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# Joint Network/Channel Decoding for Heterogeneous Multi-Source/Multi-Relay Cooperative Networks

(Invited Paper)

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## ABSTRACT

In this paper, we study joint network/channel decoding for multi-source multi-relay heterogeneous wireless networks. When convolutional and network codes are used at the physical and network layers, respectively, we show that error correction and diversity properties of the whole network can be characterized by an equivalent and distributed convolutional network/channel code. In particular, it is shown that, by properly choosing the network code, the equivalent code can show Unequal Error Protection (UEP) properties, which might be useful for heterogeneous wireless networks in which each source might ask for a different quality-of-service requirement or error probability. Using this representation, we show that Maximum-Likelihood (ML) joint network/channel decoding can be performed by using the trellis representation of the distributed convolutional network/channel code. Furthermore, to deal with decoding errors at the relays, a ML-optimum receiver which exploits side information on the source-to-relay links is proposed.

## Categories and Subject Descriptors

C.2.1 [Computed-Communication Networks]: Network Architecture and Design—*Wireless Communication*.

## General Terms

Theory, Algorithms, Performance.

## Keywords

Heterogeneous Wireless Networks, Cooperative Communications, Network Coding, Joint Network/Channel Decoding, Unequal Error Protection Coding Theory.

## 1. INTRODUCTION

Wireless networked systems arise in various communication contexts, and are becoming a bigger and integral part of our

everyday life. In today practical networked systems, information delivery is accomplished through routing: network nodes simply store and forward data, and processing is accomplished only at the end nodes. Network Coding (NC) is a recent field in electrical engineering and computer science that breaks with this assumption: instead of simply forwarding data, intermediate network nodes may recombine several input packets into one or several output packets [1]. NC offers the promise of improved performance over conventional network routing techniques. In particular, NC principles can significantly impact the next-generation wireless *ad hoc*, sensor, and cellular networks, in terms of both energy efficiency and throughput [2], [3].

However, besides the many potential advantages and applications of NC over classical routing, the NC principle is not without limitations. A fundamental problem that we need to carefully consider over wireless networks is the so-called error propagation problem: corrupted packets injected by some intermediate nodes might propagate through the network until the destination, and might render impossible to decode the original information [4], [5]. As a matter of fact, the application of NC to a wireless context needs to take into account that the wireless medium is highly unpredictable and inhospitable for adopting existing NC algorithms, which have been mostly designed by assuming wired (*i.e.*, error-free) networks as the blueprint. Furthermore, in contrast to routing, this problem is crucial in NC due to the algebraic operations performed by the nodes of the network: the mixing of packets within the network makes every packet flowing through it statistically dependent on other packets, so that even a single erroneous packet might affect the correct detection of all the other packets. On the contrary, the same error in networks using just routing would affect only a single source-to-destination path.

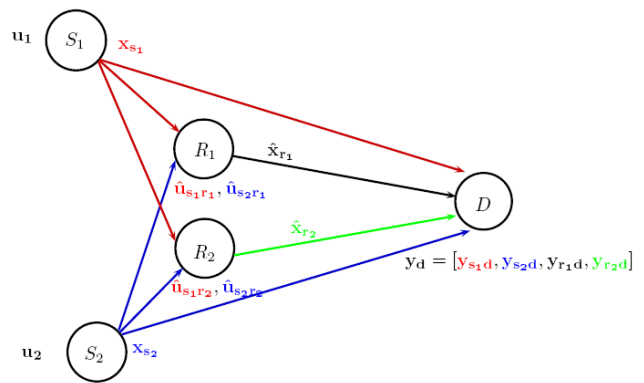
Thus, the fundamental issue to be carefully considered to understand the actual performance improvement and advantage of network-coded multi-hop/cooperative communications is to take into account that all the nodes of the network are error-prone, and that erroneous decoding and forwarding might have a significant impact on the end-to-end performance, diversity, throughput, and quality-of-service. The importance of this problem is increasing exponentially as a result of latest research achievements on the analysis of the performance of cooperative networks with NC. In fact,

recent results have highlighted that the conventional method that is often used to counteract the error propagation problem, *i.e.*, the adoption of a Cyclic Redundancy Code (CRC) check mechanism, which aims at not forwarding corrupted packets as long as being highly spectral inefficient as an entire packet is blocked if just one bit is in error [6], [7].

Among the solutions that are currently being investigated to counteract the error propagation problem [2], Joint Network/Channel Decoding (JNCD) is gaining a growing interest since its inception in [8], [9]. The basic premise of JNCD is the exploitation of the inherent redundancy of network and channel codes, in the same way as Joint Source and Channel Decoding (JSCD) exploits the inherent redundancy of source and channel codes [10]. Early results in [8], [9] have evidenced that a performance improvement can be obtained with joint decoding. Moving from these results, various studies about the performance improvement of JNCD are today available in the literature [11]–[27].

Motivated by these considerations, in this paper we aim at proposing and studying the performance of JNCD applied to heterogeneous wireless networks. In heterogeneous wireless networks, the nodes have different quality-of-service requirements, such as data rate, power consumption, reliability, and error performance. In this context, it is very important to design the network code to guaranteeing to each node of the network the requested performance, while keeping at a low complexity the operations performed at the relays and minimizing the resources (*e.g.*, time slots, frequencies) needed to deliver the data to the final destination. In [28], it has recently been shown that Unequal Error Protection (UEP) coding theory can be a viable candidate for network code design in such networks. In particular, UEP-based NC is especially useful for multi-source multi-relay cooperative networks where each source requires a different error probability. By exploiting the concept of separation vector, distributed network codes can be constructed such that the bits transmitted by each source have a different level of protection to decoding errors, which in turn provides a different minimum distance, and, thus, for independent fading channels, a different diversity gain. In this context, the error probability requirement can be mapped onto a diversity gain requirement, which provides the separation vector upon which the equivalent network code can be properly designed. In [28], it has been shown that, by exploiting a proper receiver design, the technique is robust to error-prone wireless links on the source-to-relay channels.

However, the analysis in [28] is performed by assuming an uncoded communication system, *i.e.*, no channel code is used. Thus, the aim of this paper is to extend design and analysis in [28] by including channel coding, and developing the optimal JNCD scheme for UEP-based NC. More specifically, we show that, when convolutional and network codes are used at the physical and network layers, respectively, error correction and diversity properties of the whole network can be characterized by an equivalent and distributed convolutional network/channel code. Also, it is pointed out that, by properly choosing the network code, the equivalent code can show UEP properties. Finally, we develop the Maximum-Likelihood (ML-) optimum decoder, which



**Figure 1: Two-source two-relay cooperative network.**

accounts for possible decoding errors at the relays, by exploiting side information on the source-to-relay links.

The remainder of this paper is organized as follows. In Section 2, the system model and the transmission protocol are presented. In Section 3, we show how the distributed convolutional network/channel code can be obtained. Based on this interpretation, the ML-optimum decoder is proposed. In Section 4, some simulation results are presented. Finally, Section 5 concludes the paper.

## 2. SYSTEM MODEL

In this section, we describe the transmission protocol and the notation used throughout the paper. For ease of presentation, we focus our attention on the two-source two-relay cooperative network shown in Figure 1. However, we emphasize that all the solutions can be extended to multi-source multi-relay networks.

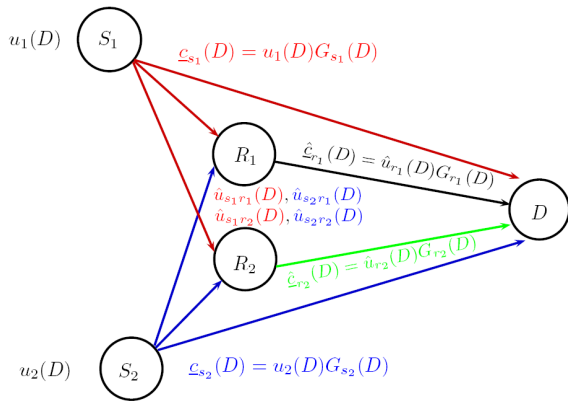
The transmission protocol is composed by two phases: i) a broadcasting phase, during which each source broadcasts its message to destination and relays; and ii) a relaying phase, during which the relays forward their messages to the destination after performing demodulation and NC on the received messages.

### 2.1 Broadcasting Phase

Each source node  $S_i$ ,  $i = \{1, 2\}$ , encodes its information message  $\mathbf{u}_i = \mathbf{u}_{s_i} = [u_{s_i}(1), \dots, u_{s_i}(K_i)]$  with  $K_i$  information bits into a codeword  $\mathbf{c}_{s_i} = [c_{s_i}(1), \dots, c_{s_i}(N_{s_i})]$  of length  $N_{s_i}$  using a binary error correcting code  $\mathcal{C}_{s_i}(N_{s_i}, K_{s_i})$  of rate  $R_{s_i} = K_{s_i}/N_{s_i}$ . Without loss of generality, we assume  $K_{s_1} = K_{s_2} = K$ . The codeword  $\mathbf{c}_{s_i}$  is modulated into  $\mathbf{x}_{s_i} = [x_{s_i}(1), \dots, x_{s_i}(N_{s_i})]$ , by using Binary Phase Shift Keying (BPSK) modulation with the mapping rule  $x = (1 - 2c)$  (*i.e.*,  $\mathcal{M} = \{0' \leftrightarrow +1, 1' \leftrightarrow -1\}$ ). Finally, the source node  $S_i$  broadcasts the coded symbols  $\mathbf{x}_{s_i}$  during the first ( $S_1$ ) and second ( $S_2$ ) time slots. The messages  $\mathbf{y}_{s_i r_j} = [y_{s_i r_j}(1), \dots, y_{s_i r_j}(N_{s_i})]$ ,  $j = \{1, 2\}$ , and  $\mathbf{y}_{s_i d} = [y_{s_i d}(1), \dots, y_{s_i d}(N_{s_i})]$ , received at relay  $R_j$  and at destination  $D$ , respectively, are given by:

$$\mathbf{y}_{s_i r_j}(n) = h_{s_i r_j}(n)x_{s_i}(n) + \eta_{s_i r_j}(n) \quad (1)$$

$$\mathbf{y}_{s_i d}(n) = h_{s_i d}(n)x_{s_i}(n) + \eta_{s_i d}(n) \quad (2)$$



**Figure 2: Coded two-source two-relay cooperative network.**

where  $h_{xy}$  is the channel coefficient from node  $x$  to node  $y$ , which takes into account path-loss and fading, and  $\eta_{xy}$  is the Additive White Gaussian Noises (AWGN) with zero-mean and variance  $N_0/2$ .

## 2.2 Relaying Phase

At the relays, we assume that a Decode-and-Forward (DF) protocol is used. In particular, the relays perform coherent ML-optimum decoding of the coded messages. Let  $\hat{\mathbf{u}}_{s_i r_j}$  be the detected information message at relay  $R_j$ , according to [28] the relays can either just relay the received data (*i.e.*, pure DF protocol) or perform NC on the received messages (*i.e.*, Decode-Network-Code-and-Forward (D-NC-F) strategy). More specifically, the information message of size  $K$ ,  $\hat{\mathbf{u}}_{r_j}$ , possibly being transmitted by relay  $R_j$  is:

$$\hat{\mathbf{u}}_{r_j} = \begin{cases} \hat{\mathbf{u}}_{s_1 r_j} \oplus \hat{\mathbf{u}}_{s_2 r_j} & \text{D-NC-F strategy at } R_j \\ \hat{\mathbf{u}}_{s_1 r_j} & \text{DF strategy for } S_1 \text{ at } R_j \\ \hat{\mathbf{u}}_{s_2 r_j} & \text{DF strategy for } S_2 \text{ at } R_j \end{cases} \quad (3)$$

where  $\oplus$  denotes XOR operations.

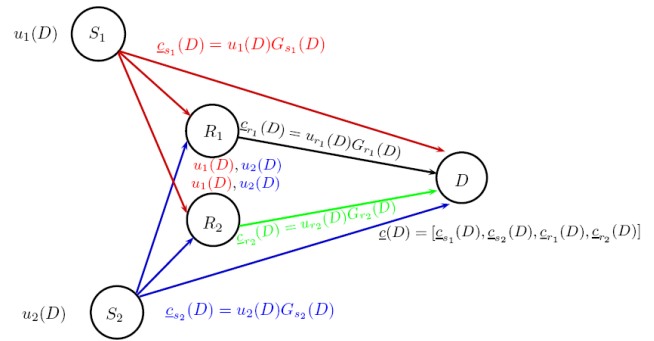
Then, the information message  $\hat{\mathbf{u}}_{r_j}$  is encoded into a codeword  $\hat{\mathbf{c}}_{r_j, d}$  of length  $N_{r_j}$  using an error correcting code of rate  $R_{r_j} = K/N_{r_j}$ . Finally, the obtained codeword is modulated into  $\hat{\mathbf{x}}_{r_j}$  by using BPSK modulation, and is transmitted to the destination during the third ( $\hat{\mathbf{u}}_{r_1}$ ) and fourth ( $\hat{\mathbf{u}}_{r_2}$ ) time slots. The message received at the destination is:

$$\mathbf{y}_{r_j, d}(n) = h_{r_j, d}(n)\hat{\mathbf{x}}_{r_j}(n) + \eta_{r_j, d}(n) \quad (4)$$

## 2.3 Detection at the Destination

After four time slots, the destination has available the vector of messages  $\mathbf{y}_d = [\mathbf{y}_{s_1 d}, \mathbf{y}_{s_2 d}, \mathbf{y}_{r_1 d}, \mathbf{y}_{r_2 d}]$ . Based on these observations, it attempts to infer both message  $\mathbf{u}_{s_1}$  and  $\mathbf{u}_{s_2}$  transmitted by  $S_1$  and  $S_2$ , respectively. For the uncoded case, three detectors have been studied in [28], and it has been shown that the maximum diversity gain is obtained when channel state information is available at the network layer. In this paper, we consider a coded system setup with convolutional codes at the physical layer. The ML-optimum decoder is developed in the next section.

## 3. JNCD BASED ON DISTRIBUTED CONVOLUTIONAL CODES



**Figure 3: Coded two-source two-relay cooperative network with perfect source-to-relay links.**

For ease of notation, we use a polynomial representation [29] as shown in Figure 2. We assume that the feed-forward convolutional codes rate  $R = 1/N$ , where  $N$  is the number of output bits of the convolutional encoder. The information and coded sequences of source  $S_i$  are denoted by  $u_{s_i}(D)$  and  $\underline{c}_{s_i}(D) = u_{s_i}(D)G_{s_i}(D)$ , respectively, where  $\underline{c}_{s_i}(D) = [c_{s_i}^{(1)}(D), \dots, c_{s_i}^{(N_i)}(D)]$  and  $G_{s_i}(D) = [g_{s_i}^{(1)}(D), \dots, g_{s_i}^{(N_i)}(D)]$  is the polynomial generator matrix of the convolutional code.

The information sequences estimated at the relays are denoted by  $\hat{u}_{s_i r_j}(D)$ , which similar to (3), can be written as:

$$\hat{u}_{r_j}(D) = \begin{cases} \hat{u}_{s_1 r_j}(D) \oplus \hat{u}_{s_2 r_j}(D) & \text{D-NC-F strategy at } R_j \\ \hat{u}_{s_1 r_j}(D) & \text{DF strategy for } S_1 \text{ at } R_j \\ \hat{u}_{s_2 r_j}(D) & \text{DF strategy for } S_2 \text{ at } R_j \end{cases} \quad (5)$$

Finally, the re-encoded sequences at the relays are given by:

$$\hat{c}_{r_j}(D) = \hat{u}_{r_j}(D)G_{r_j}(D) \quad (6)$$

where

$$\hat{c}_{r_j}(D) = [\hat{c}_{r_j}^{(1)}(D), \dots, \hat{c}_{r_j}^{(N_j)}(D)] \quad (7)$$

and

$$G_{r_j}(D) = [g_{r_j}^{(1)}(D), \dots, g_{r_j}^{(N_j)}(D)] \quad (8)$$

Finally, at the destination we have:

$$\underline{c}(D) = [\underline{c}_{s_1}(D), \underline{c}_{s_2}(D), \hat{c}_{r_1}(D), \hat{c}_{r_2}(D)] \quad (9)$$

## 3.1 Distributed Network/Channel Code

For ease of understanding, let us consider perfect source-to-relay links as shown in Figure 3. In Section 4, the numerical results are obtained for noisy source-to-relay links as well. In this case, we have  $\hat{u}_{s_i r_j}(D) = u_{s_i}(D)$ , and, thus, (5) reduces to  $\hat{u}_{r_j}(D) = u_{r_j}(D)$  with:

$$u_{r_j}(D) = \begin{cases} u_{s_1}(D) \oplus u_{s_2}(D) & \text{D-NC-F strategy at } R_j \\ u_{s_1}(D) & \text{DF strategy for } S_1 \text{ at } R_j \\ u_{s_2}(D) & \text{DF strategy for } S_2 \text{ at } R_j \end{cases} \quad (10)$$

By using the linearity property of channel coding, we have

$\hat{c}_{r_j}(D) = \underline{c}_{r_j}(D)$  with:

$$\underline{c}_{r_j}(D) = \begin{cases} u_{s_1}(D)G_{r_j}(D) \oplus u_{s_2}(D)G_{r_j}(D) & \text{D-NC-F at } R_j \\ u_{s_1}(D)G_{r_j}(D) & \text{DF for } S_1 \text{ at } R_j \\ u_{s_2}(D)G_{r_j}(D) & \text{DF for } S_2 \text{ at } R_j \end{cases} \quad (11)$$

Thus, at the destination the received vector is:

$$\underline{c}(D) = [\underline{c}_{s_1}(D), \underline{c}_{s_2}(D), \underline{c}_{r_1}(D), \underline{c}_{r_2}(D)] \quad (12)$$

Finally, by defining  $\underline{u}(D) = [u_{s_1}(D), u_{s_2}(D)]$ , we have:

$$\underline{c}(D) = \underline{u}(D)G(D) \quad (13)$$

where  $G(D)$  is the polynomial generator matrix associated with an equivalent distributed code, which takes into account both channel and network codes. The rate of this code is  $r = k/n$  with  $k = 2$  and  $n = N_{s_1} + N_{s_2} + N_{r_1} + N_{r_2}$ .

The equivalent polynomial generator matrix  $G(D)$  depends on the operations performed at the relay:

- If D-NC-F is used at  $R_1$  and DF is used at  $R_2$ , then:

$$G(D) = \begin{pmatrix} G_{s_1}(D) & 0 & G_{r_1}(D) & 0 \\ 0 & G_{s_2}(D) & G_{r_1}(D) & G_{r_2}(D) \end{pmatrix} \quad (14)$$

- If D-NC-F is used at  $R_2$  and DF is used at  $R_1$ , then:

$$G(D) = \begin{pmatrix} G_{s_1}(D) & 0 & G_{r_1}(D) & G_{r_2}(D) \\ 0 & G_{s_2}(D) & 0 & G_{r_2}(D) \end{pmatrix} \quad (15)$$

- If D-NC-F is used at  $R_1$  and  $R_2$ , then:

$$G(D) = \begin{pmatrix} G_{s_1}(D) & 0 & G_{r_1}(D) & G_{r_2}(D) \\ 0 & G_{s_2}(D) & G_{r_1}(D) & G_{r_2}(D) \end{pmatrix} \quad (16)$$

- If DF is used at  $R_1$  and  $R_2$ , then:

$$G(D) = \begin{pmatrix} G_{s_1}(D) & 0 & G_{r_1}(D) & 0 \\ 0 & G_{s_2}(D) & 0 & G_{r_2}(D) \end{pmatrix} \quad (17)$$

The equivalent matrices in (14)–(17) are a generalization of [28] for the uncoded case. In fact, the distributed network codes in [28] can be obtained by setting  $G_x(D) = 1$ . For example, if D-NC-F and DF are used at  $R_1$  and  $R_2$ , respectively, we have the (4, 2, 2) UEP-network code [32] with generator matrix:

$$G = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} \quad (18)$$

As shown by [33], UEP coding theory for block codes [30], [31] can be extended to convolutional codes as well. In fact, UEP capabilities can be expected since we have a rate  $r = k/n$  convolutional code with  $k > 1$  [33]–[35]. However, the inherent UEP properties of  $G(D)$  are closely related to the polynomials  $G_x(D)$  [34], [35]. It is important to note that the convolutional codes used at the sources should be chosen in order to avoid catastrophic convolutional codes seen at the relays [29]. Finally, we note that this scheme can be easily extended to multi-source multi-relay networks.

### 3.2 Channel-Aware Receiver Design

At the receiver, we exploit ML-optimum detection theory to estimate  $\mathbf{c} = [\mathbf{c}_{s_1}, \mathbf{c}_{s_2}, \mathbf{c}_{r_1}, \mathbf{c}_{r_2}]$  based upon the reception of  $\mathbf{y}_d = [\mathbf{y}_{s_1d}, \mathbf{y}_{s_2d}, \mathbf{y}_{r_1d}, \mathbf{y}_{r_2d}]$ , which is a noisy version of  $\mathbf{x}_d = [\mathbf{x}_{s_1}, \mathbf{x}_{s_2}, \hat{\mathbf{x}}_{r_1}, \hat{\mathbf{x}}_{r_2}]$ . With perfect channel state information at the receiver, the optimal detector is:

$$\hat{\mathbf{c}} = \arg \max_{\mathbf{c}'} p(\mathbf{y}_d | \mathbf{c}', \mathbf{h}) \quad (19)$$

where  $\mathbf{h}$  is the vector containing all the channel coefficients associated with  $\mathbf{y}_d$ , and  $p(\cdot)$  denotes probability density function. Using the memoryless property and the independence of the channels on the different links of the network, (19) can be rewritten using the channel transition probabilities and the modulated codeword  $\mathbf{x}_d$ :

$$\hat{\mathbf{x}}_d = \arg \max_{\mathbf{x}'_{s_1}, \mathbf{x}'_{s_2}} \prod_{n=1}^{N_{s_1}} p(y_{s_1d}(n) | x'_{s_1}(n), h_{s_1d}(n)) \times \prod_{n=1}^{N_{s_2}} p(y_{s_2d}(n) | x'_{s_2}(n), h_{s_2d}(n)) \times \prod_{n=1}^{N_{r_1}} p(y_{r_1d}(n) | x'_{r_1}(n), h_{r_1d}(n)) \times \prod_{n=1}^{N_{r_2}} p(y_{r_2d}(n) | x'_{r_2}(n), h_{r_2d}(n)) \quad (20)$$

If the source-to-relay links are perfect, the four terms in (20) are directly computed from the channel transition probabilities. The efficient computation of the ML-optimum decoder is obtained by using the Viterbi algorithm applied on the joint trellis given by the polynomial generator matrix  $G(D)$  [29]. The interpretation of the whole network as a distributed convolutional code is based on the assumption of perfect source-to-relay links. However, when there are decoding errors on the source-to-relay links, the two last terms in (20) are not directly given by the channel transition probabilities. More specifically,  $x_{r_j}$  can only be inferred through its noisy version  $\hat{x}_{r_j}$ . However, the decoder can take into account decoding errors at the relays through the estimation of the decoding error probability, which can be computed as follows.

Let  $\text{Pe}_j = \Pr \{ \hat{c}_{r_j}(n) \neq c_{r_j}(n) \}$  be the average coded bit error probability at relay  $R_j$ . Then, the codeword  $\hat{\mathbf{c}}_{r_j}$  can be written as:

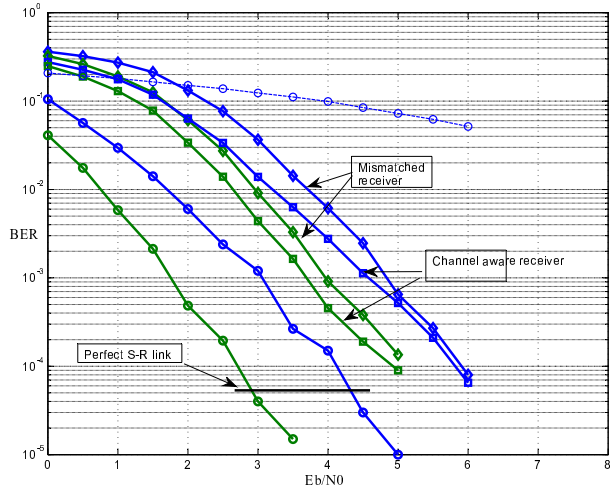
$$\hat{\mathbf{c}}_{r_j} = \mathbf{c}_{r_j} \oplus \mathbf{e}_{r_j} \quad (21)$$

where  $\mathbf{e}_{r_j}$  is an error vector that accounts for the errors at relay  $R_j$ . By assuming that the decoding errors at the relay are independent and identically distributed, we can write (conditioning on the channel is avoided for ease of notation):

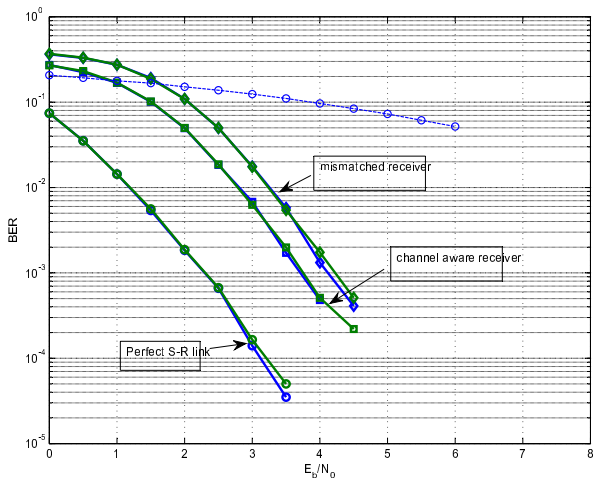
$$p(y_{r_jd}(n) | x_{r_j}(n)) = p(y_{r_jd}(n) | \hat{x}_{r_j}(n) = +1) p(\hat{x}_{r_j}(n) = +1 | x_{r_j}(n)) + p(y_{r_jd}(n) | \hat{x}_{r_j}(n) = -1) p(\hat{x}_{r_j}(n) = -1 | x_{r_j}(n)) \quad (22)$$

with

$$p(\hat{x}_{r_j}(n) | x_{r_j}(n)) = \begin{cases} \text{Pe}_j & \text{if } \hat{x}_{r_j}(n) \neq x_{r_j}(n) \\ 1 - \text{Pe}_j & \text{otherwise} \end{cases} \quad (23)$$



**Figure 4: Bit Error Rate (BER) versus  $E_b/N_0$  for the distributed convolutional code  $G(D)$  in (24).  $K = 1000$ . Blue and green curves are related to source  $S_1$  and  $S_2$ , respectively.**



**Figure 5: Bit Error Rate (BER) versus  $E_b/N_0$  for the distributed convolutional code  $G(D)$  in (25).  $K = 1000$ . Blue and green curves are related to source  $S_1$  and  $S_2$ , respectively.**

By using (22) and (23), the Viterbi algorithm can be performed on the equivalent trellis associated to  $G(D)$ .

#### 4. SIMULATION RESULTS

In this section, we provide some illustrative simulation results. We consider the performance of the distributed convolutional code over AWGN channels with the same Signal-to-Noise-Ratio (SNR) over all the wireless links. We compare the performance of the distributed coding scheme for three different configurations:

- Perfect source-to-relay links. In this case, ML-optimum

decoding of the distributed convolutional code is performed. The messages from the relays are error-free before re-transmission. This scenario provides a lower-bound of the performance of the system.

- Noisy source-to-relay links with perfect ML-optimum decoding. In this case, detection is performed by taking into account decoding errors performed at the relays and forwarded to the destination.
- Noisy source-to-relay links with mismatched ML-optimum decoding. In this case, detection is performed without knowledge about the decoding error probability on the source-to-relay links.

In Figure 4 and Figure 5, we show the Bit Error Rate (BER) by considering the  $2 \times 6$  matrices given, respectively, by:

$$G(D) = \begin{pmatrix} 23 & 33 & 0 & 0 & 37 & 0 \\ 0 & 0 & 23 & 33 & 37 & 25 \end{pmatrix} \quad (24)$$

$$G(D) = \begin{pmatrix} 23 & 33 & 0 & 0 & 37 & 25 \\ 0 & 0 & 23 & 33 & 37 & 25 \end{pmatrix} \quad (25)$$

where the polynomials are expressed in octal.

We consider rate-1/2 convolutional codes on the source-to-relay links and rate-1 convolutional codes on the relay-to-destination links. In (24),  $R_1$  and  $R_2$  use D-NC-F and DF, respectively, while in (25),  $R_1$  and  $R_2$  both use D-NC-F. The results are obtained for  $K = 1000$  information bits. Similar to [28], we can observe a UEP behavior in Figure 4. Furthermore, the decoder in (23) with side information about the source-to-relay links, *i.e.*, it knows the error probability in (23), provides better performance. Finally, we notice that decoding errors at the relays can seriously degrade the BER.

#### 5. CONCLUSION

In this paper, we have studied JNCD for multi-source multi-relay heterogeneous wireless networks. We have considered a coded communication system and developed the ML-optimum decoder for network and channel codes. Some numerical results have been shown to highlight UEP decoding properties of the proposed approach when the network code is adequately chosen. Also, we have studied the performance of the decoder when error probability information on the source-to-relay links is unavailable at the destination.

#### 6. ACKNOWLEDGMENTS

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