Sparse grids for eddy-current non-destructive testing

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Abstract. Sparse grids make possible the interpolation-based approximation of the output of computer experiments, even for a high number of independent input parameters. In this contribution, the sparse grid interpolation technique is shown to apply well to eddy-current non-destructive testing and simulations thereof. Once such interpolation is obtained –being computationally cheap–, the use of simple optimisation schemes for the inversion becomes numerically efficient. A common pattern-search algorithm illustrates this concept.

1 Introduction

Many inversion schemes in eddy-current non-destructive testing (EC-NdT) aim at fitting the output of a simulation model to the measured data by tuning the model's input parameters. This classical optimisation scheme usually means a huge computational cost due to the numerous forward simulations to perform. A novel approach of reducing the computational burden is to approximate the true simulator by a cheap-to-evaluate surrogate model (or metamodel).

A rich family of surrogate models consists in data-fitting: an interpolation and/or regression is established based on a pre-calculated set of simulation results, i.e., samples. Once the sample set is obtained, the subsequent data-fitting is far less expensive that the true electromagnetic simulation. Among the contributions in the last years, let us cite [1], where the authors combine a radial basis function (RBF) interpolation on optimally scattered samples and particle swarm optimisation (PSO) to efficiently solve EC-NdT inverse problems.

However, most of the approaches so far are limited to a small number of input parameters (i.e., defect parameters sought in the inverse problem which one is faced with) due to the "curse-of-dimensionality". In the present work, a novel scheme is proposed that enables to cope with the growth of dimensionality, consequently, a high number of defect parameters can be sought.

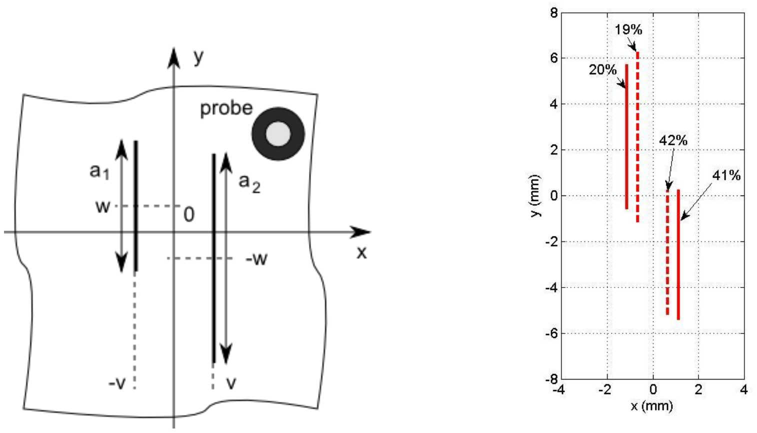

Fig. 1 *Left*: top view of the plate with the two parameterized cracks and the scanning probe. *Right*: reconstruction results in a test case. Solid/dashed lines: real/found cracks. Depths are shown in percentage of the plate thickness.

2 Sparse grid surrogate model and direct search inversion

Let each defect parameter x_i (i = 1,2,…,N) be scaled to the [0,1] interval and let them vary independently. The input space $\mathbf{X}=[0,1]^N$ consists in all conceivable parameter vectors. To obtain a data-fit surrogate model, one has to sample $\mathbf{X}$ and compute the observable signal $Z\{\mathbf{x}\}$ corresponding to each input sample. Sparse grids provide an efficient way for this sampling even for large N. A piecewise multi-linear interpolation $\hat{Z}_n\{\mathbf{x}\}$ can be established for $Z\{\mathbf{x}\}$ by the sparse tensor product of one-dimensional hierarchical bases, based on the n samples $\boldsymbol{x}_k$ (k = 1,2,…,n) in the sparse grid. The sample number n does not depend exponentially on the dimension number N which was a limitation of classical grid-based algorithms. Details will be given in the full paper; as a reference on sparse grids, see, e.g., [2]. Given the set of measured impedance variations Z_{meas}, one aims at solving the regularised inverse problem $\mathbf{x}_0 = \text{argmin} \, \|Z_{\text{meas}} - \hat{Z}_n\{\mathbf{x}\}\|$. Since $\hat{Z}_n$ is cheap-to-evaluate, even classical direct search optimisation methods are able to solve this problem, as shown in the examples.

3 Numerical example

A homogeneous, non-ferromagnetic, infinite metal plate (thickness: 1.25 mm, conductivity: 1 MS/m) is corrupted by two parallel, rectangular, ideally thin cracks, opening at the bottom plate surface (OD-type). Lengths (a_1, a_2), depths (d_1, d_2) and positions (v, w) of the centre – totally, 6 parameters– describe the defect, as shown in Fig. 1. The observed data consist in the impedance variation of an air-cored probe, driven with sinusoidal current (f = 150 kHz), recorded at 297 spatial locations in a flat rectangular domain (a surface scan) over the damaged zone. The impedance variation is calculated by a MoM-simulation [3].

A sparse grid database of n = 1457 samples is generated for this N = 6 dimensional problem based on the algorithms discussed in [4]. One evaluation of $\hat{Z}_n$ takes approx. 5 s with a CPU i3@1.90GHz. The optimisation is performed by a Sequential Quadratic Programming method which needed 10…100 function calls in the cases studied. An example is presented in Fig. 1: the performance of the crack reconstruction is shown to be quite accurate.

4 Conclusion

An efficient inversion method is developed based on the sparse grid interpolation technique. The presented example exhibits the capabilities of the scheme for the reconstruction of 6 parameters. In the full version of the paper, further examples will be presented and the adaptive generation of the sparse grid to increase the interpolation accuracy will be considered also.

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