Competition in Femtocell Networks: Strategic Access Policies in the Uplink

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***Abstract*—In emerging small cell wireless, each femtocell access point (FAP) can either service its home subscribers exclusively (i.e., closed access) or open its access to accommodate a number of macrocell users so as to reduce cross-tier interference. In this paper, we propose a game-theoretic framework that enables the FAPs to strategically decide on their uplink access policy. We formulate a noncooperative game in which the FAPs are the players that want to strategically decide on whether to use a closed or an open access policy in order to maximize the performance of their registered users. Each FAP aims at optimizing the tradeoff between reducing cross-tier interference, by admitting macrocell users, and the associated cost in terms of allocated resources. Using novel analytical techniques, we show that the game always admits a pure strategy Nash equilibrium, despite the discontinuities in the utility functions. Further, we propose a distributed algorithm that can be adopted by the FAPs to reach their equilibrium access policies. Simulation results show that the proposed algorithm provides an improvement of 85.4% relative to an optimized open access scheme in the average worst-case FAP utility.**

I. Introduction

Femtocell access points (FAPs) are low-cost, low-power base stations that can be deployed in an indoor or an outdoor environment so as to satisfy the ever-increasing needs for high wireless data rates. Overlaying existing networks with femtocells is a promising solution to increase the capacity of wireless networks as well as to deliver innovative wireless services.

The deployment of femtocell wireless networks introduces numerous technical challenges. In particular, interference management is challenging in two-tier femtocell networks due to the absence of coordination between the FAPs, which are often privately owned, and the existing macrocell base stations. Several existing works addressed the challenges of interference management in femtocell networks [1], [2].

One characteristic of femtocell networks is the ability of the FAPs to operate in three modes: closed access, open access, and hybrid access (i.e., limited open access). In a *closed access* mode, an FAP dedicates all of its resources to a specific number of registered *home* users. Hence, in this mode, access to an FAP is restricted to a handful of pre-registered subscribers (e.g., the owners of the FAP). In contrast, in an *open access* mode, the FAP can also service, along with its home users, nearby macrocell users to reduce interference and to improve the overall network performance. *Hybrid access* is a limited form of open access in which only a specific number of macrocell users is allowed to access the femtocell tier. The choice of an access policy can strongly impact the network's uplink performance as shown in [3], this work was extended to the downlink in [4]. In [5], simulation results showed that the overall throughput in a network can be enhanced when the FAP uses a hybrid access policy as opposed to open access.

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Most existing works on femtocell access modes have assumed that the FAPs can operate *exclusively* either in closed, open, or hybrid access [3]–[6]. In practice, due to their self-organization capabilities, the FAPs have an incentive to strategically adapt their access modes depending on the network status. In the uplink, the FAPs face a tradeoff when deciding on their access policy. For instance, allowing macrocell users to use the FAP would potentially reduce the interference at the FAP, but it is accompanied with a cost in terms of dedicating the FAP's own resources to these macrocell users. This gives rise to an interesting competitive scenario. On the one hand, each FAP is interested in optimizing the performance of its own registered users by dedicating the maximum resources to them. On the other hand, this FAP also has an incentive to service some macrocell users so as to reduce the potentially harmful interference. These multiple interests are often *conflicting*. Hence, it is of interest to devise a scheme that allows the FAPs to strategically decide on their preferred access policy.

The main contribution of this paper is to model and analyze the conflicting access mode preferences of the FAPs in an overlaid femtocell network. To this end, we formulate a noncooperative game between the FAPs in which the strategy of each FAP is to select an access mode to optimize the performance of its home users, given the state of the network in terms of macrocell user locations, network configuration, and others. We characterize the optimal access policies for each FAP and we show that the FAPs have an incentive to strategically select their access modes, depending on their environment. Using novel analytical techniques, we show the existence of a *pure strategy Nash equilibrium (PSNE)* for the proposed game, despite the discontinuities in the utility functions. We study this resulting Nash equilibrium (NE) which dictates the access policies that will be employed by the FAPs as captured by the amount of resources allocated to the macrocell users. To solve the game, we present a distributed algorithm that enables the FAPs to self-organize and compute their equilibrium access policies with little coordination. We study various properties of the equilibrium and show that the proposed algorithm exhibits interesting characteristics.

The rest of the paper is organized as follows. In Section II we present the system model, and we formulate the noncooperative game. We analyze the game in Section III and present a best response distributed algorithm. Simulation results are presented in Section IV, and conclusions are drawn in Section V.

II. System Model

Consider the *uplink* of a network with M FAPs overlaid on a macrocell wireless network having N users. Let the set of FAPs be $\mathcal{M} = \{1, ..., M\}$ and the set of macrocell users be $\mathcal{N} = \{1, ..., N\}$. Hereinafter, we will refer to the registered FAP users as femtocell user equipment (FUE) and to the macrocellular users as macrocell user equipment (MUE). For multiple access, we consider an OFDMA policy at both network tiers. Let K be the total number of subcarriers available to each FAP. For mathematical tractability, we consider that FUEs do not introduce interference at neighboring FAPs, and, thus, there is no uplink femtocell-to-femtocell interference. This can be achieved by assigning orthogonal frequency bands to different FAPs using dynamic OFDMA or other methods such as those in [1], [7].

We consider a Rayleigh fading channel having an exponentially distributed magnitude with unit mean. We denote the channel from the n-th MUE to the m-th FAP on the k-th subcarrier by $h_{nm}[k]$. Let $P_n[k]$ be the transmit power of the n-th MUE on the k-th subcarrier; the total transmit power of the n-th MUE is P_n, with $\sum_{k=1}^{K} P_n[k] \leq P_n$. Also, let the distance between the n-th MUE and the m-th FAP be d_{nm}. Assume that each FAP services a single FUE (our results can be readily extended to femtocells used by multiple FUEs) having transmit power P_0 distributed over the subcarriers, i.e., $\sum_{k=1}^{K} P_0[k] \leq P_0$. We denote the channel from each FUE to its corresponding FAP on the k-th subcarrier by $h_{0m}[k]$ and the distance separating them by d_{0m}. A zero-mean circular complex Gaussian noise with variance $\sigma_m^2[k]$ is added on each subcarrier at the terminal of each FAP. The signal-to-interference-plus-noise ratio at FAP m, SINR_m, is given by:

$$\text{SINR}_m[k] = \frac{\gamma_m[k]}{\sigma_m^2[k] + \sum_{n=1}^{N} \left(\prod_{\ell=1}^{M} \mathbb{1}_{\{\delta_n^\ell[k]=0\}} \right) \mu_n^m[k]},$$

where $\gamma_m[k] = |h_{0m}[k]|^2 P_0[k] L(d_{0m})^{-\beta}$ is the received signal power of the FUE on subcarrier k of FAP m. The constant $L < 1$ is the wall penetration loss, and β is the path loss exponent for indoor-to-indoor communications. Similarly, $\mu_n^m[k] = |h_{nm}[k]|^2 P_n[k] d_{nm}^{-\alpha}$ is the received signal power of the n-th MUE on the k-th subcarrier of the m-th FAP, with α being the path loss exponent for outdoor-to-indoor signalling.

Each FAP needs to decide on an access policy: closed, open, or hybrid. Although closed access reserves the resources of an FAP for its FUEs, it can potentially increase the interference in a network. Open access reduces the interference at the price of sharing the resources of the FAPs with MUEs. Hybrid access strikes a balance between both policies as it constraints the amount of resources shared with MUEs. The choice of an access policy for an FAP depends, in addition to the interference levels introduced by MUEs, on the policy choices of the other FAPs. For example, an FAP prefers to use closed access and keep its resources for the use of its FUEs solely, when other FAPs decide to serve the interfering MUEs. Thus, given the scarce radio spectrum, the FAPs become competitive when deciding on their preferred access policy.

Hence, we define a noncooperative game between the FAPs in which each FAP attempts to maximize the rate of its FUE, by choosing an appropriate access policy. The type of access employed by an FAP is captured by the resources it allocates to the interfering MUE. In other words, the *strategies* of the FAPs are the fractions of the spectrum that they can allocate to each MUE, and the *utilities* are the rates of the FUEs.

Let $\delta_n^m[k] \in \{0, 1\}$ indicate whether the k-th subcarrier of FAP m is to be assigned to the n-th MUE – $\delta_n^m[k] = 1$ indicates that the subcarrier is to be allocated to the MUE. The utility function of FAP m can be written as:

$$\tilde{\mathcal{U}}_m(\boldsymbol{\delta}_m, \boldsymbol{\delta}_{-m}) = \sum_{k=1}^{K} \prod_{\ell=1}^{M} \mathbb{1}_{\{\delta_n^\ell[k]=0\}} \cdot \log\left(1 + \text{SINR}_m[k]\right),$$

where $\mathbb{1}_{\{x=0\}} = 1$ if and only if $x = 0$. The strategy vector of FAP m is $\boldsymbol{\delta}_m = [\delta_1^m[1], ..., \delta_N^m[1], \delta_1^m[2], ..., \delta_N^m[K]]^T$, while the strategy vectors of all other FAPs are given in $\boldsymbol{\delta}_{-m} = [\boldsymbol{\delta}_1^T, ..., \boldsymbol{\delta}_{m-1}^T, \boldsymbol{\delta}_{m+1}^T, ..., \boldsymbol{\delta}_M^T]^T$.

If FAP m allocates subcarriers to an MUE, the rate of the MUE should be at least as high as a target minimum rate R_{min}^c set by the macrocell user – without loss of generality, we assume all the MUEs to have the same target rate which is known in the network a priori. Formally, we can write

$$\left(1 - \prod_{k=1}^{K} \mathbb{1}_{\{\delta_n^m[k]=0\}}\right) R_{min}^c \leq \sum_{k=1}^{K} \delta_n^m[k] \log\left(1 + \frac{\mu_n^m[k]}{\sigma_m^2[k]}\right) \qquad (1)$$

The strategy space of FAP m is therefore

$$\tilde{\mathcal{X}}_m = \left\{ \boldsymbol{\delta}_m \in \{0,1\}^{NK} : \sum_{n=1}^{N} \delta_n^m[k] \leq 1, (1) \text{ is satisfied} \right\}.$$

The first constraint in $\tilde{\mathcal{X}}_m$ ensures that a given subcarrier k at an FAP m can be accessed by only one MUE. We can now write the optimization problem to be solved by FAP m as:

$$\text{For fixed } \boldsymbol{\delta}_{-m}, \ \max \quad \tilde{\mathcal{U}}_m(\boldsymbol{\delta}_m, \boldsymbol{\delta}_{-m}) \ \text{ over } \ \boldsymbol{\delta}_m \in \tilde{\mathcal{X}}_m. \qquad (2)$$

The outcome of this noncooperative game is governed by the renowned solution concept of a Nash equilibrium. Formally:

Definition 1: A pair $(\boldsymbol{\delta}_m^\star, \boldsymbol{\delta}_{-m}^\star)$ constitutes a pure strategy Nash equilibrium (PSNE) if $\tilde{\mathcal{U}}_m(\boldsymbol{\delta}_m^\star, \boldsymbol{\delta}_{-m}^\star) \geq \tilde{\mathcal{U}}_m(\boldsymbol{\delta}_m, \boldsymbol{\delta}_{-m}^\star), \forall \boldsymbol{\delta}_m \in \tilde{\mathcal{X}}_m$.

We are interested in studying the existence of a PSNE for the above problem. However, problem (2) is challenging due the following reasons: (i) the indicator functions make the objective functions discontinuous; and (ii) the problem is combinatorial in nature and requires exponential-time complexity to be solved. We will address both of these challenges in the next section.

III. Game Formulation and Proposed Algorithm

A. Subband Allocation

Although the total number of decision variables in (2), $M \cdot N \cdot K$, grows linearly in the number of subcarriers, the complexity of the problem can be large in practice. In addition to subcarrier allocation being combinatorial in nature [8], solving for the PSNE increases the complexity as each FAP needs to consider all possible subcarrier allocations and possible deviations of other FAPs. Solving (2) is, thus, challenging.

Hereinafter, we consider that the channels are flat-fading or that they do not vary over the frequency band available to each FAP, which is a common assumption [3], [4]. Hence, OFDMA is applied per subband rather than per subcarrier. When applied over flat-fading channels, OFDMA is geared towards scheduling users rather than resolving the inter-symbol interference in the channel. We make use of this fact and formulate the problem as a subband allocation problem instead of a subcarrier allocation one. The subbands are defined as clusters of consecutive subcarriers. Hence, we assume that each FAP has a frequency band, orthogonal to the bands of other FAPs, out of which it allocates fractions to MUEs so as to maximize the rate of the FUE.

Let $0 \leq \rho_n^m \leq 1$ be the fraction of the band allocated by the m-th FAP to the n-th MUE – ρ_0^m is the frequency band fraction allocated to the FUE. Clearly, an FAP m with $\sum_{i=1}^{N} \rho_i^m = 0$ is said to employ *closed access*. An FAP m is said to employ *open access* if $\min\{\rho_1^m, ..., \rho_N^m\} > 0$ and *hybrid* access if $\exists n$ for which $\rho_n^m > 0$. In the remaining of this sequel, we will refer to both open and hybrid access by open access; it should be understood that by open access we mean either adopting all MUEs or employing limited open access, depending on the network parameters. Thus, the utility function of FAP m can be written as:

$$\mathcal{U}_m(\boldsymbol{\rho}_m, \boldsymbol{\rho}_{-m}) = (1 - \sum_{n=1}^{N} \rho_n^m) \cdot \log\left(1 + \text{SINR}_m\right), \quad (3)$$

$$\text{SINR}_m = \frac{\gamma_m}{\sigma_m^2 + \sum_{n=1}^{N} \left(\prod_{\ell=1}^{M} \mathbb{1}_{\{\rho_n^\ell = 0\}} \right) \mu_n^m},$$

where $\boldsymbol{\rho}_m = [\rho_1^m, ..., \rho_N^m]^T$, and $\boldsymbol{\rho}_{-m}$ are the decision variables of all other FAPs[1]. The other variables are as defined above with the subcarrier index k dropped. We will denote $(\boldsymbol{\rho}_m, \boldsymbol{\rho}_{-m})$ by $\boldsymbol{\rho}$. The minimum rate constraint (1) becomes:

$$\left(1 - \mathbb{1}_{\{\rho_n^m = 0\}}\right) \cdot R_{min}^c \leq \rho_n^m \cdot \log\left(1 + \frac{\mu_n^m}{\sigma_m^2}\right), \quad (4)$$

and the strategy space of FAP m is now

$$\mathcal{X}_m = \left\{ \boldsymbol{\rho}_m \in [0,1]^N : \sum_{n=0}^{N} \rho_n^m = 1, (4) \text{ is satisfied} \right\}, \quad (5)$$

where the first constraint ensures that the subband allocations are well defined. Formally, the optimization in (2) becomes:

$$\text{For fixed } \boldsymbol{\rho}_{-m}, \ \max \mathcal{U}_m(\boldsymbol{\rho}) \ \text{ over } \ \boldsymbol{\rho}_m \in \mathcal{X}_m. \quad (6)$$

We will refer to this game by the *strategic access policy (SAP)* game. With this formulation, we have overcome the complexity associated with having a large number of subcarriers. However, the objective function $\mathcal{U}_m(\boldsymbol{\rho})$ is still discontinuous. We will handle the discontinuities in the next subsection.

B. Existence of Pure Strategy NE (PSNE)

The discontinuities in the objective functions prevent us from using standard theorems of continuous-kernel noncooperative games, such as those in [9, pp. 173-179]. Here, we will apply novel analytical techniques such as those in [10] to handle discontinuities and show that the SAP game admits a PSNE.

Denote the Cartesian product of the strategy spaces of the players by $\mathcal{X} = \times_{m=1}^{M} \mathcal{X}_m$. Define the graph of the vector of payoff functions as a subset of $\mathcal{X} \times \mathbb{R}^M$ given by $\Gamma = \left\{(\boldsymbol{\rho}, \boldsymbol{\mathcal{U}}) \subseteq \mathcal{X} \times \mathbb{R}^M : \boldsymbol{\mathcal{U}} = [\mathcal{U}_1(\boldsymbol{\rho}), ..., \mathcal{U}_M(\boldsymbol{\rho})]^T\right\}$. The closure of Γ is denoted $\bar{\Gamma}$. Before studying the PSNE for the SAP game, we provide the following definitions from [10].

[1]We dropped ρ_0^m from the definition of $\boldsymbol{\rho}_m$ because $\rho_0^m = 1 - \sum_{n=1}^{N} \rho_n^m$

Definition 2: FAP m can *secure a payoff* $\mathcal{U}_m(\boldsymbol{\rho}) = \nu \in \mathbb{R}$ *at* $\boldsymbol{\rho}$ if there exists a strategy $\bar{\boldsymbol{\rho}}_m$ such that $\mathcal{U}_m(\bar{\boldsymbol{\rho}}_m, \boldsymbol{\rho'}_{-m}) \geq \nu$ for all $\boldsymbol{\rho'}_{-m}$ in some open ϵ-neighborhood U_{-m}^{ϵ} of $\boldsymbol{\rho}_{-m}$.

In view of the above definition, an FAP m can secure a certain payoff at $\boldsymbol{\rho}$ if it has a strategy that guarantees at least that payoff even if other players deviate slightly.

Definition 3: A game is *better-reply secure* if for every $(\boldsymbol{\rho}, \boldsymbol{\mathcal{U}})$ in $\bar{\Gamma}$ where $\boldsymbol{\rho}$ is a non-PSNE vector, some FAP m can *secure a payoff strictly greater* than $\mathcal{U}_m(\boldsymbol{\rho})$ that it achieves at $\boldsymbol{\rho}$.

In essence, a game is said to be better-reply secure if whenever $\boldsymbol{\rho}$ is a nonequilibrium vector achieving a utility $\boldsymbol{\mathcal{U}}$, some FAP m possesses a strategy which would provide a payoff strictly better than $\mathcal{U}_m(\boldsymbol{\rho})$ even if all other players deviate slightly from $\boldsymbol{\rho}_{-m}$.

Lemma 1: For every $\boldsymbol{\rho}_{-m}$, the utility $\mathcal{U}_m(., \boldsymbol{\rho}_{-m})$ is quasi-concave in $\boldsymbol{\rho}_m$, for all m.

Proof: See the Appendix. ■

Lemma 2: The SAP game is better-reply secure.

Proof: See the Appendix. ■

Theorem 1: The SAP game admits a PSNE.

Proof: For every m, the strategy space $\mathcal{X}_m$ is nonempty and compact (a closed and bounded subset of the Euclidean space). Also, $\mathcal{U}_m(\boldsymbol{\rho})$ is bounded for all m. Those facts coupled with the results of Lemmas 1 and 2 guarantee that Theorem 3.1 in [10], which characterizes discontinuous games possessing PSNE, holds true. Thus, the SAP game has PSNE. ■

C. Distributed Best Response Algorithm

Given the discontinuities in the utility functions, it is difficult to obtain closed-form expressions for the PSNE solutions. Thus, we propose a *distributed best response* algorithm that can be implemented by the FAPs to reach a PSNE solution while optimizing their strategies. The essence of the proposed algorithm is to enable the FAPs to update their strategies, given their view on the access modes used by all the other FAPs, at any point in time. Thus, we develop a distributed algorithm based on best response in order to find the equilibrium access policies.

The proposed algorithm is shown in Table I. The proposed algorithm uses a *parallel update* technique in which, at any iteration i, each FAP computes its optimal strategy given its observation of the network at $i-1$. The proposed algorithm starts by selecting an initial strategy vector $\boldsymbol{\rho}^{(0)}$ for the FAPs. In each iteration i, each FAP m searches for the optimal set of MUEs $\mathcal{N}_m^{(i)}$ that it can serve, given its view on the access policies (i.e., strategies) of all of the other FAPs which were obtained in the previous iteration $\boldsymbol{\rho}_m^{(i-1)}$. The set $\mathcal{N}_m^{(i)}$ is selected so as to maximize the utility of the FUE of FAP m over $\mathcal{X}_m$ (note that $\mathcal{N}_m^{(i)}$ can be $\emptyset$). The MUEs in $\mathcal{N}_m^{(i)}$ are allocated subbands as per (7), using the logic in the proof of Lemma 2.

To find its best response, each FAP needs to identify the optimal subset of MUEs to admit, if it chooses an open access strategy. To do so, the FAP needs to check its potential utility from servicing a certain subset of MUEs. In practice, instead of testing all possible sets of MUEs, which can be complex, the FAP could find this optimal subset using a branch and bound or a greedy algorithm. An FAP has, in general, a limited coverage area in which the number of MUEs is often reasonable, and, hence, identifying the

TABLE I
PROPOSED ALGORITHM

> Select a random initial strategy vector $\boldsymbol{\rho}^{(0)}$.
> For all $n \in \mathcal{N}$, $m \in \mathcal{M}$, compute $\rho_n^{m\star}$ as per (7).
> **iterate**
>   **for** $m = 1 \rightarrow M$
>   Fix $\boldsymbol{\rho}_{-m}^{(i-1)}$.
>   Select the optimal set of users $\mathcal{N}_m^{(i)}$ to be served by FAP m.
>   Set $\rho_n^{m,(i)} = \rho_n^{m\star}, \forall n \in \mathcal{N}_m^{(i)}$. Set $\rho_n^{m,(i)} = 0, \forall n \notin \mathcal{N}_m^{(i)}$.
>   **end**
>   **if** $\rho_n^{m,(i)} > 0$ for multiple values of m
>   Pair the n-th MUE to FAP j to which μ_n^j is highest.
>   Set $\rho_n^{-j,(i)} = 0$.
>   **end**
> **until convergence to a PSNE vector** $\boldsymbol{\rho}^{\star}$

best response would require an acceptable complexity. In this respect, a simple greedy algorithm in which the FAP starts by accepting the top interfering MUEs first (a similar approach was used in [3] for handoff) could be adopted for finding the optimal response while reducing complexity. At the end of each iteration, the algorithm ensures that no MUE is being served by multiple FAPs. The MUE is paired with the FAP to which it has the best channel as characterized by μ_n^m. Note that if the remaining FAPs still allocate resources to this MUE, they will be at a disadvantage as they would be wasting resources. The above steps are repeated until convergence. In general, best response based algorithms such as the one proposed in Table I have been shown to converge to an NE for many classes of noncooperative games; many modified schemes have also been proposed to ensure convergence [9].

IV. SIMULATION RESULTS

Consider a network of FAPs and MUEs that are scattered uniformly over a 250 m $\times$ 250 m square. We set the noise power added at the terminals of the FAPs to $\sigma_m^2 = -110$ dBm, for all m. The transmit power of all FUEs and MUEs is fixed at 100 mW. The wall penetration loss is set to $L = 0.5$, and the path loss factors are set to $\alpha = 3$, $\beta = 2$. All MUEs have a minimum target rate requirement of $R_{min}^c = 5$ bits. We fix $d_{0m} = 1$ m for all FAPs. All statistical results are averaged over the random channels and locations of all nodes.

We will benchmark the performance of our algorithm through comparisons with two different schemes. In the first scheme, referred to as the *all closed* scheme, all FAPs use closed access at all times. In the second scheme, all FAPs use open access while optimizing their allocated resources, as per (7), in a manner similar to our algorithm. We refer to this scheme by *optimized open access*. The latter is a particular case of our proposed scheme in which the FAPs choose to employ open access and allocate resources (if possible) without seeking equilibrium or stability; this scheme is used as the initial point for our scheme.

In Fig. 1, we show the fraction of FAPs that choose an open policy at the PSNE resulting from the proposed approach for networks with 7 MUEs and 10 MUEs as the number of FAPs varies. In this figure, we can see that the fraction of FAPs choosing open access starts by increasing because deploying more FAPs leads to more opportunities for open access. However, this fraction starts decreasing for $M \geq 6$ when $N = 7$, and for $M \geq 8$ when $N = 10$. For $N = 7$, it reaches a maximum of 64% and then starts by decreasing to reach 34.5%. For $N = 10$, it reaches a

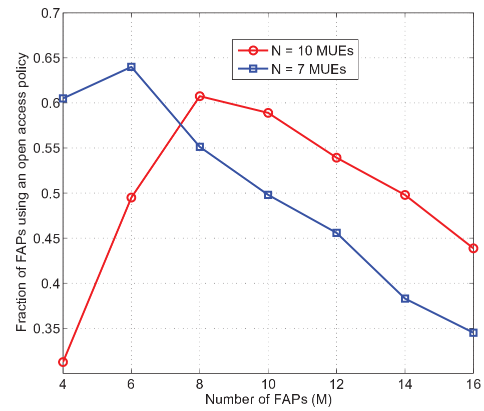

Fig. 1. Fraction of FAPs using open access as the number of FAPs varies.

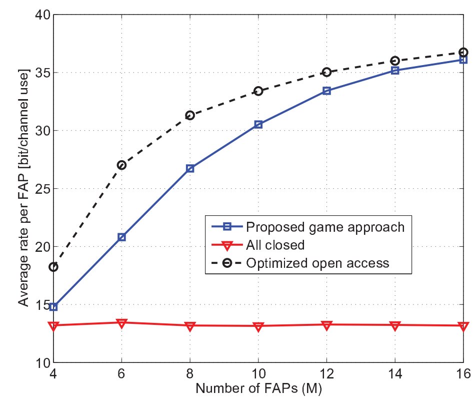

Fig. 2. Average rate per FAP resulting from the proposed algorithm as the number of FAPs M varies for a network with 10 MUEs.

maximum of 60.8% and then starts by decreasing to reach 43.9%. This is due to the fact that for a given number of MUEs, as the number of FAPs becomes much larger than the number of MUEs, the additional FAPs tend to remain closed as they rely on other FAPs to service the interfering MUEs. Clearly, most equilibria are composed of *mixed* access policies: a fraction of FAPs choosing open access and another fraction choosing closed access, with this fraction dependent on various parameters such as M or N.

In Fig. 2, we assess the performance of the PSNE resulting from the proposed algorithm by showing the average utility per FAP as the network size varies, for 10 MUEs. First, we can see that as M increases, the average utility per FAP for the proposed scheme and the optimized open access scheme increases. This is due to the fact that as more FAPs are deployed, there exists more opportunities to use open access and service highly interfering MUEs. In contrast, the all closed scheme yields an almost constant average utility at all network sizes. Fig. 2 shows that the proposed scheme yields significant gains with respect to the all closed scheme reaching up to 173.8% at $M = 16$ FAPs. This figure also highlights the interesting tradeoff between stability (PSNE) and performance. For small networks, the optimized open access scheme outperforms the PSNE state. However, this scheme is not stable, in the Nash sense. This is because the optimized open access scheme is the starting point of our algorithm, and the FAPs were able to improve their utilities by unilaterally deviating from it. Nonetheless, the performance gap

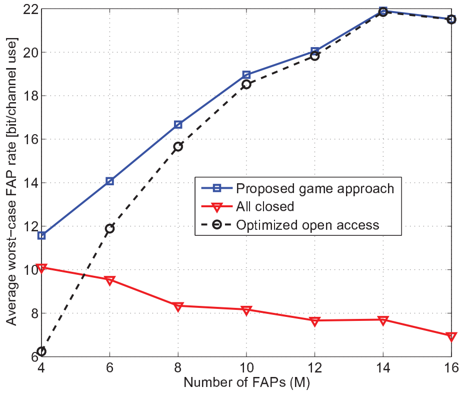

Fig. 3. Average worst-case FAP rate resulting from the proposed algorithm as the number of FAPs M varies for a network with 10 MUEs.

is reasonable. Moreover, as more FAPs are deployed, the Nash solution coincides with the optimized open access network.

In Fig. 3, we show the average worst-case utility. This demonstrates that, although our scheme has a performance gap in the average rate when compared to the optimized open access scheme as in Fig. 2, it can improve the worst-case FAP's performance. Our scheme reaches an improvement of 85.4% compared to optimized open access at $M = 4$; it also reaches an improvement of 208.9% over the all closed scheme at $M = 16$. This is a result of the selfish nature of the FAPs as captured by the PSNE solution. The PSNE ensures that no FAP can do better by unilaterally deviating from the equilibrium; hence, it is expected that, when acting strategically, no FAP will make a decision that decrease its own utility for the advantage of another, although this decision may also be detrimental to the overall welfare of the network.

V. Conclusions

In this paper, we have introduced a novel game-theoretic framework which enables the FAPs to strategically decide on their uplink access policies. Due to the absence of coordination among FAPs, we have formulated a noncooperative game in which the FAPs strategically optimize the rates of their home FUEs, given the tradeoff between reducing the cross-tier interference and the associated cost due to sharing their resources. We have applied novel analytical techniques to prove the existence of the Nash equilibrium solution for the proposed game in which the utility functions are discontinuous. Moreover, we have proposed a low-complexity distributed algorithm that can be adopted by the FAPs to reach their equilibrium access policies through parallel updates. Simulation results assessed the performance of the proposed approach in various settings.

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Appendix

A. Proof of Lemma 1: A function $f(x)$ is said to be quasiconcave if every superlevel set $S_\nu = \{x | f(x) \geq \nu\}$ is a convex set. Define the set $N_m = \{n : \rho_n^{-m} = 0\}$. When $\nu \leq 0$, $\mathcal{U}_m(\boldsymbol{\rho}) \geq 0$ implies that $\boldsymbol{\rho}_m \in \mathcal{X}_m$. The strategy space of FAP m is a convex set since it is the intersection of an $(N+1)$-dimensional simplex and halfspaces which are convex sets. Hence, the superlevel sets in this case are convex sets. When $0 < \nu \leq \log(1 + \frac{\gamma_m}{\sigma_m^2 + \sum_{n \in N_m} \mu_n^m})$, where the upper bound on ν corresponds to the payoff obtained when FAP m employs closed access, $\mathcal{U}_m(\boldsymbol{\rho}) \geq \nu$ implies that $0 \leq \mathbf{1}^T \boldsymbol{\rho}_m \leq 1 - \frac{\nu}{\log(1+\frac{\gamma_m}{\sigma_m^2})}$. The superlevel sets in this case are also convex sets because they are intersections of halfspaces. Similarly, when $\nu > \log(1 + \frac{\gamma_m}{\sigma_m^2 + \sum_{n \in N_m} \mu_n^m})$, $\mathcal{U}_m(\boldsymbol{\rho}) \geq \nu$ implies that $0 < \mathbf{1}^T \boldsymbol{\rho}_m \leq 1 - \frac{\nu}{\log(1+\frac{\gamma_m}{\sigma_m^2})}$. The superlevel sets in this case are also convex sets. Finally, if $\nu > \log(1 + \frac{\gamma_m}{\sigma_m^2})$, then $S_\nu = \emptyset$ which is also a convex set. The proof of the lemma is thus complete. ∎

B. Proof of Lemma 2: Consider a nonequilibrium vector $\boldsymbol{\rho}$. Given the constraints of (6) and the fact that $\mathcal{U}_m(\boldsymbol{\rho})$ is monotonically decreasing in $\rho_n^m > 0$, we can find the optimal subband allocation by FAP m to the n-th MUE (in open access):

$$\rho_n^{m\star} = \frac{R_{min}^c}{\log\left(1 + \frac{\mu_n^m}{\sigma_m^2}\right)}. \quad (7)$$

Let $\bar{\boldsymbol{\rho}}_m$ be the resulting strategy after transforming $\boldsymbol{\rho}_m$ as follows: **I-a)** Set $\bar{\rho}_n^m = 0$ whenever $\rho_n^{-m} > 0$. This ensures that whenever the n-th MUE is allocated a subband by an FAP, FAP m takes advantage of the fact that an MUE can only connect to one FAP and refrains from spending extra resources; **I-b)** Set $\bar{\rho}_n^m = \rho_n^{m\star}$ whenever $\rho_n^m > 0$ and $\rho_n^{-m} = 0$. This ensures that FAP m allocates the smallest possible subband to the n-th MUE. Clearly, $\mathcal{U}_m(\bar{\boldsymbol{\rho}}_m, \boldsymbol{\rho}_{-m}) > \mathcal{U}_m(\boldsymbol{\rho})$, where the assumption that $\boldsymbol{\rho}$ is nonequilibrium was crucial in finding $\bar{\boldsymbol{\rho}}_m$. Let U_{-m}^ϵ be an ϵ-neighborhood, $\epsilon > 0$, of $\boldsymbol{\rho}_{-m}$ given by $U_{-m}^\epsilon = \{\boldsymbol{\acute{\rho}}_{-m} \in \mathcal{X}_{-m} : \left|\left|\boldsymbol{\rho}_{-m} - \boldsymbol{\acute{\rho}}_{-m}\right|\right| < \epsilon\}$. We make the following two observations: **II-a)** We can always select an $\epsilon > 0$ such that if $\boldsymbol{\rho}_{-m} \ni \rho_n^\ell > 0$, it still holds that $\boldsymbol{\acute{\rho}}_{-m} \ni \acute{\rho}_n^\ell > 0$. Hence, by I-a, $\bar{\boldsymbol{\rho}}_m$ would still yield a better payoff for FAP m even if such deviations occur; **II-b)** If $\boldsymbol{\rho}_{-m} \ni \rho_n^{-m} = 0$, then $\acute{\rho}_n^{-m} > 0$. In this case, the utility of FAP m will not be affected as, by I-b, that MUE is allocated a subband at $\bar{\boldsymbol{\rho}}_m$. By II-a and II-b, we readily conclude that $\mathcal{U}_m(\bar{\boldsymbol{\rho}}_m, \boldsymbol{\acute{\rho}}_{-m}) \geq \mathcal{U}_m(\bar{\boldsymbol{\rho}}_m, \boldsymbol{\rho}_{-m})$. Hence, FAP m can *secure* a payoff $\mathcal{U}_m(\bar{\boldsymbol{\rho}}_m, \boldsymbol{\rho}_{-m})$ at $\bar{\boldsymbol{\rho}}_m$ which is strictly greater than $\mathcal{U}_m(\boldsymbol{\rho})$. Thus, the SAP game is better-reply secure. ∎