all the other scenarios, can be obtained with a similar analytical derivation.
From (19), the PEP can be computed as follows:

$$\text{PEP}^{(1)}\left(1 \to 3\right) = \int_{0^+}^{+\infty} g_{D_{1,3}}\left(\xi\right) d\xi + \frac{1}{2}\int_{0^-}^{0^+} g_{D_{1,3}}\left(\xi\right) d\xi \tag{34}$$

where $g_{D_{1,3}}\left(\cdot\right)$ is the probability density function of random variable $D_{1,3}$:

$$D_{1,3} = \sum_{i=1}^{4}\left[\left|\hat{c}_i - c_i^{(1)}\right| - \left|\hat{c}_i - c_i^{(3)}\right|\right] = \sum_{i=1}^{4}\beta_i^{(1,3)} \tag{35}$$

where $\beta_i^{(1,3)} = \left|\hat{c}_i - c_i^{(1)}\right| - \left|\hat{c}_i - c_i^{(3)}\right|$ for $i = 1, 2, 3, 4$.

By direct inspection, it is possible to show that $\beta_i^{(1,3)}$ for $i = 1, 2, 3, 4$ are independent Bernoulli distributed random variables with probability density function as follows:

$$\begin{cases} g_{\beta_1}\left(\xi\right) = \left(1 - P_{S_1 D}\right)\delta\left(\xi + 1\right) + P_{S_1 D}\delta\left(\xi - 1\right) \\ g_{\beta_2}\left(\xi\right) = \delta\left(\xi\right) \\ g_{\beta_3}\left(\xi\right) = \left(1 - P_{S_1 (R_1 R_2) D}\right)\delta\left(\xi + 1\right) + P_{S_1 (R_1 R_2) D}\delta\left(\xi - 1\right) \\ g_{\beta_4}\left(\xi\right) = \delta\left(\xi\right) \end{cases} \tag{36}$$

where $\delta\left(\cdot\right)$ is the Dirac delta function.

It is relevant to notice that $g_{\beta_2}(\xi) = g_{\beta_4}(\xi) = \delta(\xi)$, *i.e.*, $\beta_2 = \beta_4 = 0$ with unit probability, because $c_2^{(1)} = c_2^{(3)}$ and $c_4^{(1)} = c_4^{(3)}$, and, so, regardless of the estimates $\hat{c}_2$ and $\hat{c}_4$ provided by the physical layer, we always have $\left|\hat{c}_2 - c_2^{(1)}\right| - \left|\hat{c}_2 - c_2^{(3)}\right| = 0$ and $\left|\hat{c}_4 - c_4^{(1)}\right| - \left|\hat{c}_4 - c_4^{(3)}\right| = 0$. Since $\beta_i^{(1,3)}$ for $i = 1, 2, 3, 4$ are independent random variables, the probability density function of $D_{1,3}$ in (35) can be computed via the convolution operator:

$$\begin{aligned} g_{D_{1,3}}(\xi) &= \left(g_{\beta_1} \otimes g_{\beta_2} \otimes g_{\beta_3} \otimes g_{\beta_4}\right)(\xi) = \left(g_{\beta_1} \otimes g_{\beta_3}\right)(\xi) \\ &= \left[\left(1 - P_{S_1D}\right) P_{S_1(R_1R_2)D} + \left(1 - P_{S_1(R_1R_2)D}\right) P_{S_1D}\right] \delta(\xi) \\ &+ \left(1 - P_{S_1D}\right)\left(1 - P_{S_1(R_1R_2)D}\right) \delta(\xi + 2) + P_{S_1D} P_{S_1(R_1R_2)D} \delta(\xi - 2) \end{aligned} \tag{37}$$

where $\otimes$ denotes convolution.
Furthermore, by substituting (37) into (34) we can get the final result for the PEP:

$$\begin{aligned} \mathrm{PEP}^{(1)}(1 \to 3) &= (1/2)\left(1 - P_{S_1D}\right) P_{S_1(R_1R_2)D} + (1/2)\left(1 - P_{S_1(R_1R_2)D}\right) P_{S_1D} \\ &+ P_{S_1D} P_{S_1(R_1R_2)D} \end{aligned} \tag{38}$$

Finally, the APEP can be computed by simply taking the expectation of (38) and by considering that fading over all the wireless links is independent distributed:

$$\begin{aligned} \mathrm{APEP}^{(1)}(1 \to 3) &= \mathrm{E}\left\{\mathrm{PEP}^{(1)}(1 \to 3)\right\} \\ &= (1/2)\left(1 - \bar{P}_{S_1D}\right) \bar{P}_{S_1(R_1R_2)D} + (1/2)\left(1 - \bar{P}_{S_1(R_1R_2)D}\right) \bar{P}_{S_1D} \\ &+ \bar{P}_{S_1D} \bar{P}_{S_1(R_1R_2)D} \end{aligned} \tag{39}$$

We observe that (39) coincides with (30), and this concludes our proof.