Sparse grid nested sampling for model selection in eddy-current testing

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SPARSE GRID NESTED SAMPLING FOR MODEL SELECTION IN EDDY-CURRENT TESTING

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Model selection is a common problem that one can run into in non-destructive evaluations. As described in an example of eddy-current testing illustrated in Fig. 1, the measurements from two closed cracks can be very similar to those from a single crack. In this example, to determine the crack number from available measurements can be seen as a model selection problem.

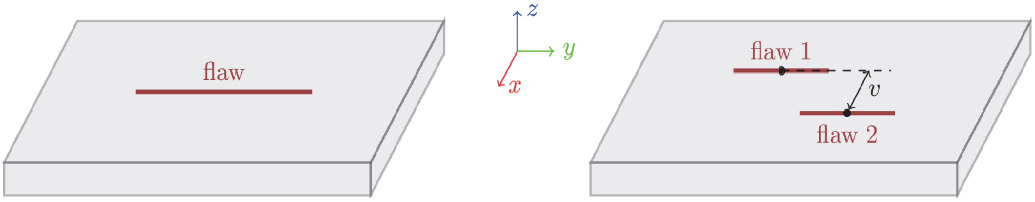

Figure 1. Model selection between a single-crack model and a two-crack model

In the Bayesian framework, selection between two models of interest, $\mathcal{M}_1$ and $\mathcal{M}_2$, can be performed based on the Bayesian factor calculated as follows

$$\begin{aligned} r(\mathcal{M}_1, \mathcal{M}_2) &= \frac{\mathrm{p}(\boldsymbol{y}|\mathcal{M}_1)}{\mathrm{p}(\boldsymbol{y}|\mathcal{M}_2)} = \frac{\mathrm{p}(\boldsymbol{x_1}|\boldsymbol{y}, \mathcal{M}_1)}{\mathrm{p}(\boldsymbol{x_2}|\boldsymbol{y}, \mathcal{M}_2)} \\ &= \frac{\int p(\boldsymbol{y}|\boldsymbol{x_1}, \mathcal{M}_1) p(\boldsymbol{x_1}|\mathcal{M}_1)\, d\boldsymbol{x_1}}{\int \mathrm{p}(\boldsymbol{y}|\boldsymbol{x_2}, \mathcal{M}_2)\, p(\boldsymbol{x_2}|\mathcal{M}_2) d\boldsymbol{x_2}} \end{aligned} \quad (1)$$

where $\boldsymbol{x_1}$, $\boldsymbol{x_2}$ are the unknown parameters in $\mathcal{M}_1$ and $\mathcal{M}_2$, $\boldsymbol{y}$ denotes the measurements. The numerator and the denominator in Eq. (1) are respectively the model evidences for $\mathcal{M}_1$ and $\mathcal{M}_2$. They are also the marginal likelihoods subject to the prior distributions. Without any further information, uniform distributions can be used as the prior models. Among the methods dedicated to model evidence estimation, Nested Sampling (NS) [1, 2, 3] is one of the most efficient one. Compared to traditional Monte-Carlo methods, it offers a good compromise between the computational cost and the ability to manage complicated objective functions. In the present work, we use an accelerated NS method. The acceleration benefits from the existing points in the database and narrows down the parameter search space at the initialisation.

One of the major difficulties in Bayesian model selection is the computational complexity in approximating the model evidences. The complexity is mainly due to the fact that thousands of forward evaluations are often required in approximating the marginal likelihood. Recent works [4, 5] have shown that the computational cost in forward model evaluation can be considerably reduced by using a data-fitting surrogate model. In this surrogate model, a database containing pairs of $\{\boldsymbol{x}^j, \boldsymbol{y}^j\}$ are trained off-line by using a method of moments [6] and the forward model can be replaced by a simple interpolation. However, for models with many unknown parameters, both the off-line database training and the on-line interpolation will be impossible because of the "curse of dimensionality".

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Sparse grids have shown great potential in dealing with the "curse of dimensionality" problem [7]. Here, we introduce sparse grids into the surrogate-model-based NS algorithm. How this method works for a two model selection problem is sketched in Fig. 2. The method can be extended easily to multiple model selections.

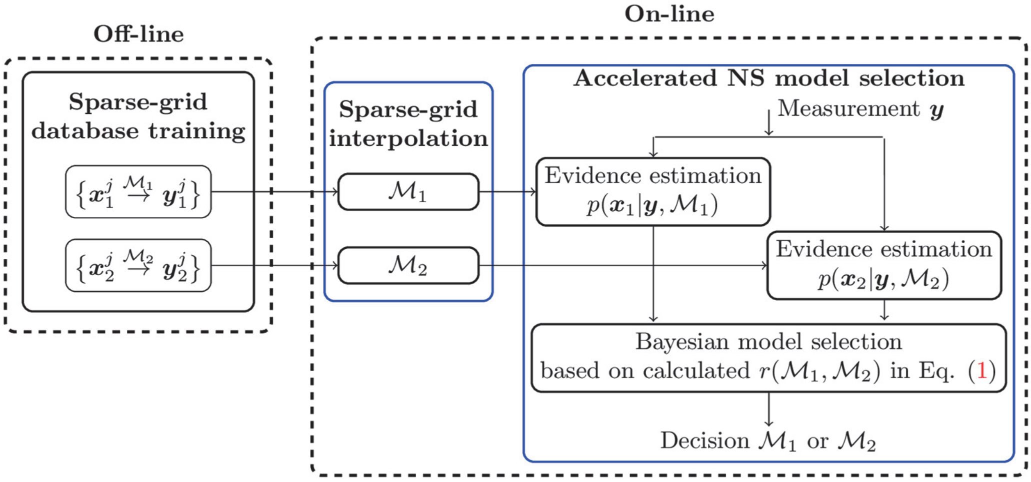

Figure 2. Sketch of the Bayesian model selection method based on sparse grid nested sampling

Simulation tests based on the same example shown in Fig. 1 have been conducted to test the performance of our algorithm. In this example, a non-ferromagnetic plate is affected by surface cracks. With a time-harmonic excitation, a surface scan of impedance variations can be measured and used for model selection between a single-crack model (2 unknown parameters) and a two-crack model (6 unknown parameters). By varying the distance between the two cracks v in the simulation, we show that this Bayesian model selection method has very high selection ability. At a typical signal-to-noise ratio of 20 dB, it is still able to distinguish the *correct* model even when $v = 0.01$ mm. Further details and more examples will be discussed in the full paper.

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