On the electromagnetic modeling of anisotropic panels

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Summary

Focus is on modeling of the electromagnetic behavior of complex dispersive, anisotropic structures, with in mind damaged fiber-based, flat composite panels as in aeronautic and automotive industries. The goal is to image these panels if damaged (defects are of many sorts, like delaminations, cracks, inclusions) via proper probes, to get reliable information on these defects. Yet, before imaging, one has to accurately and quickly compute the fields or pertinent associated quantities in ways versatile enough in terms of materials, sources, frequencies, etc. Also, one should devise both large-scale models assuming homogenized anisotropic media characterized by permittivity tensors and small-scale ones wherein each layer in the panels looks like a superposition of periodic rods (bundles of fibers) to achieve better and broader understanding. The presentation encompasses all such aspects while pinpointing also what is ahead for imaging solutions.

1. Outline

One might refer to the above summary, which introduces the theme of the work: electromagnetic behavior of anisotropic composite panels and testing thereof. These issues are studied by many since more than three decades with advance of computational tools and electromagnetic theory, notably about Green’s functions. A host of references on direct modeling of anisotropic structures with emphasis on eddy-current (mostly meaning exploration of conductive, carbon-fiber-reinforced structures in diffusive regimes) and microwave to higher frequency ranges (glass-fiber-reinforced structures being of concern in propagative regimes) is available in [14-15], publications on the small-scale model of our interest being also found in [7] —the panels are made of stacks of planar strata, each with fiber orientation parallel with the interfaces (usually differing from one to the next). As for MUSIC approaches to imaging (localization) of small volumetric defects, references are in [10], but one might refer to [1] already. Here, one is to pinpoint a handful of references, [3] [4] [6] [11-12], which illustrate challenges in nondestructive testing (but full electromagnetic testing of anisotropic composites is still in infancy today). Also, a couple of references has promises beyond imaging of small defects (or delaminations, as thin layers in otherwise sound panels). Subspace-based optimization methods (SOM) [13] and Bayesian compressive sensing (BCS) [8] appear good choices to deal with volumetric defects, SOM in a deterministic setting, BCS in a Bayesian one with sparsity enforced in addition. Yet, as underlined, one always needs an efficient direct solver (as in [15], a fast Fourier-based Method of Moments, MoM), and, also, convergence at low frequency (to tackle voids in conductive panels) might be an open question save intricate loop-tree decompositions and pre-conditioners [2] or wide scope, possibly heavy FEM [3], the solution to be envisaged here originating from [9], the so-called contraction integral equation.

2. Response of a panel to a distributed source

Take z as vertical axis orthogonal to the panel interfaces, (x, y) as horizontal axes parallel to them. Define (k_x, k_y) as lateral wave-number plane. For electrical uniaxial media with anisotropy axes along z, dispersion relationships for each plane-wave component are circular like in isotropic cases; boundary conditions (BC) are expressed in the radial wave-number k_ρ domain. For general anisotropy, or when anisotropy axes of the uniaxial media are not along z, the dispersion relations might be elliptic, and the BC are matched in the (k_x, k_y) plane. This applies to the said composite panels. The field must be decomposed into four wave modes coupled at each interface. A first-order differential (state) equation satisfied in each stratum leads to well-known propagator matrix methods, with instability due to evanescent waves fast decaying along z to cope with, via recurrence relations (RR) wherein magnitudes of evanescent components are normalized to avoid large numbers. Works and solutions thereof on this issue are numerous. However, if an active current source is as an example continuously distributed inside or outside (e.g., in air above) the panel, most of these solutions are not enough or the computational burden can become huge if one calculates the Green function at every point in the source domain and are then left to integrate it (and discretization matters, should the near-field response be needed, e.g., when constructing impedance matrices in MoM with such panels as background). In the work here, based on the eigen-analysis of the state equation and previous material on RR, novel RR are introduced. The field vector is not only decomposed into two wave modes but an extra term is introduced for the source effect and meanwhile to stabilize it. The knowledge needed is the 3-D spatial Fourier transform of the current source. So, one efficiently calculates the panel's spectral response, even in near field, dyadic Green functions being obvious byproduct.

3. Using the above for scattering by damaged panels

Volume integral (VI) equations solved via MoM are standard tools with inhomogeneous bodies in planarly-layered media. When those are isotropic, MoM impedance matrices are obtained in a fast way by accelerating the calculation of the Sommerfeld integrals (SI) involved, e.g., by discrete complex image methods. The accelerated SI is a 1-D integral arising from the 2-D inverse Fourier transform (IFT) since dispersion relations with isotropy enable to calculate the integral along the azimuth direction in closed form. Yet, when optical axes of uniaxial media (as for the panels) are parallel with the interfaces, one is confronted to full 2-D IFT. One might circumvent them, with the rectilinear mesh usual in VI methods, achieving efficient construction of impedance matrices. Leaving aside all details of tedious analyses led, inspired from pioneering [5] here, from generalized Poisson summation formula, a new relation between the continuous Fourier spectrum of a continuous function and its sampled signals on a rectilinear mesh in the spatial domain is derived. Using this relationship and applying a so-called windowing technique, with proper window functions, a fast algorithm is proposed, yielding the discrete sampled signals on a rectilinear mesh from the continuous Fourier spectrum of the original function in efficient/accurate manner. New numerical interpolation and integration methods, based on recently proposed Padua points are also implemented. Numerical simulations illustrate the above, with comparisons to known results (isotropic cases) and FEM ones (isotropic & anisotropic cases).

4. Conclusion

To proper model a damaged composite panel, with ability to manage most situations in terms of undamaged material parameters and damages (save thin cracks) in reliable and fast way is needed for non-destructive electromagnetic testing. This contribution shows the progress made and works on-going, details being left to [14-15], the small-scale model [7] as well as MUSIC imaging of small defects [10] being discussed by complementary presentations.

5. Acknowledgements

MERLION project 8-14-10, Singapore Temasek Defence Systems Institute project TDSI/10-005/1A, and DIGITEO.

6. Bibliography

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