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PERFORMANCE OF FADING MULTI-USER DIVERSITY FOR UNDERLAY COGNITIVE NETWORKS[†]

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ABSTRACT

Having multiple secondary users (SUs) can be exploited to achieve multiuser diversity and improve the throughput of the underlay secondary network. In the cognitive setting, satisfying the interference constraint is essential, and thus, a scheduling scheme is considered where some SUs are preselected based on the low interference power. From this subset, the SU that yields the highest throughput is selected for transmission. This scheduling scheme helps to lower the interference power while giving good throughput. For an independent but not identically distributed Nakagami-m fading channel, we obtain exact closed-form expressions of the capacity of this scheduling scheme. Furthermore, the scheduling time of SUs is characterized and closed-form expressions for the mean time after which a SU is selected for transmission are obtained. Numerical simulations are performed to corroborate the derived analytical results. Our results show that at low interference threshold, increasing transmit power of the SUs is not beneficial and results in reduced capacity. Furthermore, the channel idle time (i.e. time that no user is utilizing the channel) reduces with increasing the number of SUs.

Index Terms— Cognitive Radio, Multi-user Diversity, Ergodic Capacity, Fading Channels.

1. INTRODUCTION

Cognitive radio has been recently proposed as a solution to improve the utilization of the spectrum by spectrum sharing where the cognitive network transmits using the spectrum already allocated to some primary network [1, 2]. One of the approaches for spectrum sharing is the underlay approach in which the cognitive/secondary network can transmit concurrently with the primary network only if it does not cause harmful interference to the primary network.

Satisfying the interference constraint imposed by the primary network results in performance degradation of the secondary system. This performance degradation can be compensated by exploiting the best channels which become available when there are more users or mobility [3]. By selecting the user that has the best channel gives rise to a multiuser diversity effect which results in improved quality-of-service (QoS) and an increase in the overall capacity of the network [4]. Further diversity gains can be achieved by using opportunistic antennas [5]. However, a drawback of this scheme is that the users with the bad channels get selected less often, thus, these users suffer large delays in communication.

In a spectrum sharing setting, multi-user diversity can be exploited but it is different from the traditional multi-user case because of the interference power constraints imposed on the secondary user (SU). Thus, in this case, the selected SU needs to have jointly a good transmission link and a poor interference link. This idea can be achieved in various ways such as by using a precoder [6] or through the use of antennas [7]. However, in this paper, we exploit the multiple users in the secondary network. Due to the random fading, it is highly probable that some SU transmitter (SU-Tx) has the interference link (SU-Tx-to-primary user receiver (PU-Rx) link) in a deep fade and a very good SU-Tx-to-SU-Rx link. Thus, the SUs can be scheduled when the interference link is on a fade.

The effect of multi-user diversity in an underlay network with independent and identically distributed (i.i.d) Rayleigh fading was initially studied in [8]. It was shown in [8] that at sufficiently high transmit power, the selection of the user is more influenced by the interference link and the multi-user diversity gain in terms of capacity scales similar to the scaling law of the non-spectrum sharing system. The ergodic capacity of the multi-user underlay system was derived for a more generic hyper-Nakagami-m fading channel in [9]. The outage capacity as well as the effective capacity for an opportunistic cognitive broadcast channel having i.i.d Rayleigh fading was analyzed in [10]. Multi-user interference diversity which takes into account the interference from the primary network was studied in [11]. In [12], a new hybrid scheduling scheme was proposed which selects SUs for transmission in two steps; 1) First a subset of SUs is selected that have the interference link in deep fade and 2) then the SUs with the best transmis-

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sion links are selected from the pre-selected subset. The authors in [12] also gave bounds on the throughput of the hybrid scheduling scheme for a Rayleigh fading channel.

In this paper, we consider a variant of the hybrid scheduling scheme considered in [12], where only a single SU is chosen for transmission. Considering an independent but not identically distributed (i.n.i.d) Nakagami-m fading channel, we obtain closed form expression of the capacity of the scheduling scheme. In the hybrid scheduling scheme, it is possible that sometimes the initial subset is empty and thus no SU is selected for transmission. Thus, utilizing the renewal concept of a cycle, expressions for the mean time after which a SU is scheduled are derived. These expressions are useful in analyzing the feasibility of the network and adjusting network parameters. For example, the SUs in a network are usually equipped with finite buffers which might overflow if the SUs are not scheduled properly. Thus, the parameters of the network can be adjusted based on the mean scheduling time of the SU and the buffer lengths of each SU to avoid buffer overflow. Furthermore, numerical simulations are performed to corroborate the derived analytical results. Our results also show that at low interference thresholds, increasing transmit power of the SUs is not beneficial and results in reduced capacity. In addition, as the number of SUs increase the channel idle time reduces implying that for a large number SUs, there is always some SU which will be able to transmit.

2. SYSTEM MODEL

Assume that a secondary network is sharing the spectrum with a primary network. The secondary network consists of L transmitters and a receiver. There is a single primary user receiver (PU-Rx) in the surroundings of the SU-Tx. Let the channel power gain between SU-Tx and l-th SU-Rx be denoted by h_l, and the channel power gain between l-th SU-Tx and PU-Rx is denoted by g_l. The channel is assumed to be i.n.i.d with Nakagami-m fading. Therefore, h_l and g_l are Gamma distributed. For a Gamma random variable X with parameters m_a and Ω_a, the probability distribution function (PDF) is given as $f_X(x) = \frac{1}{\Gamma(m_h)} \left(\frac{m_a}{\Omega_a}\right)^{m_a} x^{m_a-1} e^{-x\frac{m_a}{\Omega_a}}$ and the cumulative distribution function (CDF) is given as $F_X(x) = \frac{\gamma\left(m_a, x\frac{m_a}{\Omega_a}\right)}{\Gamma(m_a)} = 1 - e^{-x\frac{m_a}{\Omega_a}} \sum_{k=0}^{m_a-1} \frac{x^k}{k!} \left(\frac{m_a}{\Omega_a}\right)^k$ where, the second equality assumes that m_a is an integer and $\gamma(\cdot,\cdot)$ denotes the lower incomplete gamma function [13, Eq. (8.350.1)]. The CDF and PDF of g_l is obtained by replacing $X = g_l$ and $a = g_l$. Similarly, the CDF and PDF of h_l is obtained by replacing $X = h_l$ and $a = h_l$. The noise at the nodes in the network is assumed to be zero mean complex Gaussian with variance N_0.

Selection Criteria: Let the maximum allowed interference power at the primary user be Q, then $P_l g_l < Q$, where P_l is the transmit power of the l-th SU-Tx. For simplicity, assume that the maximum transmit power allowed at each SU-Tx is P and also that the selected SU-Tx always transmits with power P. The SU-Tx for transmission is selected in two steps.

1. First a subset of SU-Tx is selected that satisfy the interference power constraint i.e.
$$T_j^v = \left\{ l : g_l < \frac{Q}{P}, \forall l \right\}, \tag{1}$$
where T_j^v denotes the j-th subset and is of size $v+1$ and contains indices of the SU-Tx whose interference link value is below $\frac{Q}{P}$ i.e. $T_j^v \in \mathcal{P}(U)$, where $U = \{SU_1, SU_2, ..SU_L\}$ is the set of all available SU-Tx and $\mathcal{P}(\cdot)$ denotes the power set of its argument.
2. In the second step the SU-Tx with the best SU forward link is selected. Let, $T_j^v(k)$ denote the k-th element of set T_j^v, then the index of SU-Tx can be obtained as
$$\iota = \arg\max_k \left\{ h_{T_j^v(k)} \right\}. \tag{2}$$

This procedure to select a SU-Tx is a special case of the scheduling scheme considered in [12].

3. ERGODIC CAPACITY

The ergodic capacity of this scheduling scheme can be obtained as
$$C = \sum_j \mathsf{Pr}\{T_j^v\} C_j, \tag{3}$$
where C_j is the ergodic capacity given that the selected subset is T_j^v and $\mathsf{Pr}\{T_j^v\}$ (probability of selecting subset T_j^v) is
$$\mathsf{Pr}\left\{T_j^v\right\} = \prod_{i_1=1, i_1 \in T_j^v}^{L} F_{g_{i_1}}\left(\frac{Q}{P}\right) \prod_{i_1=1, i_1 \notin T_j^v}^{L} \left(1 - F_{g_{i_1}}\left(\frac{Q}{P}\right)\right) \tag{4}$$
where $F_{g_k}\left(\frac{Q}{P}\right) = \mathsf{Pr}\left\{g_k < \frac{Q}{P}\right\} = \mathsf{Pr}\left\{k \in T_j^v\right\}$. For subset T_j^v, the end-to-end-SNR for the scheduling scheme is $\gamma_j = \frac{P}{N_0} \max_i \left\{ h_{T_j^v(i)} \right\}$. Using [14, Eq. (9)] the CDF of γ_j, is
$$F_{\gamma_j}(x) = 1 + \sum_{G_{v+1}} \bar{\kappa}_{n,p} e^{-x\,\bar{B}_n} x^{\bar{A}_{n,p}}, \tag{5}$$
where $\bar{\kappa}_{n,p} = \prod_{q=1}^{v+1} (-1)^{n_q} \left(\frac{m_{X_j^v(q)} N_0}{\Omega_{X_j^v(q)} P}\right)^{p_q n_q} \frac{m_{X_j^v(q)}{}^{n_q - 1}}{(p_q!)^{n_q}}$, $\bar{B}_n = \sum_{q=1}^{v+1} n_q \frac{m_{X_j^v(q)} N_0}{\Omega_{X_j^v(q)} P}$, $\bar{A}_{n,p} = \sum_{q=1}^{v+1} p_q n_q$, $\sum_{G_{v+1}}$ is a short hand notation for $\sum_{\mathbf{n} \in \theta_{v+1}, \mathbf{n} \neq 0} \sum_{p_1=0}^{m_{T_j^v(1)}-1} \sum_{p_2=0}^{m_{T_j^v(2)}-1} \cdots \sum_{p_{v+1}=0}^{m_{T_j^v(v+1)}-1}$ and θ_{v+1} is the set of all possible $v+1$ bit binary numbers and n_q is the q-th bit of the binary number $\mathbf{n} \in \theta_{v+1}$. The ergodic capacity for the subset is T_j^v is obtained using [15]
$$C_j = \vartheta \int_0^\infty \frac{1 - F_{\gamma_j}(x)}{1+x} dx = -\vartheta \sum_{G_{v+1}} \bar{\kappa}_{n,p} \int_0^\infty \frac{e^{-x\,\bar{B}_n} x^{\bar{A}_{n,p}}}{1+x} dx. \tag{6}$$
where $\vartheta = \frac{\log_2(e)}{B}$ and B denotes the bandwidth. Solving the integral using [13, Eq. (3.383.10)] one gets
$$C_j = -\frac{\log_2(e)}{B} \sum_{G_{v+1}} \bar{\kappa}_{n,p} e^{\bar{B}_n} \Gamma\left(\bar{A}_{n,p} + 1\right) \Gamma\left(-\bar{A}_{n,p}, \bar{B}_n\right). \tag{7}$$
The capacity of the multiuser scheduling scheme can be obtained by substituting (7) in (3).

$$\mathcal{I}_u = \mathsf{Pr}\left\{u - \text{th SU-Tx is selected for transmission}\right\} = \mathcal{I}_1 + \mathcal{I}_2 = \\ \mathsf{Pr}\left\{\text{Only } u \in T_j^v\right\} + \mathsf{Pr}\left\{u \text{ and additional users are} \in T_j^v\right\} \mathsf{Pr}\left\{h_u > h_{T_j^v(k)|T_j^v(k)\neq u}; \forall\, k\right\}; \forall\, T_j^v \tag{8}$$

4. ANALYSIS OF THE USER SCHEDULING TIME

4.1. Probability that u-th SU is Selected:

The probability that the u-th SU-Tx is selected for transmission can be expressed as (8), where $\mathcal{I}_2 = \mathsf{Pr}\left\{u \in T_j\right\} \chi_1$,

$$\mathcal{I}_1 = \mathsf{Pr}\left\{\text{Only } u \in T_j^v\right\} = F_{g_u}\left(\frac{Q}{P}\right) \prod_{i_1=1, i_1\neq u}^{L} \left(1 - F_{g_{i_1}}\left(\frac{Q}{P}\right)\right), \tag{9}$$

and $\chi_1 = \mathsf{Pr}\left\{T_j^v \text{ has additional users}\right\} \mathsf{Pr}\left\{h_u > h_{T_j^v(k)|T_j^v(k)\neq u}; \forall\, k\right\}$.

For a certain set T_j^v having $v+1$ elements, conditioned on the fact that $u \in T_j^v$, then T_j^v can be expressed as $T_j^v = \left\{u, X_j^v\right\}$. X_j^v is a set of indices of the SU-Tx in T_j^v excluding u. Let $X_j^v(k)$ denote the k-th element of set X_j^v. As there are L SU-Tx, so the number of possible elements in set X_j^v can vary from 1 to $L-1$, therefore $1 \leq v \leq L-1$. Furthermore, for a fixed v, X_j^v can have any of the $\binom{L-1}{v}$ possible combinations of elements of set $E = \{1, ..u-1, u+1, ..L\}$, which is a set of indices denoting each SU-Tx excluding the u-th SU-Tx. Therefore, for a fixed v, χ_1 is given by

$$\chi_1 = \sum_{v=1}^{L-1} \sum_{j=1}^{\binom{L-1}{v}} \mathsf{Pr}\left\{X_j^v\right\} \mathsf{Pr}\left\{h_u > h_{X_j^v(i)}; \forall\, i\right\}. \tag{10}$$

Conditioned on h_u, $h_{X_j^v(k)}$ are independent, thus

$$\mathsf{Pr}\left\{h_u > h_{X_j^v(i)} | h_u; \forall\, i\right\} = \prod_{i=1}^{v} F_{h_{X_j^v(i)}}(h_u). \tag{11}$$

By averaging (11) using the PDF of h_u, yields

$$\mathsf{Pr}\left\{h_u > h_{X_j^v(i)}; \forall\, i\right\} = \int_0^\infty \prod_{i=1}^{v} F_{h_{X_j^v(i)}}(h_u) f_{h_u}(h_u) dh_u. \tag{12}$$

Substituting $f_{h_u}(\cdot)$ and $F_{h_{X_j^v(i)}}(\cdot)$ into (12) and using [14, eq. (9)], (12) can be expressed as

$$\mathsf{Pr}\left\{h_u > h_{X_j^v(i)}; \forall\, i\right\} = \\ \frac{1}{\Gamma(m_{h_u})}\left(\frac{m_{h_u}}{\Omega_{h_u}}\right)^{m_{h_u}} \sum_{G_v} \kappa_{n,p} \int_0^\infty e^{-xB_n - x\frac{m_{h_u}}{\Omega_{h_u}}} x^{A_{n,p}+m_{h_u}-1} dx, \tag{13}$$

where $\kappa_{n,p} = \prod_{q=1}^{v} (-1)^{n_q} \left(\frac{m_{X_j^v(q)}}{\Omega_{X_j^v(q)}}\right)^{p_q n_q} \frac{m_{X_j^v(q)}^{n_q - 1}}{(p_q!)^{n_q}}$, $A_{n,p} = \sum_{q=1}^{v} p_q n_q$, $B_n = \sum_{q=1}^{v} n_q \frac{m_{X_j^v(q)}}{\Omega_{X_j^v(q)}}$ and $\sum_{G_v}$ is a short hand notation for $\sum_{\mathbf{n}\in\theta_v} \sum_{p_1=0}^{m_{X_j^v(1)}-1} \sum_{p_2=0}^{m_{X_j^v(2)}-1} \cdots \sum_{p_v=0}^{m_{X_j^v(v)}-1}$. Solving the integral in (13), yields

$$\mathsf{Pr}\left\{h_u > h_{X_j^v(i)}; \forall\, i\right\} = \\ \sum_{G_v} \left(\frac{m_{h_u}}{\Omega_{h_u}}\right)^{m_{h_u}} \kappa_{n,p} \frac{\Gamma\left(A_{n,p}+m_{h_u}\right)}{\Gamma(m_{h_u})} \left(B_n + \frac{m_{h_u}}{\Omega_{h_u}}\right)^{-\left(A_{n,p}+m_{h_u}\right)}. \tag{14}$$

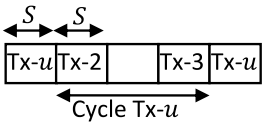


Fig. 1. Transmission Cycle.

Substituting (14) into (10) gives

$$\chi_1 = \sum_{v=1}^{L-1} \sum_{j=1}^{\binom{L-1}{v}} \sum_{G_v} \mathsf{Pr}\left\{X_j^v\right\} \left(\frac{m_{h_u}}{\Omega_{h_u}}\right)^{m_{h_u}} \kappa_{n,p} \\ \times \frac{\Gamma\left(A_{n,p}+m_{h_u}\right)}{\Gamma(m_{h_u})} \left(B_n + \frac{m_{h_u}}{\Omega_{h_u}}\right)^{-\left(A_{n,p}+m_{h_u}\right)}, \tag{15}$$

where $\mathsf{Pr}\left\{X_j^v\right\}$ is given by

$$\mathsf{Pr}\left\{X_j^v\right\} = \prod_{i_1=1, i_1\in X_j^v}^{L} F_{g_{i_1}}\left(\frac{Q}{P}\right) \prod_{i_1=1, i_1\notin X_j^v}^{L} \left(1 - F_{g_{i_1}}\left(\frac{Q}{P}\right)\right). \tag{16}$$

Substituting, $\mathcal{I}_2 = \chi_1 F_{g_u}\left(\frac{Q}{P}\right)$ in (8) yields (17). (17) gives the probability that the u-th SU-Tx is selected for transmission The probability that no SU-Tx is selected is denoted by $\mathcal{I}_0$ and can be obtained as

$$\mathcal{I}_0 = 1 - \sum_{u=1}^{L} \mathcal{I}_u. \tag{18}$$

4.2. Time Scheduling of Users

Using the renewal concept of cycle, we find the average time after which u-th SU-Tx is selected for transmission [16]. Assume that whenever a SU-Tx is selected it transmits for S seconds. A cycle is defined as the interval between two successive instances when a given SU-Tx is selected for transmission.

Fig. 1 shows an example of a cycle for the u-th SU-Tx. It can be observed in Fig. 1 that once a SU-Tx is selected, it transmits for S seconds. The empty interval implies that no SU-Tx is chosen to transmit in that interval. The inter-arrival time for SU-Tx u, is denoted by A_u, and can be expressed as

$$A_u = \sum_{l=0, l\neq u}^{L} \sum_{i=1}^{N_l^u} S = S \sum_{l=0, l\neq u}^{L} N_l^u, \tag{19}$$

where N_l^u is the number of times that the l-th SU-Tx is chosen for transmission during the cycle corresponding to the u-th SU-Tx. Note that N_l^u is a random variable. For the mean inter-arrival time, the first moment of N_l^u is required.

Assume that there are n time slots in the cycle. N_s^u is a binomial random variable having PDF

$$\mathsf{Pr}\left\{s\text{-th Tx is selected } k \text{ times from a total of } n \text{ times} | \text{Tx-}u \text{ is not selected}\right\} = \\ \mathsf{Pr}\left\{N_s^u = k | n,\ u \text{ is not selected}\right\} = \binom{n}{k} \left(\frac{\mathcal{I}_s}{1-\mathcal{I}u}\right)^k \left(1 - \frac{\mathcal{I}_s}{1-\mathcal{I}_u}\right)^{n-k}. \tag{20}$$

$$\mathcal{I}_u = F_{g_u}\left(\frac{Q}{P}\right)\left(\prod_{i_1=1,i_1\neq u}^{L}\left(1-F_{g_{i_1}}\left(\frac{Q}{P}\right)\right)+\sum_{v=1}^{L-1}\sum_{j=1}^{\binom{L-1}{v}}\sum_{G_0}\Pr\left\{X_j^v\right\}\left(\frac{m_{h_u}}{\Omega_{h_u}}\right)^{m_{h_u}}\kappa_{n,p}\frac{\Gamma\left(A_{n,p}+m_{h_u}\right)}{\Gamma(m_{h_u})}\left(B_n+\frac{m_{h_u}}{\Omega_{h_u}}\right)^{-\left(A_{n,p}+m_{h_u}\right)}\right) \tag{17}$$

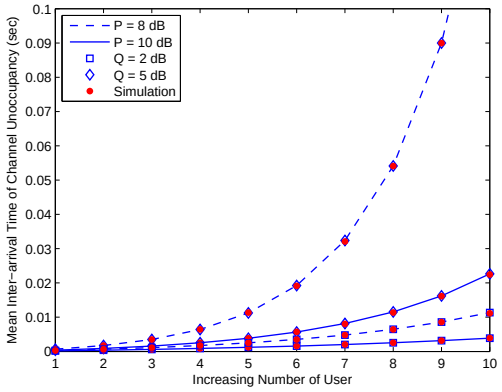


Fig. 2. Capacity of the scheduling scheme with varying peak interference power, peak transmit power and number of users, where $\Omega_{g_l} = \{1,2,1,3,1,2,1,1,1,1\}$ and $\Omega_{h_l} = \{1,1,3,1,4,4,6,1,1,1\}$.

The conditional mean of N_s^u is $E[N_s^u|n] = n\left(\frac{\mathcal{I}_s}{1-\mathcal{I}_u}\right)$, where n is a geometric random variable with PDF $f_n(n) = \mathcal{I}_u(1-\mathcal{I}_u)^n$. The unconditional mean of N_s^u is then given by

$$E[N_s^u] = \left(\frac{\mathcal{I}_s}{1-\mathcal{I}_u}\right)\sum_{n=0}^{\infty} n\,\mathcal{I}_u(1-\mathcal{I}_u)^n = \frac{\mathcal{I}_s}{\mathcal{I}_u}. \tag{21}$$

Finally, the *mean inter-arrival time* of the u-th SU-Tx is

$$E[A_u] = S\sum_{l=0,l\neq u}^{L} E[N_l^u] = S\sum_{l=0,l\neq u}^{L}\frac{\mathcal{I}_l}{\mathcal{I}_u}. \tag{22}$$

The resulting *mean inter-arrival time* of the event that no SU-Tx is selected for transmission can be deduced as

$$E[A_0] = 1 - \sum_{u=1}^{L} E[A_u]. \tag{23}$$

5. SIMULATION RESULTS

Numerical simulations were performed to evaluate the system performance and verify the derived results. In the simulations, $m_{h_l} = m_{g_l} = 2$, $B = 1$, $N_0 = 1$ and $S = 1$ ms. These parameters are fixed unless stated.

Fig. 2 shows the effect on capacity when varying the peak interference power constraint. It can be observed that as the interference constraint is relaxed, the capacity of the secondary system increases. Furthermore, the capacity increases as the number of SUs increases. At low values of Q, increasing transmit power of the SUs results in lower capacity. This is because the number of SUs selected in the initial subset decreases. However at large Q, increasing transmit power increases the capacity and follows a normal trend.

Fig. 3 (Top) shows the average time after which some SU is selected and Fig. 3 (Bottom) shows the impact of number

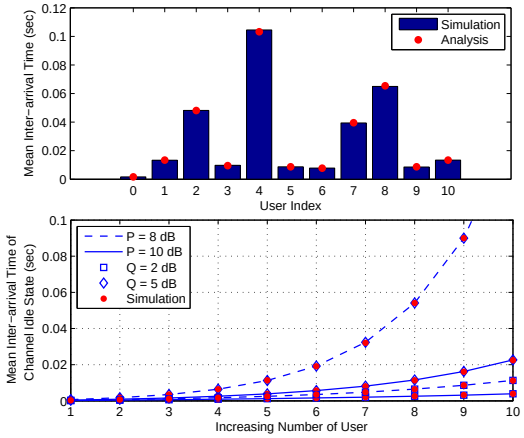


Fig. 3. *Top:* Mean inter-arrival time of users where $P = 10$ dB, $Q = 5$ dB, $\Omega_{g_l} = \{1,2,1,3,1,1,2,3,1,1\}$ and $\Omega_{h_l} = \{1,1,3,1,4,6,2,5,4,1\}$. *Bottom:* Mean inter-arrival time for channel unoccupancy with varying number of users where $m_{g_l} = m_{h_l} = 1$, and $\Omega_{g_l} = \Omega_{h_l} = 2,\ \forall l$.

of SUs in the system on the inter-arrival time of channel idle state. In Fig. 3 (Top), user index '0' indicates that the channel is idle and no SU is selected. It can be observed that the average time after which a SU is selected depends on the channel statistics. A SU with a poor channel condition has a higher mean inter-arrival time of selection. In Fig. 3 (Bottom), it can be observed that as the number of SUs increase, the inter-arrival time of the channel idle state increases. This indicates that the channel is mostly occupied by some SU and it is very rare that some SU is not transmitting. In addition, Fig. 3 shows that as the interference power constraint is relaxed, the inter-arrival time of channel idle state increases, which is obvious. Furthermore, increasing the transmit power of the SU increases the chances that the initial subset is empty and thus, the inter-arrival time of channel idle state decreases.

6. CONCLUSION

We analysed the capacity and the scheduling time of users for a multiple-user underlay secondary network. The scheduling strategy consisted of two step. First, a subset of users is selected which satisfy the interference constraint and then in the second step, the user in the subset with the best secondary user forward link was selected for transmission. We obtained exact closed-from expressions for the capacity and the mean time after which a user is selected for transmission. Our results show that as the number of users increase the channel idle time (i.e. the time that no user is utilizing the channel) reduces. In addition, at low interference power, increasing the transmit power of the users is not useful and lowers the secondary network capacity.

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