MUSIC imaging method for low-high frequency inspection of composite multi-layers

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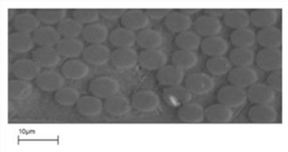
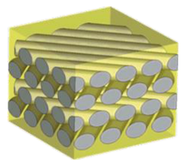
Research context

Objectives

- Electromagnetic models for multi-layer structures
- Imaging methodologies for NdT-E

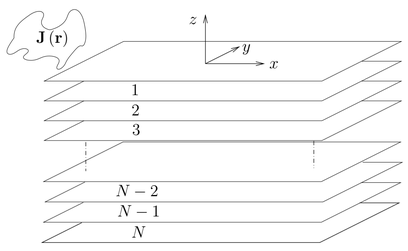




NdT-E within a broad frequency range

The model

Undamaged structure

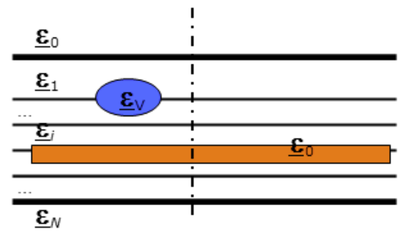


- Electric dipoles and/or magnetic coils with arbitrary position in space
- Elementary sources yielding Green's dyads
- Non-magnetic structure considered $\mu_r = 1$
- Different permittivity tensors $\bar{\bar{\epsilon}}_n$ for each layer $n = 1, \cdots, N$
- Uniaxial dielectric (glass-based) or conductive (graphite-based) media
- Large-scale hypothesis: each layer is homogeneous

$$\bar{\bar{\epsilon}}_n = \mathrm{diag}\left[\varepsilon_{n;11}; \varepsilon_{n;22}; \varepsilon_{n;22}\right] \quad \text{in the material framework}$$

Various kinds of defects

- Small with respect to λ (dielectric case) or δ (conductive case)
- Different parameters (E.M., orientation) with respect to the hosting media ones
- Uniaxial defects: 3-D (voids, fluid-filled cavities, cracks, etc.), or delaminations (thin, air-type gap)



Approximate solution of the direct problem

First-order solution

- Asymptotic formula of the field scattered by an inclusion $\bar{\bar{\epsilon}}_i = \bar{\bar{I}}\epsilon_i$ at the $\mathbf{r}_m$ position

$$\mathbf{E}^{\mathrm{sca}}(\mathbf{r}) = \mathrm{i}\omega\mu_0 \bar{\bar{G}}(\mathbf{r}, \mathbf{r}_m) \cdot \bar{\bar{\varrho}} \cdot \mathbf{E}^{\mathrm{inc}}(\mathbf{r}_m)$$

- Polarization tensor $\bar{\bar{\varrho}} = -\mathrm{i}\omega\epsilon_0 V \mathrm{diag}\left[\alpha_l, \alpha_t, \alpha_t\right]$
- $\alpha_l = \varepsilon_{n;11}\dfrac{\epsilon_i - \varepsilon_{n;11}}{\varepsilon_{n;11} + L_l\left(\epsilon_i - \varepsilon_{n;11}\right)}$, $\alpha_t = \varepsilon_{n;22}\dfrac{\epsilon_i - \varepsilon_{n;22}}{\varepsilon_{n;22} + L_t\left(\epsilon_i - \varepsilon_{n;22}\right)}$
- For a cubical inclusion of side a and volume $V = a^3$: $L_l = c\arctan\left(c/\sqrt{1+2c}\right)$ et $L_l + 2L_t = 1$, $c = \varepsilon_{n;11}/\varepsilon_{n;22}$ are found in W. S. Weiglhofer and A. Lakhtakia, *Int. J. Infrared Millim. Waves* **17** 1365-1376 (1996)

Spectral response

- Numerical solution of the state equation $\dfrac{d}{dz}\boldsymbol{\psi}(z) = \bar{\bar{A}}_n \cdot \boldsymbol{\psi}(z) + \mathbf{f}(z)$ where $\boldsymbol{\psi}(z)$ is the tangential field vector and $\mathbf{f}(z)$ is the source vector

$$\boldsymbol{\psi}(z) = \begin{bmatrix} k_x\tilde{H}_x(z) + k_y\tilde{H}_y(z) \\ k_y\tilde{H}_x(z) - k_x\tilde{H}_y(z) \\ k_x\tilde{E}_x(z) + k_y\tilde{E}_y(z) \\ k_y\tilde{E}_x(z) - k_x\tilde{E}_y(z) \end{bmatrix} \qquad \mathbf{f}(z) = \begin{bmatrix} -k_y\tilde{J}_x(z) + k_x\tilde{J}_y(z) \\ k_y\tilde{J}_y(z) + k_x\tilde{J}_x(z) \\ \dfrac{k_x^2 + k_y^2}{\omega\epsilon_{zz}}\tilde{J}_z(z) \\ 0 \end{bmatrix}$$

- ODE solution via eigenvalue decomposition of the so-called propagator matrix $\bar{\bar{A}}_n$

$$\frac{d}{dz}\boldsymbol{\psi}(z) = \bar{\bar{U}}_n \cdot \bar{\bar{\Lambda}}_n \cdot \bar{\bar{U}}_n^{-1} \cdot \boldsymbol{\psi}(z) + \mathbf{f}(z)$$

Spatial response

- The goal is to compute the I-FT of fast-oscillating functions in the $k_x - k_y$ plane

$$G(x,y) = \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} \tilde{G}_0(k_x, k_y)\mathrm{e}^{\mathrm{i}(k_x x + k_y y)}\,\mathrm{d}k_x\,\mathrm{d}k_y$$

- Interpolation of the stable part through the Pad_n degree of Padua points with scaled Chebyshev polynomials

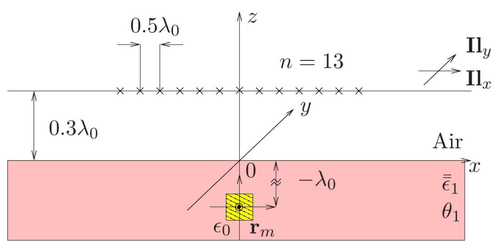
$$\mathcal{L}_{\mathrm{Pad}_n}\tilde{G}_0(k_x, k_y) = \sum_{k=0}^{\mathrm{Pad}_n}\sum_{j=0}^{k} c_{j,k-j}\hat{T}_j(k_x)\hat{T}_{k-j}(k_y) - \frac{1}{2}c_{\mathrm{Pad}_n,0}\hat{T}_{\mathrm{Pad}_n}(k_x)\hat{T}_0(k_y)$$

with the coefficients $c_{j,k-j}$ given by M. Caliari *et al.*, *Num. Alg.* **56** 45-60 (2011)

- FT of Chebyshev polynomials given by V. Domínguez *et al.*, *IMA J. Num. Anal.* **31** 1253-1280 (2011)

$$\int_{-1}^{1}\hat{T}_j(k_x)\mathrm{e}^{-\mathrm{i}k_x x}\,\mathrm{d}k_x = \sqrt{2}\int_{-1}^{1}\cos\left[j\cos^{-1}(k_x)\right]\mathrm{e}^{-\mathrm{i}k_x x}\,\mathrm{d}k_x$$

Numerical results - glass-fiber case



- $f = 6\,\mathrm{GHz}$
- $\bar{\bar{\epsilon}}_1 = \epsilon_0\mathrm{diag}\left[5.46 + \mathrm{i}2.29, 5.21 + \mathrm{i}2.08, 5.21 + \mathrm{i}2.08\right]$
- $\theta_1 = 0^\circ$ with respect to the optical axis x
- Receivers-emitters: array made of 13 electric dipoles, x and y orientation
- Inclusion: air cube of side $0.1\lambda_0$ at $\mathbf{r}_m = (0, 0, -\lambda_0)$

Observed scattered electric field. Emitter is the central dipole with x and y orientation (rows)

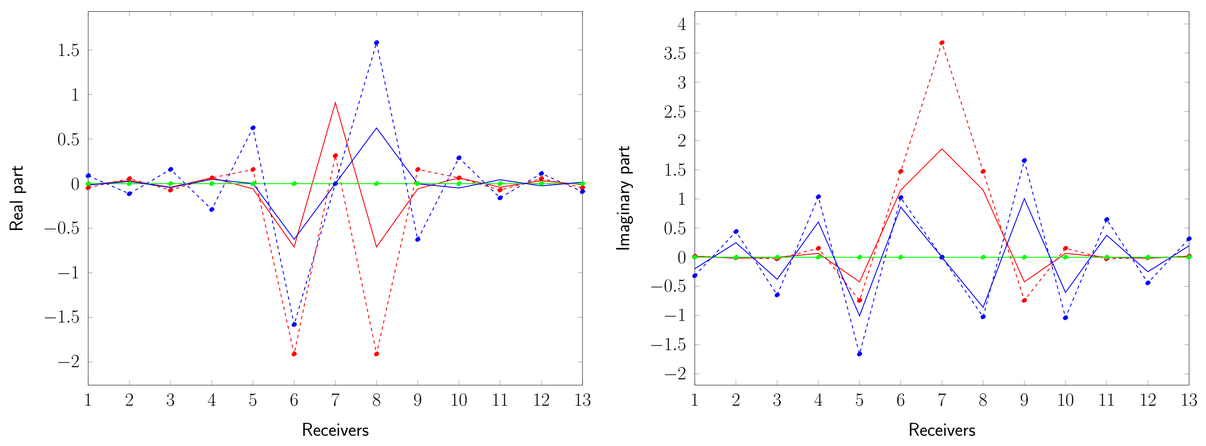


Figure: Dipole along the x axis: E_x (— MoM, -•- asymptotic), E_y (— MoM, -•- asymptotic), E_z (— MoM, -•- asymptotic).

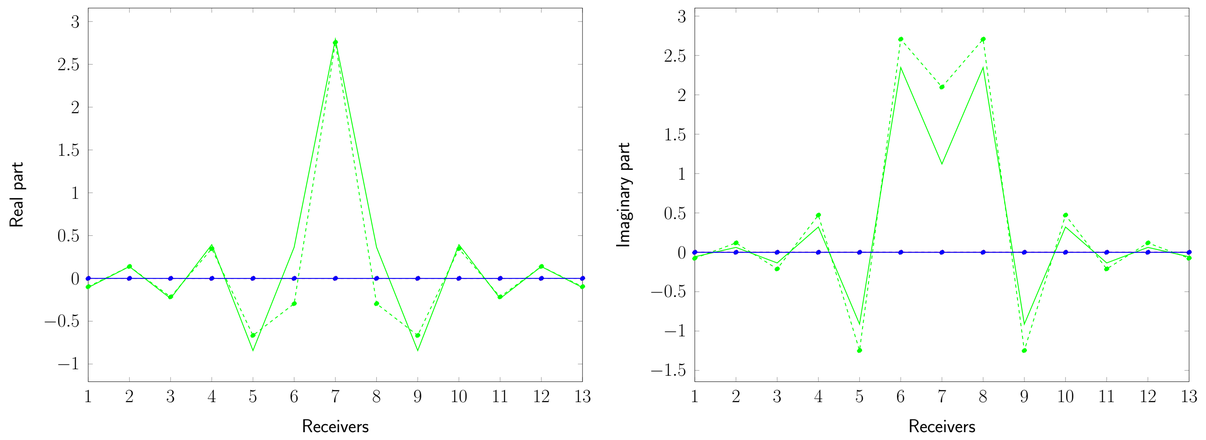
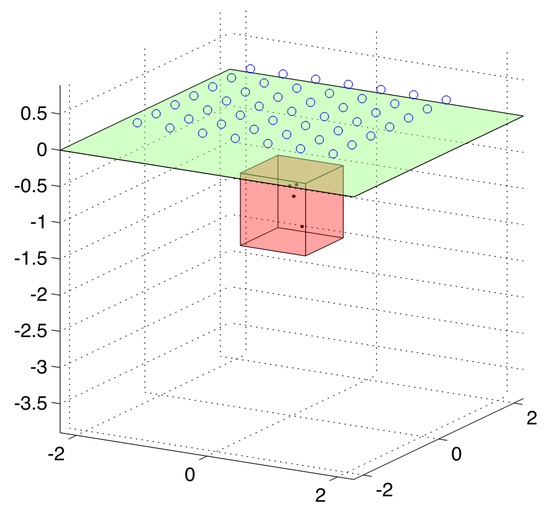


Figure: Dipole along the y axis: E_x (— MoM, -•- asymptotic), E_y (— MoM, -•- asymptotic), E_z (— MoM, -•- asymptotic).

MUSIC imaging results



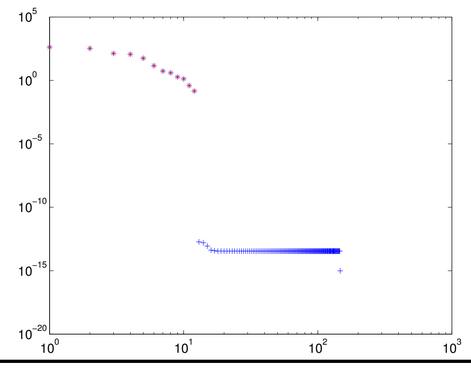
- Acquisition surface area of $L_x \times L_y$, made of 7×7 antennas $L_x = L_y = 3\lambda_0$, $L_{\mathrm{acquisition}} = 0.3\lambda_0$
- Background: fiber-glass $\bar{\bar{\epsilon}}_1 = \epsilon_0\mathrm{diag}\left[5.46 + \mathrm{i}2.29, 5.21 + \mathrm{i}2.08, 5.21 + \mathrm{i}2.08\right]$ $\theta_1 = 0^\circ$
- Air cube inclusions with dimensions $0.1\lambda_0 \times 0.1\lambda_0 \times 0.1\lambda_0$ and permittivity $\bar{\bar{\epsilon}}_{\mathrm{inclusion}} = \bar{\bar{I}}\epsilon_0$

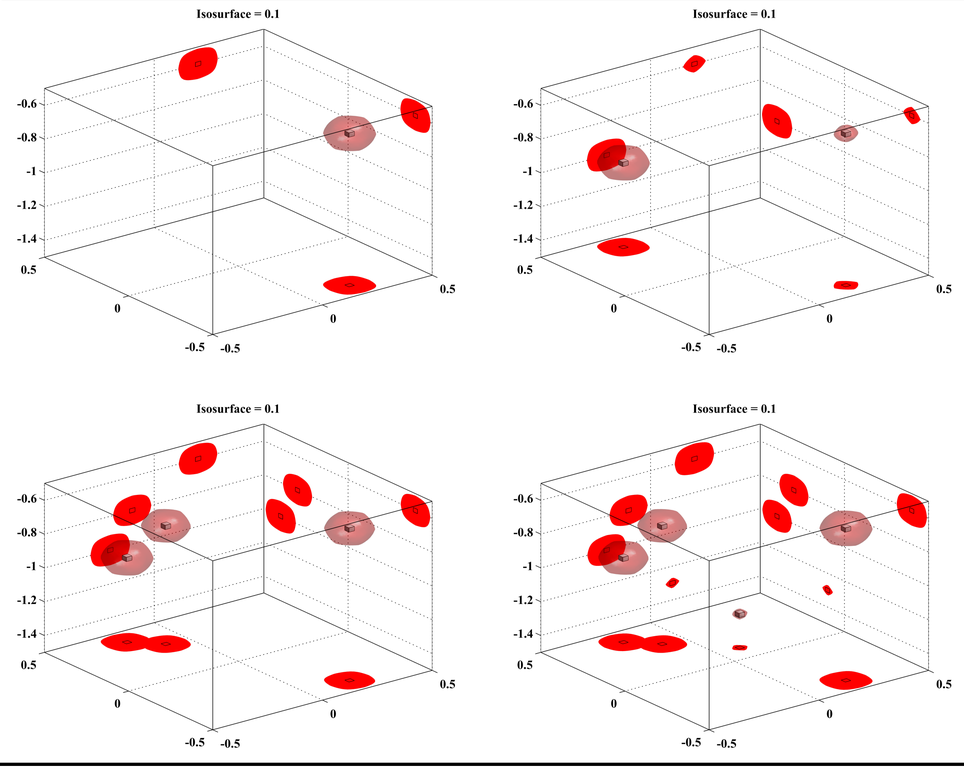
- Standard MUSIC H. Ammari *et al.*, *SIAM J. Sci. Comp.* **29** 674-709 (2007)
- MultiStatic Response matrix

$$\bar{\bar{A}} = \bar{\bar{G}}^{\mathrm{ee}}(\mathbf{r}_o, \mathbf{r}_m) \cdot \bar{\bar{\varrho}} \cdot \bar{\bar{G}}^{\mathrm{ee}}(\mathbf{r}_m, \mathbf{r}_s)$$

- SVD: $\bar{\bar{A}} = \bar{\bar{U}}\bar{\bar{\Sigma}}\bar{\bar{V}}^*$
- Imaging function with the s non-zero singular values in $\bar{\bar{\Sigma}}$

$$\mathbf{W}(\mathbf{x}) = \frac{1}{\left\|\left(\bar{\bar{I}} - V_s V_s^*\right) \cdot \left(\bar{\bar{G}}^{\mathrm{ee}}(\mathbf{x}, \mathbf{r}_s) \cdot \mathbf{a}\right)\right\|^2}$$





Conclusions and perspectives

- Numerical integration method based on the Padua-Domínguez interpolation-integration technique is proposed to efficiently compute integration of fast oscillating functions
- The approach is validated by comparisons with a Method of Moments (MoM) in Y. Zhong *et al.*, *IEEE Trans. Antennas Prop.*, submitted (2014)
- Numerical examples with multi-layer structures made of glass & carbon fiber materials
- Uniaxial defects with various orientations
- Validation by comparisons with experimental data