

A study of dynamic thermal expansion using a laser-generated ultrasound 1-d model

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ABSTRACT

The laser-ultrasound 1-d model is used to describe phenomenologically the behavior of a non-conductive infinite plate subjected to different types of temperature raises: uniform (in z) and instantaneous (in t), uniform and non-instantaneous, non-uniform (exponential) and instantaneous. We show that this simple model can be used to understand the basics of dynamic thermal expansion.

INTRODUCTION

Laser-ultrasonics is a technique in which mechanical displacements associated with acoustic waves are both produced and detected by optical means. It is often used in the thermoelastic regime, in which the optical absorption of the laser pulse produces a localized temperature elevation, which produces a localized thermal expansion which, in turn, generates ultrasound. In the most general case, the form of the detected ultrasound signal depends on a large number of parameters. A large numerical model [1] is needed if one wants to reproduce every detail of the experimental signal. For the last few years, a one-dimensional model has been found useful to simulate and understand the physical phenomena happening during a laser-induced heating [2,3]. In the case of non-conductive materials, this 1-d model is extremely simple, since optical absorption is, in that case, the only source of temperature elevation. That 1-d approach cannot reproduce exactly what is happening, since it puts aside a number of phenomena: thermal diffusion, shear stress, transverse waves, anisotropy, etc.... However, the advantage of such a model is precisely to put aside those phenomena; this makes it easier to assign to each effect its real cause.

THE 1-D MODEL

In this one-dimensional (the z axis) model, the normal stress is expressed simply by

$$\sigma(z, t) = C \frac{\partial u}{\partial z}(z, t) - \lambda \Delta T(z, t) \quad (1)$$

where C is the diagonal component of the rigidity tensor, u is the normal displacement, λ a compound thermal stress coefficient and ΔT the temperature raise. We solve the 1-d Christoffel

equation: $\rho \frac{\partial^2 u}{\partial t^2} = C \frac{\partial^2 u}{\partial z^2} - \lambda \frac{\partial \Delta T}{\partial z}$ (2) using the initial conditions ($u = 0$ and $\frac{\partial u}{\partial t} = 0$ at $t = 0$) and the boundary conditions ($\sigma = 0$ at $z = 0$ and $z = L$ for all t).

UNIFORM, NEARLY-INSTANTANEOUS TEMPERATURE FIELD

With this simple 1-d model, one can obtain the answer to the following question: what happens if a stress-free infinite plate of a non-conductive material is heated uniformly and very

rapidly? For instance, one can assume a temperature raise of the form $\Delta T = \Delta T_f * (1 - e^{-t/\tau}) * (1 + t/\tau)$ where τ is a characteristic rising time ($\Delta T \approx \Delta T_f$ at $t = 5\tau$), with τ very small ($\tau \ll L/v$, L being the finite thickness of the plate and v the longitudinal wave velocity).

This temperature raise induces a thermal stress $-\lambda\Delta T$ inside the plate. At the surfaces $z = 0$ and $z = L$, the total stress must be zero, and therefore (equation (1)) a displacement gradient $\partial u/\partial z = \lambda\Delta T/C$ is produced at those two locations. The existence of a displacement gradient at $z = 0$ and $z = L$ gives rise to a stress differential at $z = 0^+$ and at $z = L^-$, and this stress differential gives rise to a displacement gradient, etc... Therefore, there are two waves of gradient $\partial u/\partial z$, one produced at $z = 0$ propagating from left to right and the other, produced at $z = L$, propagating in the opposite direction (Figure 1). At $z = L$, the displacement variation $\partial u/\partial t$ is given simply by $\partial u/\partial t = v \partial u/\partial z$.

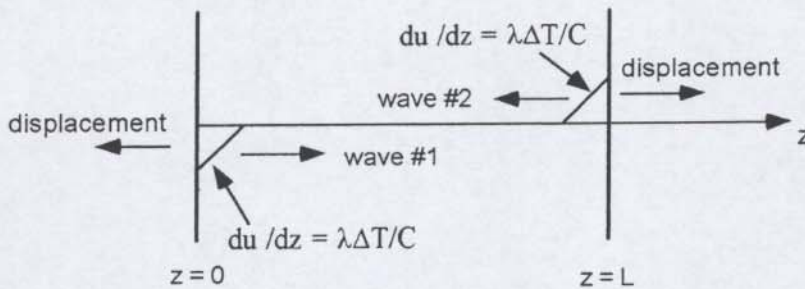


Figure 1.

The thermal expansion is given directly by the diagram of the displacement $u(z=L, t)$ versus the time (Figure 3a). The result is the superposition of the waves produced at $z = 0$ (wave #1) and $z = L$ (wave #2) and of their multiple reflections. At the beginning, u increases until $t = L/v$, the time of arrival of wave #1. The addition of wave #1, of its reflection on the $z = L$ face, and of wave #2 explains the first decrease of u for $L/v < t < 2L/v$. When $t = 2L/v$, the reflection of wave #2 on the $z = 0$ face arrives at $z = L$ and is superposed to the previous result, and so forth. The global result is an oscillation of u around an equilibrium position.

We can reproduce experimentally this simulated result, using laser-generated ultrasounds. To obtain a quasi-uniform and very rapid heating, we submitted a weakly-absorbing glass sample to a laser pulse. The result (Figure 2) is similar to what we predicted above. The main difference is the regular decrease of the "triangles". This can be attributed to a 2-d effect: it is caused by the radial expansion of the sample, which gives rise to a decrease in normal displacement. The final result can be reproduced with the help of a two-dimensional ($r-z$) model, like the one described by Lafond et al[4].

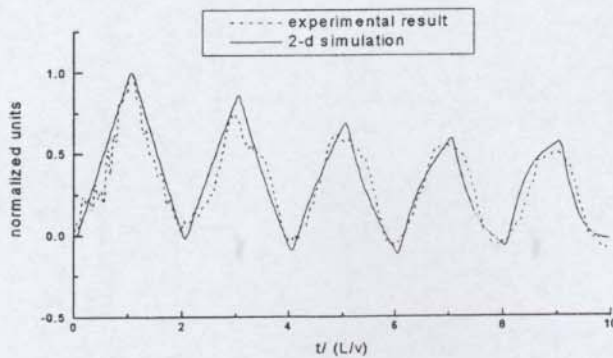


Figure 2.

UNIFORM, NON INSTANTANEOUS TEMPERATURE FIELD

If the temperature raise is still uniform, but not nearly instantaneous (τ is in the scale of L/v), the basic physical phenomena remain the same. A gradient $\partial u/\partial z = \lambda \Delta T/C$ is still produced at both faces, but this gradient is no longer a near-constant with time. Furthermore the temperature does not have the time to reach its final level before wave #1 arrives at the $z = L$ face, which is why the displacement u does not reach its maximum right away. Also, when wave #1 arrives at the back-face (at $t = L/v$), the gradient associated with it is still small compared with the gradient associated with wave #2; wave #1 and its reflection cannot compensate for wave #2 right away. This is why the first peak happens at $t > L/v$. If $\tau \gg L/v$ (Figure 3c), the oscillations of u around the equilibrium position become small and the plate reaches slowly its final equilibrium position. Figure 3b is similar to the results obtained by Tang et al[5,6]. In those papers, the authors observe the same kind of oscillations of the back-face, which they explain by introducing a relaxation of the thermal expansion coefficient α . Although there are some experimental differences between their work and ours, we believe those oscillations may be attributed to the propagation and reflection of stress waves.

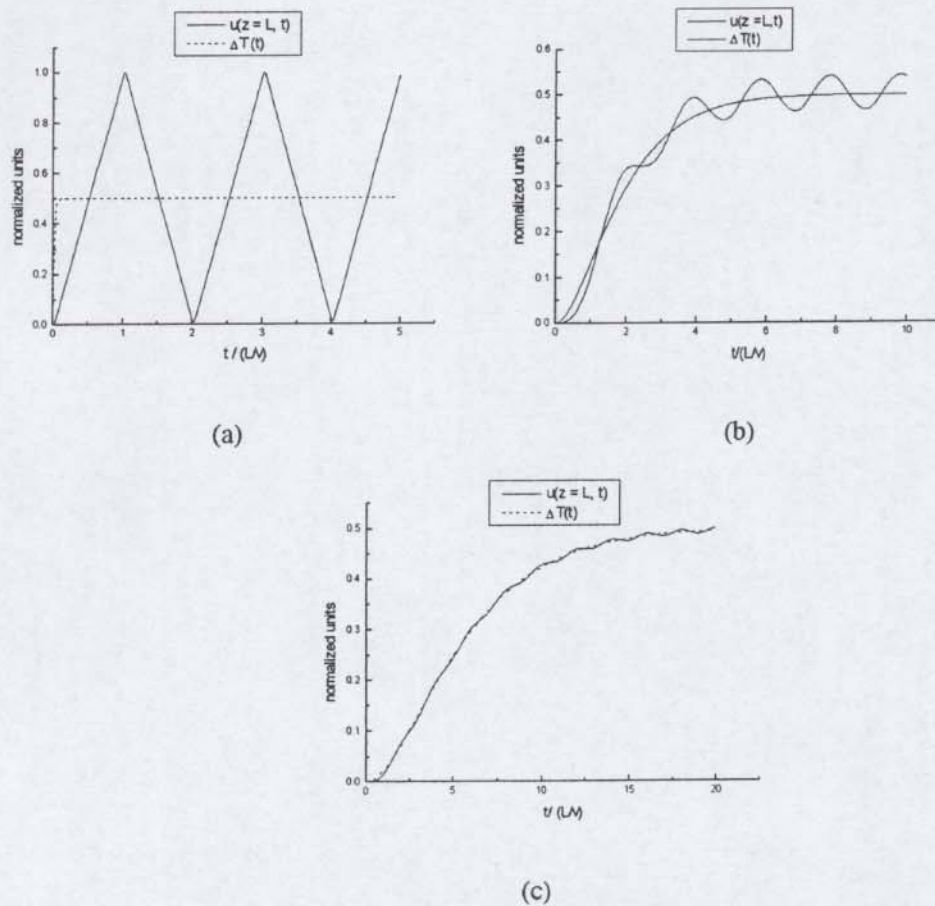


Figure 3. $u(z=L, t)$ vs normalized time. (a) $\tau/(L/v) = 0.01$. (b) $\tau/(L/v) = 1$. (c) $\tau/(L/v) = 3$.

NON-UNIFORM, NEARLY INSTANTANEOUS TEMPERATURE FIELD

We may also include in our analysis the case where the temperature raise decreases exponentially

with z : $\Delta T(z, t) = \Delta T_0 \left[1 - e^{-t/\tau} \left(1 + \frac{t}{\tau} \right) \right] e^{-\beta z}$. This temperature field corresponds to the optical absorption of a laser-pulse by a non-conductive material. The temperature distribution (in

z) can be modeled as a series of infinitesimal temperature steps (Figure 4a). The continuity of the normal stress inside the material implies the creation of a $\partial u/\partial z$ wave at each one of those steps (Figure 4b). The final result (Figure 4c), which was already shown, for instance, in [7], is thus easily explained and understood.

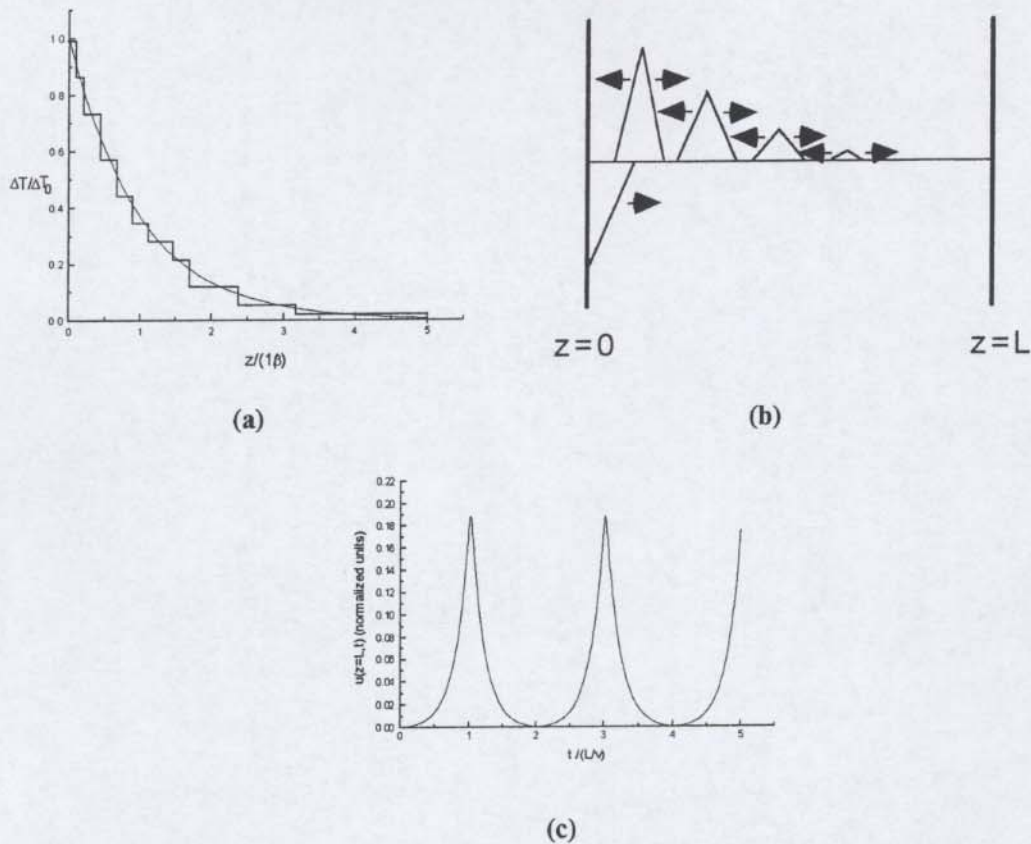


Figure 4. (a) exponential temperature distribution. (b) $\partial u/\partial z$ waves. (c) simulated result ($\beta L = 5$).

CONCLUSION

We have used a simple 1-d model to understand the basics of dynamic thermal expansion of a non-conductive infinite plate of finite thickness subjected to three different types of temperature raises: uniform (in z) and instantaneous (in t), uniform and non-instantaneous, non-uniform (exponential) and instantaneous. Our analysis show that the behavior of the plate will be the result of the creation of $\partial u/\partial z$ waves where there is a thermal stress differential and of the propagation and reflection of those waves. The final result will be a function of the absorption coefficient β (if the temperature raise is exponential) and of the ratio $\tau/(L/v)$ (i.e. characteristic time of temperature raise / time of travel of the longitudinal wave).

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