Fast Calculation of Response of Scatterers in Uniaxial Laminates

Yu Zhong[1], Xudong Chen[2], Pingping Ding[3], Marc Lambert[3], and Dominique Lesselier[3]

[1] *Department of Electronics and Photonics*
*Institute of High Performance Computing (IHPC), A*STAR, Singpoare*
[2] *Department of Electrical and Computer Engineering*
National University of Singapore (NUS), Singapore
[3] *Laboratoire des Signaux et Systèmes, Département de Recherche en Electromagnétisme*
UMR8506 (CNRS, Supélec, University Paris-Sud), France

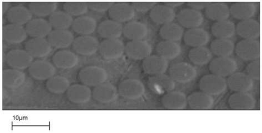
August 28th 2014 / PIERS 2014

Outline

Introduction

- Layered structures are of great interest in non-destructive testing (NDT) community due to their vast practical implementations in many areas.
- Nowadays, more and more composite materials have been used to construct such layered structures due to their lightness and robustness, such as the carbon fiber reinforced materials.
- However, due to the intrinsic anisotropy of some composite materials, the NDT problems with these materials become challenging.

A schematic of fibres in composite materials from [A. Galehdar, *et. al.*, Proc. the 40th European Microwave Conference, pp. 882-885, 2010, Paris, France.]



Introduction

- A proper and efficient modeling of the layered anisotropic media is essential for tackling detection problems. The volume integral equation method is a good candidate.
- The full vectorization of the propagator matrix method will be introduced to give the spectral response of anisotropic laminates due to any bounded active source within the laminates.
- In order to achieve good efficiency and accuracy, a new windowing technique is introduced. Furthermore, interpolation and integration algorithms based on the Padua points are implemented.

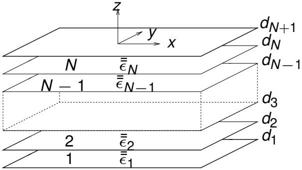
State Equation

- Uniaxial permittivity tensors:

  $\bar{\bar{\epsilon}}'_n = \mathrm{diag}\left[\epsilon_{11}^{(n)}, \epsilon_{22}^{(n)}, \epsilon_{22}^{(n)}\right]$

  with the optical axes always rotating in the $x - y$ plane.
- In the Fourier domain, the field vector is defined as

$$\bar{\varphi}(k_x, k_y, z) = \begin{bmatrix} k_x \tilde{H}_x(k_x, k_y, z) + k_y \tilde{H}_y(k_x, k_y, z) \\ k_y \tilde{H}_x(k_x, k_y, z) - k_x \tilde{H}_y(k_x, k_y, z) \\ k_x \tilde{E}_x(k_x, k_y, z) + k_y \tilde{E}_y(k_x, k_y, z) \\ k_y \tilde{E}_x(k_x, k_y, z) - k_x \tilde{E}_y(k_x, k_y, z) \end{bmatrix}$$



- The state equation reads (for each (k_x, k_y))

$$\frac{d}{dz}\bar{\varphi}(z) = \bar{\bar{A}}_n \cdot \bar{\varphi}(z) + \bar{f}(z)$$

  with $\bar{\bar{A}}_n = \bar{\bar{U}} \cdot \bar{\bar{\Sigma}} \cdot \bar{\bar{U}}^{-1}$ for the n^{th} layer (can be constructed analytically), and $\bar{f}(z)$ the source term.

Solution of the state equation

- The solution reads

$$\bar{\varphi}(d_{n+1}) = e^{\bar{\bar{A}}_n(\delta_n)} \cdot \bar{\varphi}(d_n) + \int_{d_n}^{d_{n+1}} e^{\bar{\bar{A}}_n \delta_n'} \cdot \bar{f}(z')\, dz'$$

- Cannot be directly calculated due to the numerical instability, since $e^{\bar{\bar{A}}_n(\delta_n)} = \bar{\bar{U}} \cdot e^{\bar{\bar{\Sigma}}(\delta_n)} \cdot \bar{\bar{U}}^{-1}$, and $e^{\bar{\bar{\Sigma}}(\delta_n)}$ could explode when
  1. Large thickness δ_n.
  2. Large lateral spatial frequency k_x or k_y (representing a fast changing evanescent wave).

- For such numerical instability, wave mode decomposition method has been proposed to deal with source free problems [1] by expanding the field vector as

$$\bar{\varphi}(d_n) = \bar{\bar{\Omega}}_n \cdot [\alpha_n, \beta_n]^{\mathrm{T}}, N+1 > n > 0$$

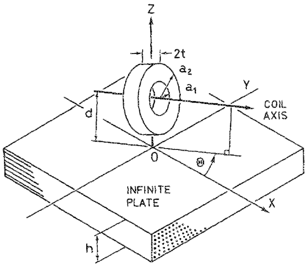
1. H. D. Yang, IEEE Trans. Antennas Propagat., vol. 45, pp. 520-526, 1997.

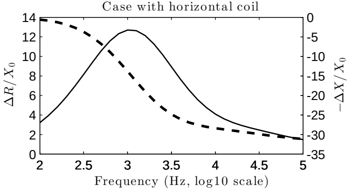
Stable solution for problems with sources

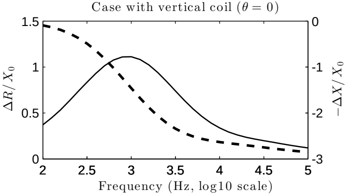
- If there is an active source embedded inside the layered media (or in the outer half space), the field transformation is different from the case without source.
- One needs to write the field vector after the transformation as $\overline{\varphi}(d_{n+1}) = \overline{\overline{\Omega}}_{n+1} \cdot [\alpha_{n+1}, \beta_{n+1}]^{\mathrm{T}} + \overline{h}$.
  The difference is that the constant term $\overline{h}$ is added.
- Keys to generate an accurate source term $\overline{h}$:
  - To have the Fourier spectrum of the distribution of the current density.
  - To follow a stable transforming scheme similar to the one for the source-free case to avoid the stability issue.
- This constructs the spectral responses of the laminates to the current basis used in MoM.

A test

A numerical example from [1] is reproduced:







1. S. Burke, J. Appl. Phys., vol. 68, pp. 3080-3090, 1990.

Spatial responses of uniaxial laminates

- The most time-consuming part of using MoM is to construct the impedance matrix from the spectral responses.
- The convention is to use the Sommerfeld integral to calculate the responses of the current basis at each testing point (suppose the delta function is used as testing function).
- When dealing with volume integral equation, one usually has periodic discretization, i.e., the rectilinear meshing.
- Using such meshing, one may be able to directly construct the discrete spectrum of the impedance matrix without the Sommerfeld integral.

Spatial responses of uniaxial laminates

- Using IFT, the response of a current basis at some test point can be obtained as

  $$\eta_{u,v;p;p'}(m,n) = \mathrm{IFT}\{\tilde{\eta}_{u,v;p;p'}(k_x,k_y)\}|_{x=x_m,y=y_n}$$

  where the $\tilde{\eta}_{u,v;p;p'}(k_x,k_y)$ is the spectral response of the layers due to the current basis.

- Using the periodic property of the testing point, one can construct it in another way, by generalized Poisson summation formula:

  $$\hat{\eta}_{u,v;p;p'}(\alpha,\beta) = \iint_{-\infty}^{+\infty} \tilde{\eta}_{u,v;p;p'}(k_x,k_y)\tilde{Q}_{\alpha,\beta}(k_x,k_y)dk_x dk_y$$

  where $\tilde{Q}_{\alpha,\beta}(k_x,k_y)$ is the superposition of the spectrums of some window function.

Spatial responses of uniaxial laminates

- The $\tilde{Q}_{\alpha,\beta}(k_x, k_y)$ is found to be

  $$\tilde{Q}_{\alpha,\beta}(k_x, k_y) \propto \sum_{l_1=-\infty}^{+\infty} \sum_{l_2=-\infty}^{+\infty} \tilde{W}\left[\frac{2\pi}{\Delta x}(\frac{\alpha}{2M-1} + l_1) - k_x, \frac{2\pi}{\Delta y}(\frac{\beta}{2N-1} + l_2) - k_y\right]$$

  where $\hat{W}(k_x, k_y)$ is the spectrum of the chosen window function.

- By only integrating areas where the main lobes of the window spectrum cover, the efficiency of such an integral can be significantly increased.

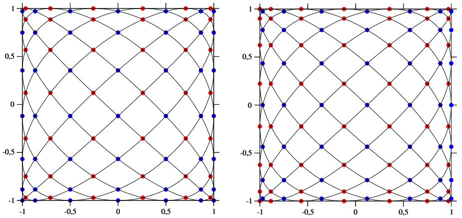
- The computational efficiency ratio between the proposed method and IFT is (if the same numerical integration scheme adopted)

  $$\gamma = \frac{16}{(2M-1)(2N-1)}$$

  where M and N are the total meshing cells along x and y directions.

Numerical integration

- The Padua point based numerical integration method will be used.
- Make an unisolvent point set (unique interpolating polynomial) with minimal growth of their Lebesgue constant [1].
- specifically designed partition of integration area in spectral domain to fully reuse all spectral samplings.
- Computational cost $16 \times \sum_{j=1}^{M_t N_t} O\left(N_j^2 \log N_j\right)$ for all meshing points in one plane, with N_j the order of Chebyshev Polynomial used in the j^{th} partition.



$(-\cos[(n+1)t], -\cos(nt))$, $t \in [0, \pi]$, for $n = 12$ and $n = 13$.

1. L. Bos, M. Caliari, M. Vianello, S. De Marchi, and Y. Xu, J. Approx. Theory, vol. 143, pp. 15-25, 2006.

Numerical test 1

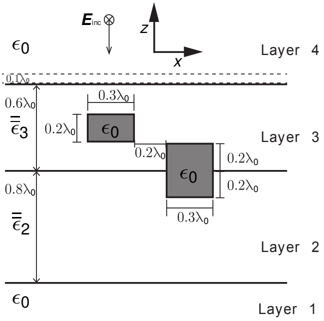
Uniaxial layered media with two air cubes inside:

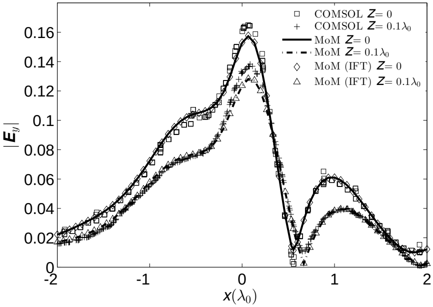
$\bar{\bar{\epsilon}}_2 = \mathrm{diag}\,[3, 2, 2]\,\epsilon_0$ with 30 degree rotation angle.

$\bar{\bar{\epsilon}}_3 = \mathrm{diag}\,[4, 2.5, 2.5]\,\epsilon_0$, with 60 degree rotation angle.

thickness of both scatterers along the **y** direction is 0.3 λ_0.

plane wave normal incidence with *y* polarization.





Numerical test 2

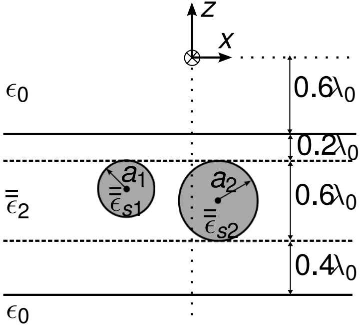
Uniaxial layered media with two spheres inside:

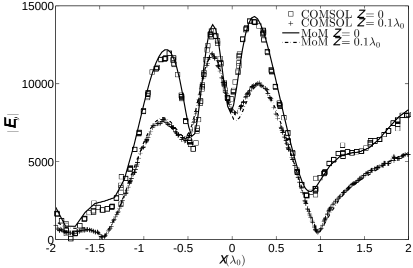
$\bar{\bar{\epsilon}}_2 = \mathrm{diag}\,[5, 2, 2]\,\epsilon_0$ with 60 degree rotation angle.

$\bar{\bar{\epsilon}}_{s1} = \mathrm{diag}\,[1, 1, 1]\,\epsilon_0$ and $\bar{\bar{\epsilon}}_{s2} = \mathrm{diag}\,[3, 1, 1]\,\epsilon_0$ with 30 degree rotation angle.

two spheres are of radii $0.2\lambda_0$ and $0.3\lambda_0$.

y-oriented dipole at $[0, 0, 0.6]\lambda_0$ illuminating the scatterers.





Conclusion

- A fast solution to solve scattering problems from inhomogeneities embedded in uniaxial layered media is presented, which includes
  1. An efficient and stable method to construct the spectral responses of the uniaxial laminates with optical axes in $x - y$ plane;
  2. A fast and accurate method to calculate the spatial responses of the laminates on a rectilinear mesh.
- Next step is for NdT problems involving uniaxial materials: either for inspection problems or for imaging problems.
- Further information can be found in our recently papers:
  1. Y. Zhong, M. Lambert, D. Lesselier, and X. Chen, IEEE Trans. Antennas Propagat., Vol. 62, pp. 247-256, 2014.
  2. Y. Zhong, M. Lambert, P. Ding, D. Lesselier, and X. Chen, submitted to IEEE Trans. Antennas Propagat, 2014.

Institute of High Performance Computing A*STAR NUS National University of Singapore L2S

Thank you!

For more information, please email

Yu Zhong (*zhongyu@ihpc.a-star.edu.sg*)
or
Xudong Chen (*elechenx@nus.edu.sg*)