RANDOM MATRIX THEORY RATE LEARNING FOR COGNITIVE SMALL CELLS

Abla Kammoun[1], *Romain Couillet*[2], *Mérouane Debbah*[1]

Alcatel Lucent Chair, Supélec[1], Supélec[2]

ABSTRACT

This paper addresses the problem of maximum data rate learning in small cells networks. Considering a shared carrier deployment, small cell users have to adapt their energy in such a way to not disturb macro-cellular communications. In such a context, small cell users would probably undergo unacceptable levels of interference, thereby considerably affecting their performance. The objective of our work is to propose a method for fast prediction of these events and their corresponding maximum achievable data rates. This can help small cell users to select the optimal transmission strategy.

***Index Terms*—** Mutual information estimation, G-estimation, Random matrix theory.

1. INTRODUCTION

With the growing demands for wireless high data rate applications, current 3G cellular networks are experiencing real difficulties for meeting capacity needs. Given the fast rate at which applications evolved, a considerable increase in data traffic is predicted, [1].

Despite its interesting features, Long Term Evolution (LTE), the significantly faster technology, suffers like its predecessors from serious restrictions, namely, a limited coverage and a reduced capacity on the cell edge. The principal reason can be attributed to the macro-cellular structure itself, in which a macro cell covers a large area and essentially ensures the coverage for outdoor users. On the contrary, indoor users which are estimated to contribute to 90% of all data traffics [11] suffer from a weak coverage.

To circumvent these limitations, an attractive solution consists in shrinking current cells by deploying micro or pico cells with a reduced coverage range of ten to several hundreds of meters, thereby creating a massively dense network of several small cells. Although proposed initially to solve the problem of weak indoor coverage, small cells can be deployed either in outdoor (pico-cells) or indoor environments (femto-cells). Clearly, if we consider their ease of integration to the existing infrastructure as well as their fully compatibility with macro-cells, small cell networks remain among the most promising techniques.

However, when it comes to small cells deployment, a major issue is represented by the way spectrum resources are allocated to small cells networks. Three main options exist [4]:

- Separate carrier deployment: Macro-cell and small cell networks are dedicated separate spectrum.
- Shared carrier deployment: Small cell networks are allowed to share the same spectrum as the macro-cells
- Partially shared carrier deployment: Small cell networks are allowed to share only a fraction of the spectrum.

Obviously, shared and partially shared carriers deployments are less expensive and allow to reach better spectral efficiency, the price to be paid is a high inter-tier interference between small cell and macro-cell users. This calls for interference management strategies that will principally serve to reduce the interference caused by small cell users on macro-cell users. Among the various types of interference management strategies, we distinguish two classes, namely, interference cancellation techniques and interference avoidance techniques [7]. In the context of small cells, interference cancellation methods are in general discarded in favor of interference avoidance based techniques, for they assume knowledge of the interfering signal and are more difficult to implement. On the other hand, interference avoidance techniques rely on power control methods in which, each user in the small cell network perform self-adaption of its power in such a way to sufficiently protect the macro-cellular network [7]. In this line of research, several works have proposed optimal power policies that guarantees sufficient protection of the macro-cellular network, [8, 5]. These works suppose in general a perfect estimation of the power level of the interfering signal, a hypothesis which rarely holds especially when the length of the observation window is reduced.

Taking a look on the previous research on this field reveals that there is a special focus on how interference affect the over-all system capacity [6, 2]. In this paper, we will consider a different point of view. More explicitly, we assume that each user in the small cell adjusts its power according to an interference avoidance based algorithm in such a way that it guarantees little interference to the macro-cell users. Communication between small cell users should be preceded by a short sensing period, in which they estimate their mutual information and adapt accordingly their rate. Obviously, the

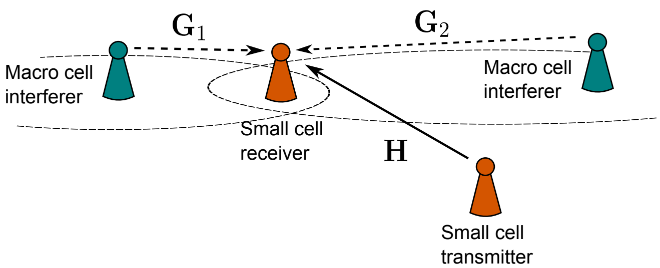

Fig. 1: System Model

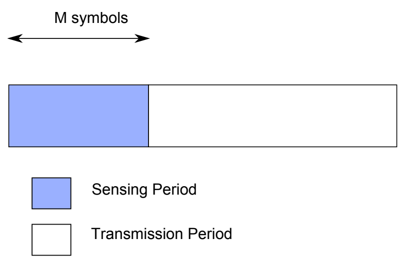

Fig. 2

length of the sensing period will play a key role in the estimation of the maximum achievable rate. Since the sensing period does not correspond to data transmission, the small cell transmitter should make it as short as possible while ensuring an acceptable estimation quality of its mutual information. The objective of our work is to propose efficient techniques that provide accurate maximum rate estimates even for short sensing periods compared to the spatial dimensions, and to compare their performance with that of the commonly used techniques.

This paper is organized as follows: we describe in section 2 the system model. Then we present in section 3 and section 4 the first order results of the proposed and traditional estimators. Finally, we provide in section 5 numerical simulations that support the accuracy of the derived results.

2. SYSTEM MODEL

Consider a communication link between two users of the small cell network, a receiver equipped with N antennas and a transmitter. Also assume that the communication link is disturbed by the presence of K interferers of the macro-cell network. and that, each interferer is equipped with n_k transmitting antennas.

Figure 1 describes this scenario in the case of two interfering macro-cell users. We assume that the transmission phase is preceded by a sensing period where the transmitter probes its surrounding environment(see fig.2).

Let $\mathbf{H}$ denote $N \times n_0$ multiple-input multiple output (MIMO) channel matrix between the small cell transmitter and the small cell receiver, and $\mathbf{G}_k$ the MIMO channel between the k^{th} macro cell transmitter and the small cell receiver. It is natural to assume that the small cell receiver knows perfectly $\mathbf{H}$ but knows nothing on $\mathbf{G}_k$.

Denote by M the length of the sensing period. During this period, the received signal correspond to the incurred interference and noise. Concatenating M signal received vectors, the received matrix writes as:

$$\mathbf{Y} = \sum_{k=1}^{K} \mathbf{G}_k \mathbf{X}_k + \sigma \mathbf{W}$$

where $\mathbf{X}_k$ and $\mathbf{W}$ stand for the signal emitted by the k^{th} macro-cell users and the noise matrix.

The maximum achievable rate I for a noise power σ^2 reads:

$$I = \frac{1}{N} \log\det\left(\mathbf{I}_N + \frac{1}{\sigma^2}\left(\mathbf{G}\mathbf{G}^{\text{H}} + \mathbf{H}\mathbf{H}^{\text{H}}\right)\right) - \frac{1}{N}\log\det\left(\mathbf{I}_N + \frac{1}{\sigma^2}\mathbf{G}\mathbf{G}^{\text{H}}\right) \quad (1)$$

where $\mathbf{G} = [\mathbf{G}_1, \cdots, \mathbf{G}_K]$.

If M, the number of available observations is too large as compared to the number of receiver antennas, the unknown interference plus noise covariance matrix can be substituted by its standard empirical estimate $\frac{1}{M}\mathbf{Y}\mathbf{Y}^{\text{H}}$. Hence, the following mutual information estimator :

$$\hat{I} = \frac{1}{N}\log\det\left(\frac{1}{M}\mathbf{Y}\mathbf{Y}^{\text{H}} + \mathbf{H}\mathbf{H}^{\text{H}}\right) - \frac{1}{N}\log\det(\frac{1}{M}\mathbf{Y}\mathbf{Y}^{\text{H}})$$

is consistent, i.e, converges rapidly to I as $M \to \infty$.

However, in practice, the sensing period should be made as short as possible, thereby implying M is of the same order as N. Based on tools borrowed from random matrix theory, we provide in this paper a consistent estimate for the mutual information in the regime $N, M, n = \sum_{k=1}^{K} n_k, n_0 \to \infty$ with :

$$\begin{cases} 0 < \liminf \frac{N}{n} \leq \limsup \frac{N}{n} < +\infty, \\ 1 < \liminf \frac{M}{N} \leq \limsup \frac{M}{N} < +\infty, \\ 0 < \liminf \frac{n_0}{N} \leq \limsup \frac{n_0}{N} < +\infty. \end{cases} \quad (2)$$

This regime will be simply referred to as $M, N \to +\infty$.

But before that, we shall study the behavior of the standard mutual information estimator $\hat{I}$. In particular, we will prove that it is biased and compute its asymptotic bias.

3. TRADITIONAL ESTIMATOR

Under the asymptotic regime (2), the traditional estimator is biased. Its asymptotic bias will depend on the deterministic quantity κ that we define in the following lemma:

Lemma 3.1 ([10]). *The following equation:*

$$\kappa = \frac{1}{M}\operatorname{tr}\left(\left(\mathbf{G}\mathbf{G}^{H} + \sigma^2\mathbf{I}_N\right)\left(\frac{\mathbf{G}\mathbf{G}^{H} + \sigma^2\mathbf{I}_N}{1+\kappa} + \mathbf{H}\mathbf{H}^{H}\right)^{-1}\right)$$

admits a unique positive solution κ.

The asymptotic bias is thus defined as:

Theorem 3.1 ([10]). *Let* $\mathbf{T} = \left(\mathbf{H}\mathbf{H}^{H} + \frac{\mathbf{G}\mathbf{G}^{H}+\sigma^2\mathbf{I}_N}{1+\kappa}\right)$. *Then,*

$$\hat{I} - V \xrightarrow[M,N\to+\infty]{a.s.} 0$$

where

$$V = -\frac{1}{N}\log\det(\mathbf{T}) + \frac{M}{N}\log(1+\kappa) - \frac{M}{N}\frac{\kappa}{1+\kappa}.$$

4. CONSISTENT ESTIMATION OF THE MAXIMUM DATA RATE

As aforementioned, the traditional estimator is biased. Finding a consistent estimator for the maximum mutual information is the main objective of our work. Our study is based on the use of random matrix theory whose results are accurate once the dimensions are of the same order of magnitude. This situation is frequently encountered in digital communications, domain where these techniques have been successfully applied since the nineties.

We assume that the secondary receiver has perfect knowledge of $\mathbf{H}$. It is then natural to assume that our estimator includes this knowledge. This makes our study more tricky since the first term in (1) depends jointly on the eigenvectors of $\mathbf{HH}^{\mathrm{H}}$ and $\mathbf{GG}^{\mathrm{H}}$, in addition to their corresponding eigenvalues. To overcome these difficulties, we will use the approach of [9], more appropriate for dealing with the cases where eigenvectors and eigenvalues have to be taken into account. The following theorem presents the obtained estimator:

Theorem 4.1. *Consider the quantity:*

$$\hat{I}_G = \frac{1}{N}\log\det\left(\mathbf{I}_N + \hat{y}_N\mathbf{H}\mathbf{H}^{H}\left(\frac{1}{M}\mathbf{Y}\mathbf{Y}^{H}\right)^{-1}\right) + \frac{M-N}{N}\left[\log\left(\frac{M\hat{y}_N}{M-N}\right)+1\right] - \frac{M\hat{y}_N}{N}$$

where $\hat{y}_N$ *is the unique real positive solution of the following equation:*

$$\hat{y}_N = \frac{\hat{y}_N}{M}\operatorname{tr}\mathbf{H}\mathbf{H}^{H}\left(\hat{y}_N\mathbf{H}\mathbf{H}^{H} + \frac{1}{M}\mathbf{Y}\mathbf{Y}^{H}\right)^{-1} + \frac{M-N}{M}.$$

Then under some mild assumptions, the following holds true:

$$\hat{I}_G - I \xrightarrow[M,N\to+\infty]{a.s.} 0.$$

Throughout this paper, we refer to the proposed estimator as the G-estimator in reference to G-estimation methods introduced by Girko [3]. To assess the performance of the G-estimator, we study hereafter its fluctuations around the exact value of the mutual information. In particular, we provide the expression of its variance and show that it has asymptotically a Gaussian behavior. More explicitly, we prove the following.

Theorem 4.2. *Let* y_N^* *be the deterministic quantity:*

$$y_N^* = 1 - \frac{1}{M}\operatorname{tr}\left((\mathbf{G}\mathbf{G}^{H} + \sigma^2\mathbf{I}_N)(\mathbf{H}\mathbf{H}^{H} + \mathbf{G}\mathbf{G}^{H} + \sigma^2\mathbf{I}_N)^{-1}\right)$$

Under some mild assumptions, the G-estimator verify the following:

$$\frac{N}{\sqrt{\theta_N}}\left(\hat{I}_G - I\right) \xrightarrow[M,N\to+\infty]{} \mathcal{N}(0,1).$$

where θ_N *is given by* (3)*:*

5. SIMULATIONS

We present in this section the results of the carried out simulations. We consider the case where the small cell receiver is equipped with $N = 4$ antennas and collects during $M = 15$ samples data stemming from $n_0 = 4$ antennas. We assume that the communication link with the transmitter is degraded by the interference of $K = 8$ mono-antenna users. Fig. 3 displays the theoretical and empirical normalized error variance of the G-estimator with respect to the $\mathrm{SNR} = \frac{1}{\sigma^2}$ given respectively by:

$$\mathrm{MSE}_{\mathrm{th}} = \frac{\theta_N}{I^2}$$

$$\mathrm{MSE}_{\mathrm{g,emp}} = \frac{1}{P}\sum_{i=1}^{P}\frac{N^2(\hat{I}_G^i - I)^2}{I^2}$$

where $\hat{I}_G^i$ is the G-estimator at the i-th Monte Carlo iteration with P is the total number of iterations. We also display in the same graph the empirical normalized mean square error of the traditional estimator defined as:

$$\mathrm{MSE}_{\mathrm{emp}} = \frac{1}{P}\sum_{i=1}^{P}\frac{N^2(\hat{I}^i - I)^2}{I^2}.$$

We note that the proposed estimator exhibits better performance for the whole SNR range. Finally to assess the Gaussian behavior of the proposed estimator, we represent in fig. 4 its corresponding histogram when the SNR is set to 10dB. We note a good fit between theoretical and empirical results although the system dimensions are small.

6. CONCLUSION

We have proposed in this work an efficient method for fast rate estimation of the mutual information in small cells networks. We have shown in particular that our method exhibits

$$\theta_N = 2\log(My_N^*) - \log\left((M-N)\left(M - \mathrm{tr}\left(\left(\mathbf{I}_N + \mathbf{H}\mathbf{H}^{\mathrm{H}}\left(\mathbf{G}\mathbf{G}^{\mathrm{H}} + \sigma^2\mathbf{I}_N\right)^{-2}\right)\right)\right)\right) \tag{3}$$

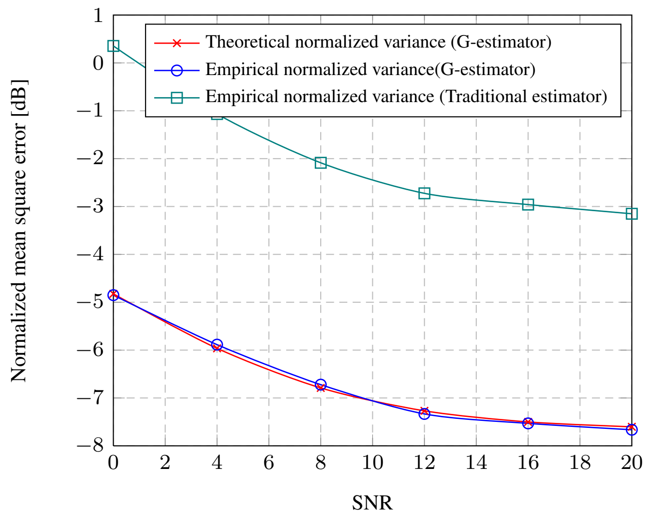

Fig. 3: Theoretical and empirical variances with respect to the SNR

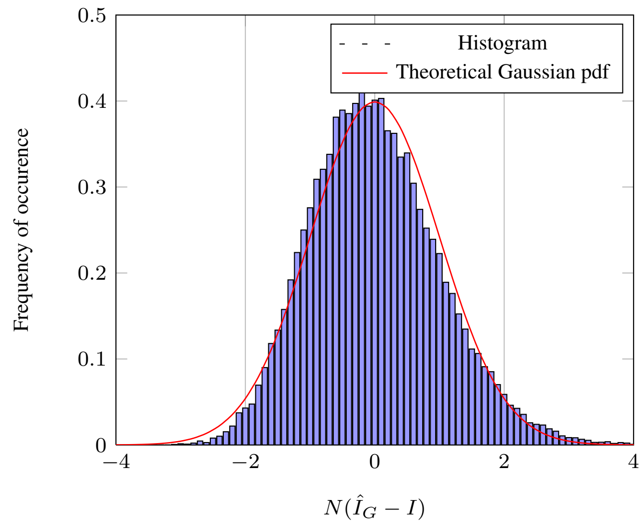

Fig. 4: Histogram of $N(\hat{I}_G - I)$.

higher performance than the traditional estimation technique. Numerical simulations have been provided and strongly support the accuracy of our results even for usual system dimensions.

7. REFERENCES

[1] Cisco. Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2009-2014.

[2] D. L. Perez and A. Valcarce and G. De la Roche and E. Liu and J. Zhang. Access Methods to WiMAX Femtocells : A Downlink System-Level Case Study. *ICCS*, 2008.

[3] V. L. Girko. Ten Years of General Statistical Analysis, unpublished.

[4] J. D. Hobby and H. Claussen. Deployment options for femtocells and their impact on existing macroceullar networks. *Bell Labs Technical Journal*, 13(4):145–160, 2009.

[5] K. Han and Y. Choi and D. Kim and M. Na and S. Choi and K. Han. Optimization of Femtocell Network Configuration under Interference Constraints. *International Symposium on Modeling and Optimization in Mobile AdHoc and Wireless Networks*, June 2009.

[6] S. P. Yeh and S. Talwar and S. C. Lee and H. Kim. Wimax Femtocells: A perspective on Network Architecture, Capacity and Coverage. *IEEE Communications Magazine*, October 2008.

[7] T. Zahir and K. Arshad and A. Nakata and K. Moessner. Interference Management in Femtocells. *IEEE Communiations Surveys and Tutorials*, 2012.

[8] T. Zahir and K. Arshad and Y. Ko and K. Moessner. A Downlink Power Control Scheme for Interference Avoidance in Femtocells. *IWCMC*, July 2011.

[9] P. Vallet and P. Loubaton. A G-estimator of the MIMO Channel Ergodic Capacity. *IEEE International Symposium on Information Theory*, pages 774–778, June 2009.

[10] W. Hachem and P. Loubaton and J. Najim. Deterministic Equivalents for Certain Functionals of Large Random Matrices. *Annals of Applied Probability*, 17(3):875–930, 2007.

[11] J. Zhang and G. De la Roche. *Femtocells, Technologies and Deployments*. Wiley, 2010.