GOODNESS-OF-FIT TESTS IN RADIATED SUSCEPTIBILITY TESTS

Florian Monsef, Andrea Cozza

GOODNESS-OF-FIT TESTS IN RADIATED SUSCEPTIBILITY TESTS

Florian Monsef (1), Andrea Cozza (1)

(1) Laboratoire des Signaux et Systèmes,UMR8506, Univ Paris-Sud SUPELEC CNRS, 3 rue Joliot Curie, Gif-sur-Yvette, France, Email:florian.monsef@lss.supelec.fr / andrea.cozza@supelec.fr

ABSTRACT

Goodness-of-fit (GoF) tests are commonly used to assess the degree of perfection of a reverberation chamber to fulfill the overmoded condition. One may expect the outcome of GoF tests to provide some hints on the value of practical interest. The present paper shows that no correlation is found between the outcome of GoF tests and the range of values taken by the maximum power which is of practical interest in radiated susceptibility tests.

1. Introduction

It is commonly accepted today that an electric field in an over-moded reverberation chamber (RC) should fit standard statistical models, namely, that the real and imaginary parts of a field component should follow a Normal probability density distribution (pdf); or, equivalently, that the power along a field component follows an exponential distribution.

The adoption of a common statistical distribution for a given field parameter has an important role in the statistics of quantities of practical interest for EMC susceptibility-test purposes and for standardization test methods [1]. For instance, the statistical distribution related to the maximum electric-field level applied to a device under test in a radiated susceptibility test, ensues directly from the electric-field statistics as shown in [2] with corresponding related moments.

In order to check in practice whether an empirical cumulative distribution function (ECDF) belongs to a given cumulative distribution function (CDF), goodness-of-fit (GoF) tests must be used as done in [3][4][5][6][7].

From a RC user point of view, a simple question may arise on the degree of information provided by GoF outcomes on quantities of practical interest. In others terms, one may wonder if a correlation may exist between a GoF outcome and the acceptable standard deviation of the maximum electromagnetic field stressing an equipment under test (EUT).

The present paper aims at studying this question in various scenarios in order to see, whether or not, GoF tests applied to the electric power along a field component can be regarded as a useful tool for predicting a compliancy or non-compliancy of the maximum stress applied onto a EUT.

This work will be presented as follows. The first section recalls usual GoF tests and justifies the choice of the one used in our study. The second section aims at presenting the statistics of the maximum power expected in a radiated susceptibility test (RST). After a brief description of the experimental setup, we will proceed to a statistical analysis aiming at investigating the existence of a correlation between the outcome of a GoF test and the values of the maximum power.

2. Goodness-of-fit tests

Three GoF tests have been mainly used in the EMC community: the χ^2 test [5], the Kolmogorov-Smirnov (KS) test [8] [9] and the Anderson-Darling (AD) test [3][4][6][7]. Each of them has certain advantages. The χ^2 test, for instance, is well adapted to the case of discontinuous CDF and has the ability to adapt the statistic for the case when parameters of the CDF must themselves be estimated from the sample. The KS test in its basic version is well-adapted to continuous CDF but needs the null hypothesis to be fully specified, i.e., needs the statistical moments of the CDF of reference to be known. The AD test is more powerful than the two tests previously mentioned, in the statistical sense, since it has the ability of assessing the belonging of an ECDF to a reference CDF in its finest details. This feature tended to make it popular during the last few years in the EMC community.

Our concern is to define a criterion able to guide our choice. The power of the AD test may be tempting. However, if one recalls that the Gaussian distribution of the real and imaginary parts of an electric-field component is just an asymptotic model, there is no need to check extreme values of the ECDF with care; worse, it would give an important weight to a part of the pdf where the asymptotic model will probably not hold. Accordingly, standard GoF methods, i.e., the χ^2 and KS test, seem more adequate and less conservative for the purpose of our study. The CDF of reference being continuous, the KS test will be chosen. In order to cope with the case when parameters of the CDF must themselves be estimated from the sample, a derived version of the KS test, namely, the Liliefors GoF test (or composite KS test) will be used [10] [11].

The principle of a GoF test starts by expressing a null hypothesis H_0. In the present case, H_0 states that the samples belong to a CDF of reference. In the composite KS-test case, the first step consists in computing the Kolmogorov statistics defined by D_{KS} as

$$D_{KS} = \max_x \left| S(x) - F(x) \right| \quad (1)$$

where $S(x)$ is the ECDF estimated from the vector sample x and $F(x)$ is the CDF of reference with mean and standard deviation equal to the mean and standard deviation of the sample.

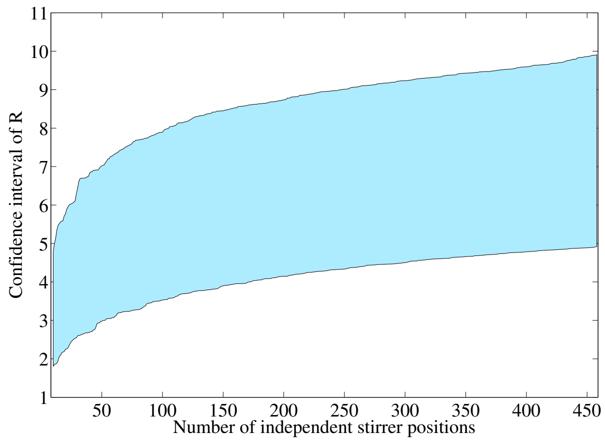

Figure 1. 95 % confidence interval of the normalized maximum power R (blue shaded area) as a function of the number N_s of stirrer positions.

In the present study, the samples are related to the power received by an linearly-polarized antenna; the reference CDF is then an exponential law. The distance D_{KS} will be compared to a critical value which depends on the number of independent samples and the significance level α chosen for the test. In order to properly take into account the fact that parameters of the CDF are estimated from the sample, modified critical values expressions are used [12]. If the distance D_{KS} is greater than the critical value, the null hypothesis H0 is rejected, i.e., the test states that the sample CDF does probably not belong to the CDF of reference.

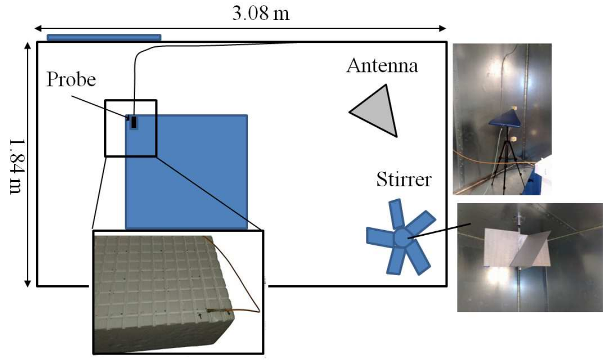

Figure 2. The experimental setup takes place in Supelec's RC. The measurements are based on the use of a log-periodic antenna and of an optical-link electric-field probe placed on a Styrofoam support (lower frame). The stirrer can provide 100 steps.

3. Quantity of interest: maximum received power

Under RST conditions, the quantity of practical interest is the maximum power available to stress EUTs [13]. When a RST is performed, the EUT is stressed for N different stirrer positions [1]. The typical number of positions can run from a few tens to a few hundreds. This provides a number of realizations of the electric-field power which is regarded as a random variable following asymptotically an exponential distribution [13] [1] [2].

The maximum power applied onto a EUT is an unknown quantity which can only be estimated as it behaves as a random variable whose asymptotic distribution ensues from the pdf of the electric-field power.

The aim here is not to derive the statistical distribution of the maximum power, but rather to focus on the confidence interval (CI) obtained in the case of a perfect RC, i.e., in a RC in which the electric-field related quantities would fit the standard distributions previously mentioned. The idea will be to check in the next sections, if the maximum power may fall into the satisfactory range although the RC is not perfect, i.e., although the null hypothesis is rejected.

Before computing this range, we introduce the normalized power quantity, referred to as R, defined as,

$$R = \frac{P_{Max}}{P_m}, \quad (2)$$

where P_{Max} and P_m refer to the maximum and the average power applied onto the EUT, respectively.

In order to compute the CI of the normalized power, we use a Monte Carlo simulation. In order to reduce the dispersion around the values bounding the confidence interval taken at 95%, we assume 2500 independent samples. The number of stirrer positions is regarded as a variable running from 10 to 450. Fig. 1 shows the resulting bounding values of the CI as a function of the number of stirrer positions.

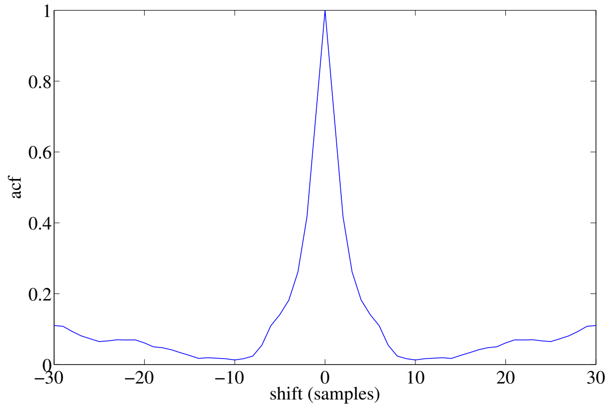

Figure 3. Real part of the complex autocorrelation function of the x-component of the electric field.

These limits will be regarded as a satisfactory range of reference for which we expect the *R* variables to fall into when assessed in practice from experimental measurements.

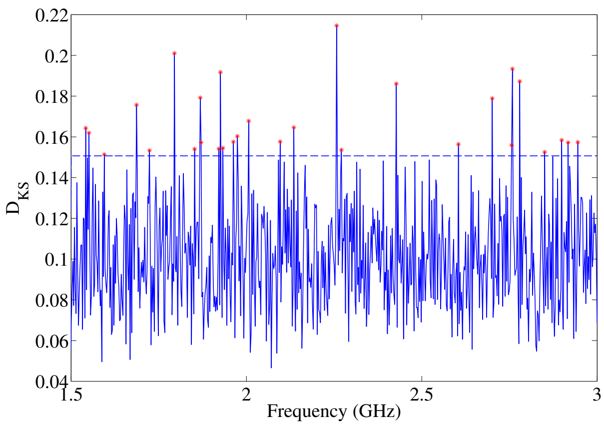

Figure 4. Kolmogorov statistics (solid line) obtained for each frequency. The critical value (dashed line) is 0.1507. Markers highlight the frequencies for which the null hypothesis is rejected.

4. Experimental setup

Tests and measurements were carried out in Supelec's RC. This latter corresponds to a rectangular cavity (13.3 m^3) including a mechanical stirrer (100 positions) capable of ensuring compliancy with the IEC standards [1] for frequencies above 550 MHz. The excitation was performed using a log-periodic antenna whose main lobe was directed towards the stirrer, as shown in Fig. 2, away from the center of the chamber. In the central volume of the cavity, the electric-field was sampled by means of an optical-link electric-field probe, mounted on a Styrofoam support. This EFS-105 probe, manufactured by Enprobe, is linearly polarized and presents a flat response of the frequency range of interest, namely, from 700 MHz to 3 GHz, for which 1001 points were uniformly sampled. The use of a vector network analyzer allows accessing to the phase of the electric-field.

The chamber was operated in two cases: a loaded and unloaded case. For the loaded scenario, a 4-piece pyramidal absorber was placed into the chamber. Each piece was about 30-cm high and was placed at the center of the cavity, on the floor. The average composite quality factor was assessed to be around 2000 at 1 GHz and twice this value at 2 GHz. A comparison between the modal bandwidth, turning out to be around 500 kHz, and the frequency spacing of 2.3 MHz between each measured point, allows to consider each frequency sample as independent.

5. Statistical characterization

It is important to emphasize that GoF tests must be used with independent samples. In order to measure the independence of the observed data, we need to estimate the autocorrelation function (*acf*) of the electric-field components. Given that the real and imaginary parts of the electric-field components follow (asymptotically) a Normal law, decorrelation implies independence. The use of a phase-sensitive probe allows computing the complex *acf* of an electric-field component.

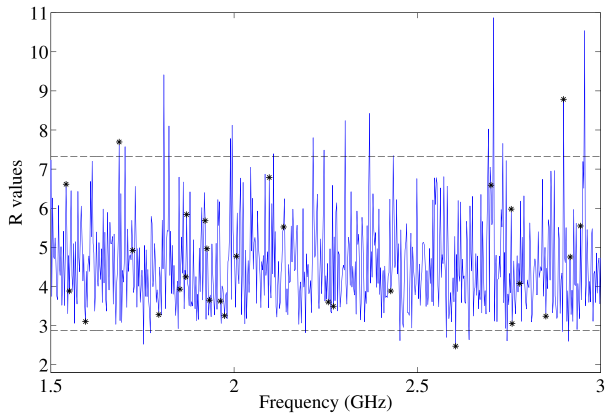

Figure 5. The maximum normalized power value R of the unloaded case are plotted as a function of frequency (solid line). Bounding values of the CI (dashed line) are taken from Fig. 1 and markers highlight the frequencies that did not pass the GoF test.

In order to estimate the number of independent samples we compute the *acf* in the unloaded case. Fig. 3 shows the real part of the *acf* obtained around 700 MHz in the unloaded scenario. It shows that a shift of 5 samples leads to a correlation coefficient lower than 15 %. This induces a decimation factor of 5: 20 of the 100 samples obtained by using the stirrer can fruitfully be used. A similar approach shows that a decimation factor of about 3 must be considered between 1 GHz and 1.5 GHz, and a decimation factor of 2 beyond 1.5 GHz. Accordingly, to use GoF tests in a simpler way, we will restrict the frequency range from 1.5 GHz to 3 GHz with a constant number of 50 independent samples. This has the benefit to place our study in a frequency range where the RC can be regarded as overmoded according to [1] and where GoF tests should not be then expected to be too conservative.

Although the decimation factor is assessed on the base of the *acf* of the E-field component, the GoF is applied to the power of an electric-field component which should follow an exponential law.

For such a law and a significance level α of 5% the Liliefors test provides a critical value of $1.065/\sqrt{N}$s. Fig. 4 shows the Kolmogorov statistics D_{KS} (solid line) for each frequency of the 1.5 to 3 GHz range; a critical value (dashed line) of 0.1507 is obtained for 50 samples. Markers are placed at frequencies, at which the

GoF test has rejected the null hypothesis, i.e., at which the D_{KS} parameter is greater than the critical value. We can see that although the study is performed in a frequency range in which the RC is commonly considered as overmoded, a certain number of frequencies do not pass the test.

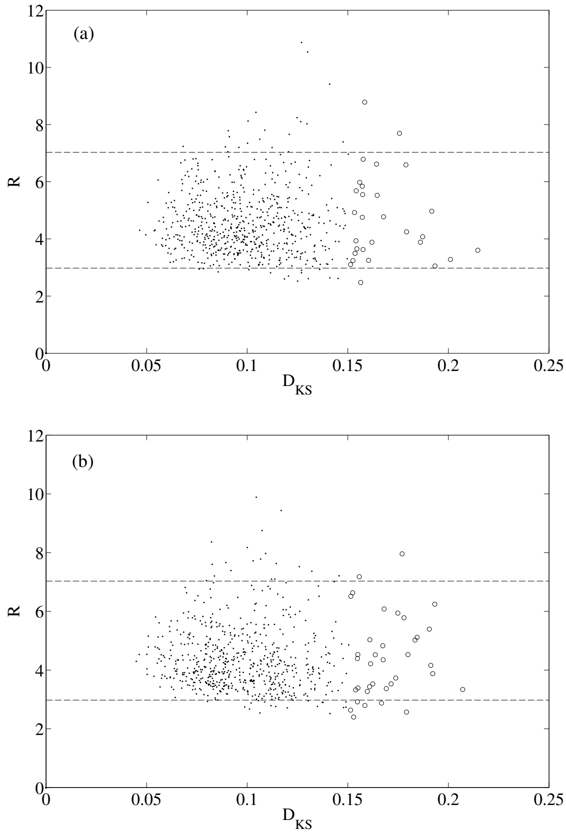

Figure 6. Scatter plot of R versus D_{KS} for (a) the unloaded scenario (b) the loaded scenario. Black-colored dots represent the cases for which the null hypothesis H_0 is accepted; black circles represent the cases for which H_0 is rejected. Bounding values of the CI are also reported (dashed lines).

In order to see if the outcomes of the GoF test may be regarded as a reliable clue on the value of the maximum normalized power that will be applied onto an EUT, the maximum value over the 50 independent samples of the electric-field component is stored, and this, at each frequency. The results are shown (solid line) on Fig. 5. Bounding values of the confidence interval (dashed line) are also plotted and are taken from Fig. 1. Frequencies which did not pass the GoF test are highlighted by markers.

We can clearly see that the rejection of the null hypothesis at certain frequencies does not imply that the maximum power of the related samples will not fall into the CI resulting from the asymptotical law for which the RC is supposedly perfect.

In order to point out more clearly this fact, we show in Fig. 6a a scatter plot linking the Kolmogorov statistics D_{KS} to the maximum normalized power *R*: frequencies that did not pass the test (black circles) and those that did pass the test (black dots) are shown with the bounding values (dashed line) of the CI taken from Fig. 1.

We can see that there is no real link between the outcome of the GoF test and the range value of *R*. In other terms, the scatter plot shows that no correlation exists between the rejection of H_0 and the possible unsatisfactory value of *R*.

In order to see if the presence of losses may affect this trend, we proceeded in the same way for the loaded scenario, i.e., the case for which losses are introduced into the RC. The insertion of losses raises the modal overlap. Accordingly, the sample should fit more closely the asymptotic laws and the GoF test could be expected to be more accurate in its overmodedness assessment. This could make one expect a better correlation with the CI value of *R* in practice. Fig. 6b shows the resulting scatter plot. As one can notice, the dispersiveness of *R* is lower than in the unloaded case, which is in accordance with a larger modal overlap; however, the correlation between the GoF outcome and the CI of *R* is not enhanced.

6. CONCLUSION

The use of high-power GoF tests these last few years, aim at assessing the overmodedness condition but is based on a need that the samples generated for different stirrer positions should fit as perfectly as possible asymptotic statistical laws.

In the present study a composite-KS GoF test has been applied to the maximum normalized power *R* along an electric-field component. Although the KS-GoF is only of moderated power the results presented herein show that the outcomes are already too conservative when the output quantity of interest, such as the maximum power for RST application, is observed. Indeed, independently of the outcome, for different frequencies, the normalized maximum power is always found to be in accordance with the 95% CI resulting from asymptotic laws. The absence of correlation between GoF outcomes and the CI of *R* may put into perspective what can really be provided by GoF tests in terms of real hints on quantities of practical interest in RST.

7. REFERENCES

[1] *Reverberation chamber test methods*, International Electrotechnical Commission (IEC), Std. 61 000-4-21, 2003.

[2] T. Lehman and G. Freyer, “Characterization of the maximum test level in a reverberation chamber,” in

Electromagnetic Compatibility, 1997. IEEE 1997 International Symposium on. IEEE, 1997, pp. 44–47.

[3] C. Lemoine, P. Besnier, and M. Drissi, “Investigation of reverberation chamber measurements through high-power goodness-of-fit tests,” *Electromagnetic Compatibility, IEEE Transactions on*, vol. 49, no. 4, pp. 745–755, 2007.

[4] R. Serra, F. Leferink, and F. Canavero, ““good-but-imperfect” electromagnetic reverberation in a virc,” in *Electromagnetic Compatibility (EMC), 2011 IEEE International Symposium on*. IEEE, 2011, pp. 954–959.

[5] J. Kostas and B. Boverie, “Statistical model for a mode-stirred chamber,” *Electromagnetic Compatibility, IEEE Transactions on*, vol. 33, no. 4, pp. 366–370, 1991.

[6] F. Moglie and V. Primiani, “Evaluation of uncorrelation and statistics inside a reverberation chamber in presence of two independent stirrers,” in *Electromagnetic Compatibility (EMC), 2010 IEEE International Symposium on*. IEEE, 2010, pp. 515–519.

[7] V. Mariani Primiani and F. Moglie, “Numerical simulation of reverberation chamber parameters affecting the received power statistics,” *Electromagnetic Compatibility, IEEE Transactions on*, no. 99, pp. 1–11.

[8] P. Corona, G. Ferrara, and M. Migliaccio, “Reverberating chambers as sources of stochastic electromagnetic fields,” *Electromagnetic Compatibility, IEEE Transactions on*, vol. 38, no. 3, pp. 348–356, 1996.

[9] A. Coates, H. Sasse, D. Coleby, A. Duffy, and A. Orlandi, “Validation of a three-dimensional transmission line matrix (TLM) model implementation of a mode-stirred reverberation chamber,” *Electromagnetic Compatibility, IEEE Transactions on*, vol. 49, no. 4, pp. 734–744, 2007.

[10] H. Lilliefors, “On the kolmogorov-smirnov test for normality with mean and variance unknown,” *Journal of the American Statistical Association*, pp. 399–402, 1967.

[11] ——, “On the kolmogorov-smirnov test for the exponential distribution with mean unknown,” *Journal of the American Statistical Association*, pp. 387–389, 1969.

[12] M. Stephens, “EDF statistics for goodness of fit and some comparisons,” *Journal of the American Statistical Association*, pp. 730–737, 1974.

[13] M. Hoijer, “Maximum power available to stress onto the critical component in the equipment under test when performing a radiated susceptibility test in the reverberation chamber,” *Electromagnetic Compatibility, IEEE Transactions on*, vol. 48, no. 2, pp. 372–384, 2006.