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Abstract

There has been substantial interest over the last decade in developing low complexity decentralized scheduling algorithms in wireless networks. In this context, the queue-length based Carrier Sense Multiple Access (CSMA) scheduling algorithms have attracted significant attention because of their attractive throughput guarantees. However, the CSMA results rely on the mixing of the underlying Markov chain and their performance under fading channel states is unknown.

In this work, we formulate a partially decentralized randomized scheduling algorithm for a two transmitter receiver pair set up and investigate its stability properties. Our work is based on the Fast-CSMA (FCSMA) algorithm first developed in [1] and we extend its results to a signal to interference noise ration(SINR) based interference model in which one or more transmitters can transmit simultaneously while causing interference to the other. In order to improve the performance of the system, we split the traffic arriving at the transmitter into *schedule based queues* and combine it with the FCSMA based scheduling algorithm. We theoretically examine the performance our algorithm in both non-fading and fading environment and characterize the set of arrival rates which can be stabilized by our proposed algorithm.

I. INTRODUCTION

We consider the problem of decentralized channel access for the two user interference channel. Classical information theoretic approach assumes that the transmitters are always saturated with information bits. However, in this work, we consider the randomness in arrival of information bits and hence account for the queuing backlog at the transmitters. We consider that the transmission rates of each transmitter-receiver pair are a function of the signal to interference ration (SINR) at the receiver.

The work in this paper is comparable to the stream of works related to scheduling algorithms in wireless networks which operate at the packet level and assume that a fixed number of packets can be transmitted per time slot . The task then is to schedule a set of non-conflicting links for transmission (*conflict graph* based interference model) in order to ensure the long term stability of the associated queues in the network. The authors in the seminal work of [2] developed a maximum-weight based scheduling strategy which is proved to be throughput-optimal. However, the max-weight based algorithms are centralized in nature and suffer from high computational complexity. Subsequently low-complexity, decentralized, and possibly suboptimal scheduling algorithms were developed in series of works [3],[4],[5] with varying complexities and performances. In particular, recently a class of randomized scheduling algorithms namely the CSMA-based scheduling algorithms ([6],[7],[8]) have received a lot of attention because of their attractive throughput guarantees. However, the CSMA based scheduling algorithms rely on the mixing of the underlying Markov chain which cannot be guaranteed in a fading environment. Hence their performance in fading environment is not known.

Specifically, we develop a FCSMA (Fast-CSMA) based scheduling algorithm that extends the earlier results to the SINR-based interference model. The FCSMA operation has advantage over the CSMA based scheduling algorithms under fading conditions in that it quickly reaches one of the favorable schedules and sticks to it rather than relying on the convergence of the underlying Markov chain . Hence, the FCSMA based algorithm can perform well under fading environment as well.

We first note that the straightforward application of FCSMA to the SINR based interference model has a low performance. In order to improve the performance of this scheme, we formulate a dynamic rule to split the incoming traffic into *schedule based queues* at the transmitters and combine it with the FCSMA scheduling. By favorably tuning the control parameter of the traffic splitting rule, we prove that the FCSMA based algorithm along with the appropriate traffic splitting rule can provide a good performance.

Finally, we would like to mention reference [9] a decentralized queue-length dependent probabilistic scheduling algorithm for the two user multi-access channel. However, the analysis of the algorithm is done assuming that the channel realization stays constant through out and hence assumes a non-fading scenario. In contrast, we analyze our system under fading environment as well.

II. System Model

We consider a set up in which two transmitters (Tx) are trying to communicate to their respective receivers (Rx) over a common frequency band. We assume that the system operates in a time slotted fashion. We denote $A_i[t]$ as the amount of information bits that flow into the Tx$_i$ during each time slot t. The arrival process is assumed to be independent across users and independently and identically distributed over time slots with a rate of $\lambda_i, \; i = 1, 2,$ and $A_i[t] \leq K, \; \forall t$. Accordingly, there is queue associated with Tx$_i$ whose queue-length at time slot t is denoted by the notation $Q_i[t]$. Let $S_i[t]$ denote the number of information bits served from the queue of Tx$_i$ during the time slot t. The equation for the queue-length evolution is given by

$$Q_i[t+1] = Q_i[t] + A_i[t] - S_i[t] + U_i[t] \tag{1}$$

where $U_i[t]$ denotes the unused service, , $0 < U_i[t] \leq 1$ if $Q_i[t] \leq 1$ and is selected for service, else $U_i[t] = 0$. We say that a queue is stable if $\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbf{E}\left[Q_i[t]\right] < \infty$.

We consider the SINR based interference model in which one or more transmitters can transmit simultaneously. In this case, the maximum achievable transmission rate for any Tx-Rx depends on the SINR at the Rx. In general, the transmission rate for a Tx-Rx pair during any time slot can be chosen from a continuous set. However, in order to simplify the analysis, we allow two levels of rates for every Tx-Rx pair. First, a rate of R_i when only one two transmitters transmitting (while the other transmitter is turned off) and a rate r_i when both the transmitters transmitting simultaneously (in which case, they cause interference to each other). These rates correspond to the three possible scheduling decisions in the set $\mathbf{\Omega} = \{\omega_1, \omega_2, \omega_3\}$ where the rates obtained in the three scheduling decisions are given by $\{R_1, 0\}, \{0, R_2\}, \{r_1, r_2\}$ respectively. A reasonable assumption is that the maximum achievable rate is an increasing function of the SINR. Hence, we assume that the rates $r_1 \leq R_1$ and $r_2 \leq R_2$. The stability region for this system can be given as the convex hull of the possible transmission rates.

$$\mathbf{\Lambda} = \Big\{\lambda_1 < \pi_1 R_1 + \pi_3 r_1, \quad \lambda_2 < \pi_2 R_2 + \pi_3 r_2 \\ \sum_{i=1}^{3} \pi_i \leq 1, \; \pi_i \geq 0\Big\}$$

The stability region of the system is shown in Figure 1. Additionally, we note the condition $\frac{r_1}{R_1} + \frac{r_2}{R_2} \geq 1$, which ensures that the stability region goes beyond the time sharing region.

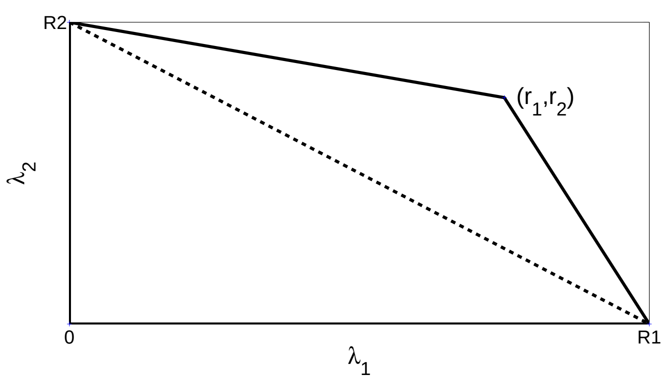

Fig. 1. Stability region for the 2 User System

The objective of this work is to design a decentralized throughput optimal scheduling algorithm in which the transmitters cannot exchange the full CSI of the UTs.

III. FCSMA Algorithm Description

The FCSMA based scheduling algorithm operates in the following way. At the beginning of time slot t, each Tx independently generates two timers whose values are an exponentially distributed random variable with mean $Q_i[t]R_i$ and $Q_i[t]r_i$ respectively. These timers correspond to the respective scheduling decisions in which the Tx$_i$ can achieve a non zero rate. We assume that each Tx maintains a one bit index for the timers associated with it. Let us assume that the index of 0 corresponds to the timer $Q_i[t]R_i$ and an index of 1 indicates that timer $Q_i[t]r_i$.

The system has four timers. Without the loss of generality, assume that one of the timers associated with Tx_1 expires first among the four timers. The algorithm operates in the following manner. Tx_1 immediately suspends its second timer (which has not yet expired) and starts to transmit bits from its queue at the appropriate rate (rate R_1 if timer 0 expires or a rate r_1 if timer 1 expires). Tx_1 communicates the index of timer which has expired to Tx_2. Upon receiving the index bit, Tx_2 also suspends both its timers. We assume the following pre-agreed protocol between the two Txs. Upon reception of the index 0, the Tx_2 keeps silent during corresponding time slot t. Upon reception of the index 1, the Tx_2 transmits from its queue at the rate r_2. We ignore the overhead associated with communicating the bit between the two Txs. The state diagram for the FCSMA based scheduling algorithm is shown in Figure 2. The probabilities

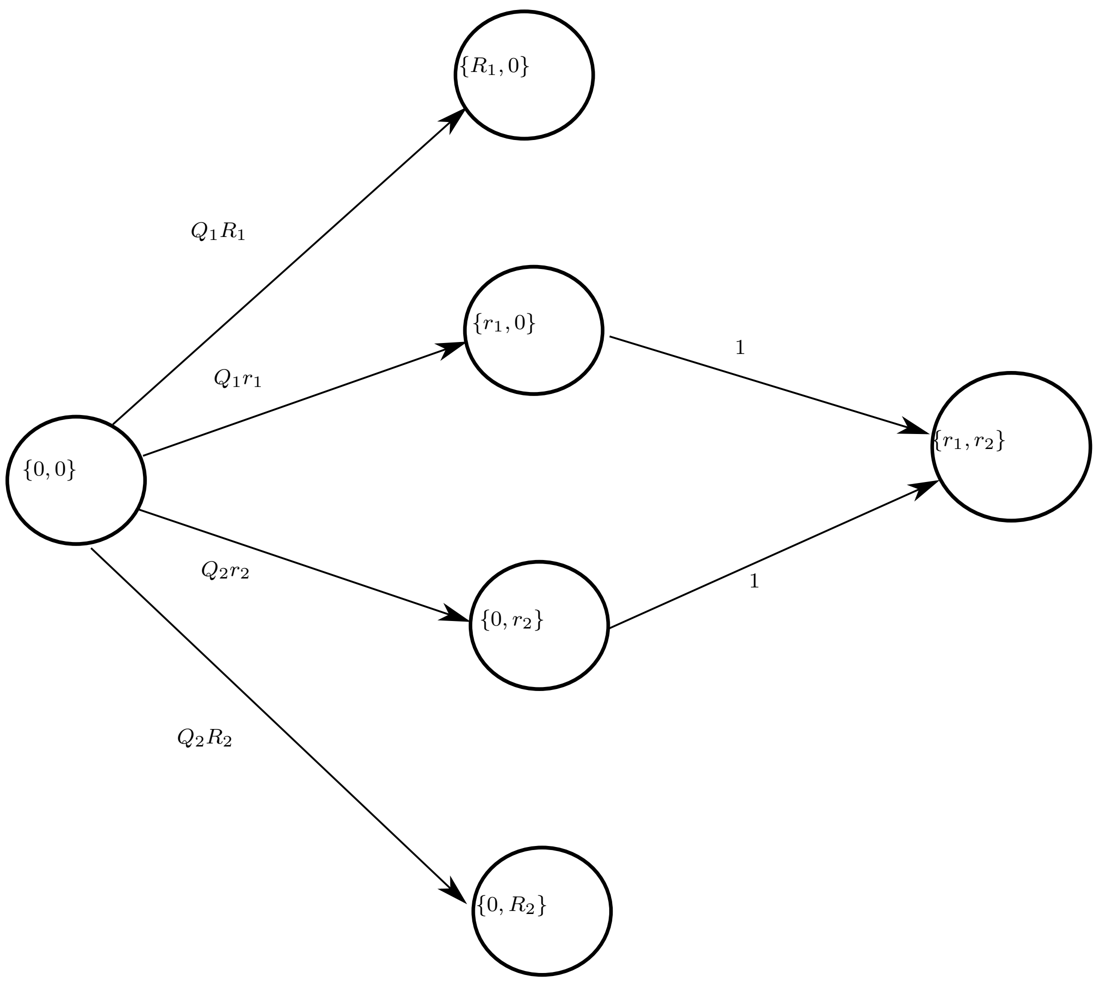

Fig. 2. FCSMA State Diagram

of reaching each of the three possible schedules during a time slot t is given by the following expressions.

$$\mathbb{P}_{\omega_1}(\mathbf{Q}) = \frac{R_1 Q_1}{\sum_{i=k}^{2} Q_k (R_k + r_k)}, \quad \mathbb{P}_{\omega_2}(\mathbf{Q}) = \frac{R_2 Q_2}{\sum_{k=1}^{2} Q_k (R_k + r_k)}$$
$$\mathbb{P}_{\omega_3}(\mathbf{Q}) = \frac{\sum_{k=1}^{2} r_k Q_k}{\sum_{k=1}^{2} Q_k (R_k + r_k)} \tag{2}$$

Additionally, the expected value of service rate for the queue at Tx$_i$ during the time slot t can be given by

$$\mathbf{E}\left[S_i[t] \middle| \mathbf{Q}[t] = \mathbf{Q}\right] = \frac{R_i^2 Q_i + r_i \sum_{k=1}^{2} r_k Q_k}{\sum_{i=k}^{2} Q_k (R_k + r_k)}, \; i = 1, 2 \tag{3}$$

Proposition 1. *Consider a 2-user perfectly symmetric network in which* $R_1 = R_2 = 1$ *and* $r_1 = r_2 = \alpha$. *(Note* $0 \leq \alpha \leq 1$.*) When the mean rate of the arrival process into the two Txs are the same (i.e.,* $\lambda_1 = \lambda_2 = \lambda$*), the maximum arrival rate which can be supported by the FCSMA scheduling algorithm is given by*

$$\lambda < \frac{\alpha^2}{\alpha + 1} + \frac{1}{2(\alpha + 1)} \tag{4}$$

Proof: The proof proceeds by considering a quadratic Lyapunov function of the form

$$V(\mathbf{Q}[t]) = \frac{1}{2} \sum_{i=1}^{2} Q_i^2[t]$$

and examining the value of λ for which the Lyapunov drift is negative outside a bounded set. Here, we only provide the essential technical arguments of the proof analyze the term

$$\dot{V}(\mathbf{Q}[t]) = \sum_{i=1}^{2} Q_i \dot{Q}_i[t], \qquad \dot{Q}_i[t] = \lambda - \mathbf{E}\left[S_i[t] \middle| \mathbf{Q}[t] = \mathbf{Q}\right]$$

which loosely represents the Lyapnov drift in continuous time. We examine the range of λ for the which this quantity is negative.

$$\begin{aligned}\dot{V}(\mathbf{Q}[t]) &= \sum_{i=1}^{2} Q_i \dot{Q}_i[t] \\ &= \sum_{i=1}^{2} Q_i \left(\lambda - \frac{Q_i + \alpha^2 \sum_{k=1}^{2} Q_k}{\sum_{i=k}^{2} Q_k (1 + \alpha)} \right)\end{aligned}$$

$$= \frac{\lambda(1+\alpha)\left(\sum_{i=1}^{2} Q_i\right)^2 - \left(\sum_{i=1}^{2} Q_i^2 + \alpha^2 \left(\sum_{k=1}^{2} Q_k\right)^2\right)}{\sum_{i=k}^{2} Q_k(1+\alpha)}$$

$$= \frac{\lambda((1+\alpha)-\alpha^2)\left(\sum_{i=1}^{2} Q_i\right)^2 - \left(\sum_{i=1}^{2} Q_i^2\right)}{\sum_{i=k}^{2} Q_k(1+\alpha)} \tag{5}$$

$$\overset{(a)}{\leq} 0 \qquad \text{for} \qquad \frac{\alpha^2}{\alpha+1} \leq \lambda \leq \frac{\alpha^2}{\alpha+1} + \frac{1}{2(\alpha+1)} \tag{6}$$

where (a) follows from the following inequality. For $x, y, \beta_1, \beta_2 \geq 0$,

$$\begin{aligned} \beta_1(x+y)^2 - \beta_2(x^2+y^2) &= (\beta_1-\beta_2)x^2 + (\beta_1-\beta_2)y^2 + 2\beta_1 xy \\ &\leq (\beta_1-\beta_2)x^2 + (\beta_1-\beta_2)y^2 + \beta_1(x^2+y^2) \\ &= (2\beta_1-\beta_2)(x^2+y^2) \end{aligned} \tag{7}$$

Let us denote the numerator term of (5) as $g(\mathbf{Q}) \triangleq \lambda((1+\alpha)-\alpha^2)\left(\sum_{i=1}^{2} Q_i\right)^2 - \left(\sum_{i=1}^{2} Q_i^2\right)$. The condition $\beta_1 > 0$ implies that $\lambda \geq \frac{\alpha^2}{\alpha+1}$. Note that $Q_1 \geq 0$ and $Q_2 \geq 0$. Rearranging the term inside the brackets of (5), it can be verified that

$$g(\mathbf{Q}) \leq 0 \text{ for } \lambda \leq \frac{\alpha^2}{\alpha+1} + \frac{1}{2(\alpha+1)} \tag{8}$$

Combining with the condition $\lambda \geq \frac{\alpha^2}{\alpha+1}$, we have

$$g(\mathbf{Q}) \leq 0 \text{ for } \frac{\alpha^2}{\alpha+1} \leq \lambda \leq \frac{\alpha^2}{\alpha+\delta} + \frac{1}{2(\alpha+1)} \tag{9}$$

Notice that the range of λ specified in (8) is just a sufficient condition $g(\mathbf{Q}) \leq 0$. We now claim that

$$g(\mathbf{Q}) \leq 0 \qquad \text{for} \qquad 0 \leq \lambda \leq \frac{\alpha^2}{\alpha+1} + \frac{1}{2(\alpha+1)} \tag{10}$$

We justify our claim in the following way. Let us define the upper bound on λ in (10) as $\lambda_{\max}$. Notice that $g(\mathbf{Q})$ is an increasing function of λ for a fixed value of the queue-lengths Q_1, Q_2. Therefore, $g(\mathbf{Q})\big|_{\lambda} \leq g(\mathbf{Q})\big|_{\lambda=\lambda_{\max}} \leq 0$ for $\lambda \leq \lambda_{\max}$ and hence the claim of (10).

Notice that the bound of (38) was obtained considering a fixed value of Q_1 and Q_2. However, the argument is true for any positive value of Q_1 and Q_2. Hence repeating the arguments for any Q_1 and Q_2, we conclude that the Lyapunov drift is negative for all positive values of queue-lengths and $\lambda \leq \lambda_{\max}$. We hence claim that the algorithm can stabilize the traffic whose arrival rate is less than $\lambda_{\max}$. The exact arguments and the connection to the Foster Lyapunov theorem is deferred till the proof of Theorem 2. ∎

Remarks on the FCSMA algorithm: The FCSMA based scheduling algorithm described above is a partially decentralized algorithm in which the Txs exchange one bit information (index of the timer that expires first). This calls for a substantially less overhead of information exchange between the Txs as compared to exchanging the full CSI. From the plot of the stability region in Figure 1, notice that the maximum achievable rate in the symmetric case is $\lambda_1 = \lambda_2 = \lambda$ is $\lambda < \alpha$. However, the bound specified in (4) is lesser than α. In what follows, we overcome this problem by combining the FCSMA scheduling scheme with a dynamic traffic splitting algorithm.

IV. FCSMA WITH DYNAMIC TRAFFIC SPLITTING ALGORITHM

In this section, we introduce the concept of *schedule based queues* to split the input traffic arriving into the Txs. Each Tx maintains two different queues one for each scheduling decision. For the Tx$_i$, the queue Q_{ii} corresponds to the first scheduling decision in which the Tx$_i$ can transmit at the higher rate R_i. When selected for service, this queue gets a service rate of R_i. Let us define $\bar{i} = \mod (i, 2) + 1$. The second queue $Q_{i\bar{i}}$ corresponds to the scheduling decision in which both the Txs have joint access to the channel and when selected for service, gets a rate of r_i. The traffic splitting policy can be described as follows. During the time slot t, each transmitter compares the current queue-lengths $Q_{ii}[t]$ and $\delta_i Q_{i\bar{i}}[t]$ where $\delta_i \geq 0$ is a scaling factor. If $Q_{ii}[t] < \delta_i Q_{i\bar{i}}[t]$, the information bits arriving in the respective slot enter the queue Q_{ii} and vice versa. Accordingly,

$$\lambda_{ii} = \mathbf{E}[A_{ii}[t]] = \begin{cases} \lambda_i & \text{if} \quad \delta_i Q_{i\bar{i}}[t] > Q_{ii}[t] \\ 0 & \text{else} \end{cases}$$

$$\lambda_{i\bar{i}} = \mathbf{E}[A_{i\bar{i}}[t]] = \begin{cases} \lambda_i & \text{if} \quad \delta_i Q_{i\bar{i}}[t] \leq Q_{ii}[t] \\ 0 & \text{else} \end{cases} \tag{11}$$

The scheduling algorithm is exactly the same as the FCSMA algorithm described in Section II B except that the two timers associated with the Tx$_i$ are exponential random variables with mean $Q_{ii}[t]R_i$ and $Q_{i\bar{i}}[t]r_i$ respectively (note that the queue-length values associated with the two mean values are different). The probabilities of each scheduling decision in this case are

$$\mathbb{P}_{\omega_1}\left(\mathbf{Q}\right)=\frac{Q_{11}R_{11}}{\sum_{k=1}^{2}Q_{kk}R_k+Q_{k\bar{k}}r_k},\mathbb{P}_{\omega_2}\left(\mathbf{Q}\right)=\frac{Q_{22}[t]R_{22}}{\sum_{k=1}^{2}Q_{kk}R_k+Q_{k\bar{k}}r_k}$$

$$\mathbb{P}_{\omega_3}\left(\mathbf{Q}\right)=\frac{\sum_{k=1}^{2}Q_{k\bar{k}}r_k}{\sum_{k=1}^{2}Q_{kk}R_k+Q_{k\bar{k}}r_k} \tag{12}$$

Also, the expected service rate for each queue is given by

$$\mathbf{E}\left[S_{ii}[t]\middle|\mathbf{Q}[t]=\mathbf{Q}\right]=\frac{Q_{ii}R_i^2}{\sum_{k=1}^{2}\left(Q_{kk}R_i+Q_{k\bar{k}}r_k\right)}$$

$$\mathbf{E}\left[S_{i\bar{i}}[t]\middle|\mathbf{Q}[t]=\mathbf{Q}\right]=\frac{r_i\sum_{k=1}^{2}Q_{k\bar{k}}r_k}{\sum_{k=1}^{2}\left(Q_{kk}R_k+Q_{k\bar{k}}r_k\right)},\qquad i=1,2$$

Having defined a dynamic traffic splitting policy described above, the next task is to examine theoretically the set of arrival rate which can be can be stabilized by our algorithm. To do the same, we define a Lyapunov function and examine its properties for different values of the queue-lengths. In order to make things more amenable for theoretical analysis, we restrict our proofs to a perfectly symmetric system model.

Theorem 2. *Consider a 2-user perfectly symmetric network described in Proposition 1. When the mean rate of the arrival process into the two transmitters are the same (i.e., $\lambda_1=\lambda_2=\lambda$), the maximum arrival rate which can be supported by the traffic splitting policy described in equation* (11) *followed by the FCSMA scheduling algorithm is given by*

$$\lambda<\frac{\alpha^2}{\alpha+\delta}+\frac{\delta}{2(\alpha+\delta)} \tag{13}$$

Proof: Consider the Lyapunov function given by

$$V(\mathbf{Q}[t])=\frac{1}{2}\sum_{i=1}^{2}\left(Q_{ii}^2[t]+\delta Q_{i\bar{i}}^2[t]\right) \tag{14}$$

where $\bar{i}=\mod(i,2)+1$. Our approach to finding the maximum supportable rate is to examine the drift of the Lyapunov function and determine the maximum value of the arrival rate λ for which the Lyapunov drift is negative outside a bounded region around the origin. In doing so, we bound the Lyapunov function by a series of upper bounds and take the most restrictive condition on the arrival rate λ.

The Lyapunov drift in discrete time is given by

$$\Delta V(\mathbf{Q}[t]) = \mathbf{E}\left[V(\mathbf{Q}[t+1]) - V(\mathbf{Q}[t]) | \mathbf{Q}[t] = \mathbf{Q}\right]$$

where $\mathbf{Q} = [Q_{11}, Q_{12}, Q_{21}, Q_{22}]^T$. Applying mean value theorem, considering $R_{ij}[t]$ between $Q_{ij}[t]$ and $Q_{ij}[t+1]$,

$$\begin{aligned}
\Delta V(\mathbf{Q}[t]) &= \sum_{i=1}^{2} \mathbf{E}\left[R_{ii}[t](Q_{ii}[t+1] - Q_{ii}[t]) + \delta R_{i\bar{i}}[t]\left(Q_{i\bar{i}}[t+1]) - Q_{i\bar{i}}[t]\right) \Big| \mathbf{Q}(t) = \mathbf{Q}\right] \\
&= \sum_{i=1}^{2} \mathbf{E}\left[R_{ii}[t](\dot{Q}_{ii}[t] + U_{ii}[t]) + \delta R_{i\bar{i}}[t](\dot{Q}_{i\bar{i}}[t] + U_{i\bar{i}}[t]) \Big| \mathbf{Q}(t) = \mathbf{Q}\right]
\end{aligned}$$

$$= \underbrace{\sum_{i=1}^{2} \mathbf{E}\left[R_{ii}[t]U_{ii}[t] + \delta R_{i\bar{i}}[t]U_{i\bar{i}}[t] \Big| \mathbf{Q}(t) = \mathbf{Q}\right]}_{\Delta V_1(\mathbf{Q}[t])} \tag{15}$$

$$+ \underbrace{\sum_{i=1}^{2} \mathbf{E}\left[R_{ii}[t]\dot{Q}_{ii}[t] + \delta R_{i\bar{i}}[t]\dot{Q}_{i\bar{i}}[t] \Big| \mathbf{Q}(t) = \mathbf{Q}\right]}_{\Delta V_2(\mathbf{Q}[t])} \tag{16}$$

Let us denote the term in equation (15) as $\Delta V_1(\mathbf{Q}[t])$ and (16) as $\Delta V_2(\mathbf{Q}[t])$. Consider

$$\Delta V_1(\mathbf{Q}[t]) = \sum_{i=1}^{2} \mathbf{E}\left[R_{ii}[t]U_{ii}[t] + \delta R_{i\bar{i}}[t]U_{i\bar{i}}[t] \Big| \mathbf{Q}(t) = \mathbf{Q}\right] \tag{17}$$

We would like the bound the terms of $\Delta V_1(\mathbf{Q}[t])$. First note that if $Q_{ij}[t] = Q_{ij} > 1$ then $U_{ij}[t] = 0$. Else if $Q_{ij}[t] = Q_{ij} < 1$ and is selected for service then $0 < U_{ij} \leq 1$. In this case $Q_{ij}[t+1] < K+1$ (because $A_{ij}[t+1] < K$)and hence

$$\Delta V_1(\mathbf{Q}[t]) \leq b_1 K \tag{18}$$

where b_1 is a bounded positive constant. Now consider the terms of $\Delta V_2(\mathbf{Q}[t])$. Rewriting, we have,

$$\begin{aligned}
\Delta V_2(\mathbf{Q}[t]) = &\underbrace{\sum_{i=1}^{2} \mathbf{E}\left[R_{ii}[t]\dot{Q}_{ii}[t] + \delta R_{i\bar{i}}[t]\dot{Q}_{i\bar{i}}[t] \Big| \mathbf{Q}(t) = \mathbf{Q}\right] \mathbf{1}_{\mathbf{Q} \leq \mathbf{M}}}_{\Delta V_3(\mathbf{Q}[t])} \\
+ &\underbrace{\sum_{i=1}^{2} \mathbf{E}\left[R_{ii}[t]\dot{Q}_{ii}[t] + \delta R_{i\bar{i}}[t]\dot{Q}_{i\bar{i}}[t] \Big| \mathbf{Q}(t) = \mathbf{Q}\right] \mathbf{1}_{\mathbf{Q} > \mathbf{M}}}_{\Delta V_4(\mathbf{Q}[t])}
\end{aligned}$$

where $\mathbf{1}_{(.)}$ is the indicator function. Note that since $A_{ij}[t] \leq K$, we can also conclude that $|A_{ij}[t] - S_{ij}[t]| \leq K$, therefore,

$$\Delta V_3(\mathbf{Q}[t]) \leq b_4(K+M)K \tag{19}$$

In order to bound the terms of $\Delta V_4(\mathbf{Q}[t])$, first note that for a sufficiently large value of $Q_{ij}[t] = Q_{ij} > M$, we have $\left|\frac{R_{ij}[t]}{Q_{ij}} - 1\right| < \epsilon$ and therefore,

$$(1-\epsilon)Q_{ij} \leq R_{ij}[t] \leq (1+\epsilon)Q_{ij}$$

Thus, we have

$$\begin{aligned}
&R_{ij}[t]\dot{Q}_{ij}[t] = R_{ij}[t]\left(A_{ij}[t] - S_{ij}[t]\right) \\
&= R_{ij}[t]\left((A_{ij}[t] - S_{ij}[t])_+ - (A_{ij}[t] - S_{ij}[t])_-\right) \\
&< (1+\epsilon)Q_{ij}(A_{ij}[t] - S_{ij}[t])_+ - (1-\epsilon)Q_{ij}(A_{ij}[t] - S_{ij}[t])_- \\
&= Q_{ij}(A_{ij}[t] - S_{ij}[t]) + \epsilon Q_{ij}[t]\Big|A_{ij}[t] - S_{ij}[t]\Big| \\
&\leq Q_{ij}\dot{Q}_{ii}[t] + \epsilon K Q_{ij}
\end{aligned} \tag{20}$$

where $(x)_+ = \max\{x, 0\}$, $(x)_- = -\min\{x, 0\}$ and $\Big|A_{ij}[t] - S_{ij}[t]\Big| \leq A_{ij}[t] \leq K$. Therefore, we have

$$\sum_{i=1}^{2} R_{ii}[t]\dot{Q}_{ii}[t] + \delta R_{i\bar{i}}[t]\dot{Q}_{i\bar{i}}[t] \leq \sum_{i=1}^{2} Q_{ii}\dot{Q}_{ii}[t] + \delta Q_{i\bar{i}}\dot{Q}_{i\bar{i}}[t] + K\epsilon \sum_{i=1}^{2}(Q_{ii} + \delta Q_{i\bar{i}}) \tag{21}$$

We focus on the first term on the right hand side of equation (21). Let us denote

$$\Delta V_5[t] \triangleq \sum_{i=1}^{2} \mathbf{E}\left[Q_{ii}\dot{Q}_{ii}[t] + \delta Q_{i\bar{i}}\dot{Q}_{i\bar{i}}[t]\Big|\mathbf{Q}(t) = \mathbf{Q}\right] \tag{22}$$

where

$$\begin{aligned}
\Delta V_5[t] =& Q_{11}\left(\mathbf{E}[A_{11}(t)] - \frac{Q_{11}}{B(\mathbf{Q})}\right) + \delta Q_{12}\left(\mathbf{E}[A_{12}(t)] - \frac{\alpha^2(Q_{12}+Q_{21})}{B(\mathbf{Q})}\right) \\
&+ Q_{22}\left(\mathbf{E}[A_{22}(t)] - \frac{Q_{22}}{B(\mathbf{Q})}\right) + \delta Q_{21}\left(\mathbf{E}[A_{21}(t)] - \frac{\alpha^2(Q_{12}+Q_{21})}{B(\mathbf{Q})}\right)
\end{aligned} \tag{23}$$

$B(\mathbf{Q}) = Q_{11} + \alpha(Q_{12} + Q_{21}) + Q_{22}$. Depending on the relationship between the queue lengths, we need to consider the four cases for the Lyapunov function (see equation (11)). These four cases throw up a series of bounds on λ under which the right hand side of equation (23) is negative. We take the most restrictive condition of all the bounds as the upper bound on the maximum supportable rate.

Case1: $Q_{11} \leq \delta Q_{12}; Q_{22} \leq \delta Q_{21}$.

In this case,

$$\begin{aligned}
(23) = & Q_{11}\left(\lambda - \frac{Q_{11}}{B(\mathbf{Q})}\right) + \delta Q_{12}\left(\frac{\alpha^2(Q_{12}+Q_{21})}{B(\mathbf{Q})}\right) \\
& + Q_{22}\left(\lambda - \frac{Q_{22}}{B(\mathbf{Q})}\right) + \delta Q_{21}\left(\frac{\alpha^2(Q_{12}+Q_{21})}{B(\mathbf{Q})}\right) \\
= & \frac{f_1(\mathbf{Q})}{B(\mathbf{Q})}
\end{aligned}$$

where

$$f_1(\mathbf{Q}) = \lambda\left(Q_{11}+Q_{22}\right)\left(Q_{11}+\alpha(Q_{12}+Q_{21})+Q_{22}\right) - \left(Q_{11}^2 + \delta\alpha^2(Q_{12}+Q_{21})^2 + Q_{22}^2\right)$$

Let us examine the behavior of the function $f_1(\mathbf{Q})$ with respect to the variables Q_{12} and Q_{21} for a fixed value of Q_{11} and Q_{22}. Writing the gradients of the function $f_1(\mathbf{Q})$ with respect to the variables Q_{12} and Q_{21},

$$\begin{aligned}
\frac{\partial f_1(\mathbf{Q})}{\partial Q_{12}} &= \alpha\lambda\left(Q_{11}+Q_{22}\right) - 2\alpha^2\delta(Q_{12}+Q_{21}) \\
&\leq \alpha\lambda\delta\left(Q_{12}+Q_{21}\right) - 2\alpha^2\delta(Q_{12}+Q_{21}) \\
&= Q_{12}\left(\alpha\lambda\delta - 2\alpha^2\delta\right) + Q_{21}\left(\alpha\lambda\delta - 2\alpha^2\delta\right) \\
&\leq 0 \qquad \text{for} \qquad \lambda \leq 2\alpha
\end{aligned} \tag{24}$$

Similarly taking the gradients with respect to Q_{21}, we have,

$$\frac{\partial f_1(\mathbf{Q})}{\partial Q_{21}} \leq 0 \qquad \text{for} \qquad \lambda \leq 2\alpha \tag{25}$$

Therefore, $f_1(\mathbf{Q})$ is a decreasing function of both Q_{12} and Q_{21}. For a given value of Q_{11} and Q_{22}, the function $f_1(\mathbf{Q})$ is maximized when $\delta Q_{12} = Q_{11}$ and $\delta Q_{21} = Q_{22}$ (hitting the boundary conditions of case 1). Therefore,

$$\begin{aligned}
f_1(\mathbf{Q}) &\leq f_1(\mathbf{Q})\Big|_{\delta Q_{12}=Q_{11}, \delta Q_{21}=Q_{22}} \qquad \text{for} \qquad \lambda \leq 2\alpha \\
&= \lambda(Q_{11}+Q_{22})\left(\left(1+\frac{\alpha}{\delta}\right)(Q_{11}+Q_{22})\right) - \left(Q_{11}^2 + Q_{22}^2 + \alpha^2\delta\left(\frac{(Q_{11}+Q_{22})^2}{\delta^2}\right)\right) \\
&= (Q_{11}+Q_{22})^2\left(\lambda\left(1+\frac{\alpha}{\delta}\right) - \frac{\alpha^2}{\delta}\right) - (Q_{11}^2+Q_{22}^2) \\
&\overset{(a)}{\leq} \left(2\left(\lambda\left(1+\frac{\alpha}{\delta}\right) - \frac{\alpha^2}{\delta}\right) - 1\right)(Q_{11}^2+Q_{22}^2)
\end{aligned} \tag{26}$$

where (a) follows from the inequality (7). The condition $\beta_1 \geq 0$ implies that $\lambda \geq \frac{\alpha^2}{\alpha+\delta}$. Note that $Q_{11} \geq 0$ and $Q_{22} \geq 0$. Rearranging the term inside the brackets of (26), it can be verified that

$$f_1(\mathbf{Q}) \leq 0 \text{ for } \lambda \leq \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)} \tag{27}$$

Combining with the condition $\lambda \geq \frac{\alpha^2}{\alpha+\delta}$, we have

$$f_1(\mathbf{Q}) \leq 0 \text{ for } \frac{\alpha^2}{\alpha+\delta} \leq \lambda \leq \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)} \tag{28}$$

Notice that the range of λ specified in (27) is just a sufficient condition $f_1(\mathbf{Q}) \leq 0$. We now claim that

$$f_1(\mathbf{Q}) \leq 0 \qquad \text{for} \qquad 0 \leq \lambda \leq \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)} \tag{29}$$

We justify our claim in the following way. Let us define the upper bound on λ in (29) as $\lambda_{\max}$. Notice the expression on the right hand side of equation (26) is an increasing function of λ for a fixed value of the queue-lengths Q_{11}, Q_{22}. Therefore, $f_1(\mathbf{Q})\big|_{\lambda} \leq f_1(\mathbf{Q})\big|_{\lambda=\lambda_{\max}} \leq 0$ for $\lambda \leq \lambda_{\max}$ and hence the claim of (29).

Also, it can be verified that

$$\frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)} \leq 2\alpha$$

(the bound of $\lambda \leq 2\alpha$ is obtained from (25)). Hence, in this case, we have that for a given value of Q_{11} and Q_{22},

$$\Delta V_5[t] \leq 0 \qquad 0 \leq \lambda \leq \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)} \tag{30}$$

Notice that the result of (30) is true for any value of Q_{11} and Q_{22}. Hence repeating the argument for any Q_{11} and Q_{22}, we conclude that (30) is negative for all values of positive value of queue-length and the range of λ specified.

Case2: $Q_{11} \geq \delta Q_{12}; Q_{22} \geq \delta Q_{21}$. In this case,

$$\begin{aligned}
(23) =& Q_{11}\left(\frac{Q_{11}}{B(\mathbf{Q})}\right) + \delta Q_{12}\left(\lambda - \frac{\alpha^2(Q_{12}+Q_{21})}{B(\mathbf{Q})}\right) \\
&+ Q_{22}\left(\frac{Q_{22}}{B(\mathbf{Q})}\right) + \delta Q_{21}\left(\lambda - \frac{\alpha^2(Q_{12}+Q_{21})}{B(\mathbf{Q})}\right) \\
=& \frac{f_2(\mathbf{Q})}{B(\mathbf{Q})}
\end{aligned}$$

where

$$f_2(\mathbf{Q}) = \lambda\delta\left(Q_{12}+Q_{21}\right)\left(Q_{11}+\alpha(Q_{12}+Q_{21})+Q_{22}\right) - \left(Q_{11}^2+\delta\alpha^2(Q_{12}+Q_{21})^2+Q_{22}^2\right)$$

Once again, we would like to examine the behavior of the function $f_2(\mathbf{Q})$ for fixed values of Q_{11} and Q_{22}. Writing the gradients of the function $f_2(\mathbf{Q})$ with respect to the variables Q_{12} and Q_{21},

$$\begin{aligned}\frac{\partial f_2(\mathbf{Q})}{\partial Q_{12}} &= \lambda\delta\left(Q_{11}+Q_{22}\right)+(2\alpha\lambda\delta-2\alpha^2\delta)(Q_{12}+Q_{21}) \\ &\geq \lambda\delta^2\left(Q_{12}+Q_{21}\right)+(2\alpha\lambda\delta-2\alpha^2\delta)(Q_{12}+Q_{21}) \\ &\geq 0 \qquad \text{for} \qquad \lambda \geq \frac{2\alpha^2}{2\alpha+\delta} \end{aligned} \tag{31}$$

The function $f_2(\mathbf{Q})$ is an increasing function of Q_{12} for $\lambda \geq \frac{2\alpha^2}{2\alpha+\delta}$. Similarly it can be shown that

$$\frac{\partial f_2(\mathbf{Q})}{\partial Q_{21}} \geq 0 \qquad \text{for} \qquad \lambda \geq \frac{2\alpha^2}{2\alpha+\delta} \tag{32}$$

$f_2(\mathbf{Q})$ is an increasing function of both Q_{12} and Q_{21} for the range of λ specified. Therefore for a given value of Q_{11} and Q_{22}, the function $f_2(\mathbf{Q})$ is maximized when $\delta Q_{12} = Q_{11}$ and $\delta Q_{21} = Q_{22}$. Once again, repeating the arguments like that of case 1 (equation (26)), we have

$$f_2(\mathbf{Q}) \leq 0 \qquad \text{for} \qquad \lambda \leq \frac{\alpha^2}{\alpha+\delta}+\frac{\delta}{2(\alpha+\delta)} \tag{33}$$

From the analysis of Case 2, there are two bounds on λ(from equations (31) and (33)). It can be verified that for $\delta \geq 0$,

$$\frac{2\alpha^2}{2\alpha+\delta} \leq \frac{\alpha^2}{\alpha+\delta}+\frac{\delta}{2(\alpha+\delta)}$$

and hence,

$$\Delta V_5[t] \leq 0 \qquad \text{for} \qquad \frac{2\alpha^2}{2\alpha+\delta} \leq \lambda \leq \frac{\alpha^2}{\alpha+\delta}+\frac{\delta}{2(\alpha+\delta)} \tag{34}$$

Once again, we can make arguments similar to that of case 1 and extend the inequality in (34) for all positive values of the queue-lengths.

Case3: $Q_{11} \geq \delta Q_{12}; Q_{22} \leq \delta Q_{21}$.

In this case,

$$
\begin{aligned}
(23) = & Q_{11}\left(\lambda - \frac{Q_{11}}{B(\mathbf{Q})}\right) + \delta Q_{12}\left(\frac{\alpha^2(Q_{12}+Q_{21})}{B(\mathbf{Q})}\right) \\
& + Q_{22}\left(\frac{Q_{22}}{B(\mathbf{Q})}\right) + \delta Q_{21}\left(\lambda - \frac{\alpha^2(Q_{12}+Q_{21})}{B(\mathbf{Q})}\right) \\
= & \frac{f_3(\mathbf{Q})}{B(\mathbf{Q})}
\end{aligned}
$$

where

$$
f_3(\mathbf{Q}) = \lambda\left(Q_{11}+\delta Q_{21}\right)\left(Q_{11}+\alpha(Q_{12}+Q_{21})+Q_{22}\right) - \left(Q_{11}^2+\delta\alpha^2(Q_{12}+Q_{21})^2+Q_{22}^2\right)
$$

Writing the gradients of the function $f_3(\mathbf{Q})$ with respect to the variables Q_{12} and Q_{21},

$$
\begin{aligned}
\frac{\partial f_3(\mathbf{Q})}{\partial Q_{12}} &= \lambda\delta Q_{11} + \lambda(\delta+\alpha)Q_{22} + (2\alpha\lambda\delta - 2\alpha^2\delta)Q_{12} + (\alpha\lambda\delta - 2\alpha^2\delta)Q_{21} \\
&\geq \lambda\delta^2 Q_{21} + \lambda(\delta+\alpha)\delta Q_{21} + (2\alpha\lambda\delta - 2\alpha^2\delta)Q_{12} + (\alpha\lambda\delta - 2\alpha^2\delta)Q_{21} \\
&\geq 0 \qquad \text{for} \qquad \lambda \geq \frac{2\alpha^2}{2\alpha+\delta}
\end{aligned}
$$

Therefore, $f_3(\mathbf{Q})$ is an increasing function of Q_{12} for the range of λ specified and is maximized when $\delta Q_{12} = Q_{22}$. Examining the behavior of $f_3(\mathbf{Q})$ with respect to Q_{21},

$$
\begin{aligned}
\frac{\partial f_3(\mathbf{Q})}{\partial Q_{21}} &= (\alpha\lambda\delta - 2\alpha^2\delta)Q_{12} - 2\alpha^2\delta Q_{21} + \lambda\alpha Q_{22} \\
&\leq (\alpha\lambda\delta - 2\alpha^2\delta)Q_{12} + (\alpha\lambda\delta - 2\alpha^2\delta)Q_{21} \\
&\leq 0 \qquad \text{for} \qquad \lambda \leq 2\alpha
\end{aligned}
$$

$$
\frac{\partial f_3(\mathbf{Q})}{\partial Q_{21}} \leq 0 \qquad \text{for} \qquad \lambda \leq 2\alpha
$$

$f_3(\mathbf{Q})$ is a decreasing function of Q_{21} and hence is maximized when $\delta Q_{21} = Q_{22}$. Therfore for a given value of Q_{11} and Q_{22}, the function $f_3(\mathbf{Q})$ is maximized when $\delta Q_{12} = Q_{11}$ and $\delta Q_{21} = Q_{22}$.

$$
f_3(\mathbf{Q}) \leq 0 \qquad \text{for} \qquad 0 \leq \lambda \leq \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)} \tag{35}
$$

Once again, we have that

$$
\Delta V_5[t] \leq 0 \qquad \text{for} \qquad \frac{2\alpha^2}{2\alpha+\delta} \leq \lambda \leq \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)} \tag{36}
$$

Case4: $Q_{11} \geq \delta Q_{12}; Q_{22} < \delta Q_{21}$.

Case4 can be analyzed exactly similar to Case3. Once again it can be shown that the function is maximized when $\delta Q_{12} = Q_{11}$ and $\delta Q_{21} = Q_{22}$.

Summary: From the analysis of cases 1- 4, we have shown that

$$\Delta V_5[t] \leq 0 \qquad \text{for} \qquad \frac{2\alpha^2}{2\alpha+\delta} \leq \lambda \leq \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)} \tag{37}$$

for any positive values of the queue-lengths. Notice that the range of λ specified in (37) is just a sufficient condition for the Lyapunov drift to be negative. We now claim that

$$\Delta V_5[t] \leq 0 \qquad \text{for} \qquad 0 \leq \lambda \leq \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)} \tag{38}$$

We justify our claim in the following way. Notice that

$$\Delta V_5[t] = 0 \qquad \text{for} \qquad \lambda = \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)}$$

Also notice from equation (23) that $\Delta V_5[t]$ is an increasing function of λ for a fixed value of the queue-lengths ($Q_{11}, Q_{12}, Q_{21}, Q_{22}$). Hence

$$\Delta V_5[t]\Big|_{\lambda} \leq \Delta V_5[t]\Big|_{\lambda = \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)}} \qquad \text{for} \qquad \lambda \leq \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)} \tag{39}$$

This is true for a given value of queue-lengths. But recall that this argument can be extended for any value of the value of the queue-lengths. Therefore, (39) is true of all values of the queue-lengths and hence the claim of (38). Also note that

$$\Delta V_5[t] < 0 \qquad \text{for} \qquad 0 \leq \lambda < \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)}$$

In other words, for $\mathbf{Q} \in \mathbb{R}^+$ and $0 \leq \lambda < \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)}$,

$$\lambda \left(Q_{11}\mathbf{1}_{Q_{11}\leq \delta Q_{12}} + \delta Q_{12}\mathbf{1}_{Q_{11}\geq \delta Q_{12}} + \delta Q_{21}\mathbf{1}_{Q_{22}\leq \delta Q_{21}} + Q_{22}\mathbf{1}_{Q_{22}\geq \delta Q_{12}}\right)\left(Q_{11} + \alpha(Q_{12}+Q_{21}) + Q_{22}\right) - \left(Q_{11}^2 + \delta\alpha^2(Q_{12}+Q_{21})^2 + Q_{22}^2\right) < 0$$

Denoting

$$g(\mathbf{Q}) \triangleq \left(Q_{11}\mathbf{1}_{Q_{11}\leq \delta Q_{12}} + \delta Q_{12}\mathbf{1}_{Q_{11}\geq \delta Q_{12}} + \delta Q_{21}\mathbf{1}_{Q_{22}\leq \delta Q_{21}} + Q_{22}\mathbf{1}_{Q_{22}\geq \delta Q_{12}}\right) \tag{40}$$

Therefore, there exists a $\eta > 0$ such that,

$$\left(Q_{11}^2 + \delta\alpha^2(Q_{12}+Q_{21})^2 + Q_{22}^2\right) \geq \lambda(1+\eta)g(\mathbf{Q})\left(Q_{11} + \alpha(Q_{12}+Q_{21}) + Q_{22}\right) \tag{41}$$

From (21) and (41), we have

$$\Delta V_4(\mathbf{Q}[t]) \le (-\eta + K\epsilon) g(\mathbf{Q}) \tag{42}$$

By choosing ϵ sufficiently small, we can have $\gamma = -\eta + K\epsilon > 0$, such that

$$\Delta V_4(\mathbf{Q}[t]) \le -\gamma g(\mathbf{Q}) \tag{43}$$

Combining the result of (18),(19) and (43), we have

$$\Delta V(\mathbf{Q}[t]) = -\gamma g(\mathbf{Q}) \mathbf{1}_{\mathbf{Q}>\mathbf{M}} + C \qquad \text{for} \qquad \lambda \le \frac{\alpha^2}{\alpha+\delta} + \frac{\delta}{2(\alpha+\delta)}$$

where $C < \infty$ is a bounded positive constant. By Foster-Lyapunov theorem [10], implies that the queue-lengths are bounded for the FCSMA algorithm with $\lambda \le \lambda_{\max}$. ■

Remarks on the dynamic traffic splitting algorithm: Note that the result of Theorem 2 can be generalized to the case when the data rates are $\{R, 0\}, \{r, r\}, \{0, R\}$. In this case any rate $\lambda < \frac{r^2}{r+\delta R} + \frac{\delta R^2}{2(r+\delta R)}$ can be stabilized. The FCSMA based algorithm along with the dynamic traffic splitting algorithm provides us with a tunable parameter δ which can be varied in order to achieve better performance. Specifically when $\lambda_1 = \lambda_2 = \lambda$, by setting $\delta = 0$, from the result of Theorem 2 that any rate $\lambda < \alpha$ can be stabilized by the system (which is also the maximum achievable rate in the symmetric arrival case from the plot of the stability region). The parameter δ can be calculated based on the point inside the stability region in which we are operating. Our theoretical analysis was limited to the case of symmetric arrivals $(\lambda_1 = \lambda_2 = \lambda)$. Based on the analysis we can conjecture that by suitably varying δ, any rate point inside the stability region can be stabilized by our algorithm. The proof is left for future investigation.

V. Fading Channels

Now consider a symmetric block fading model where the channel realization is fixed during the time slot but changes after every time slot t. The set of channels in the network can assume a state $s = \{1, \ldots, S\}$ according to stationary probability p_s. We denote the cardinality of the set by $|S|$ and $\sum_{s=1}^{|S|} p_s = 1$. In each time slot t, the achievable rate for the three possible scheduling decisions are $\{\{R_s, 0\}, \{r_s, r_s\}, \{0, R_s\}\}$ if the network is in fading state s at time slot t. In this scenario, when $\lambda_1 = \lambda_2 = \lambda$, the maximum rate that can be stabilized by the FCSMA policy along with traffic splitting algorithm is given by

$$\lambda < \sum_{s=1}^{|S|} p_s \left(\frac{r_s^2}{r_s + \delta R_s} + \frac{\delta R_s^2}{2(r_s + \delta R_s)} \right) \tag{44}$$

Proof: Consider the quadratic Lyapunov function given by $V(\mathbf{Q}[t]) = \sum_{i=1}^{2} \frac{1}{2} \left(Q_{ii}^2 + \delta Q_{i\bar{i}}^2[t] \right)$. We once again analyze only the following expression of the Lyapunov drift given by

$$\begin{aligned}\dot{V}(\mathbf{Q}) = \sum_{i=1}^{2} Q_{ii} \left(\mathbf{E}[A_{ii}(t)] - \sum_{s=1}^{|S|} p_s \left(\frac{R_s^2 Q_{ii}}{B_s(\mathbf{Q})} \right) \right) \\ + \delta Q_{i\bar{i}} \left(\mathbf{E}[A_{i\bar{i}}(t)] - \sum_{s=1}^{|S|} p_s \left(\frac{r_s^2 \left(\sum_{k=1}^{2} Q_{k\bar{k}} + Q_{k\bar{k}} \right)}{B_s(\mathbf{Q})} \right) \right)\end{aligned} \tag{45}$$

where $B_s(\mathbf{Q}) = R_s Q_{11} + r_s(Q_{12} + Q_{21}) + R_s Q_{22}$. Let us denote the maximum supportable arrival rate in the fading case by the notation $\lambda_{\max}$. We will first analyze the Laypunov drift term at $\lambda = \lambda_{\max}$. Notice that $\lambda_{\max}$ can be written as a convex combination of λ^s (where λ^s are some rate points inside the stability region on the line $\lambda_1 = \lambda_2$) and hence $\lambda_{\max} = \sum_{s=1}^{|S|} p_s \lambda^s$. Therefore, we can rewrite the Lyapunov drift as

$$\begin{aligned}\dot{V}(\mathbf{Q}) = \sum_{s=1}^{|\mathcal{S}|} p_s \sum_{i=1}^{2} \lambda^s (Q_{ii} \mathbf{1}_{Q_{ii} \leq \delta Q_{i\bar{i}}} + \delta Q_{i\bar{i}} \mathbf{1}_{Q_{ii} \geq \delta Q_{i\bar{i}}}) \\ - \left(\frac{R_s^2 Q_{ii} + r_s^2 \left(\sum_{k=1}^{2} Q_{k\bar{k}} + Q_{k\bar{k}} \right)}{B_s(\mathbf{Q})} \right)\end{aligned} \tag{46}$$

From the proof of Theorem 2 , we have proved that each of the terms inside the summation for is negative (every channel state) as long as

$$\lambda^s < \frac{r_s^2}{r_s + \delta R_s} + \frac{\delta R_s^2}{2(r_s + \delta R_s)}$$

and hence from the above observation and $\lambda_{\max} = \sum_{s=1}^{|S|} p_s \lambda^s$, we have the result of (44). Also note that (45) is an increasing function of λ for a given value of queue-lengths. Hence, $\dot{V}(\mathbf{Q})\big|_{\lambda \leq \lambda_{\max}} < \dot{V}(\mathbf{Q})\big|_{\lambda = \lambda_{\max}}$ for $\lambda < \lambda_{\max}$. Also, this argument holds for any value of the queue-lengths. Therefore, $\dot{V}(\mathbf{Q}) \leq 0$ for $\lambda < \lambda_{\max}$ and for all values of queue-length and hence any rate $\lambda < \lambda_{\max}$ is stabilizable. ■

VI. Conclusion

In this work, we have formulated a partially decentralized randomized scheduling algorithm for a two user set up under a SINR based interference model. In our algorithm, the transmitters have to exchange only one bit information between themselves. Our algorithm has advantage over existing scheduling algorithms since it is decentralized in nature and can perform well under fading conditions.

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