Scaling solution and n dependance of the eddy current distribution in a flat superconductor

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***Abstract*—In this paper we propose an approximate analytical solution of the problem of nonlinear diffusion of the current density in a HTS superconducting plate with current transport. It is obtained by the technique of the self-similar solution. The construction of this solution highlights a characteristic time of penetration T_p whose limit for large n is the model of Bean. We compare our solution to the ones obtained using COMSOL multiphysics. We study the influence of variation of the magnetic induction on the time penetration and the influence of the n factor on the time penetration.**

***Index Terms*—Self-similar solution, dimensional analysis, ordinary differential equation (ODE), High Temperature Superconductors(HTS).**

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I. Introduction

THE superconductors offer new development prospects for many applications. Indeed, we have an increase of the use of new conductors like MgB_2 or $YBCO$ coated conductors. Therefore, it is important to determine precisely their properties to design some application like superconducting motors or superconducting fault current limiter.
The characterization of the superconductors is made by experimentations and the principals critical values are deduced from. These caracteristics are used to determine the current density and the electric field by a numerical or analytical calculation. They determine the penetration of the induced fields in the superconductors. Unfortunately the experimentals processes plays an important role in their determination, and it became useful to have approximations rules which allow to decribe the effects due to the changing in their values.

In this paper we present an analytical and very easy to implement solution of the penetration of the current density in a superconducting plate taking into account a variation of n factor. This calculation is also important for the problem of trapped flux or magnetic screening, because we can determine easily the time penetration versus the factor n whatever its value.

We study a superconducting plate subjected to an external field. The use of a power constitutive law (1) to describe HTS superconducting materials, leads to non linear vector differential equations (2) which are very difficult to solve.

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$$\frac{\overrightarrow{E}}{E_c} = \left|\frac{\overrightarrow{J}}{J_c}\right|^{n-1}\frac{\overrightarrow{J}}{J_c} \quad (1)$$

$$\overrightarrow{\triangle}\overrightarrow{E} = \mu_0\frac{\partial\overrightarrow{J}}{\partial t} \quad (2)$$

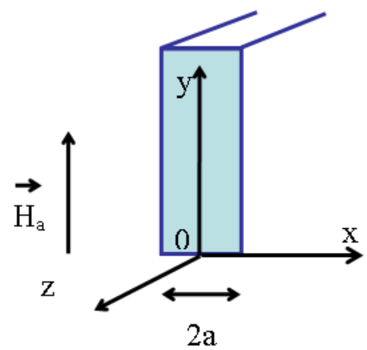

Fig. 1: A superconducting plate which is subjected to an external magnetic field.

This equation (2) is solved by a search of invariant solution by change of scale [3]. It is the search of self-similar solution [4]. This method was used with success to describe magnetic induction in the case of a semi infinite superconducting plate [5]. It is based on the existence of a self-similar variable without dimension :

$$\xi(x,t) = \frac{x}{t^m} \quad (3)$$

With this new variable (3), and equation (2), an ordinary differential equation is constructed. When the magnetic field at the border of the domain has the following form $B = t^p$, with $p \geq 0$, the solution is proportional to $t^p f(\xi(x,t))$ and $f(\xi(x,t))$ is a solution of an ordinary differential equation. This problem has been studied and is well-known [6].

We can consider for a superconducting plate two cases for studies.

- ▷ For a superconducting plate subjected to an external magnetic field the distribution of the current density is asymmetric and the total current is null.
- ▷ For a superconducting plate with current transport, the distribution of the current density is symmetric.

The calculations are made with an electric field which is imposed at the border based on a potential.

Our objective is to compute the electric field induced in a finite superconducting plate subjected to a magnetic field of the following form :

$$\frac{\partial B}{\partial t} = t^p \tag{4}$$

This form for the variation of the magnetic field allows the simulation of different rates of rise of the magnetic field.

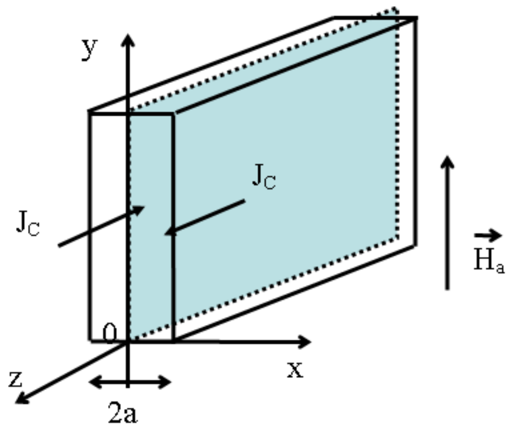

Fig. 2: A symetric distribution of current density in a superconducting plate.

As we can see in Fig.2, the current density has just one component, thus the electric field has just one component: $\overrightarrow{E} = E\overrightarrow{e}_z$. Consequently, the equation (2) is scalar.

$$\triangle E = \mu_0 \frac{\partial J}{\partial t} \tag{5}$$

We adopt the following notations: $v = \dfrac{J}{J_c}$ and $c = \dfrac{\mu_0 J_c}{E_c}$. With these notations, the previous equation becomes :

$$\triangle v^n = c\frac{\partial v}{\partial t} \tag{6}$$

We propose to solve this equation by an analytical way. This choice allows an easy to implement solution with instantaneous calculation.

II. Analytical resolution

We suppose that

$$\frac{\partial B}{\partial t} = \pm V_B t^p \tag{7}$$

Thus, we express our problem

$$\begin{cases} c\dfrac{\partial v}{\partial t} - \dfrac{\partial^2 v^n}{\partial x^2} & = 0 \\ \dfrac{\partial v^n}{\partial x}|_{x=\pm a} & = \pm \frac{V_B}{E_c} t^p \\ v(x,0) & = 0 \end{cases} \tag{8}$$

The main idea of the resolution is to transform this partial differential equation in an ordinary differential equation by a change of variable. We present in the first part the construction of the differential equation and in the second part the solve of the previous equation.

A. A ordinary differential problem

In this part, with the previous partial differential equation of the system (8), we build an ordinary differential equation. This work began by the definition of a dimensionless variable. With new variable we determine a new function and we apply boundary and initial condition to obtain the new differential equation.

1) *Dimensionless variable*

Firstly, our goal is to determine the dimensionless variable. For that, we made a dimensional analysis of the equations to solve :

$$\frac{[v]^n}{[x]^2} = [c]\frac{[v]}{[t]} \quad \text{and} \quad \frac{[v]^n}{[x]} = \left[\frac{V_B}{E_c}\right][t]^p$$

From these two equations, we deduced:

$$[v]^{n-1} = [c]\frac{[x]^2}{[t]} \quad \text{and} \quad [v]^n = [x]\left[\frac{V_B}{E_c}\right][t]^p$$

And we simplify these two previous equation,

$$[v] = [c]^{\frac{1}{n-1}}[x]^{\frac{2}{n-1}}[t]^{\frac{1}{n-1}} \quad \text{and} \quad [v] = [x]^{\frac{1}{n}}\left[\frac{V_B}{E_c}\right]^{\frac{1}{n}}[t]^{\frac{p}{n}}$$

So we deduce that the quantity $\dfrac{[c]^{\frac{1}{n-1}}[x]^{\frac{n+1}{n(n-1)}}}{\left[\frac{V_B}{E_c}\right]^{\frac{1}{n}}[t]^{\frac{n+p(n-1)}{n(n-1)}}}$ is dimensionless. From this result, we choose the following shape for the dimensionless variable :

$$[\xi] = \frac{[c]^{\frac{n}{n+1}}}{\left[\frac{V_B}{E_c}\right]^{\frac{n-1}{n+1}}}\frac{[x]}{[t]^{\frac{n+p(n-1)}{n+1}}} \tag{9}$$

Finally, from this general form, we obtain the following relationship for our variable :

$$\xi(x,t) = \frac{a-|x|}{bt^m} \tag{10}$$

with

$$m = \frac{n+p(n-1)}{n+1} \tag{11}$$

$$b = \left(\frac{V_B}{E_c}\right)^{\frac{n-1}{n+1}} c^{-\frac{n}{n+1}} \tag{12}$$

We need an absolute value for the variable x to reflect the symmetry of the system. A general property of self similar solution gives the following form for the solution $v(x,t)$:

$$v(x,t) = Kt^k g(\xi) \tag{13}$$

Where K, k and function g have to be determined.

2) *The ordinary differential equation*

Now, we develop each term of the system (8) :

$$\frac{\partial^2 v^n}{\partial x^2} = K^n t^{nk} \left(\frac{\partial \xi}{\partial x}\right)\left(\frac{\partial \xi}{\partial x}\right)\frac{\partial^2 g^n}{\partial \xi^2} \tag{14}$$

$$\frac{\partial v}{\partial t} = K\left[kt^{k-1}g(\xi) + t^k \frac{\partial \xi}{\partial t}\frac{\partial g}{\partial \xi}\right] \tag{15}$$

And we have the following expressions for the derivatives of $\xi(x,t)$, when $x \in [-a, 0[\bigcup] 0, a]$ and $t > 0$:

$$\frac{\partial \xi}{\partial x} = -b^{-1}t^{-m} sign(x) \tag{16}$$

$$\frac{\partial \xi}{\partial t} = -m\frac{\xi}{t} \tag{17}$$

After, we replace these equations in the first equation of the system (8). Thus, we have now the following equation to solve :

$$kg(\xi) - m\xi\frac{\partial g}{\partial \xi} = \frac{K^{n-1}}{b^2 c} t^{kn-2m-k+1}\frac{\partial^2 g^n}{\partial \xi^2} \tag{18}$$

The term t in the equation (18) induces a very complicated non linearity. So, the scaling properties of the self similar solution allow us to suppress this term. Therefore we have a choice for the value of k :

$$k = \frac{2m-1}{n-1} \tag{19}$$

With this choice, we obtain a simplification of equation (20).

$$\frac{K^{n-1}}{b^2 c}\frac{\partial^2 g^n}{\partial \xi^2} + m\xi\frac{\partial g}{\partial \xi} - kg(\xi) = 0 \tag{20}$$

We need to calculate the initial and boundary conditions.

3) *Initial condition*
After determining the form of ordinary differential equation, we must calculate the initial condition and the conditions at the borders.

We have $v(x,0) = 0$, because at the initial time the distribution of current is equal to zero in the superconductor. At the initial time the dimensionless variable becomes :

$$\lim_{t \mapsto 0}\xi(x,t) = \lim_{t\mapsto 0}\frac{a-|x|}{bt^m} \longrightarrow \infty \tag{21}$$

As, we have $v(x,t) = Kt^k g(\xi)$, therefore, $v(x,0) = 0$ is equivalent to $g(\infty) = 0$.

4) *Boundary condition*
After initial condition, we need to calculate boundary condition. The beginning of the calculation is the second relation of the system (8)

$$\frac{\partial v^n}{\partial x}|_{x=\pm a} = \pm\frac{V_B}{E_c}t^p$$

So,

$$\frac{\partial v^n}{\partial x}|_{x=\pm a} = -sign(x)\frac{K^n t^{kn-m}}{b}\frac{\partial g^n}{\partial \xi}|_{x=\pm a} \tag{22}$$

With the two previous relations, we obtain

$$\frac{\partial g^n}{\partial \xi}|_{\xi(\pm a,t)} = \pm\frac{1}{sign(x)}\frac{V_B}{E_c}t^{p+kn-m}\frac{b}{K^n} \tag{23}$$

In the same way that for (18), we need to suppress the non linearity induced by the term t. This is realised thanks to the relations (11) and (19) which lead to $p = kn - m$.
Therefore, we have

$$\frac{\partial g^n}{\partial \xi}|_{\xi(\pm a,t)} = -\frac{V_B}{E_c}\frac{b}{K^n} \tag{24}$$

Finally, we have the whole system:

$$\begin{cases} \frac{K^{n-1}}{b^2 c}\frac{\partial^2 g^n}{\partial \xi^2} + m\xi\frac{\partial g}{\partial \xi} - kg(\xi) = 0 \\ \frac{\partial g^n}{\partial \xi}|_{\xi(\pm a,t)} = -\frac{V_B}{E_c}\frac{b}{K^n} \\ g(\infty) = 0 \end{cases} \tag{25}$$

To simplify this expression, we notice $\lambda = \dfrac{K^{n-1}}{b^2 c}$.
We obtain

$$\begin{cases} \lambda\frac{\partial^2 g^n}{\partial \xi^2} + m\xi\frac{\partial g}{\partial \xi} - kg(\xi) = 0 \\ \frac{\partial g^n}{\partial \xi}|_{\xi(\pm a,t)} = -\lambda^{-\frac{n}{n-1}} \\ g(\infty) = 0 \end{cases} \tag{26}$$

B. *Resolution of the ordinary differential problem*

The second step of this work is to solve the system (26). Our work is based on a modification of a calculation made by Mayergoysz [6]. The main idea is to use a particular case of the equation which has an exact solution to obtain the asymptotic behavior of the general solution of the system (26). We write a new formulation the equation that we have to solve introducing ξ_m coefficient.

First equation of system (26) become

$$\lambda\frac{\partial^2 g^n}{\partial \xi^2} + m\xi_m\frac{\partial g}{\partial \xi} - m(\xi_m - \xi)\frac{\partial g}{\partial \xi} - kg(\xi) = 0 \tag{27}$$

First, we calculate a particular solution and after, we present the calculation of the asymptotic behavior of the general solution.

1) *Solution for $m = 1$*
For the case where $m = 1$, we have an exact solution of the equation which is :

$$g(\xi) = \begin{cases} d_1(\xi_1 - \xi)^{\frac{1}{n-1}} & 0 \le \xi \le \xi_1 \\ 0 & \xi \ge \xi_1 \end{cases} \tag{28}$$

And

$$d_1 = \left(\frac{(n-1)\xi_1}{\lambda n}\right)^{\frac{1}{n-1}} \tag{29}$$

We do not calculate ξ_1 for the moment; we make it in a most general case with the boundary condition in the next part.

2) *Solution for $m \neq 1$*
The general solution of our problem has an asymptotic form which looks like the particular solution

$$g(\xi) \propto d_m(\xi_m - \xi)^{\frac{1}{n-1}} \tag{30}$$

The uses of this expression (30) and its introduction in equation (27) give the following expression for d_m.

$$d_m = \left(\frac{m(n-1)}{n}\frac{\xi_m}{\lambda}\right)^{\frac{1}{n-1}} \tag{31}$$

The approximate solution is built like the N terms of series expansion around its asymptotic expression

$$g(\xi) = d_m(\xi_m - \xi)^{\frac{1}{n-1}} \sum_{i=0}^{N} f_i(\xi_m - \xi)^i \tag{32}$$

Let's consider $f_0 = 1$. We limit our calculation to the first term. We show in the presentation of the results that it is sufficient. Thus, we just need to calculate f_1. To make this calculation, we use equation (32). The introduction of $g(\xi)$ in the ODE (27) is taken as a linear combination of $d_m(\xi_m - \xi)^{\gamma+i-1}$:

$$\Phi_0(\xi_m - \xi)^{\gamma-1} + \Phi_1(\xi_m - \xi)^{\gamma} + \Phi_2(\xi_m - \xi)^{\gamma+1} + ... = 0 \tag{33}$$

with $\gamma = \dfrac{1}{n-1}$. It forms a family of a space vector of dimension $N+1$ and it is a base only if all term Φ_i are equal to zero. As we limit our development to the first term, the coefficient Φ_0 and Φ_1 are the following :

$$\Phi_0 = \frac{n}{(n-1)^2}\lambda d_m^n - \frac{m}{n-1}d_m\xi_m \tag{34}$$

$$\begin{aligned}\Phi_1 = {} & \frac{n^2}{(n-1)^2}d_m^n\lambda f_1 + \frac{2n^2}{n-1}\lambda d_m^n f_1 - m\xi_m d_m f_1 \\ & - \frac{md_m}{n-1}\xi_m f_1 + \frac{m}{n-1}d_m - kd_m\end{aligned} \tag{35}$$

From $\Phi_1 = 0$, we obtain

$$f_1 = \frac{k(n-1) - m}{2m\xi_m n(n-1)} \tag{36}$$

Now, it is necessary to calculate the term ξ_m. To this end, we use the boundary condition

$$\frac{\partial g^n}{\partial \xi}|_{\xi(\pm a,t)} = -\lambda^{-\frac{n}{n-1}}$$

We use the following notation :

$$G_0(\xi) = \sum_{i=0}^{N} f_i(\xi_m - \xi)^i \tag{37}$$

$$G_1(\xi) = \sum_{i=1}^{N} i f_i(\xi_m - \xi)^{i-1} \tag{38}$$

So, we obtain the following expression for the boundary conditions

$$\begin{aligned}\frac{\partial g^n}{\partial \xi} = -\left[\frac{1}{n-1}G_0(\xi) + (\xi_m - \xi)\, G_1(\xi)G_0^{n-1}(\xi)\right] \\ \times n d_m^n(\xi_m - \xi)^{\frac{1}{n-1}}\end{aligned} \tag{39}$$

For $x = \pm a$ we have $\xi(x = \pm a, t) = 0$. Therefore, we obtain the following relation

$$\begin{aligned}n d_m^n(\xi_m)^{\frac{1}{n-1}}\left[\frac{1}{n-1}G_0(0) + \xi_m G_1(0)G_0^{n-1}(0)\right] \\ = -\lambda^{-\frac{n}{n-1}}\end{aligned} \tag{40}$$

As, we limit our calculation to the first order :

$$G_0(0) = 1 + \xi_m f_1 \tag{41}$$

$$G_1(0) = f_1 \tag{42}$$

Finally, we obtain

$$\begin{aligned}n d_m^n(\xi_m)^{\frac{1}{n-1}}\left[\frac{1 + \xi_m f_1}{n-1} + \xi_m f_1(1 + \xi_m f_1)^{n-1}\right] \\ = -\lambda^{-\frac{n}{n-1}}\end{aligned} \tag{43}$$

The final result for ξ_m is given by this expression

$$\xi_m = \frac{n}{m^n(n-1)}^{\frac{1}{n-1}}\left[1 + q + (n-1)q(1+q)^{n-1}\right]^{-\frac{n-1}{n+1}} \tag{44}$$

with $q = \dfrac{k(n-1) - m}{2mn(n-1)}$. The general solution can be written as

$$\begin{aligned}g(\xi) \approx \lambda^{-\frac{1}{n-1}}\left(\frac{m(n-1)\xi_m}{n}\right)^{\frac{1}{n-1}}(\xi_m - \xi)^{\frac{1}{n-1}} \\ \times\left(1 + \frac{q}{\xi_m}(\xi_m - \xi)\right)\end{aligned} \tag{45}$$

We have calculated the general solution of our system. The last point consists to determine the current distribution.

C. *Calculation of the current density*

We have $v(x,t) = Kt^k g(\xi)$ and $\xi = \dfrac{a - |x|}{bt^m}$. So, with the previous results

$$\begin{aligned}v(x,t) = Kt^k\lambda^{-\frac{1}{n-1}}\left(\frac{m(n-1)\xi_m}{n}\right)^{\frac{1}{n-1}}\left(\xi_m - \left[\frac{a-|x|}{bt^m}\right]\right)^{\frac{1}{n-1}} \\ \times\left(1 + \frac{q}{\xi_m}\left(\xi_m - \left[\frac{a-|x|}{bt^m}\right]\right)\right)\end{aligned} \tag{46}$$

and $K\lambda^{-\frac{1}{n-1}} = K\left(\dfrac{K^{n-1}}{b^2c}\right) = b^2c$.

So, we have the complete solution for the system (8)

$$\begin{cases} c = \dfrac{\mu_0 J_c}{E_c} \\ b = \dfrac{V_B}{E_c}^{\frac{n-1}{n+1}} c^{-\frac{n}{n-1}} \\ m = \dfrac{n+p(n-1)}{n+1} \\ k = \dfrac{2m-1}{n-1} \\ q = \dfrac{k(n-1)-m}{2mn(n-1)} \\ \xi_m = \dfrac{n}{m^n(n-1)}^{\frac{1}{n-1}} \left[1+q+(n-1)q(1+q)^{n-1}\right]^{-\frac{n-1}{n+1}} \\ \xi = \dfrac{a-|x|}{bt^m} \\ d_m = \left(\dfrac{m(n-1)\xi_m}{n}\right)^{\frac{1}{n-1}} \\ \dfrac{J}{J_c} = \left(b^2c\right)^{\frac{1}{n-1}} t^k (\xi_m-\xi)^{\frac{1}{n-1}} \left[1+\dfrac{q}{\xi_m}(\xi_m-\xi)\right] \end{cases} \quad (47)$$

Now, we can consider two cases, the complete penetration and the incomplete penetration.The limit for both cases is obtained as soon as only $v(x=0,t)$ is equal to zero. We note that $v(x=0,t)=0$ when the dimensionless variable $\xi(x=0,t)$ reaches at $\xi_m : \xi(x=0,t) = \dfrac{a}{bt^m} = \xi_m$. This equality enable us to define the penetration time T_p :

$$T_p = \left(\frac{a}{b\xi_m}\right)^{\frac{1}{m}} \quad (48)$$

1) *Incomplete penetration*
When $t < T_p$, the proposed solution is suitable for study of a superconducting plate when :
 - ▷ An external magnetic field such as $\dfrac{\partial B}{\partial t}|_{x=\pm a} = V_B t^p$ is applied. In this case, for $x \in [-a,a]$, we have $v(-x,t) = -v(x,t)$.
 - ▷ A transport current such as $\dfrac{\partial B}{\partial t}|_{x=\pm a} = \pm V_B t^p$ is imposed. In this case, for $x \in [-a,a]$, we have $v(-x,t) = v(x,t)$.

Both cases, the penetration is characterized by position $x_f(t)$ and speed $\dfrac{dx_f}{dt}$ of the fronts of the current density. We remark that $\xi(x,t) \in [0,\xi_m]$ leads to $a - \xi_m bt^m \le |x| \le a$. This allow us to deduce :

$$x_f(t) = a - \xi_m bt^m \quad (49)$$

$$\frac{dx_f}{dt}(t) = m\xi_m bt^{m-1} \quad (50)$$

2) *Complete penetration*
When $t > T_p$, we have $\xi > \xi_m$. The proposed solution needs additional considerations :
 - ▷ In applied magnetic field, we need to impose $v(x=0,t)=0$.
 - ▷ In transport current, current density becomes rapidly independent of the position x. We consider that $v(x,t) \approx v(t)$. By using the Ampere's law $\dfrac{\partial B}{\partial x} = \mu_0 J_c v(t)$, and the assumption such as $v(t) = \underbrace{v(t) - v(T_p)}_{t>T_p} + \underbrace{v(x=0,T_p)}_{t=T_p}$, we propose the following expression for the complete penetration :

$$v(t) = \frac{V_B}{(p+1)\mu_0 J_c a}\left(t^{p+1} - T_p^{p+1}\right) + \left(b^2c\right)^{\frac{1}{n-1}} T_p^k \times (\xi_m)^{\frac{1}{n-1}} [1+q] \quad (51)$$

III. Results

First, we need to choose the size and characteristics of the superconducting plate and the rate. These characteristics are summarized in the following table.

Parameters	Values	Units
$2a$	10	mm
E_c	10^{-4}	$V.mm^{-1}$
J_c	100	$A.mm^{-2}$
n	20	
p	0	
V_B	1	$T.s^{-1}$

TABLE I: parameters for the simulation

With these data, we can calculate the time for the full penetration in the superconductor : $T_p = 0.72s$

A. Comparison between analytical and numerical results

We present in Fig.3 and Fig.4 the comparison between analytical and numerical results obtain by Comsol [7] for two times. We notice a very good correlation. These results validate our method and the choice of one term in the serial expansion.

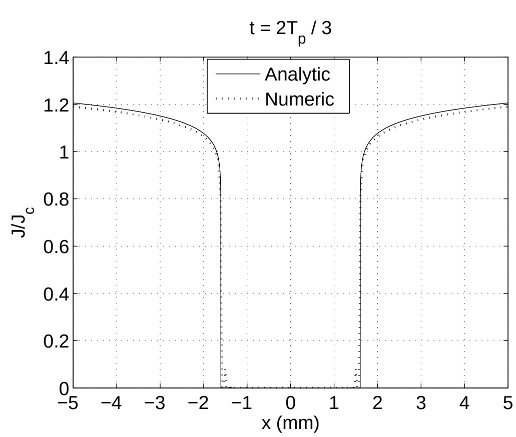

Fig. 3: Comparison of analytical and numerical profiles in partial penetration at $t = 0.48s$

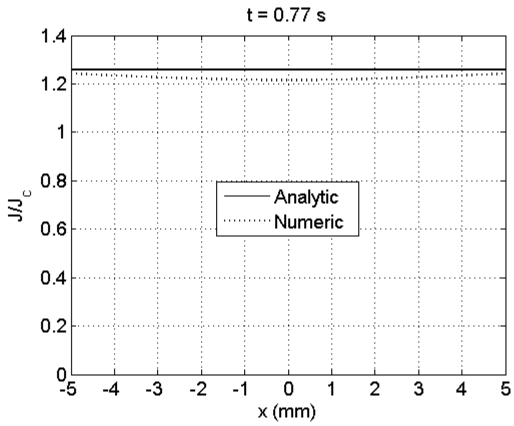

Fig. 4: Comparison of analytical and numerical profiles in full penetration at $t = 0.77s$

B. Influence of $n-$factor on the penetration time

We can show in Fig.5 the variation of the time penetration versus $n-$factor. It is important to remark that this method is suitable to calculate time penetration for high value of $n-$factor.

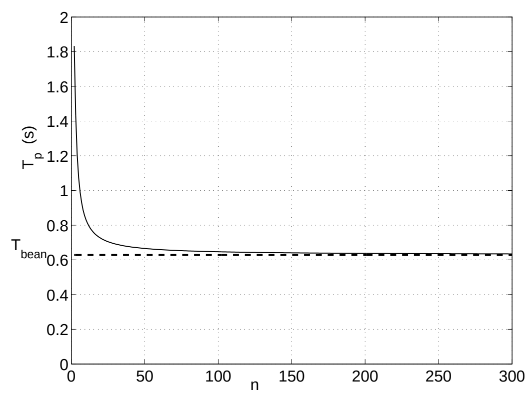

Fig. 5: $n-$factor versus time penetration

We notice that the time of penetration decrease when the n factor increases and it is constant when the n factor is greater than 50. In this case, it coincides with the Bean model.

C. Influence of the rise of magnetic flux on the penetration time

In this part, we study the influence of increased time of magnetic flux on the penetration of the current density in the superconductors.

When V_B is small and n increases, the penetration of J is slower. The position of fronts and the penetration time grow to reach the higher limits given by the Bean model as shown in Fig.6. The Bean model overestimates x_f and T_p. When V_B is large and n increases, the penetration of J is faster. The position of fronts and the penetration time decreases to the lower limits of the Bean model as shown in Fig.7. The Bean model underestimates x_f and T_p.

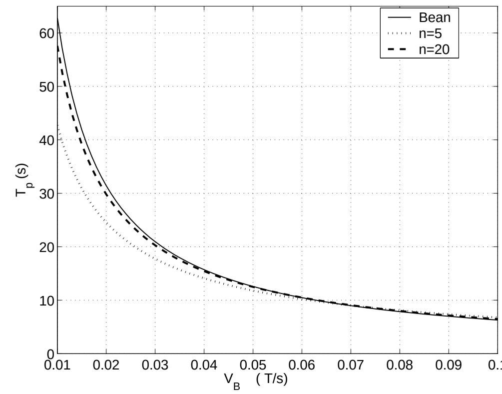

Fig. 6: Time penetration for small values of V_B.

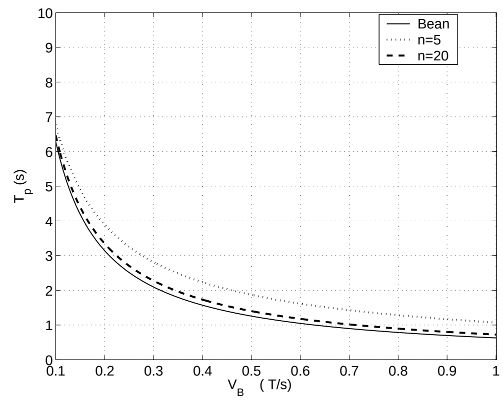

Fig. 7: Time penetration for large values of V_B.

IV. Conclusion

We present in this paper an extension of the method developed for the first time by Mayegoysz. Our calculation is able to determine the diffusion of current density in a finite superconducting plate for for all values of the $n-$factor. We show also the influence of the rise of the magnetic flux on the time penetration and the influence of $n-$factor on the time penetration.

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