Hierarchical Control Strategy based on Robust MPC and Integral Sliding mode - Application to a Continuous Photobioreactor

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Abstract: This paper proposes the design of a hierarchical control strategy formed by a two-level controller: a Linearized Robust MPC (LRMPC) and an Integral Sliding Mode (ISM) control laws. The proposed strategy guarantees robustness towards parameters mismatch for a macroscopic continuous photobioreactor model, obtained from mass balance based modelling. Firstly, as a starting point, this work focuses on classical robust nonlinear model predictive control law under model parameters uncertainties implying solving a basic min-max optimization problem for setpoint trajectory tracking. We reduce this problem into a regularized optimization problem based on the use of linearization techniques, to ensure a good trade-off between tracking accuracy and computation time. Secondly, in order to eliminate the static error due to the fact that the nonlinear model is approximated through linearization in the LRMPC law, an ISM controller is synthesized relying on the knowledge of the nonlinear model of the system. Finally, the efficiency of the developed hierarchical approach is illustrated through numerical results and robustness against parameter uncertainties is discussed for the worst case model mismatch.

Keywords: Robust predictive control, Min-max optimization problem, Integral Sliding Mode, Bioprocesses, Uncertain systems.

1. INTRODUCTION

The industrial success of the microalgae cultivation is due to its biochemical characteristics. The microalga is of particular interest for the growing demand of organic products intended to a large number of industrial applications: food, pharmacology, chemistry and cosmetics production with more recent applications in sustainable environment, such as wastewater treatment or decomposition of different classes of toxic compounds (Spolaore et al., 2006). Hence, to increase significantly the process performances, new challenges emerge related to the control of the biological variables. Biochemical processes are systems where nonlinear effects are significant enough to justify the use of nonlinear model. In addition, generally the process model is identified and uncertain parameters are estimated with evaluated confidence intervals, which motivates the development of robust control laws in the presence of modelling uncertainties. In the literature of microalgae cultivations, several nonlinear control laws have been developed (Masci et al., 2010; Ifrim et al., 2013; Toroghi et al., 2013; Tebbani et al., 2014). They however do not specifically focus on robustness features.

The objective of this study is to design a robust controller which would be able to elaborate an adequate feeding strategy in order to guarantee that the process will yield the desired amount of biomass along the cultivation period under model parameter uncertainties. Here, the challenge is to lay down a stable real time operation, insensitive to various disturbances, then, close to a certain state or desired profile. This requires the application of advanced optimal control strategies to ensure the bioprocess efficiency, among them predictive control is a good candidate. The key advantage of the Model Predictive Control law (MPC) (Camacho and Bordons, 2004) is that it allows the current control input to be determined, while taking into account the future system behavior. This is achieved by optimizing the control profile over a finite time horizon, but applying only the current control input. However, the performances of the NMPC law usually decrease when the true plant evolution deviates significantly from the one predicted by the model. Robust variants of NMPC (RNMPC) (Kerrigan and Maciejowski, 2004; Limon et al., 2004) are able to take into account set bounded disturbance. The RNMPC can be formulated as a nonlinear min-max optimization problem which tends to become too complex to be solved online numerically. Consequently, in order to reduce as much as possible the calculation time, the proposed solution in this study aims at transforming the min-max problem into a robust regularized least squares problem. The original problem is converted into a scalar minimization problem using a model linearization technique (first order Taylor series expansion) at each sampling time along the nominal trajectory which is defined based on the nominal parameter values and the current operating point. The main advantage of LRMPC is to be computationally tractable in calculating the optimal control reducing the computation load. In order to reduce the difference between the dynamics of the nominal model and the current evolution of the state, which is due to the model approximation through linearization and the model uncertainties, the proposed approach consists in using a hierarchical control structure (Rubagotti et al., 2011) which can also be regarded as a way to combine the use of the LRMPC with the ISM (Utkin and Shi, 1996) that guarantees its robustness with respect to model uncertainties. The choice of the ISM is motivated by the fact that this strategy is able to cope with time-varying disturbance terms coming from the linearization step.

The paper is structured as follows. Section 2 examines the class of nonlinear systems that will be considered. In order to regulate the biomass concentration at a desired value, by manipulating the dilution rate chosen as a control variable, Section 3 presents a hierarchical control strategy: LRMPC, based on the linearization technique combined with the ISM. An illustrative example (Droop dynamic model of a continuous photobioreactor) is presented in Section 4. Moreover, numerical results are provided in Section 5 to compare LRMPC and the proposed strategy performances in case of model mismatch. Conclusions and perspectives end this paper in Section 6.

2. PROBLEM STATEMENT

Consider a system described by an uncertain continuous time nonlinear model:

$$\begin{cases} \dot{x}(t) = f(x(t),u(t),\theta), \;\; x(t_0=0) = x_0 \\ y(t) = Hx(t) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state vector and x_0 its initial value, $y \in \mathbb{R}^{n_y}$ is the measured output, f the nonlinear process dynamics, f is of class C^1 with respect to all its arguments, $u \in U \subset \mathbb{R}^{n_u}$ represents the control input with U the set of admissible controls and $\theta \in \mathbb{R}^{n_\theta}$ is the vector of uncertain parameters that are assumed to lie in the admissible region $\Theta = [\theta^-, \theta^+]$. The measurement matrix is given by $H \in \mathbb{R}^{n_y \times n_x}$. Exogenous inputs can act on system (1). They are omitted in the notation for simplification.

The control input u is parametrized using a piecewise-constant approximation ($u(\tau) = u(k)$, $\tau \in [kT_s,(k+1)T_s[$) over a time interval $[t_k,t_{k+1}] \triangleq [kT_s,(k+1)T_s]$ considering a constant sampling time T_s. Let us define the discrete state trajectory g by the solution at time t_{k+1} of system (1) with initial state x_0, and $u_{t_0}^{t_k}$ the control sequence from the initial time instant t_0 to the time instant t_k:

$$x_{k+1} = g(t_0,t_{k+1},x_0,u_{t_0}^{t_k},\theta) \tag{2}$$

where x_{k+1} is the state at t_{k+1}, k is the time index, x_k and y_k are the discrete state vector and the sampled measurement at time k, respectively.

This paper aims at designing a control strategy such that the output signal y_k tracks the reference signal $\bar{y}_k$ without steady state error and with optimized closed-loop performance.

3. CONTROLLER DESIGN

The predictive controller uses a nonlinear dynamic model to predict the behavior of the plant over a finite receding horizon of length N_pT_s starting from the current state. At each time t_k, the optimal control sequence is computed by minimizing a cost function expressed as a quadratic criterion based on the tracking error while making sure that all constraints are respected. This optimal control sequence is implemented until the next measurement becomes available. The optimization problem is solved again at the next sampling time according to the receding horizon principle.

In practice the parameter vector θ is often uncertain. The parameters values are nevertheless assumed to belong to the known region Θ. In this case, robust predictive control strategy (RNMPC) implying a min-max optimization problem (Kerrigan and Maciejowski, 2004) can be defined as follows :

$$\tilde{u}_k^{k+N_p-1} = arg \min_{u_k^{k+N_p-1}} \max_{\theta\in\Theta} \Pi(u_k^{k+N_p-1},\theta) \tag{3}$$

where the cost function is defined as

$$\Pi(u_k^{k+N_p-1},\theta) = ||u_k^{k+N_p-1} - \bar{u}_k^{k+N_p-1}||_V^2 + ||\hat{y}_{k+1}^{k+N_p} - \bar{y}_{k+1}^{k+N_p}||_W^2 \tag{4}$$

with

$||z||_P^2 = z^\top Pz$, the Euclidean norm weighted by P.

$u_k^{k+N_p-1} = [u_k^\top,\ldots,u_{k+N_p-1}^\top]^\top$ the optimization variable,

$\bar{u}_k^{k+N_p-1} = [u_k^{r\top},\ldots,u_{k+N_p-1}^{r\top}]^\top$ the reference control sequence,

$$\hat{y}_{k+1}^{k+N_p} = \begin{bmatrix} Hg(t_k,t_{k+1},x_k,u_k,\theta) \\ Hg(t_k,t_{k+2},x_k,u_k^{k+1},\theta) \\ \vdots \\ Hg(t_k,t_{k+N_p},x_k,u_k^{k+N_p-1},\theta) \end{bmatrix} \text{ the predicted output,} \tag{5}$$

and $\bar{y}_{k+1}^{k+N_p} = [y_{k+1}^{r\top},\ldots,y_{k+N_p}^{r\top}]^\top$ the setpoint values.

$V \geq 0$ and $W > 0$ are tuning diagonal matrices.

The optimal control sequence is determined so that the maximum deviation for all trajectories over all possible data scenarii is minimized. Nevertheless, the min-max optimization problem is time consuming. In the sequel, it will be simplified, in order to reduce the online computational burden.

3.1 Linearized Robust Model Predictive controller

In this paper, we propose a new formulation of RNMPC law. From (2), the predicted state for time t_{k+j}, starting from state at t_k, is linearized around the reference trajectory given by the reference control sequence $\bar{u}_k^{k+N_p-1}$ and for the nominal parameters, $\theta_{nom} = (\theta^+ + \theta^-)/2$. A first order Taylor series expansion for $j = \overline{1,N_p}$ is used:

$$\begin{aligned} g(t_k,t_{k+j},x_k,u_k^{k+j-1},\theta) &\approx g_{nom}(t_{k+j}) + \\ \nabla_u g(t_{k+j})(u_k^{k+j-1} - \bar{u}_k^{k+j-1}) &+ \nabla_\theta g(t_{k+j})(\theta-\theta_{nom}) \end{aligned} \tag{6}$$

with

$$g_{nom}(t_{k+j}) = g(t_k,t_{k+j},x_k,\bar{u}_k^{k+j-1},\theta_{nom}) \tag{7}$$

$$\nabla_\theta g(t_{k+j}) = \frac{\partial g(t_k,t_{k+j},x_k,u_k^{k+j-1},\theta)}{\partial\theta}\Bigg|_{\substack{u_k^{k+j-1}=\bar{u}_k^{k+j-1} \\ \theta=\theta_{nom}}} \tag{8}$$

$$\nabla_u g(t_{k+j}) = \frac{\partial g(t_k,t_{k+j},x_k,u_k^{k+j-1},\theta)}{\partial u_k^{k+j-1}}\Bigg|_{\substack{u_k^{k+j-1}=\bar{u}_k^{k+j-1} \\ \theta=\theta_{nom}}} \tag{9}$$

Different approaches are possible for determining the sensitivity functions defined in (8). The most precise method involves analytical derivation (Dochain, 2008): the dynamics of sensitivity function with respect to θ can be computed for time $t \in [t_k,t_{k+N_p}]$ by solving numerically the following differential equation (from (1 and 2)):

$$\frac{d}{dt}(\nabla_\theta g) = \frac{\partial f(x,u,\theta_{nom})}{\partial x}\nabla_\theta g + \frac{\partial f(x,u,\theta)}{\partial\theta}\Big|_{\theta=\theta_{nom}} \tag{10}$$

with as an initial condition:

$$\nabla g_\theta(t_k) = 0_{n_x\times n_\theta} \tag{11}$$

where $0_{n\times m} \in \mathbb{R}^{n\times m}$ is the zero matrix.

In order to simplify the calculation of the gradient $\nabla_u g$, finite differences are used to approximate numerically the derivative $\nabla_u g(t_{k+j})$ for each control u_j, $j \in [k, k+N_p-1]$.

From (5) and (6), it comes:

$$\begin{aligned}\hat{y}_{k+1}^{k+N_p} \approx\ & \bar{G}_{nom,k+1}^{k+N_p} + \bar{G}_{u,k}^{k+N_p-1}(u_k^{k+N_p-1} - \bar{u}_k^{k+N_p-1}) \\ & + \bar{G}_{\theta,k+1}^{k+N_p}(\theta - \theta_{nom})\end{aligned} \quad (12)$$

with $\bar{G}_{nom,k+1}^{k+N_p^\top} = [Hg_{nom}(t_{k+1}), \ldots, Hg_{nom}(t_{k+N_p})]$, the vector containing the predicted output for the nominal case.
$\bar{G}_{u,k+1}^{k+N_p^\top} = [H\nabla_u g(t_{k+1}), \ldots, H\nabla_u g(t_{k+N_p})]$, the vector of Jacobian matrices related to the control sequence.
$\bar{G}_{\theta,k+1}^{k+N_p^\top} = [H\nabla_\theta g(t_{k+1}), \ldots, H\nabla_\theta g(t_{k+N_p})]$, the vector of Jacobian matrices related to the parameters.

Assuming that the uncertain parameters are uncorrelated, then the bounded parametric error can be expressed by:

$$\theta - \theta_{nom} = \gamma \delta\theta_{max} \quad (13)$$

with

$$\delta\theta_{max} = (\theta^+ - \theta^-)/2 \text{ and } ||\gamma|| \leq 1 \quad (14)$$

Matrix norm $||A||$ is given by $||A|| = \sqrt{\bar{\sigma}(A^\top A)}$ with $\bar{\sigma}(A)$ the maximum eigenvalue of A.
The min-max optimization problem (3) is converted into a robust regularized least squares problem when applying (12-14) in the presence of uncertain data (Sayed et al., 2002).
Let us consider the following optimization problem:

$$\tilde{z} = arg \min_z \max_{\delta A, \delta b} ||z||_V^2 + ||(A+\delta A)z - (b+\delta b)||_W^2 \quad (15)$$

where δA denotes a perturbation matrix to the nominal matrix A and δb a perturbation vector to the nominal vector b which are assumed to satisfy the following factorized form:

$$\begin{cases} \delta A = C\Delta E_a \\ \delta b = C\Delta E_b \end{cases} \quad (16)$$

where Δ denotes an arbitrary contraction with $||\Delta|| \leq 1$.
$C \neq 0$, E_a and E_b are known quantities of appropriate dimensions.
The uncertainties in matrix A and vector b are modelled with an unknown perturbation vector y:

$$Cy = \delta Az - \delta b \quad (17)$$

Using (17), we rewrite the optimization problem (15) as follows:

$$\min_z \max_{||y|| \leq \pi(z)} ||z||_V^2 + ||Az - b + Cy||_W^2 \quad (18)$$

The nonnegative function $\pi(z)$ is assumed to be a known bound on the perturbation y and is a function of z only.
From (16) and (17), it comes:

$$\pi(z) = ||E_a z - E_b|| \quad (19)$$

Introducing the Lagrange multiplier λ, the problem (18) becomes equivalent to (Sayed et al., 2002):

$$\min_{\lambda \geq ||C^\top WC||} \min_z z^\top Vz + (Az-b)^\top W(\lambda)(Az-b) + \lambda\pi(z)^2 \quad (20)$$

where the minimizer z must satisfy the equation

$$(V + A^\top W(\lambda)A)z + \frac{1}{2}\lambda\nabla\pi^2(z) = A^\top W(\lambda)b \quad (21)$$

The modified weighting matrix $W(\lambda)$ is obtained from W via:

$$W(\lambda) = W + WC(\lambda I - C^\top WC)^\dagger C^\top W \quad (22)$$

The solution of equation (21) becomes

$$z(\lambda) = E(\lambda)^{-1}B(\lambda) \quad (23)$$

with

$$\begin{cases} E(\lambda) = V(\lambda) + A^\top W(\lambda)A + \lambda E_a^\top E_a \\ B(\lambda) = A^\top W(\lambda)b + \lambda E_a^\top E_b \end{cases} \quad (24)$$

where the modified weighting matrix $V(\lambda)$ is obtained from V via:

$$V(\lambda) = V + \lambda E_a^\top E_a \quad (25)$$

The invertibility of $E(\lambda)$ is guaranteed by the positive definiteness of V.
The nonnegative scalar parameter $\lambda^o \in \mathbb{R}$ solution of (20), is computed from the following unidimensional minimization:

$$\begin{aligned}\lambda^o = arg \min_{\lambda \geq \lambda_l}\ & ||z(\lambda)||_V^2 + \lambda||E_a z(\lambda) - E_b||^2 + \\ & ||Az(\lambda) - b||_{W(\lambda)}^2\end{aligned} \quad (26)$$

The lower bound on λ is denoted by λ_l, with: $\lambda_l = ||C^\top WC||$.
For any value of λ in the semi-open interval $[\lambda_l, +\infty[$, the matrix $W(\lambda)$ is nonnegative definite so that criterion (26) is nonnegative for $\lambda \geq \lambda_l$.
Finally, the problem has a unique global minimum z^o given by (from (23-26)):

$$\tilde{z} = z^o(\lambda^o) = E(\lambda^o)^{-1}B(\lambda^o) \quad (27)$$

For more details see Sayed et al. (2002).
The robust nonlinear predictive problem which is defined by (3-4), is written in the form (15-16) with:

$$\begin{cases} z = u_k^{k+N_p-1} - \bar{u}_k^{k+N_p-1}, A = \bar{G}_{u,k}^{k+N_p-1}, b = \hat{y}_{k+1}^{k+N_p} - \bar{G}_{nom,k+1}^{k+N_p} \\ C = \bar{G}_{\theta,k+1}^{k+N_p}, \Delta = \gamma, E_a = 0, E_b = -\delta\theta_{max} \end{cases} \quad (28)$$

The application of (24-27) provides the solution of (3-4):

- λ^o is computed from the following minimization problem:

$$\lambda^o = arg \min_{\lambda \geq ||\bar{G}_{\theta,k+1}^{k+N_p^\top} W \bar{G}_{\theta,k+1}^{k+N_p}||} G(\lambda) \quad (29)$$

  where the function $G(\lambda)$ is defined by:

$$\begin{aligned}G(\lambda) =\ & ||\bar{G}_{u,k}^{k+N_p-1} z(\lambda) - \hat{y}_{k+1}^{k+N_p} + \bar{G}_{nom,k+1}^{k+N_p}||_{W(\lambda)}^2 + \\ & ||z(\lambda)||_V^2 + \lambda||\delta\theta_{max}||^2\end{aligned} \quad (30)$$

  with

$$\begin{aligned}z(\lambda) =\ & [V + \bar{G}_{u,k}^{k+N_p-1^\top} W(\lambda) \bar{G}_{u,k}^{k+N_p-1}]^{-1} \\ & [\bar{G}_{u,k}^{k+N_p-1^\top} W(\lambda)(\hat{y}_{k+1}^{k+N_p} - \bar{G}_{nom,k+1}^{k+N_p})]\end{aligned} \quad (31)$$

  and

$$W(\lambda) = W + W\bar{G}_{\theta,k+1}^{k+N_p}(\lambda I - \bar{G}_{\theta,k+1}^{k+N_p^\top} W \bar{G}_{\theta,k+1}^{k+N_p})^\dagger \bar{G}_{\theta,k+1}^{k+N_p^\top} W \quad (32)$$

- The control sequence is derived from (27):

$$\begin{aligned}\tilde{u}_k^{k+N_p-1} =\ & \bar{u}_k^{k+N_p-1} + [V + \bar{G}_{u,k}^{k+N_p-1^\top} W(\lambda^o) \bar{G}_{u,k}^{k+N_p-1}]^{-1} \\ & [\bar{G}_{u,k}^{k+N_p-1^\top} W(\lambda^o)(\hat{y}_{k+1}^{k+N_p} - \bar{G}_{nom,k+1}^{k+N_p})]\end{aligned} \quad (33)$$

  with $W(\lambda^o)$ given in (32) for $\lambda = \lambda^o$.

The minimum λ^o of the unidimensional function $G(\lambda)$ is found using the golden section search algorithm.
As a conclusion, the predictive controller consists in solving online an unidimensional optimization problem (29-30) at each sampling time, instead of solving min-max problem (3-4). In the sequel, this predictive control law will be called as linearized robust model predictive controller (LRMPC).

3.2 Integral Sliding Mode

To go further with model uncertainties and linearization drawback, the idea is to use the hierarchical control scheme (fig. 1), as similar to the one proposed in (Rubagotti et al., 2011). The control strategy is formed by an Integral Sliding mode (ISM) controller and a LRMPC law. The reason for the choice of the ISM is that this strategy can eliminate the static error due to the approximation of the model through the linearization and the model mismatch with guaranteed stability. Details related to ISM can be found in (Utkin and Shi, 1996).

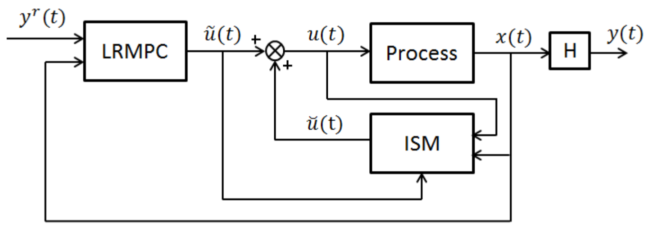

Fig. 1. Scheme of the hierarchical control strategy.

For the considered application (Section 4), the system has a single output y and single input u and is control-affine, which is a special case of (1). We will then consider for the ISM development, the following model:

$$\begin{cases} \dot{x} = f_x(x,\theta) + f_u(x,\theta)u, \ \forall t > t_0, \ x(t_0) = x_0 \\ y = Hx \end{cases} \tag{34}$$

At each time instant $t = kT_s$, the goal is to complete the optimal control law $\tilde{u}(t)$ obtained from the predictive controller (33) by an Integral Sliding mode control law $\breve{u}(t)$ in order to cancel the error between the prediction model output and the system output. In this study, the ISM design is performed in continuous-time, then discretized for implementation.

The sliding mode control design consists in choosing the control input in such a way to drive the system to reach a sliding manifold and maintain there for all future time. The goal is to track the predicted output $\hat{y}$ in order to cancel the difference between the model prediction output and the system output. The local attractivity of the sliding surface ϕ can be expressed by the condition:

$$\forall x \in \mathbb{R}^{n_x} : \dot{\phi}(x,t)\phi(x,t) < 0 \tag{35}$$

Let us define the modelling error variables (Toroghi et al., 2013) for $t \in [t_{k-1}, t_k]$:

$$\begin{cases} Z_1(t) = \displaystyle\int_{t_{k-1}}^{t} (y(\tau) - \hat{y}(\tau))\,\mathrm{d}\tau \\ Z_2(t) = \xi_1^{-1}(y(t) - \hat{y}(t) - \xi_2 Z_1(t)) \end{cases} \tag{36}$$

where

$$\hat{y}(t) = Hg(t_{k-1}, t, x_{k-1}, u_{k-1}, \theta_{nom}) \tag{37}$$

$\hat{y}(t)$ represents the model prediction resulting from the application of the previous control input.
Differentiating (36) with respect to time, we obtain:

$$\begin{cases} \dot{Z}_1(t) = \xi_1 Z_2 + \xi_2 Z_1 \\ \dot{Z}_2(t) = \xi_1^{-1}(\dot{y}(t) - \dot{\hat{y}}(t) - \xi_2 \dot{Z}_1(t)) \end{cases} \tag{38}$$

A time varying sliding surface $\phi(x,t)$ is defined in the state space $\mathbb{R}^{n_x}$ as

$$\phi(x,t) = Z_2(t) + \xi_3 Z_1(t) \tag{39}$$

The ISM control law needs to be designed so that the invariance of the sliding manifold is satisfied:

$$\forall x \in \mathbb{R}^{n_x}, \phi(x,t) = 0 \tag{40}$$

From (38), (39) and (40), it comes:

$$\dot{Z}_1(t) = (\xi_2 - \xi_1\xi_3)Z_1(t) \tag{41}$$

In order to assure the convergence of Z_1, the following condition must be satisfied:

$$\xi_2 - \xi_1\xi_3 < 0 \tag{42}$$

Consequently, differentiating the sliding surface vector (39), we obtain:

$$\dot{\phi}(x,t) = \ \xi_1^{-1}(\dot{y}(t) - \dot{\hat{y}}(t) - (\xi_2 - \xi_1\xi_3)(\xi_1 Z_2(t) + \xi_2 Z_1(t))) \tag{43}$$

The system output is obtained by the application of the previous control input u_{k-1} combined with the sliding mode control law $\breve{u}(t)$ and the predicted output is generated by applying only the previous input u_{k-1} as follows:

$$\begin{cases} \dot{y}(t) = H\left(f_x(x,\theta) + f_u(x,\theta)(u_{k-1} + \breve{u}(t))\right) \\ \dot{\hat{y}}(t) = H\left(f_x(\hat{x},\theta_{nom}) + f_u(\hat{x},\theta_{nom})u_{k-1}\right) \end{cases} \tag{44}$$

where the difference between the system output and the nominal model prediction output is due only to parameters uncertainties:

$$\dot{y}(t) - \dot{\hat{y}}(t) = \varphi + \chi u_{k-1} + \eta\breve{u}(t) \tag{45}$$

with

$$\begin{cases} \varphi = H(f_x(x,\theta) - f_x(\hat{x},\theta_{nom})), \\ \chi = H(f_u(x,\theta) - f_u(\hat{x},\theta_{nom})), \\ \eta = Hf_u(x,\theta) \end{cases} \tag{46}$$

Hence, the sliding surface (39) is made attractive by choosing:

$$\dot{\phi}(x,t) = -K_s sign(\phi(x,t)) \tag{47}$$

where the switching gain K_s is a strictly positive constant.
From (43-47), it is deduced that the control law can be accordingly found as:

$$\begin{aligned} \breve{u}(t) = \ & \eta^{-1}(-\varphi - \chi u_{k-1} - \xi_1 K_s sign(\phi(x,t)) \\ & +(\xi_2 - \xi_1\xi_3)(\xi_1 Z_2(t) + \xi_2 Z_1(t))) \end{aligned} \tag{48}$$

Note that the term η must be regular.
Then, the attractive equation which implies that the distance to the sliding surface decreases along all system trajectories is satisfied since (from 43-48):

$$\dot{\phi}(x,t)\phi(x,t) = -K_s|\phi(x,t)| < 0 \tag{49}$$

Here, in order to eliminate chattering phenomenon, a hyperbolic function is used instead of the switching function $sign(\phi(x,t))$. Finally, with (48) evaluated at $t = t_k$, the control input is obtained as the sum of two parts, given by

$$u(t_k) = \tilde{u}(t_k) + \breve{u}(t_k) \tag{50}$$

The component $\tilde{u}(t_k)$ (the first value of the optimal control sequence) is generated by the LRMPC controller, while $\breve{u}(t_k)$ is generated by the Integral sliding mode controller as shown in the following algorithm:

Step 1: Initialisation.
Step 2: Update x_k.
Step 3: Compute $\bar{G}_u$, $\bar{G}_{nom}$, $\bar{G}_\theta \longleftarrow x_k$, $\bar{u}$, $\bar{y}$, θ_{nom}.
Step 4: Determine $\lambda^o \longleftarrow \bar{G}_u$, $\bar{G}_{nom}$, $\bar{G}_\theta$, $\bar{y}$, V, W, $\delta\theta_{max}$.
Step 5: Compute $\tilde{u}_k \longleftarrow \bar{G}_u$, $\bar{G}_{nom}$, $\bar{G}_\theta$, $\bar{y}$, $\bar{u}$, V, W, λ^o
Step 6: Compute $\breve{u}_k \longleftarrow x_k$, θ_{nom}, u_{k-1}.
Step 7: Apply $u_k \longleftarrow \tilde{u}_k + \breve{u}_k$.
Step 8: Go to Step 2.

4. ILLUSTRATIVE EXAMPLE

The process under consideration here is a continuous photobioreactor (medium withdrawal flow rate equals its supply one, leading to a constant effective volume), without any additional biomass in the feed, and neglecting the effect of gas exchanges. The nonlinear model is represented in the state-space formalism (34), with:

$$\begin{cases} x = \begin{bmatrix} X \\ Q \\ S \end{bmatrix}, f_x = \begin{bmatrix} \mu(Q,I)X \\ \rho(S) - \mu(Q,I)Q \\ -\rho(S)X \end{bmatrix}, f_u = \begin{bmatrix} -X \\ 0 \\ (S_{in} - S) \end{bmatrix} \\ \theta = [\rho_m \;\; K_s \;\; \bar{\mu} \;\; K_Q \;\; K_{sI} \;\; K_{iI}]^\top, \; u = D, \; y = X \end{cases} \tag{51}$$

where D represents the dilution rate (d^{-1}, d: day).
The specific uptake rate, $\rho(S)$, and the specific growth rate, $\mu(Q,I)$, are given by:

$$\rho(S) = \rho_m \frac{S}{S + K_s} \;\; , \;\; \mu(Q,I) = \bar{\mu}(1 - \frac{K_Q}{Q}) \frac{I}{I + K_{sI} + \frac{I^2}{K_{iI}}} \tag{52}$$

To simplify notations, the exogenous inputs (S_{in}, I) are omitted but are applied to the model. In the sequel, the light intensity I is either set at its optimal value $I_{opt} = \sqrt{K_{sI}K_{iI}}$ or is time varying, modelling the night/day cycle. The parameters of the model used in this study are displayed in Table 1 (Goffaux and Vande Wouwer, 2008; Munoz-Tamayo et al., 2014).

Table 1. Model parameters.

	Paramet.	Value	Unit
maximal specific growth rate	$\bar{\mu}$	2	d^{-1}
maximal specific uptake rate	ρ_m	9.3	$\mu mol \; \mu m^{-3} \; d^{-1}$
minimal cell quota	K_Q	1.8	$\mu mol \; \mu m^{-3}$
substrate half saturation constant	K_s	0.105	$\mu mol \; L^{-1}$
light saturation constant	K_{sI}	150	$\mu E \; m^{-2} \; s^{-1}$
light inhibition constant	K_{iI}	2000	$\mu E \; m^{-2} \; s^{-1}$
inlet substrate concentration	S_{in}	100	$\mu mol \; L^{-1}$
optimal light intensity	I_{opt}	547	$\mu E \; m^{-2} \; s^{-1}$

Details related to the modelling can be found in (Benattia et al., 2014a). The main objective of this study is to regulate the biomass concentration X to a reference value X^r, while the dilution rate D is constrained to track the reference D^r. The dilution rate reference trajectory is computed from the knowledge of the targeted setpoint at each time instant as detailed in (Benattia et al., 2014b).

5. RESULTS AND DISCUSSION

In this section, the efficiency of the proposed control strategy is validated in simulation. The performances of the above mentioned algorithms are compared for a worst uncertain parameters case. The worst case biomass prediction can be approximated using parameter bounds $\{\theta^-, \theta^+\}$ only, rather than by exploring the full parameter space (Goffaux and Vande Wouwer, 2008). The parameters values of the system are chosen on the parameter subspace border ($\theta_{real} = [\rho_m^+, K_s^-, \bar{\mu}^+, K_Q^-, K_{sI}^-, K_{iI}^+]$) and correspond to one of the 4 worst-case model mismatches (Benattia et al., 2014b), where the uncertain parameters subspace $[\theta^-, \theta^+]$ is given by $[0.8\theta_{nom}, 1.2\theta_{nom}]$. The initial biomass concentration value is set close to the setpoint in order to cancel the transient effect and to focus only on the behavior during setpoint changes (rising and falling edge respectively), with a maximal admissible dilution rate D_{max} equal to 1.6 d^{-1}. The simulation time T_f and the sampling time T_s are chosen equal to 1 day and 20 min respectively. The inlet substrate concentration S_{in} is assumed to be perfectly known. The light intensity is assumed to be non measured online. The controllers tuning parameters are determined by a trial-and-error technique (see Table 2).

Table 2. Controllers tuning parameters.

	LRMPC			ISM			
Paramet.	N_p	V	W	ξ_1	ξ_2	ξ_3	K_s
Value	5	I_5	I_5	0.1	-10	0.1	1

Figure 2 illustrates the comparison of LRMPC and LRMPC-ISM.

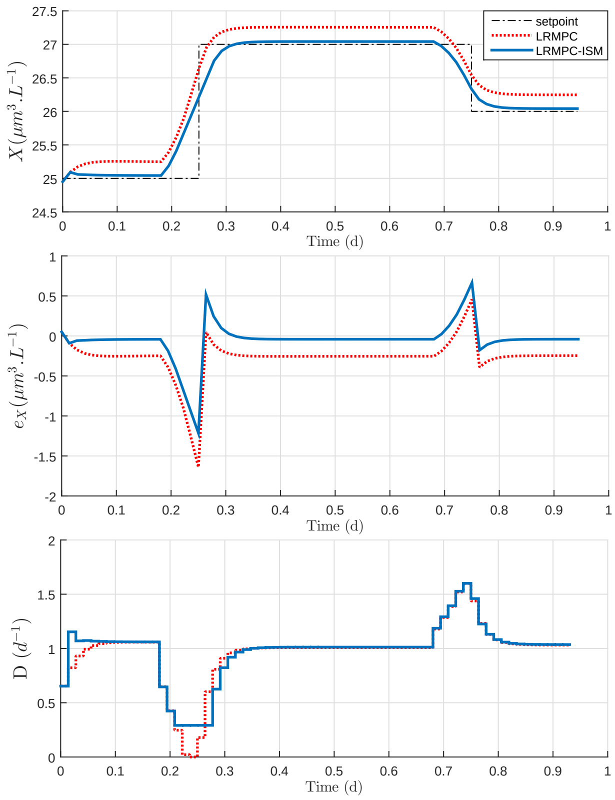

Fig. 2. Biomass concentration, tracking error and dilution rate evolutions with time in the case of model mismatch.

In the case of LRMPC law, it can be noticed the anticipation of a setpoint change due to the prediction of the model behavior in the moving horizon with a static error which is due to the superposition of two phenomena: the approximation of the model through linearization and the model mismatch. It should be mentionned that the static error could be further reduced by decreasing T_s. On the other side, the LRMPC-ISM law allows reducing the static error in comparison with LRMPC law as shown in Fig.2. Moreover, it can be observed that the control input is non zero during setpoint change (rising edge) due to the fact that the ISM control is not canceled. The LRMPC, thanks to it predictive property, cancels the dilution rate so that the growth is maximized, whereas, the ISM does not take into account future reference evolution.
The light intensity was set constant in the previous simulations (equal to I_{opt}). Now, a day/light variation is considered, modelled as the square of sinusoidal function (Masci et al., 2010):

$$I(t) = I_{opt}(max\{0, sin(2\pi t)\})^2 \tag{53}$$

where the time t is in days. The light intensity modeling is assumed mismatched (as shown in Fig.3). Figure 3 illustrates the performances of the LRMPC law and the proposed strategy for a constant reference trajectory ($X^r = 25\mu m^3/L$).

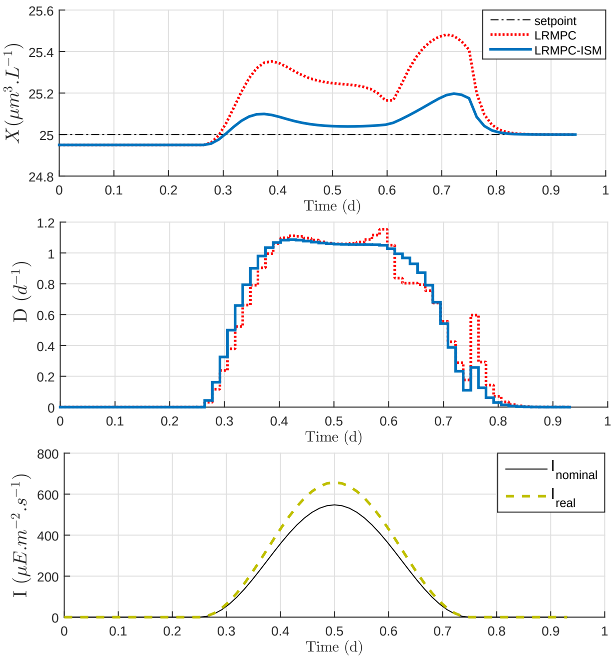

Fig. 3. Biomass concentration tracking error and dilution rate in the case of model mismatch with light intensity evolution.

When the setpoint is higher than the biomass concentration, as soon as the light vanishes (i.e. at night, $t \in [0, 0.25]$), the dilution rate is cancelled and cellular concentration remains constant at its previous value, leading to a non zero static error. In contrast, when the reference is smaller than the biomass concentration and the light begins sharply to decline, the dilution rate is slightly increased, causing a reduction of the biomass concentration in order to cancel the tracking error. Then, the dilution rate is cancelled when it is dark. Both LRMPC and proposed approach counter the effect of fluctuations in light intensity. It can be observed that the hierarchical approach (LRMPC-ISM) reduces the fluctuations of biomass concentration better than LRMPC. In conclusion, the proposed approach is robust against parameter uncertainties and less sensitive to light variations than LRMPC.

6. CONCLUSION

In this paper, a hierarchical control strategy formed by an ISM controller and a robust MPC law is proposed for biomass tracking trajectory. The min-max problem consists in determining the optimal control sequence so that the maximum deviation for all trajectories over all possible data scenarii is minimized. Firstly, The original problem is converted into a simple scalar minimization problem through linearization of the predicted trajectory. Secondly, the ISM control strategy is added to the optimal controller in order to reduce the gap between the model prediction and the system behaviour. Finally, tests in simulation show the efficiency and good performance of the proposed strategy in the worst case of model uncertainties. It is observed that the input setpoint is computed for the equilibrium resulting in the desired output which depends on the nominal model. In future work, an improvement will consist on considering the control increments instead of the input setpoint in order to overcome the previous drawback. An interesting perspective may be the determination of sufficient conditions ensuring robust stability of the overall control scheme in case of bounded uncertainties and/or trajectory constraints.

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