Radiative transfer within porous media and couplings

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1 Introduction

Radiation is a determinant transfer mode in applications involving porous media at high temperature. It is generally coupled to conduction and convection transfer. Such a coupling generally requires development of a radiation model not only for the whole porous medium but for each phase separately, more often development of coupled radiation models for the different phases. In the case of a porous medium with opaque and transparent phases, the radiation model is based on the interfacial temperature field T_i, which a priori differs from the opaque solid temperature T_S and the transparent fluid one T_F. In many cases, when the Biot number Bi is small compared to 1, T_i is equal to T_S. For a porous medium with two semi transparent phases 1 and 2 (one of them can be transparent), the model is based on two temperatures T_1 and T_2.

The statistical approach developed in the companion paper[1] allows the extinction, scattering and absorption properties of the different phases to be exhaustively characterized by statistical radiative distribution functions $G_{\nu\, ext}$, $P_{\nu\, a}$ and $P_{\nu\, sc}$ and a general phase function p_ν from high resolution tomography data or from the exact knowledge of the geometrical configuration of the porous medium. A key problem is then to model emission. For any real semi transparent phase characterized by an absorption coefficient κ_ν the emission term is based on κ_ν. For a homogenized phase i associated with opaque interfacial elements this emission term depends on both the properties of the temperature field and on the features of the porous medium. Different approaches are then used:

i) Beerian homogenized interfacial phases i are simply characterized by absorption coefficients. In these conditions, the emission terms are based on these absorption coefficients and a classical RTE can be applied in any part of porous medium. All the classical methods of radiative transfer can be used.

ii) Non Beerian interfacial homogenized phases i have, in principle, to be modeled by the GRTE, but generalized absorption coefficients at equilibrium K_ν can be used for characterizing emission in precise conditions, corresponding to Local Thermal Equilibrium of the radiation

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field (see Sec.4.2 of the companion paper[1]): The temperature field must weakly vary at a spatial scale δ over which the phase is optically thick. In these particular conditions, the GRTE degenerates into a classical RTE and the radiative transfer is then modeled by a perturbation technique, which directly leads to a radiative conductivity tensor, associated with a radiative Fourier's law. Note that, in the case of a Beerian homogenized phase, a perturbation technique of the RTE can also be implemented, under the same assumptions, for obtaining a radiative conductivity tensor.
But, for both Beerian and non Beerian media, this model is not valid in the vicinity of the boundaries of the porous medium, where the distance of a current point to the nearest boundary is not optically thick. Moreover, for non Beerian homogenized interfacial phases i, the radiative statistical distribution functions are truncated, depend on the radiative conditions at the boundary and are strongly non homogeneous. A GRTE based on these radiative statistical functions has then to be solved, as in the following case iii).
In a particular case of weak relative variations of the temperature in the boundary region another type of linearization can be undertaken. It is a basic model, leading to different results. But, often the strongest temperature gradients occur in the vicinity of the boundary.

iii) For a statistically non homogeneous, and consequently non Beerian, homogenized interfacial phase i, emission cannot be modeled on the basis of an absorption coefficient. It can be done by using a GRTE or coupled GRTEs and the Reciprocity Theorem[2]. This approach has also to be followed for a homogenized interfacial phase i:
- In the vicinity of the boundaries of a porous medium, even if it is statistically homogeneous in its core, as previously discussed.
- In the core of the porous medium, if the validity conditions of the Fourier's law precised in Sec.2.1 are not fulfilled.

Section 2 deals with a common model valid in the core of a porous medium, optically thick at the spatial scale of temperature: This diffusion model is based on a radiative Fourier's law, which will be introduced for a medium with opaque and transparent phases (OT), opaque and semi transparent phases (OST), two semi transparent phases (ST2) or when a phase is transparent (STT). The validity conditions of this approach will first be discussed.
The general expressions of the GRTE(s) are developed for the OT, OST, STT and ST2 cases in Sec.3. This general formalism must be used for non Beerian homogenized phases, when the radiative Fourier's law is not valid. The specific case of a statistically non homogeneous interfacial homogenized phase i is also studied in this Section.
The last Section is devoted to coupling with other heat transfer modes.

2 Radiative Fourier's law (statistically homogeneous phases)

In many applications, the radiative flux $\mathbf{q}^R$ within the core of a statistically homogeneous phase can simply be expressed by a radiative Fourier's law, under some precise conditions. These conditions are defined in Sec.2.1. When these conditions are fulfilled, the GRTE degenerates into a classical RTE, based on generalized extinction, absorption and scattering coefficients at equilibrium (see Secs. 4.1-2 of the companion paper[1]): These RTEs are introduced for OT, OST, STT and ST2 cases in Sec.2.2. The radiative Fourier's laws based on radiative conductivity tensors are then derived from the corresponding RTEs, for the different cases, in Sec.2.3.

In a more general case, radiative transfer is directly based on the resolution of a Generalized Radiative Transfer Equation (see Sec.3).

2.1 Validity conditions of the radiative Fourier's law

A classical homogeneous and isotropic semi transparent medium, which is then Beerian, is characterized by the classical Radiative Transfer Equation (RTE)

$$\frac{\mathrm{d}I_\nu(\mathbf{u})}{\mathrm{d}s} + \beta_\nu\, I\nu(\mathbf{u}) = \kappa_\nu\, n_\nu^2\, I_\nu^\circ(T) + \frac{\sigma_\nu}{4\pi}\int_0^{4\pi} p_\nu(\mathbf{u}_1.\mathbf{u})\, I_\nu(\mathbf{u}_1)\,\mathrm{d}\Omega_1 \tag{1}$$

where β_ν, κ_ν and σ_ν are the extinction, absorption and scattering coefficients, and $p_\nu(\mu_{sc})$ the scattering phase function, only dependent on the scattering angle cosine $\mu_{sc} = \mathbf{u}_1.\mathbf{u}$ in an isotropic space. In the considered case, the scattering is characterized by a simple scalar, the scattering asymmetry parameter g, defined by[3, 4]

$$g = \frac{1}{2}\int_{-1}^{1} p_\nu(\mu)\,\mu\,\mathrm{d}\mu \tag{2}$$

The medium is also characterized by its albedo: $\omega_\nu = \sigma_\nu/\beta_\nu$.

In many applications, the radiative flux can be expressed, under precise conditions, by a radiative Fourier's law, *i.e.*

$$\mathbf{q}^R = -k^R\,\mathbf{grad}(T), \tag{3}$$

where k^R is a scalar radiative conductivity for this isotropic medium. A precise criterion of

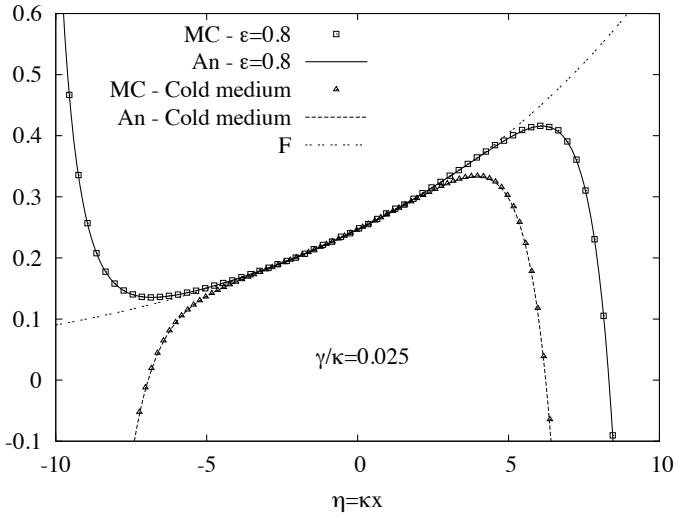

Figure 1: NOrmalized P_F^R (F), P_{MC}^R (MC) and analytical P^R (An) for a purely absorbing medium[5], enclosed between planes distant of 2L, with $2\kappa L = 20$, in different benchmark thermal conditions of Ref.[5].

the validity of the radiative Fourier's law for a Beerian semi transparent medium, *far from the medium boundaries*, has only recently be published by Gomart and Taine[5]. These authors have systematically studied the relative discrepancy ζ between radiative power values calculated by the radiative Fourier's law P_F^R and an exact Monte Carlo method P_{MC}^R, vs key parameters. An example of results is given in Fig.1 for a purely absorbing medium for which a analytical solution exists.

The results of this study have been tabulated vs the logarithmic derivative of the temperature $\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}x}$ and an effective absorption coefficient accounting for scattering $\kappa^{eff}(\omega, g)$, where ω and g

are the albedo and the asymmetry parameter of the semi transparent medium. More precisely, for a given relative discrepancy ζ, the temperature field has to fit the following inequality

$$\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}x} < \epsilon(\zeta)\,\kappa^{eff}(\omega,g). \tag{4}$$

The functions $\epsilon(\zeta)$ and $\kappa^{eff}(\omega,g)$ are tabulated in Ref.[5].

Estimate typical orders of magnitude. For instance, for $\zeta = 10^{-2}$, ϵ is equal to 0.033. For an albedo equal to 0.5 and a value of g equal to -0.3, typical values of backscattering by a porous medium, the criterion corresponds to $\kappa^{eff} = 1.70\,\kappa$. For a typical pore size of $100\,\mu$m and a κ value of 50 cm^{-1}, the radiative Fourier's law is valid if: $\frac{\mathrm{d}T}{\mathrm{d}x} < 2.8\,T$ K/cm. This criterion is generally validated at high temperature. At 2000 K, the temperature variation has to be smaller than 56 K for this pore size of $100\,\mu$m and, at 500K, smaller than 14 K. Nevertheless, the Fourier's law is, for instance, not valid for a flame front smaller than the pore size.

A second important limitation, for any semi transparent medium, is the non validity of the radiative Fourier's law in the *radiative boundary layers, i.e.* in the vicinity of the medium boundaries. This fact is well illustrated in Fig.1 issued from Ref.[5]: The radiative boundary layer effects characterize any semi transparent medium and strongly depend on ω and g, in the general case.
Moreover, for any homogenized non Beerian phase of a porous medium, which is statistically homogeneous in the core of the medium, it has been shown in Sec.4.1 of the companion paper[1], that generalized extinction, absorption and scattering coefficients cannot be defined in a point M such that its distance z to a boundary is not optically thick. Indeed, in these conditions the radiative statistical distribution functions of the phase at M are truncated at the boundary and strongly depend on the distance to the boundary: The total source term, in equilibrium conditions, then depends on z. The medium is statistically non homogeneous in the radiative boundary layers. In these conditions, the radiative flux can only be determined from the GRTE.

In many applications, the largest temperature gradients are encountered in the vicinity of the boundaries. Nevertheless, in some particular cases, the temperature variations are weak along a distance to boundaries which is optically thin. The temperature field can then be linearized: An effective conductivity could be used, but it would not be given by the previous model.

2.2 RTE associated with an optically thick homogeneous phase

A homogenized phase is assumed optically thick at a spatial scale δ, such that it can be considered as practically isothermal: The precise conditions of Sec.2.1 are assumed fulfilled. Consequently, the variations of the fields of temperature, intensity and the different source terms occur at a spatial scale larger than δ and characterized by the macroscopic coordinates $\mathbf{X}$. It has been shown in Sec.4.2 of the companion paper [1] that, in these conditions, the GRTEs degenerate into RTEs.

2.2.1 OT case

In the OT case, the only homogenized phase, associated with interface elements i, is characterized by a general phase function $p_{\nu\,i}(\mathbf{u}_1,\mathbf{u})$ and generalized extinction, scattering and absorption coefficients at equilibrium given by Eqs.13 of Ref.[1], *i.e.*

$$B_i(\mathbf{u}) = \frac{1}{\int_0^\infty [1-G_{ext\,i}(\mathbf{u},v)]\mathrm{d}v},\ \Sigma_{\nu\,i}(\mathbf{u}) = P_{sc\,\nu\,i}(\mathbf{u},\infty)\,B_i(\mathbf{u}),\ K_{\nu\,i}(\mathbf{u}) = B_i(\mathbf{u}) - \Sigma_{\nu\,i}(\mathbf{u}). \tag{5}$$

where $P_{sc\nu i}(\mathbf{u},\infty)$ is the phase albedo. The corresponding RTE writes within the propagation medium of refractive index n_ν[6]

$$\begin{aligned}\Pi\frac{\mathrm{d}I_\nu}{\mathrm{d}s}(\mathbf{u},\mathbf{X}) &+ \Pi\,B_i(\mathbf{u})\,I_\nu(\mathbf{u},\mathbf{X}) \\ &= \Pi\,K_{\nu i}(\mathbf{u})n_\nu^2\,I_\nu^\circ[T_i(\mathbf{X})] + \Pi\frac{1}{4\pi}\int_{4\pi}\Sigma_{\nu i}(\mathbf{u}_1)p_{\nu i}(\mathbf{u}_1,\mathbf{u})I_\nu(\mathbf{u}_1,\mathbf{X})\mathrm{d}\Omega_1,\end{aligned} \tag{6}$$

where s is the coordinate at the considered point $\mathbf{X}$ in the direction $\mathbf{u}$, I_ν and $\mathrm{d}\Omega_1$ are the intensity and the current elementary solid angle within the propagation phase, respectively. As shown in Sec.4.3.1 of the companion paper[1], the scattering source term can simply be written

$$\begin{aligned}&\mathrm{d}\Omega(\mathbf{u})\,\Pi\int_{4\pi}\Sigma_{\nu i}(\mathbf{u}_1)\,\frac{p_{\nu i}(\mathbf{u}_1,\mathbf{u})}{4\pi}\,I_\nu(\mathbf{u}_1,\mathbf{X})\,\mathrm{d}\Omega_1 \\ =\;& \mathrm{d}\Omega(\mathbf{u})\;\Pi\Sigma_{\nu i}(\mathbf{u})\int_{4\pi}\frac{p_{\nu i}(-\mathbf{u},-\mathbf{u}_1)}{4\pi}\,I_\nu(\mathbf{u}_1,\mathbf{X})\,\mathrm{d}\Omega_1(-\mathbf{u}_1)\end{aligned} \tag{7}$$

In these conditions, the RTE simply writes[6]

$$\begin{aligned}\Pi\frac{\mathrm{d}I_\nu}{\mathrm{d}s}(\mathbf{u},\mathbf{X}) &+ \Pi\,B_i(\mathbf{u})\,I_\nu(\mathbf{u},\mathbf{X}) \\ &= \Pi\,K_{\nu i}\,I_\nu^\circ[T_i(\mathbf{X})] + \Pi\,\Sigma_{\nu i}(\mathbf{u})\,\frac{1}{4\pi}\int_{4\pi}p_{\nu i}(-\mathbf{u},-\mathbf{u}_1)I_\nu(\mathbf{u}_1,\mathbf{X})\mathrm{d}\Omega_1,\end{aligned} \tag{8}$$

Note that the temperature involved in Eq.8 is the interfacial temperature T_i, which a priori differs from the temperatures of the opaque solid phase and the transparent fluid phase.

2.2.2 OST case

A typical example of OST case is a porous medium with an opaque solid phase and a semi transparent one, water vapor and droplets[6]. In practice, two phases of same fraction per unit volume Π, occupy the propagation volume:

i) A real global semi transparent phase, of refractive index 1, associated with water vapor (v) and droplets (d) and characterized by Beerian absorption and scattering coefficients $\kappa_{\nu v}+\kappa_{\nu d}$, $\sigma_{\nu d}$ respectively and a phase function $p_{\nu d}(\mu_{sc})$, only dependent on the cosine μ_{sc} of the scattering angle (Mie theory);

ii) A homogenized non Beerian phase associated with the opaque interfaces i and characterized, as in the OT case, by generalized extinction, scattering and absorption coefficients at equilibrium $B_i(\mathbf{u})$, $K_{\nu i}(\mathbf{u})$ and $\Sigma_{\nu i}(\mathbf{u})$ respectively and a general phase function $p_{\nu i}(\mathbf{u}_1,\mathbf{u})$.

There is only one RTE for this complex system, with three emission source terms due to vapor, droplets and interfaces of temperatures T_v, T_d and T_i respectively, three associated absorption terms and two scattering terms due to droplets and interfaces. The RTE is here written in the same medium v of refractive index 1 for all the contributions. Consequently, after the same modification of the scattering interfacial source term as for the OT case, the complete RTE writes in the medium v [6]

$$\begin{aligned}\frac{\mathrm{d}I_{\nu v}}{\mathrm{d}s}(\mathbf{u},\mathbf{X}) =\;& -\;[B_i(\mathbf{u})+\kappa_{\nu v}+\kappa_{\nu d}+\sigma_{\nu d}]\,I_{\nu v}(\mathbf{u},\mathbf{X}) \\ &+\;K_{\nu i}(\mathbf{u})\,I_\nu^\circ[T_i(\mathbf{X})]+\kappa_{\nu v}\,I_\nu^\circ[T_v(\mathbf{X})]+\kappa_{\nu d}\,I_\nu^\circ[T_d(\mathbf{X})] \\ &+\;\Sigma_{\nu i}(\mathbf{u})\int_{4\pi}\frac{p_{\nu i}(-\mathbf{u},-\mathbf{u}_1)}{4\pi}\,I_{\nu v}(\mathbf{u}_1,\mathbf{X})\mathrm{d}\Omega_1 \\ &+\;\sigma_{\nu d}(\mathbf{u})\int_{4\pi}\frac{p_{\nu d}(\mu_{sc})}{4\pi}I_{\nu v}(\mathbf{u}_1,\mathbf{X})\mathrm{d}\Omega_1.\end{aligned} \tag{9}$$

2.2.3 ST2 case

A typical example of a medium with two semi transparent phases is a set of fibers within a matrix. Emission, absorption and scattering are volume phenomena associated with the two real phases. Moreover, four scattering phenomena are also associated with reflection within the two phases and transmission from any phase to the other one.

The real phases 1 and 2, homogeneous and isotropic at local scale, are characterized by fractions by unit volume Π_j, refractive indices $n_{\nu\,j}$, extinction, absorption and scattering coefficients $\beta_{\nu\,j}$, $\kappa_{\nu\,j}$ and $\sigma_{\nu\,j}$ respectively and phase functions $p_{\nu\,j}(\mu_{sc})$, $(j = 1, 2)$. The homogenized phases 1 and 2 are characterized by the same quantities. The homogenized phase associated with interfacial elements are characterized by four scattering cumulative probabilities $P_{\nu\,sc\,ijj}$, reflection within j at local scale, and $P_{\nu\,sc\,ijk}$, transmission from j to k at local scale, and the corresponding phase functions $p_{\nu\,ijj}(\mathbf{u_1}, \mathbf{u})$ and $p_{\nu\,ijk}(\mathbf{u_1}, \mathbf{u})$.

Under the assumptions of an optically thick medium far from the boundaries of the porous medium, made in the present Section, the GRTEs degenerate into two coupled RTEs[7], in which appear the generalized scattering coefficients at equilibrium $\Sigma_{\nu\,ijj}(\mathbf{u})$ and $\Sigma_{\nu\,ijk}(\mathbf{u})$. These coupled RTEs are here expressed in Eqs 10,12, in which $I_{\nu\,1}$ and $I_{\nu\,2}$ are the intensities within the media of refractive indices $n_{\nu\,1}$ and $n_{\nu\,2}$, respectively. The scattering source term associated with internal reflection has been modified as for OT and OST cases. The scattering source term associated with transmission from 2 to 1 is detailed in the first equation, *i.e.*

$$\begin{aligned}
&\Pi_1 \frac{\mathrm{d}I_{\nu\,1}}{\mathrm{d}s}(\mathbf{u}_1, \mathbf{X})\,\mathrm{d}\Omega_1 \,+\, \Pi_1\,[\beta_{\nu\,1} \,+\, \Sigma_{\nu\,i\,11}(\mathbf{u}_1) \,+\, \Sigma_{\nu\,i\,12}(\mathbf{u}_1)]\, I_{\nu\,1}(\mathbf{u}_1, \mathbf{X})\,\mathrm{d}\Omega_1 \\
=\;& \Pi_1\, n_{\nu\,1}^2\, \kappa_{\nu\,1}\, I_\nu^\circ[T_1(\mathbf{X})]\,\mathrm{d}\Omega_1 \,+\, \Pi_1\, \frac{\sigma_{\nu\,1}}{4\pi}\,\mathrm{d}\Omega_1 \int_{4\pi} p_{\nu\,1}(\mu_{sc})\, I_{\nu\,1}(\mathbf{u}_1', \mathbf{X})\,\mathrm{d}\Omega_1' \\
+\;& \Pi_1\, \frac{\Sigma_{\nu\,i\,11}(\mathbf{u}_1)}{4\pi}\,\mathrm{d}\Omega_1 \int_{4\pi} p_{\nu\,i\,11}(\mathbf{u}_1'', \mathbf{u})\, I_{\nu\,1}(\mathbf{u}_1'', \mathbf{X})\,\mathrm{d}\Omega_1'' \\
+\;& \Pi_2\, \frac{1}{4\pi}\,\mathrm{d}\Omega_1 \int_{4\pi} \Sigma_{\nu\,i\,21}(\mathbf{u}_2)\, p_{\nu\,i\,21}(\mathbf{u}_2, \mathbf{u}_1)\, I_{\nu\,2}(\mathbf{u}_2, \mathbf{X})\,\mathrm{d}\Omega_2. \qquad (10)
\end{aligned}$$

As shown and discussed in Sec. 4.3.2 of the companion paper[1] the last term of the second member of Eq.10 simply writes

$$\Pi_1\,(\frac{n_{\nu\,1}^2}{n_{\nu\,2}^2})\,\frac{\Sigma_{\nu\,i\,12}(\mathbf{u}_1)}{4\pi}\,\mathrm{d}\Omega_1 \int_{4\pi} p_{\nu\,i\,12}(-\mathbf{u}_1, -\mathbf{u}_2) I_{\nu\,2}(\mathbf{u}_2, \mathbf{X})\,\mathrm{d}\Omega_2, \qquad (11)$$

and the second coupled equation similarly writes, after some simplifications,

$$\begin{aligned}
&\frac{\mathrm{d}I_{\nu\,2}}{\mathrm{d}s}(\mathbf{u}_2, \mathbf{X}) \,+\, [\beta_{\nu\,2} \,+\, \Sigma_{\nu\,i\,22}(\mathbf{u}_2) \,+\, \Sigma_{\nu\,i\,21}(\mathbf{u}_2)]\, I_{\nu\,2}(\mathbf{u}_2, \mathbf{X}) \\
=\;& n_{\nu\,2}^2\, \kappa_{\nu\,2}\, I_\nu^\circ[T_2(\mathbf{X})] \,+\, \frac{\sigma_{\nu\,2}}{4\pi} \int_{4\pi} p_{\nu\,2}(\mu_{sc})\, I_{\nu\,2}(\mathbf{u}_2', \mathbf{X})\,\mathrm{d}\Omega_2' \\
+\;& \frac{\Sigma_{\nu\,i\,22}(\mathbf{u}_2)}{4\pi} \int_{4\pi} p_{\nu\,i\,22}(\mathbf{u}_2'', \mathbf{u}_2) I_{\nu\,2}(\mathbf{u}_2'', \mathbf{X})\mathrm{d}\Omega_2'' \\
+\;& \frac{\Sigma_{\nu\,i\,21}(\mathbf{u}_2)}{4\pi}\,(\frac{n_{\nu\,2}^2}{n_{\nu\,1}^2}) \int_{4\pi} p_{\nu\,i\,21}(-\mathbf{u}_2, -\mathbf{u}_1) I_{\nu\,1}(\mathbf{u}_1, \mathbf{X})\mathrm{d}\Omega_1. \qquad (12)
\end{aligned}$$

2.2.4 STT case

A typical example of a medium with a semi transparent phase and a transparent one is a set of fibers within air [7]. Emission and absorption are only volume phenomena within the fibers.

Moreover, there are, as in Sec.2.2.3, four scattering phenomena associated with reflections within the two phases and transmission from any phase to the other one. The results are deduced from the previous ones by setting:

$$\beta_{\nu\,2} \,=\, \sigma_{\nu\,2} \,=\kappa_{\nu\,2} \,=\, 0;\; p_{\nu\,2} \,=0.$$

2.3 Derivation of the radiative Fourier's law

The OT case will be detailed. Only the structure of the results and some references will be given for the other cases.

2.3.1 OT case

a) General case

Under the precise conditions of Sec.2.1 the GRTE degenerates in the OT case into the classical RTE, given by Eq.8, which writes by introducing $s^+ = s/\delta$

$$\begin{aligned} \Pi \quad & (\frac{1}{\delta\, B_i(\mathbf{u})})\frac{\mathrm{d}I_\nu}{\mathrm{d}s^+}(\mathbf{u},\mathbf{X}) + \Pi\, I_\nu(\mathbf{u},\mathbf{X}) \\ = \quad & \Pi\frac{K_{\nu\,i}(\mathbf{u})}{B_i(\mathbf{u})}\, I_\nu^\circ[T_i(\mathbf{X})] + \Pi[1 \,-\, \frac{K_{\nu\,i}(\mathbf{u})}{B_i(\mathbf{u})}] \int_{4\pi} \frac{p_{\nu\,i}(\mathbf{u}_1,\mathbf{u})}{4\pi} I_\nu(\mathbf{u}_1,\mathbf{X})\mathrm{d}\Omega_1. \end{aligned} \tag{13}$$

The first term of the first member, which is the only term which connects two adjacent cells of the medium and is called the transport term, is proportional to $\mathrm{Kn}^R = \frac{1}{\delta\, B_i(\mathbf{u})}$ and consequently it is small, at the exponential sense, compared to the other terms. If this term was zero, a cell of the medium would be completely insulated, condition of ideal equilibrium of the radiation field. As the term is only small, the radiation in the cell is in *Local Thermal Equilibrium*, associated with a value of the radiative Knudsen number Kn^R small compared to unity. The intensity field $I_\nu(\mathbf{u},\mathbf{X})$ can then be determined by a perturbation technique vs the perturbation parameter Kn^R, as for the resolution, in the same conditions, of the Boltzmann's equation[8].

It is easy to show that $I_\nu^{(0)}(\mathbf{u},\mathbf{X})$, the unique zeroth order solution of Eq.13, *i.e.* independent of Kn^R, is the equilibrium intensity: $I_\nu^{(0)}(\mathbf{u},\mathbf{X}) \,=\, n_\nu^2(\mathbf{u})\, I_\nu^\circ[T_i(\mathbf{X})]$.

The first order solution, associated with the terms of Eq.13 proportional to Kn^R, is solution of the implicit equation

$$I_\nu^{(1)}(\mathbf{u}) = -\frac{1}{B_i(\mathbf{u})}\,\frac{\mathrm{d}I_\nu^{(0)}}{\mathrm{d}s}(\mathbf{u}) + \mathcal{L}_\nu\left[I_\nu^{(1)}(\mathbf{u}_1)\right](\mathbf{u}), \tag{14}$$

where $\mathcal{L}_\nu$ is a linear functional defined by

$$\mathcal{L}_\nu\left[X(\mathbf{u}_1)\right](\mathbf{u}) \,=\, \frac{\Sigma_{\nu\,i}(\mathbf{u})}{4\pi\, B_i(\mathbf{u})} \int_{4\pi} p_{\nu\,i}(\mathbf{u}_1,\mathbf{u}) X(\mathbf{u}_1)\,\mathrm{d}\Omega_1. \tag{15}$$

Equation 14 is solved by applying k times the functional $\mathcal{L}_\nu$

$$I_\nu^{(1)}(\mathbf{u}) \,=\, -\sum_{k=0}^{\infty} \mathcal{L}_\nu^k\,[\frac{1}{B_i(\mathbf{u}_1)}\,\frac{\mathrm{d}I_\nu^{(0)}}{\mathrm{d}s}(\mathbf{u}_1)]\,(\mathbf{u}), \tag{16}$$

$$\text{with :} \qquad \mathcal{L}_\nu^0 \,=\, \frac{1}{B_i(\mathbf{u}_1)}\,\frac{\mathrm{d}I_\nu^{(0)}}{\mathrm{d}s} \tag{17}$$

$I_\nu^{(1)}(\mathbf{u})$ is then proportional to $-\,\mathrm{d}I_\nu^{(0)}/\mathrm{d}s$[1], in fact, in tensorial notations, to $-\,(\mathrm{d}I_\nu^{(0)}/\mathrm{d}T)\,u_n\,\mathrm{d}T/\mathrm{d}X_n$. $I_\nu^{(1)}(\mathbf{u})$ is then proportional to the opposite of temperature gradient, typical of a Fourier's law.

Due to the symmetries, there is obviously no flux is generated by the zeroth order solution associated with LTE. The radiative flux vector is then, in tensorial notations,

$$q_m^R \;=\; \int_0^\infty \int_{4\pi} I_\nu^{(1)}(\mathbf{u})\,u_m\,\mathrm{d}\Omega\,\mathrm{d}\nu \;=\; -\,k_{mn}\,\frac{\partial T}{\partial X_n}, \tag{19}$$

where k_{mn} is the radiative conductivity tensor, here directly deduced from the radiative statistical distribution functions. More details are given in Ref.[6].

For instance in the case of degraded rod bundles of Sec.3.2.1.c) of the companion paper[1], the conductivity tensor is diagonal, characterized by an axial component k_z^R and a radial one k_r^R in directions perpendicular to the rod axes. The results are normalized by $4\,\Pi^2\,\sigma T_i^3/A$, where A is the specific area A of the medium: They are plotted in Fig.2 vs its porosity Π. The dispersion of these results is due to the local non homogeneity of this degraded medium. It is paradoxical that this rather advanced approach leads to this extremely simple results. Nevertheless, this

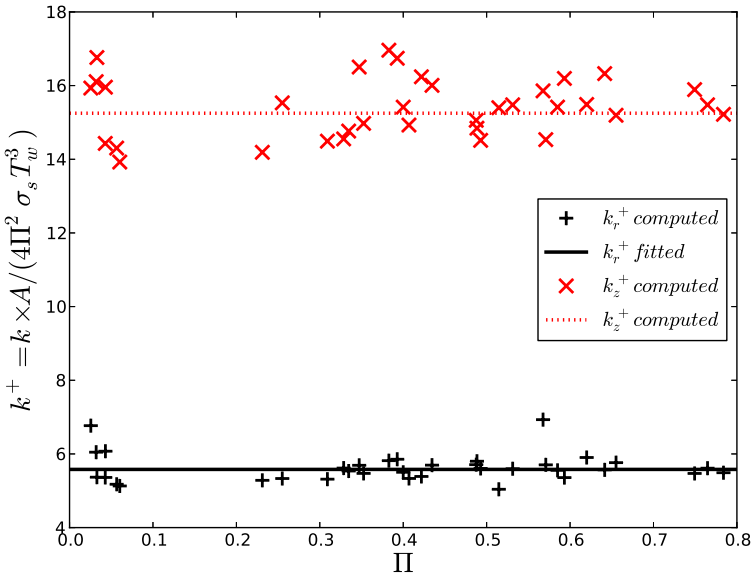

Figure 2: Radial (k_r^R) and axial (k_z^R) components of the radiative conductivity tensor associated with the degraded rod bundle of experiment FPT1 (IRSN/CEA)[6].

approach is not valid close to the walls of the reactor enclosure, *i.e.* in the radiative boundary layers of the medium. Note that a suggested approach is developed, for this region, in Sec. 3.3 of the companion paper[1].

[1]Equation 14 can also be written

$$I_\nu^{(1)}(\mathbf{u}) \;-\; \mathcal{L}_\nu\left[I_\nu^{(1)}(\mathbf{u}_1)\right](\mathbf{u}) \;=\; -\,\frac{1}{B_i(\mathbf{u})}\,\frac{\mathrm{d}I_\nu^{(0)}}{\mathrm{d}s}(\mathbf{u}), \tag{18}$$

where $I_\nu^{(1)} \;-\; \mathcal{L}[I_\nu^{(1)}]$ is a linear operator of $I_\nu^{(1)}$. Consequently, after inversion of Eq.18, it appears that $I_\nu^{(1)}(\mathbf{u})$ also is proportional to
$-\frac{1}{B_i(\mathbf{u})}\,\frac{\mathrm{d}I_\nu^{(0)}}{\mathrm{d}s}(\mathbf{u})$.

b) Case of a statistically homogeneous and isotropic homogenized phase

A simplified approach has been developed for statistically homogeneous and isotropic, Beerian or non Beerian, homogenized phases[9, 4]. In this case the phase function $p_\nu(\mu_{sc})$ only depends on the scattering angle cosine μ_{sc}. As the medium is isotropic, the radiative conductivity is simply a scalar given by

$$k^R(T_w) = \frac{4\pi\,\Pi}{3}\int_0^\infty \frac{n_\nu^2}{K_\nu + \Sigma_\nu(1-g_\nu)}\,\frac{\mathrm{d}I_\nu^\circ}{\mathrm{d}T_w}(T_w)\,\mathrm{d}\nu, \tag{20}$$

in which g_ν is the asymmetry factor of scattering given by Eq.2. A set of DOTS (see Sec.3.2.1 b of the companion paper[1]) is an interesting model of foams. A dimensionless generalized extinction coefficient at equilibrium $B^+(\mathbf{u}) = 4\Pi\,B(\mathbf{u})/A$ has been normalized by the extinction coefficient at the optically thin limit $A/(4\Pi)$, where A is the specific area of the porous medium, *i.e.* the interfacial area $\mathrm{d}S$ per unit volume $\mathrm{d}V$ of the whole porous medium. For DOTS, A is linked to the distribution of the radii R by [10]

$$A = \frac{3 < R^2 >}{< R^3 >}\,(1-\Pi)\,\ln(\frac{1}{1-\Pi}), \tag{21}$$

where $< R^2 >$ and $< R^3 >$ are the average values of R^2 and R^3 .

In the common case of a diffuse reflection law of the opaque phase, characterized by the hemispherical absorptivity α_ν^h, the isotropic radiative conductivity writes

$$k^{R\,diff}(T_w,\Pi,A) = [\frac{\Pi^2}{A}\frac{1}{B^+(\Pi)}]\,\frac{16\pi}{3}\int_0^\infty \frac{n_\nu^2}{\alpha_\nu^h + (1-\alpha_\nu^h)(1-g^{diff})}\,\frac{\mathrm{d}I_\nu^\circ}{\mathrm{d}T_w}(T_w)\,\mathrm{d}\nu, \tag{22}$$

and if the opaque interfaces are assumed gray

$$k^{R\,diff}(T_w,\Pi,A) = [\frac{\Pi^2}{A}\frac{1}{B^+(\Pi)}]\,\frac{64\pi}{3}\,\frac{n^2\,\sigma\,T_w^3}{\alpha^h + (1-\alpha^h)(1-g^{diff})}. \tag{23}$$

Results related to specular reflection are given in Ref.[9].

More generally, for porous media of OT type, the key spatial scale is Π/A, and the order of magnitude of an extinction coefficient B is then A/Π. As the radiative conductivity is both proportional to $1/B$ (see Ref.[4]) and to Π, as the radiation field only occupies the transparent phase, a general result is that the radiative conductivity is proportional to Π^2/A.

2.3.2 OST case

A complete study of the radiative conductivity for a porous medium with an opaque phase and real semi transparent scattering and absorbing one has been published in Ref.[6] by generalizing, at the first perturbation order, the approach of Sec. 2.3.1. The conditions of this work have been defined in Sec.2.2.2. The corresponding application is the study of the reflooding of a nuclear core, degraded during a severe accident.

Note that, in these conditions, important energy exchanges occur at the zeroth order of perturbation between the interfaces i, the gas (water vapor v) and the water droplets d, which are at the same macroscopic point $\mathbf{X}$ at different temperatures T_i, T_v and T_d, respectively. By

using all the notations introduced in Sec.2.2.2, the radiative power per unit volume associated with a phase c (i, v or d) simply writes

$$\mathcal{P}_c^{(0)} = \int_0^\infty \int_{4\pi} \kappa_{\nu\, c} \left[I_{\nu\, c}^{(0)} - \Pi\, n_{\nu\, v}^2\, I_\nu^\circ(T_c) \right] \mathrm{d}\Omega\, \mathrm{d}\nu, \tag{24}$$

where $\kappa_{\nu\, c}$ represents $K_{\nu\, i}(\mathbf{u})$, $\kappa_{\nu\, v}$ or $\kappa_{\nu\, d}$. In the considered conditions, the different phases $c = i, v, d$ are at different temperatures. The zeroth order solution, in LTE conditions of a phase c, considered within the gaseous phase v, is obtained for this OST case[6] by an approach similar to the one of Sec.2.3.1

$$I_{\nu\, c}^{(0)}(\mathbf{u}) = \Pi\, n_{\nu\, v}^2\, I_\nu^\circ(T_c) \sum_{k=0}^{\infty} \mathcal{L}_\nu^k \left[\frac{\kappa_{\nu\, c}}{B_i(\mathbf{u}_1) + \kappa_{\nu\, v} + \kappa_{\nu\, d} + \sigma_{\nu\, d}} \right](\mathbf{u}). \tag{25}$$

2.3.3 ST2 case

Radiative Fourier's laws can also be derived from the coupled RTEs of Sec.2.2.3, devoted to two semi transparent phases[7]. Equations 10 and 12 become at the zeroth order of perturbation two coupled equations vs $I_{\nu\, 1}^{(0)}$ and $I_{\nu\, 2}^{(0)}$

$$\begin{aligned} I_{\nu\, 1}^{(0)}(\mathbf{u}_1, \mathbf{X}) \;=\; & \frac{1}{[\beta_{\nu\, 1} \,+\, \Sigma_{\nu\, i\, 11}(\mathbf{u}_1) \,+\, \Sigma_{\nu\, i\, 12}(\mathbf{u}_1)]} \qquad \Big[n_{\nu\, 1}^2\, \kappa_{\nu\, 1}\, I_\nu^\circ[T_1(\mathbf{X})] \\ +\; & \frac{\sigma_{\nu\, 1}}{4\pi} \int_{4\pi} p_{\nu\, 1}(\mu_{sc})\, I_{\nu\, 1}^{(0)}(\mathbf{u}_1', \mathbf{X})\, \mathrm{d}\Omega_1' \,+\, \frac{\Sigma_{\nu\, i\, 11}(\mathbf{u}_1)}{4\pi} \int_{4\pi} p_{\nu\, i\, 11}(\mathbf{u}_1'', \mathbf{u}_1)\, I_{\nu\, 1}^{(0)}(\mathbf{u}_1'', \mathbf{X})\, \mathrm{d}\Omega_1'' \\ +\; & \qquad (\frac{n_{\nu\, 1}^2}{n_{\nu\, 2}^2})\, \Sigma_{\nu\, i\, 12}(\mathbf{u}_1) 4\pi \int_{4\pi} p_{\nu\, i\, 12}(-\mathbf{u}_1, -\mathbf{u}_2'') I_{\nu\, 2}^{(0)}(\mathbf{u}_2'', \mathbf{X})\, \mathrm{d}\Omega_2'' \Big]. \end{aligned} \tag{26}$$

and

$$\begin{aligned} I_{\nu\, 2}^{(0)}(\mathbf{u}_2, \mathbf{X}) \;=\; & \frac{1}{[\beta_{\nu\, 2} \,+\, \Sigma_{\nu\, i\, 22}(\mathbf{u}_2) \,+\, \Sigma_{\nu\, i\, 21}(\mathbf{u}_2)]} \qquad \Big[n_{\nu\, 2}^2\, \kappa_{\nu\, 2}\, I_\nu^\circ[T_2(\mathbf{X})] \\ +\; & \frac{\sigma_{\nu\, 2}}{4\pi} \int_{4\pi} p_{\nu\, 2}(\mu_{sc})\, I_{\nu\, 2}^{(0)}(\mathbf{u}_1', \mathbf{X})\, \mathrm{d}\Omega_2' \,+\, \frac{\Sigma_{\nu\, i\, 22}(\mathbf{u}_2)}{4\pi} \int_{4\pi} p_{\nu\, i\, 22}(\mathbf{u}_2'', \mathbf{u}_2)\, I_{\nu\, 2}^{(0)}(\mathbf{u}_2'', \mathbf{X})\, \mathrm{d}\Omega_2'' \\ +\; & \qquad (\frac{n_{\nu\, 2}^2}{n_{\nu\, 1}^2})\, \frac{\Sigma_{\nu\, i\, 21}(\mathbf{u}_2)}{4\pi} \int_{4\pi} p_{\nu\, i\, 21}(-\mathbf{u}_2, -\mathbf{u}_1'') I_{\nu\, 2}^{(0)}(\mathbf{u}_1'', \mathbf{X})\, \mathrm{d}\Omega_1'' \Big]. \end{aligned} \tag{27}$$

At the first order of perturbation two coupled equations vs $I_{\nu\, 1}^{(1)}$ and $I_{\nu\, 2}^{(1)}$ are also deduced from Equations 10 and 12

$$\begin{aligned} I_{\nu\, 1}^{(1)}(\mathbf{u}_1, \mathbf{X}) \;=\; & \frac{1}{[\beta_{\nu\, 1} \,+\, \Sigma_{\nu\, i\, 11}(\mathbf{u}_1) \,+\, \Sigma_{\nu\, i\, 12}(\mathbf{u}_1)]} \qquad \Big[-\, \frac{\mathrm{d}I_{\nu\, 1}^{(0)}}{\mathrm{d}s}(\mathbf{u}_1, \mathbf{X}) \\ +\; & \frac{\sigma_{\nu\, 1}}{4\pi} \int_{4\pi} p_{\nu\, 1}(\mu_{sc})\, I_{\nu\, 1}^{(1)}(\mathbf{u}_1', \mathbf{X})\, \mathrm{d}\Omega_1' \,+\, \frac{\Sigma_{\nu\, i\, 11}(\mathbf{u}_1)}{4\pi} \int_{4\pi} p_{\nu\, i\, 11}(\mathbf{u}_1'', \mathbf{u}_1)\, I_{\nu\, 1}^{(1)}(\mathbf{u}_1'', \mathbf{X})\, \mathrm{d}\Omega_1'' \\ +\; & \qquad (\frac{n_{\nu\, 1}^2}{n_{\nu\, 2}^2})\, \Sigma_{\nu\, i\, 12}(\mathbf{u}_1) 4\pi \int_{4\pi} p_{\nu\, i\, 12}(-\mathbf{u}_1, -\mathbf{u}_2'') I_{\nu\, 2}^{(1)}(\mathbf{u}_2'', \mathbf{X})\, \mathrm{d}\Omega_2'' \Big]. \end{aligned} \tag{28}$$

and

$$\begin{aligned} I_{\nu\, 2}^{(1)}(\mathbf{u}_2, \mathbf{X}) \;=\; & \frac{1}{[\beta_{\nu\, 2} \,+\, \Sigma_{\nu\, i\, 22}(\mathbf{u}_2) \,+\, \Sigma_{\nu\, i\, 21}(\mathbf{u}_2)]} \qquad \Big[-\, \frac{\mathrm{d}I_{\nu\, 2}^{(0)}}{\mathrm{d}s}(\mathbf{u}_1, \mathbf{X}) \\ +\; & \frac{\sigma_{\nu\, 2}}{4\pi} \int_{4\pi} p_{\nu\, 2}(\mu_{sc})\, I_{\nu\, 2}^{(1)}(\mathbf{u}_1', \mathbf{X})\, \mathrm{d}\Omega_2' \,+\, \frac{\Sigma_{\nu\, i\, 22}(\mathbf{u}_2)}{4\pi} \int_{4\pi} p_{\nu\, i\, 22}(\mathbf{u}_2'', \mathbf{u}_2)\, I_{\nu\, 2}^{(1)}(\mathbf{u}_2'', \mathbf{X})\, \mathrm{d}\Omega_2'' \\ +\; & \qquad (\frac{n_{\nu\, 2}^2}{n_{\nu\, 1}^2})\, \frac{\Sigma_{\nu\, i\, 21}(\mathbf{u}_2)}{4\pi} \int_{4\pi} p_{\nu\, i\, 21}(-\mathbf{u}_2, -\mathbf{u}_1'') I_{\nu\, 1}^{(1)}(\mathbf{u}_1'', \mathbf{X})\, \mathrm{d}\Omega_1'' \Big]. \end{aligned} \tag{29}$$

The solutions at zeroth and first perturbation orders $I^{(0)}_{\nu\,1}$, $I^{(0)}_{\nu\,2}$, $I^{(1)}_{\nu\,1}$ and $I^{(1)}_{\nu\,2}$ are in both cases expressed vs different coupled functionals. For the sake of simplicity, the corresponding algorithms are defined in Appendix A.

The radiative flux vector in a phase $\gamma = 1, 2$ is always determined by Eq.19, from the expression of $I^{(1)}_{\nu\,\gamma}(\mathbf{u}_\gamma, \mathbf{X})$ which now depends on the temperature gradients in the two phases $\mathrm{d}T_1/\mathrm{d}X_k$ and $\mathrm{d}T_2/\mathrm{d}X_k$ (in tensorial notations for j and γ')

$$q^R_{\gamma\,i}(X_k) = -k^R_{\gamma'\,\gamma\,i\,j}\frac{\partial T_{\gamma'}}{\partial X_j}(X_k) \qquad \gamma = 1,\,2; \qquad \gamma' = 1,\,2. \tag{30}$$

2.3.4 STT case

The results of this case are simply deduced from the previous one, by setting: $\beta_{\nu\,2} = \sigma_{\nu\,2} = \kappa_{\nu\,2} = 0$; $p_{\nu\,2} = 0$. Note that, in this simpler case, the solution at the zeroth perturbation order is for both phases $I^\circ_\nu[T_1(\mathbf{X})]$.
The solution at the first perturbation order is similar to the one of ST2 case. But, as the phase 2 is transparent, the radiative flux vector in a phase $\gamma = 1, 2$ only depends on $\frac{\partial T_1}{\partial X_j}$, the temperature gradient in phase 1, *i.e.* in tensorial notations

$$q^R_{\gamma\,i}(X_k) = -k^R_{\gamma\,i\,j}\frac{\partial T_1}{\partial X_j}(X_k) \qquad \gamma = 1,\,2. \tag{31}$$

3 GRTE based radiative transfer

This section is devoted to radiative transfer within statistically homogeneous or non homogeneous media, by using the most general approach, based on GRTE(s). Statistically anisotropic porous media and even, in some cases, statistically isotropic media (DOTS or foams of porosity less than 0.6, for instance) do not follow the Beer's law. When the conditions of application of a radiative Fourier's law are not fulfilled (see Sec.2.1) GRTE(s) must then be used, even in the core of the homogenized phases. For statistically non homogeneous media, it is the unique convenient model.
The precise expressions of the GRTE or of the coupled GRTEs are given in Sec.3.1. The principles of implementation of a (G)RTE by a Monte Carlo method are studied in Sec.3.2. A specific model for the emission source term of a homogenized interfacial phase is developed in Sec.3.3.

3.1 Expressions of the GRTEs in different cases

3.1.1 OT case

The GRTE dedicated to a statistically homogeneous medium with opaque and transparent phases has been given in Sec.2.3 of the companion paper[1]. In this case, the homogenized phase represents the interfacial elements i and occupies the same physical volume as the transparent propagation phase.

A non homogeneous medium is characterized by a non uniform fraction per unit volume field $\Pi(s)$ along the considered direction. The GRTE associated with this statistically non

homogeneous medium, expressed in the propagation phase of refractive index 1, writes

$$\begin{aligned}\frac{\mathrm{d}(\Pi\, I_\nu)}{\mathrm{d}s'}(s',\mathbf{u})\,\mathrm{d}\Omega &= [S_{sc\,\nu\,i}(s',\mathbf{u}) + S_{e\,\nu\,i}(s',\mathbf{u})]\,\mathrm{d}\Omega \\ &- \int_{s_b}^{s'} [S_{sc\,\nu\,i}(s,\mathbf{u}) + S_{e\,\nu\,i}(s,\mathbf{u})\, C(s,s'-s,\mathbf{u})]\, \frac{\mathrm{d}G_{ext\,\nu\,i}}{\mathrm{d}s'}(s,\mathbf{u},s'-s)\,\mathrm{d}s\,\mathrm{d}\Omega \\ &- \Pi(s_b)\, I_{b\,\nu}(\mathbf{u}) \frac{\mathrm{d}G_{ext\,\nu\,i}}{\mathrm{d}s'}(s_b,\mathbf{u},s'-s_b)\,\mathrm{d}\Omega \end{aligned} \tag{32}$$

In Eq.32, the associated scattering source term $S_{sc\,\nu\,i}(s',\mathbf{u})$ of Sec. 2.2 of the companion paper[1] becomes

$$\begin{aligned} S_{sc\,\nu\,i}(s',\mathbf{u})\;\mathrm{d}\Omega &= \mathrm{d}\Omega \left(\int_{4\pi} \int_{s_{1b}}^{s_1'} [S_{sc\,\nu\,i}(s_1,\mathbf{u}_1) + S_{e\,\nu\,i}(s_1,\mathbf{u}_1)\, D(s,s_1'-s_1,\mathbf{u}_1)] \right. \\ &\quad \mathrm{x} \quad \frac{\mathrm{d}P_{sc\,\nu\,i}}{\mathrm{d}s_1'}(s_1,\mathbf{u}_1,s_1'-s_1) \frac{p_\nu(\mathbf{u}_1,\mathbf{u})}{4\pi} \mathrm{d}s_1\;\mathrm{d}\Omega_1 \\ &\quad + \left. \int_{4\pi} I_{b\nu}(s_{1b},\mathbf{u}_1)\, \frac{\mathrm{d}P_{sc\,\nu\,i}}{\mathrm{d}s_1'}(s_{1b},\mathbf{u}_1,s_1'-s_{1b}) \frac{p_\nu(\mathbf{u}_1,\mathbf{u})}{4\pi} \mathrm{d}s_1\;\mathrm{d}\Omega_1 \right). \end{aligned} \tag{33}$$

In Eqs.32 and 33, $C(s,s'-s,\mathbf{u})$ and $D(s,s'-s,\mathbf{u})$ are correlation factors between emission and transmission[2], due to the non Beerian radiative properties associated with interfacial homogenized phases i. These key factors are introduced and discussed in Sec.3.3 and are directly obtained in a Monte Carlo calculation. Note that:
- It is assumed that there is no correlation between scattering and transmission, due to the summation over 4π steradians within the scattering source term.
- The fraction per unit volume $\Pi(s)$ of an emission cell is included in any emission term $S_{e\,\nu\,i}(s,\mathbf{u})$ and implicitly in any scattering term $S_{sc\,\nu\,i}(s_1,\mathbf{u}_1)$.

The extremely implicit Equation 32 can be solved by a Monte Carlo method. But the emission source term $S_{e\,\nu\,i}(s',\mathbf{u})$, in this OT case, cannot be expressed by introducing an absorption coefficient, which has no physical meaning in the considered conditions. A specific approach is developed in Sec.3.3.

3.1.2 OST case

The GRTE dedicated to a medium with opaque and semi-transparent phases is a generalization of the GRTE associated with the OT case. We consider, as previously, that the semi transparent medium is water vapor and scattering droplets characterized by $\kappa_{\nu\,v}$, $\kappa_{\nu\,d}$, $\sigma_{\nu\,d}$, $p_{\nu\,d}(\mu_{scd})$ and a refractive index $n_{\nu\,v}$, equal to 1. In this case, the homogenized phase represents the interfacial elements i and occupies the same physical volume as the semi transparent propagation phase. As for the OT case, the GRTE is expressed in the propagation phase of refractive index 1. It writes in the case of a statistically non homogeneous medium

$$\begin{aligned} \frac{\mathrm{d}(\Pi I_\nu)}{\mathrm{d}s'}(s',\mathbf{u})\mathrm{d}\Omega &= \left[S_{sc\,\nu}(s',\mathbf{u}) + n_{\nu\,v}^2 \Pi(s') \Big(\kappa_{\nu\,v} I^\circ[T_v(s')] + \kappa_{\nu\,d} I^\circ[T_d(s')] \Big) + S_{e\,\nu\,i}(s,\mathbf{u}) \right] \mathrm{d}\Omega \\ - \int_{s_b}^{s'} & \left[S_{sc\,\nu}(s,\mathbf{u}) + n_{\nu\,v}^2 \Pi(s) \Big(\kappa_{\nu\,v} I^\circ[T_v(s)] + \kappa_{\nu\,d} I^\circ[T_d(s)] \Big) + S_{e\,\nu\,i}(s,\mathbf{u}) E(s,s'-s,\mathbf{u}) \right] \\ & \qquad \mathrm{x} \quad \frac{\mathrm{d}G_{\nu extg}}{\mathrm{d}s'}(s,\mathbf{u},s'-s)\,\mathrm{d}s\,\mathrm{d}\Omega \\ - \Pi_{s_b} I_{b\,\nu}(\mathbf{u}) & \frac{\mathrm{d}G_{\nu\,ext\,g}}{\mathrm{d}s'}(s_b,\mathbf{u},s'-s_b)\mathrm{d}\Omega, \end{aligned} \tag{34}$$

where $E(s, s'-s, \mathbf{u})$, is also a correlation factor between emission and transmission within a non Beerian homogenized phase. From Eq.10 of the companion paper[1], the global extinction cumulative distribution function $G_{ext\,g}(s, \mathbf{u}, s'-s)$ and its derivative are given by

$$G_{\nu\,ext\,g}(s, \mathbf{u}, s'-s) \;=\; 1 - \,[1 - G_{ext\,i}(s, \mathbf{u}, s'-s)]\exp[-\,(\kappa_{\nu\,v} \,+\, \kappa_{\nu\,d} \,+\, \sigma_{\nu\,d})\,(s'-s)]. \tag{35}$$

$$\begin{aligned}\frac{\mathrm{d}G_{\nu\,ext\,g}}{\mathrm{d}s'}(s, \mathbf{u}, s'-s) \;&=\; \exp[-\,(\kappa_{\nu\,v} + \,\kappa_{\nu\,d} \,+\, \sigma_{\nu\,d})\,(s'-s)] \\ \mathrm{x}\quad &\left((\kappa_{\nu\,v} \,+\, \kappa_{\nu\,d} \,+\, \sigma_{\nu\,d})\,[1 \,-\, G_{ext\,i}(s, \mathbf{u}, s'-s)] \,+\, \frac{\mathrm{d}G_{ext\,i}}{\mathrm{d}s'}(s, \mathbf{u}, s'-s)\right).\end{aligned} \tag{36}$$

The global scattering source term $S_{sc\,\nu}(s', \mathbf{u})$ writes

$$\begin{aligned}&S_{sc\,\nu}(s', \mathbf{u}) \;= \\ &\int_{4\pi}\int_{s_{1b}}^{s'_1}\left(S_{sc\,\nu}(s_1, \mathbf{u}_1) + n^2_{\nu\,v}[\kappa_{\nu\,v} I^\circ_\nu(T_v) + \kappa_{\nu\,d} I^\circ_\nu(T_d)] + S_{e\,\nu\,i}(s_1, \mathbf{u}_1) F(s_1, s'-1-s_1, \mathbf{u})\right) \\ &\qquad \mathrm{x}\quad \exp[-\,(\kappa_{\nu\,v} \,+\, \kappa_{\nu\,d} \,+\, \sigma_{\nu\,d})\,(s'_1 - s_1)]\quad \frac{\mathrm{d}P_{sc\,\nu\,i}}{\mathrm{d}s'_1}(s_1, \mathbf{u}_1, s'_1 - s_1)\frac{p_{\nu\,i}(\mathbf{u}_1, \mathbf{u})}{4\pi}\mathrm{d}s_1\;\mathrm{d}\Omega_1 \\ +\quad &\int_{4\pi}\int_{s_{1b}}^{s'_1}\left(S_{sc\,\nu}(s_1, \mathbf{u}_1) + n^2_{\nu\,v}[\kappa_{\nu\,v} I^\circ_\nu(T_v) + \kappa_{\nu\,d} I^\circ_\nu(T_d)] + S_{e\,\nu\,i}(s_1, \mathbf{u}_1) G(s_1, s'_1 - s_1, \mathbf{u})\right) \\ &\qquad\qquad \mathrm{x}\quad [1 - G_{\nu\,ext\,g}(s_1, \mathbf{u}_1, s'_1 - s_1)]\sigma_{\nu\,d}\frac{p_{\nu\,d}(\mu_{sc\,d})}{4\pi}\mathrm{d}s_1\;\mathrm{d}\Omega_1 \\ +\quad &\int_{4\pi}\Pi\, I_{b\nu}(s_{1b}, \mathbf{u}_1)\,[1 - G_{\nu\,ext\,g}(s_{1b}, \mathbf{u}_1, s'_1 - s_{1b})]\sigma_{\nu\,d}\frac{p_{\nu\,d}(\mu_{sc\,d})}{4\pi}\,\mathrm{d}\Omega_1 \\ +\quad &\int_{4\pi}\Pi\, I_{b\nu}(s_{1b}, \mathbf{u}_1)\,\exp[-\,(\kappa_{\nu\,v} \,+\, \kappa_{\nu\,d} \,+\, \sigma_{\nu\,d})\,(s'_1 - s_1)]\,\frac{\mathrm{d}P_{sc\,\nu\,i}}{\mathrm{d}s'_1}(s_1, \mathbf{u}_1, s'_1 - s_1)\frac{p_{\nu\,i}(\mathbf{u}_1, \mathbf{u})}{4\pi}\,\mathrm{d}\Omega_1.\end{aligned} \tag{37}$$

The physical origin of the correlation factors $F(s, s'-s, \mathbf{u})$ and $G(s, s'-s, \mathbf{u})$, which appear in this equation and are similar to $C(s, s'-s, \mathbf{u})$, $D(s, s'-s, \mathbf{u})$ and $E(s, s'-s, \mathbf{u})$ in the previous ones, is explained in Sec. 3.3.

3.1.3 ST2 case

The GRTE of the statistically non homogeneous semi transparent phase 1 of fraction per unit volume $\Pi_1(s)$ along the considered direction, expressed in medium 1, writes[7][2]

$$\begin{aligned}\frac{\mathrm{d}(\Pi_1\, I_{\nu\,1})}{\mathrm{d}s'}(s', \mathbf{u})\,\mathrm{d}\Omega_1 \;&=\; [S_{sc\,\nu\,1}(s', \mathbf{u}) \,+\, \Pi_1(s')\,\kappa_{\nu\,1} n^2_{\nu\,1} I^\circ_\nu(T_1)]\,\mathrm{d}\Omega_1 \\ &-\; \mathrm{d}\Omega_1 \int_{s_b}^{s'}[S_{sc\,\nu\,1}(s, \mathbf{u}) + \Pi_1(s)\,\kappa_{\nu\,1} n^2_{\nu\,1} I^\circ_\nu(T_1)]\,\frac{\mathrm{d}G_{ext\,\nu\,1}}{\mathrm{d}s'}(s, \mathbf{u}, s'-s)\,\mathrm{d}s \\ &-\; \Pi_1(s_b)\, I_{b\,\nu}(\mathbf{u})\frac{\mathrm{d}G_{ext\,\nu\,1}}{\mathrm{d}s'}(s_b, \mathbf{u}, s'-s_b)\,\mathrm{d}\Omega_1.\end{aligned} \tag{38}$$

with

$$\frac{\mathrm{d}G_{ext\,\nu\,1}}{\mathrm{d}s'}(s, \mathbf{u}, s'-s) = \frac{\mathrm{d}P_{a\,\nu\,1}}{\mathrm{d}s'}(s, \mathbf{u}, s'-s) + \frac{\mathrm{d}P_{sc\,\nu\,11}}{\mathrm{d}s'}(s, \mathbf{u}, s'-s) + \frac{\mathrm{d}P_{sc\,\nu\,12}}{\mathrm{d}s'}(s, \mathbf{u}, s'-s) \tag{39}$$

[2]The notations in use in the scattering source terms are defined in Fig.2 of the companion paper[1].

Note that $\mathbf{u}$ is associated with the direction of the axis along which the GRTE is written: It is the same for the media 1 and 2; $\mathrm{d}\Omega_1$ is an elementary solid angle expressed in medium 1 of refractive index $n_{1\,\nu}$, etc.

The scattering source term $S_{sc\,\nu\,1}(s', \mathbf{u})$, expressed in medium 1, is the sum of two partial source terms associated with reflection within 1 and transmission from 2 to 1, *i.e.*

$$
\begin{aligned}
& S_{sc\nu 1}(s', \mathbf{u}) = \\
& \int_{4\pi} \int_{s_{1b}}^{s'_1} \frac{\mathrm{d}P_{sc\nu 11}}{\mathrm{d}s'_1}(s_1, \mathbf{u}_1, s'_1 - s_1) \frac{p_{\nu 11}(\mathbf{u}_1, \mathbf{u})}{4\pi} [S_{sc\nu 1}(s_1, \mathbf{u}_1)\mathrm{d}\Omega_1 + \Pi_1(s_1)\kappa_{\nu 1} n^2_{\nu 1} I^\circ_\nu(T_1)\mathrm{d}\Omega_1]\mathrm{d}s_1 \\
+ & \int_{4\pi} \int_{s_{2b}}^{s'_2} \frac{\mathrm{d}P_{sc\nu 21}}{\mathrm{d}s'_2}(s_2, \mathbf{u}_2, s'_2 - s_2) \frac{p_{\nu 21}(\mathbf{u}_2, \mathbf{u})}{4\pi} [S_{sc\nu 2}(s_2, \mathbf{u}_2)\mathrm{d}\Omega_2 + \Pi_2(s_2)\kappa_{\nu 2} n^2_{\nu 2} I^\circ_\nu(T_2)\mathrm{d}\Omega_2]\mathrm{d}s_2. \\
+ & \int_{4\pi} \Pi_1(s_{1b})\, I_{b\nu}(s_{1b}, \mathbf{u}_1) \frac{\mathrm{d}P_{sc\nu 11}}{\mathrm{d}s'_1}(s_{1b}, \mathbf{u}_1, s'_1 - s_{1b}) \frac{p_{\nu 11}(\mathbf{u}_1, \mathbf{u})}{4\pi} \mathrm{d}\Omega_1 \\
+ & \int_{4\pi} \Pi_2(s_{2b})\, I_{b\nu}(s_{2b}, \mathbf{u}_2) \frac{\mathrm{d}P_{sc\nu 21}}{\mathrm{d}s'_2}(s_{2b}, \mathbf{u}_2, s'_2 - s_{2b}) \frac{p_{\nu 21}(\mathbf{u}_2, \mathbf{u})}{4\pi} \mathrm{d}\Omega_2. \qquad (40)
\end{aligned}
$$

Note that $\mathbf{u}_1$ and $\mathbf{u}_2$ are dummy quantities, associated with the current propagation directions in media 1 and 2 before scattering.

The GRTE associated with the medium 2 is simply obtained by permuting 1 and 2 in the different equations.

3.1.4 STT case

If the medium 2 is transparent, the associated results are obtained[7] by setting:
$\kappa_{\nu\,2} = \sigma_{\nu\,2} = 0$, $\mathrm{d}P_{\nu\,a\,2}/\mathrm{d}s' = \mathrm{d}P_{\nu\,sc\,2}/\mathrm{d}s' = 0$.

3.2 Monte Carlo method applied to GRTE (explicit emission source term)

If the GRTEs introduced in Sec.3.1 seem at first glance complex, they are easy to implement in a Monte Carlo approach, in so far as they are expressed in terms of cumulative distribution functions. Only the principles of Monte Carlo methods specific to porous media are given here. Many works are more generally devoted to many variants of the Monte Carlo method[11]. In particular, the interest of a Monte Carlo method is to associate with results standard deviations on these results[11, 4].

In this Section, an emission source term is assumed to be explicitly expressed from an absorption coefficient in the ST2, STT and OST cases for the real semi transparent medium, and under some precise conditions (Beerian homogenized interfacial phases i in the core of the porous medium) in the OT and OST cases.

All thermophysical properties and the temperature fields in all phases are assumed known and are entry data. The results are the radiative power per unit volume within any volume cell and the radiative flux at any surface cell. These quantities allow new temperature fields to be determined, in an iterative scheme, after possibly accounting for other heat transfer modes.

The proposed method is based on some main ideas:

- An emission source term accounts for all emitted photons, *i.e.* photons which are self absorbed by the emitting elementary cell are also accounted for in the emission term (true emission). Self absorption will be accounted for, in this scheme, as absorption by the emitting cell.

- Bundles of power emitted by all the emission sources (volume or surface cells) are followed up to absorption by a cell, after possibly a large number of scattering phenomena.

- At the limit of large numbers of emitted bundles by all sources, which respect the real power emitted by any cell, the balance of emitted and absorbed powers in any cell tends to the physical value. The method associates to any result a standard deviation.

- The application of the reciprocity theorem between any emission and any absorption cell accelerates the convergence of the method. It also allows the validity of the code to be checked.

3.2.1 Emission modeling

The power bundle

- which is emitted in an elementary solid angle $\mathrm{d}\Omega(\theta,\varphi)$ by a volume element $\Pi\,\mathrm{d}V_K$ of a cell K of temperature T_k, around a point $M_k(s_k)$ of a homogenized phase of refractive index n_ν,

- and which is extinguished, by absorption or scattering, at a point $M_j(s'_j)$ of a cell J, in a volume element of the same phase or at an interface with another phase, simply writes

$$\mathrm{d}\mathcal{P}^e_{\nu\,K} \;=\; \kappa_{\nu\,k}\, n_\nu^2\, I^\circ_\nu(T_k)\,\mathrm{d}V_k\,\Pi\,[1-G_{\nu\,ext}(s_k,\mathbf{u},s'_j\;-\;s_k)]\,\mathrm{d}\Omega\,\mathrm{d}\nu, \tag{41}$$

where s'_j is the abscissa of the first extinction point of this shot. There is, under the assumptions of this section, no correlation between the emitted flux and its transmissivity $1-G_{\nu\,ext}$. This power bundle will then be

- either absorbed in the cell J: Its power will be added to the power absorbed by the cell $\mathcal{P}^a_J$,

- or scattered at the point j of the cell J, case studied in the following.

If emission is isotropic, the total flux emitted by the cell K writes [3]

$$\mathcal{P}^e_K \;=\; 4\pi\,\delta V_k\,\Pi\int_0^\infty \kappa_{\nu\,k}\, n_\nu^2\, I^\circ_\nu(T_k)\,\mathrm{d}\nu, \tag{44}$$

and the emission probability at the point M_k of the cell K

$$\begin{aligned}\mathrm{d}\,\mathrm{Prob}^e_{\nu\,k} &= \left(\frac{\kappa_{\nu\,k}\, n_\nu^2\, I^\circ_\nu(T_k)\,\mathrm{d}\nu}{\int_0^\infty \kappa_{\nu\,k}\, n_\nu^2\, I^\circ_\nu(T_k)\,\mathrm{d}\nu}\right)\left(\frac{\sin\theta\mathrm{d}\theta}{2}\right)\left(\frac{\mathrm{d}\varphi}{2\pi}\right)\left(\frac{\mathrm{d}V}{\Pi V}\right),\\ &= F_1(\nu)\mathrm{d}\nu\; F_2(\theta)\mathrm{d}\theta\; F_3(\varphi)\mathrm{d}\varphi\;\frac{\mathrm{d}V}{\Pi V},\end{aligned} \tag{45}$$

where $F_1(\nu)$, $F_2(\theta)$ and $F_3(\varphi)$ are the independent distribution functions of ν, θ and φ, in this simple case.

In a Monte Carlo method ν, θ and φ are determined, for any emission power bundle shot, from the shots of three random numbers a_1, a_2 and a_3 in the range $[0,1]$. In these conditions, the values associated with a shot t are

$$\nu_t \;=G_1^{-1}(a_1),\;\theta_t \;=G_2^{-1}(a_2),\;\varphi_t \;=G_3^{-1}(a_3), \tag{46}$$

where $G_i^{-1}(a_m)$ is the reciprocal function of the cumulative distribution function associated with a quantity m.

[3] The power emitted by an interfacial homogenized phase i writes

$$\mathcal{P}^e_{i\,K} \;=\; \delta V_k\,\Pi\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} K_{\nu\,k}(\theta,\varphi)\, n_\nu^2\, I^\circ_\nu(T_k)\sin\theta\,\mathrm{d}\varphi\,\mathrm{d}\theta\,\mathrm{d}\nu \tag{42}$$

and the emission probability at the point M_k of the cell K writes if, for instance, $K_{\nu\,k}(\theta,\varphi) \;=\; K_{1\,k}(\nu)\,K_2(\theta,\varphi)$

$$\mathrm{d}\,\mathrm{Prob}^e_{\nu\,K} \;=\; \left(\frac{K_{1\,k}(\nu)\, n_\nu^2\, I^\circ_\nu(T_k)\,\mathrm{d}\nu}{\int_0^\infty K_{1\,k}(\nu)\, n_\nu^2\, I^\circ_\nu(T_k)\,\mathrm{d}\nu}\right)\left(\frac{K_2(\theta,\varphi)\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi}{\int_0^{2\pi}\int_0^{\pi} K_2(\theta,\varphi),\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi}\right)\left(\frac{\mathrm{d}V}{\Pi V}\right), \tag{43}$$

where appears a joined distribution function of φ and θ, etc.

The shot point M_k is the center of the cell K if it is optically thin or is randomly obtained by a special algorithm. N_K emission power bundles are shot from a cell K. Their total power is exactly $\mathcal{P}^e_K$.

3.2.2 Following a bundle

The first extinction point of a given shot t is obtained from a new random number b_1 in the range $[0,1]$ applied to transmissivity $\tau_{\nu\,ext}(s_k, \mathbf{u}, s'_j - s_k) = 1 - G_{\nu\,ext}(s_k, \mathbf{u}, s'_j - s_k)$; An equivalent formulation is that the extinction length $l_{t\,1} = (s'_j - s_k)_1$ is given by

$$l_{t\,1} = \tau^{-1}_{\nu\,ext}(s_k, \mathbf{u}, b_1) \tag{47}$$

The direct formulation of the GRTE in terms of cumulative distribution functions is a great advantage for this approach.

The bundle is either absorbed or scattered in J. The absorption probability is simply

$$\mathrm{P}_{abs} = \left[\frac{\mathrm{d}P_{\nu\,a}(s_k, \mathbf{u}, s' - s_k)}{\mathrm{d}s'} \Big/ \frac{\mathrm{d}G_{\nu\,ext}(s_k, \mathbf{u}, s' - s_k)}{\mathrm{d}s'}\right]_{s' - s_k = l_t}, \tag{48}$$

with

$$\mathrm{d}G_{\nu\,ext}(s_k, \mathbf{u}, s' - s_k) = \mathrm{d}P_{\nu\,a}(s_k, \mathbf{u}, s' - s_k) + \mathrm{d}P_{\nu\,sc}(s_k, \mathbf{u}, s' - s_k) \tag{49}$$

quantity which has previously been determined. If a new random number b_{1a} is smaller than P_{abs}, the emission power bundle is absorbed and this power is attributed to the absorbing cell (volume or surface element). If $b_{1a} > \mathrm{P}_{abs}$, the bundle is scattered. In this case, different scattering phenomena can occur, characterized for instance by $P_{\nu\,sc\,1}(s_k, \mathbf{u}, s' - s_k)$, $P_{\nu\,sc\,11}(s_k, \mathbf{u}, s' - s_k)$ and $P_{\nu\,sc\,12}(s_k, \mathbf{u}, s' - s_k)$, such that

$$\mathrm{d}P_{\nu\,sc}(s_k, \mathbf{u}, s' - s_k) = \mathrm{d}P_{\nu\,sc\,1}(s_k, \mathbf{u}, s' - s_k) + \mathrm{d}P_{\nu\,sc\,11}(s_k, \mathbf{u}, s' - s_k) + \mathrm{d}P_{\nu\,sc\,12}(s_k, \mathbf{u}, s' - s_k). \tag{50}$$

The associated probabilities are

$$\mathrm{P}_{sc\,1} = \left[\frac{\mathrm{d}P_{\nu\,sc\,1}(s_k, \mathbf{u}, s' - s_k)}{\mathrm{d}s'} \Big/ \frac{\mathrm{d}G_{\nu\,ext}(s_k, \mathbf{u}, s' - s_k)}{\mathrm{d}s'}\right]_{s' - s_k = l_t} \quad \text{etc.} \tag{51}$$

In these conditions

- If $\mathrm{P}_{abs} < b_{1\,a} < \mathrm{P}_{abs} + \mathrm{P}_{sc\,1}$, the emission power bundle is scattered with the phase function $p_{\nu\,1}$;
- If $\mathrm{P}_{abs} + \mathrm{P}_{sc\,1} < b_{1\,a} < \mathrm{P}_{abs} + \mathrm{P}_{sc\,1} + \mathrm{P}_{sc\,11}$, the emission power bundle is scattered with the phase function $p_{\nu\,11}$;
- If $\mathrm{P}_{abs} + \mathrm{P}_{sc\,1} + \mathrm{P}_{sc\,11} < b_{1\,a} < 1$, the emission power bundle is scattered with the phase function $p_{\nu\,12}$.

Note that:

- If a phenomenon ph is Beerian, the associated variation of the cumulative distribution function, including $\mathrm{d}G_{\nu\,ext}$ simply writes

$$\mathrm{dP}_{ph} = [1 - G_{\nu\,ext}]\,\chi_{\nu\,ph}\,\mathrm{d}s'; \qquad \chi_{\nu\,ph} = \beta_\nu,\, \kappa_\nu,\, \sigma_\nu. \tag{52}$$

- If all phenomena are Beerian, the probabilities simply are

$$\mathrm{P}_{abs\,1} = \frac{\kappa_{\nu\,1}}{\beta_{\nu\,1}}; \quad \mathrm{P}_{sc\,1} = \frac{\sigma_{\nu\,1}}{\beta_{\nu\,1}}; \quad \mathrm{P}_{sc\,11} = \frac{\sigma_{\nu\,11}}{\beta_{\nu\,1}}; \quad \mathrm{P}_{sc\,112} = \frac{\sigma_{\nu\,11}}{\beta_{\nu\,1}}, \tag{53}$$

$$\text{with :} \qquad \beta_{\nu\,1} \,=\, \kappa_{\nu\,1}\,\sigma_{\nu\,1} \,+\, \sigma_{\nu\,11} \,+\, \sigma_{\nu\,12}. \tag{54}$$

- If, in the OT case, the reflection law of the opaque interfaces is diffuse, characterized by an absorptivity α_ν, we simply obtain, even for a statistically non homogeneous medium

$$\mathrm{P}_{abs} \,=\, \alpha_\nu; \quad \mathrm{P}_{sc} \,=\, 1 \,-\, \alpha_\nu. \tag{55}$$

The scattering direction $\mathbf{u}_1(\theta_1, \varphi_1)$ is determined from the phase function, which generally is a joined cumulative distribution function of θ_1 and φ_1, by using two new random number shots. In this classical formulation all the power of the bundle is scattered.

A second extinction point of the shot t is obtained from a new random number b_2 in the range $[0, 1]$ applied to transmissivity $1 - G_{\nu\,ext}(s_j, \mathbf{u}_1, s'_n - s_j)$ as previously, and the method is iterated up to absorption.

The theorem of reciprocity[4], associated with a time reversing, allows the power emitted by any absorbing cell and absorbed by the initially emitting cell to be immediately expressed

$$\mathrm{d}\mathcal{P}_{\nu\,J\,K}^{\;e\;\;a} \,=\, \frac{I^\circ_\nu(T_J)}{I^\circ_\nu(T_K)}\,\mathrm{d}\mathcal{P}_{\nu\,K\,J}^{\;e\;\;a}. \tag{56}$$

The symmetry of the $P_{\nu\,J\,K}^{\;e\;\;a}$ has to be verified. The use of the theorem accelerates the convergence of the Monte Carlo method by symmetrizing this matrix. The radiative power in a boundary surface or volume cell K is then

$$\mathcal{P}^R_{\nu\,K} \,=\, \sum_J \left(\mathcal{P}_{\nu\,J\,K}^{\;e\;\;a} \,-\, \mathcal{P}_{\nu\,K\,J}^{\;e\;\;a} \right) \tag{57}$$

The set of the $\mathcal{P}^R_{\nu\,K}$ are the entry data of a coupled heat transfer calculation, generally involving other heat transfer modes and leading after iteration to the determination of the temperature and flux fields.

Many variants of the Monte Carlo methods have been developed. In a common hybrid version, based on the statistical independence of absorption and scattering, absorption along a ray is calculated in a deterministic manner: At any exit boundary of a cell, the part of the power of the shot absorbed in this cell K is added to $\mathcal{P}^a_{\nu\,K}$. Instead of extinction lengths, scattering lengths are obtained from $P_{\nu\,sc}(s, \mathbf{u}, s' - s)$

$$l_{t\,sc} \,= P^{-1}_{\nu\,sc}(s, \mathbf{u}, c), \tag{58}$$

where c is a random number belonging to the range $[0, P_{\nu\,sc}(s, \mathbf{u}, \infty)]$. The calculation is stopped when the residual power of a bundle is smaller than a given criterion.

3.3 Emission by a homogenized interfacial phase i

In the previous approach, as shown in Eq.41, transmission from s to s' is non correlated to emission around s: This equation is based on the product of the transmissivity from s to s' by an absorption coefficient in s'.

But in OT and OST cases, more precisely in the case of emission by a homogenized phase i associated with opaque interfacial elements which is not Beerian, strong correlated effects appear between emission and transmission: Indeed, extinction and absorption coefficients have then no physical meaning. For instance, in the case of Fig. 3 issued from Ref.[2], the correlation factor between:
- Emission, in the direction defined by μ, by the cell J enclosed between two planes $(x_{j+1} - dx; x_{j+1})$,

- And transmission to the plane k of abcissa $x_k = [x_{j+1} + \mu(s' - s)]$ (case of Fig. 3)
is defined by

$$F_{J,k}(\mu) \;=\; \frac{\Phi^c_{J,k}(\mu)}{\Phi^{nc}_{J,k}(\mu)} \tag{59}$$

where:

- $\Phi^c_{J,k}(\mu)$ is the *correlated* flux emitted by J and transmitted to k, obtained by a Monte Carlo method based on the reciprocity theorem as defined in the following;

- $\Phi^{nc}_{J,k}(\mu)$ is the flux emitted by J and transmitted to k *without correlation* between emission and transmission, *i.e.*

$$\Phi^{nc}_{J,k}(\mu) \;=\; \Phi^c_{J,j+1}(\mu)\,[1 \,-\, G_{ext}(x_{j+1},\, \mu,\, x_k)], \tag{60}$$

where $\Phi^c_{J,j+1}(\mu)$ is the flux emitted by the cell J at its boundary $j+1$.

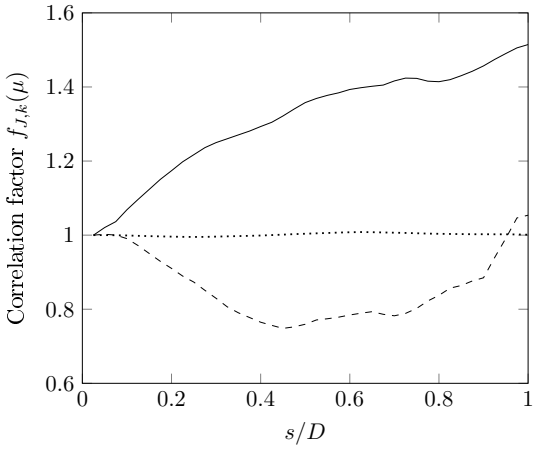

Figure 3: Correlation factor between emission, in the direction defined by μ, by the cell J between two planes $(x_{j+1} - dx; x_{j+1})$ and transmission to the plane k of abcissa $x_k = [x_{j+1} + \mu(s' - s)]$[2] with: $dx = 2.5\,10^{-2} D$ (D: sphere diameter); $x_{j+1} = D$, $\mu = -1$ (solid curve); $x_{j+1} = D$, $\mu = -0.5$ (dashed curve); $x_{j+1} = 5D$, $\mu = 0.05$ (dotted curve).

For this type of medium, the reciprocity theorem allows the power emitted by a cell K and incident on a boundary surface element of a cell J to be deduced from the power emitted by this boundary surface element, *assumed to be a black body*, and absorbed by the cell K[2]. Indeed, the model of the companion paper[1] easily allows this absorbed power to be determined, by accounting for correlation between transmission and absorption. Two other justifications of this approach are that:

- There is no correlation between these phenomena and emission by a black body;

- A flux incident over a surface element, of any radiative property, is equal to the flux absorbed by this surface, assumed to be a black body.

For the sake of simplicity, the model is here summarized under the following assumptions:

- The medium is enclosed between two parallel infinite planes;

- The temperature field is one-dimensional along x, axis normal to the planes;

- A current cell J is enclosed between the planes x_j and x_{j+1} and characterized by the total area S_J and normal unit vectors $\mathbf{n}_j$ oriented towards J;

- The reflection law of the opaque interfaces within any cell J is characterized by the gray absorptivity α_J.

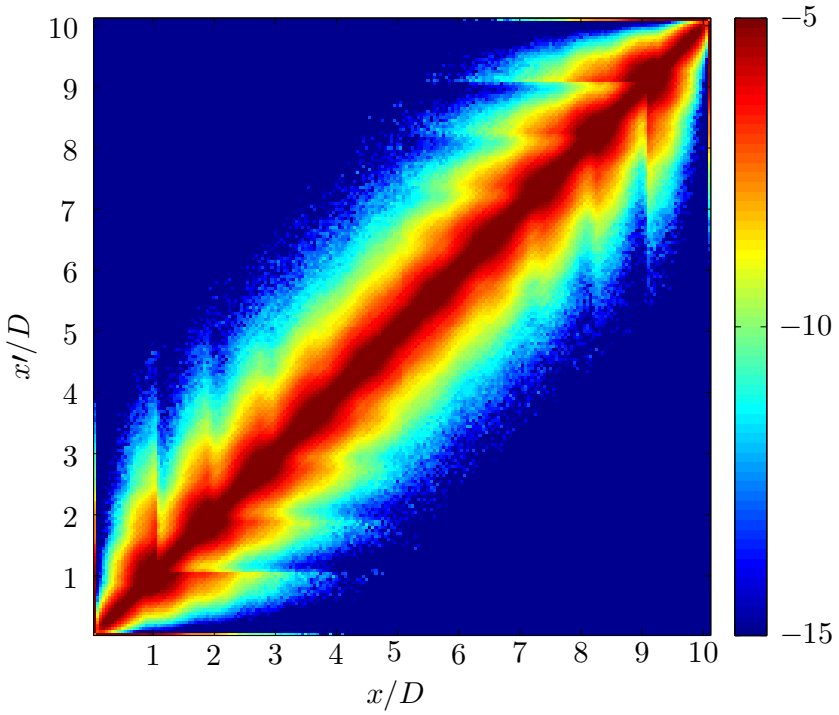

Figure 4: Typical example of matrix $\mathcal{P}*^{e\,a}_{I\,J}$; $N = 5\,10^5$ realizations per cell; number of cells: $N_C = 200$; Size of the slab: $L = 10D$; $\omega = 0$ and $\varepsilon_w = 1$. [2].

By using the reciprocity theorem applied to emission and absorption and the following reciprocity equation applied to the transmissivities established in Ref.[2]

$$\Pi_i \left[1 - G_{ext}(M_i, \mathbf{u}_{ij}, M_j)\right] = \Pi_j \left[1 - G_{ext}(M_j, \mathbf{u}_{ji}, M_i)\right]. \tag{61}$$

the flux emitted by the cell J and incident, in the elementary solid angle $\Omega_{ji}(M_j)$ around the direction $\mathbf{u}_{ji}$, on a point M_i of the plane x_i, boundary of a cell I, has been expressed as[2]

$$\mathrm{d}\Phi^{e,inc}_{J\,M_i} = \alpha_J I^\circ(T_J) \oint_{S_J} \Pi_j [1 - G_{ext}(M_j, \mathbf{u}_{ji}, M_i)](\mathbf{n}_j.\mathbf{u}_{ji})\mathrm{d}\Omega_{ji}(M_j)\mathrm{d}S_j. \tag{62}$$

A simple balance of all fluxes, given by Eq.62 applied to the two faces x_i and x_{i+1} of the cell I, with normal unit vectors oriented towards the cell, then leads to the flux emitted by J and absorbed by I. An example of an associated $\mathcal{P}^{e\,a}_{I\,J}$ matrix of the normalized powers emitted by I and absorbed by J is shown in Fig.4. It is obviously symmetrical.

This approach can be generalized to other types of geometrical configurations and reflection laws.

4 Coupling with other heat transfer modes

A few number of works are devoted to coupling, at the scale of a phase, of radiation with other heat transfer modes. This Section is limited to the principles of coupling and to the compatibility of coupling, of the previous radiation statistical approach with other modes. It is mainly based on a theoretical paper of Leroy et al.[12], in which the popular Volume Averaging Method (VAM)[13, 14, 15, 16, 17] is used for conduction and convection. In fact this topic today mainly remains an open research field.

4.1 Scales associated with the transfer modes

4.1.1 VAM scales

The considered homogenization of the radiation field is statistical. On the contrary the Volume Averaging Method is quasi deterministic. Nevertheless, these approaches can be coupled if their homogenization scales are compatible. Only some elements, of interest for the coupling, are briefly summarized here:

- The homogenization is carried out in a Representative Volume Element (REV), of characteristic size l_m, in which averaged values of the fields are defined, for instance $< T >$.

- The average fields, for instance the average temperature $< T > (\mathbf{X})$, are continuously defined, at macro scale characterized by the coordinate vector $\mathbf{X}$. Averaged values of the thermophysical properties are associated with these fields.

- Fluctuations of the fields, at a smaller scale, for instance T' are accounted for in complementary microscopic models, which lead to complementary effective properties at macro scale associated with thermal conduction and convection.

Finally, the model leads to macroscopic coupled equations expressed in average fields, as $< T > (\mathbf{X})$, and depending on the averaged thermophysical properties and effective ones issued from the microscopic models.

4.1.2 Radiative Fourier scales

Note that the Volume Averaging Method is similar to the perturbation method in use for deriving the radiative Fourier's law: The radiative flux is finally expressed vs $< T > (\mathbf{X})$ by using an effective conductivity tensor depending on the medium properties at micro scale. The characteristic length l_F, at which the radiative Forier's law is valid can be deduced from the Gomart's criterion[5] introduced in Sec.2.1: It depends both on effective radiative properties (ω and κ^{eff}) and on the gradient of the macroscopic temperature field $\mathbf{grad}[< T > (\mathbf{X})]$.

In both cases, the models are not valid in the vicinity of the porous medium boundaries, if the distances from the considered point to these boundaries are smaller than l_m (VAM) or do not correspond to an optically thick medium (radiation).

4.1.3 GRTE scales

As the radiative statistical method is based on complete distribution functions, the scale associated with radiative properties associated with absorption, scattering, extinction and emission, even in the case of emission by a homogenized interfacial phase, is much smaller than the typical size of the Representative Volume, in use for the determination of G_{ext}, P_{abs}, P_{sc}. This scale l_{rp} has in fact no theoretical limit. Limitations result from the resolution of the tomography or of the numerical scheme in use for determining the radiative distribution functions.

In fact the principal limitation is linked to the smallest scale l_T at which the temperature field can be defined, at the probabilistic sense of the radiative homogenization.

- In some applications, the temperature field is one-dimensional and the system presents two infinite directions, or it is two-dimensional with one infinite direction. In these two cases, if it is assumed that there is no temperature variation in the infinite direction(s), a slice as thin as l_{rp} and infinite, *i.e.* a slice of transverse size sufficiently large for being representative, is sufficient for the temperature definition.

- Another interesting case is a periodic system, for instance a regular rod bundle, which could be modeled as a strongly non homogeneous porous medium as at the end of Sec.3.3 of the

companion paper[1].In this case also, the temperature can be defined in arbitrary thin slices, when porosity is not equal to 1 (alleys).

4.2 Methods of coupling

Thermal conduction and convection generally occur at local scale (continuous physics based on diffusion). On the contrary, radiation is in the general case, *i.e.* by using RTEs or GRTEs, characterized by long range interactions (ballistic regime), even in continuous semi transparent media.
Under some precise conditions defined in Sec.2, the radiative flux is expressed by a radiative Fourier's law, exactly as the thermal conduction, after homogenization from molecular scale by the Chapman Enskog theory[8]. The radiative conductivity associated with a real semi transparent phase and the thermal conductivity can only be added if the homogenization scales are similar (model in use in the core of a molten glass bath). But it is generally not the case for a porous medium. Moreover, a radiative conductivity associated with a virtual homogenized phase i of opaque interfaces only allows a radiative flux between opaque interfaces or between opaque interfaces and a semi transparent phase to be determined. Its nature completely differs from the radiative conductivity associated with a real semi transparent phase.

A consequence is that radiation can generally be coupled to other heat transfer modes only by an *iterative scheme*. It is a general result for any application of radiation: See, for instance, couplings of radiation and turbulent convection or turbulent combustion [18, 19, 20, 21], etc. In the cited cases, radiation modeled by a Monte Carlo method is easily coupled to turbulent convection modeled by LES or DNS.

At each iteration step, input data for the radiation transfer model are the temperature fields and radiative thermophysical properties, often depending on temperature.

Output data are:
- Radiative power per unit volume $P^R_{\gamma\,J}$ in any cell J for any semi transparent phase γ, source term of the energy balance equation of the phase in the thermal model;
- Radiative interfacial flux $\varphi^R_{i\,I}$ in any cell I, if the medium presents opaque interfaces i, characterized by a temperature field $T_i(\mathbf{X})$ which a priori differs from the temperature fields of the phases. In many cases, the temperature field $T_i(\mathbf{X})$ is assumed to be equal to the field $T_s(\mathbf{X})$ of a solid phase: An associated radiative power per unit volume $P^R_{\gamma\,J}$ is then added in any cell J for the opaque solid phase.
- Radiative flux per unit area $\varphi^R_{B\,K}$ at any cell K of the boundaries of the whole porous medium.

Leroy et al.[12] have developed a complete model of coupling between the Volume Averaging Model of Whitaker and Quintard[13, 14, 15, 16, 17, 22], with a radiative Fourier's law or GRTEs, in the OT, OST, STT and ST2 cases. This Section introduces the different types of radiative source terms and boundary radiative fluxes, output data of the radiation part of an iterative scheme of coupled calculation.

4.2.1 Coupling by using the Fourier's law

As discussed in Sec.2.1, the radiative Fourier's law is only valid in the core of the porous medium under precise conditions. A radiative power per unit volume $P^R_\gamma(X_m)$ can be determined, for any phase γ, at any point M of macroscopic coordinates X_m. It writes, in tensorial notations,

$$P^R_\gamma(X_m) = -\frac{\partial}{\partial X_j} q^R_{\gamma\,j}(X_m) \tag{63}$$

where $q^R_{\gamma\, j}\,(X_m)$ is the radiative flux

$$q^R_{\gamma\, j}\,(X_m) \;=\; -\,k^R_{\gamma'\,\gamma\, j\, k}\,\frac{\partial T_{\gamma'}}{\partial X_k}(X_m). \tag{64}$$

where $k^R_{\gamma'\,\gamma\, j\, k}$ is a tensor, generally diagonal, associated with coupled radiative transfer between the phases γ and γ'. $P^R_\gamma\,(X_m)$ is a source term of the energy balance equation associated with the phase γ.

4.2.2 Coupling by using RTE(s) or GRTE(s)

Whatever the type(s) of homogenized phase(s) γ within the porous medium, a radiative power per unit volume $P^R_\gamma(X_m)$, is determined, for any phase γ, at any point X_m within its volume

$$P^R_\gamma(X_m) \;=\; P^a_\gamma(X_m) \;-\; P^e_\gamma(X_m), \tag{65}$$

where $P^a_\gamma(X_m)$ and $P^e_\gamma(X_m)$ are the powers per unit volume absorbed and emitted by the phase γ at the point X_m, respectively. In all cases, the total radiative power $\mathcal{P}^R_\gamma(X_m)$ is, in fact, determined by using the reciprocity theorem

$$\begin{aligned}\mathcal{P}^R_\gamma(X_m) \;&=\; \int_0^\infty \left(\sum_J \sum_{\gamma'} [\mathcal{P}_{\nu\, \overset{e}{J}\gamma'\, \overset{a}{M}\,\gamma} \;-\; \mathcal{P}_{\nu\, \overset{e}{M}\gamma\, \overset{a}{J}\,\gamma'}] \;+\; \sum_L [\mathcal{P}_{\nu\, \overset{e}{L}\gamma'\, \overset{a}{M}\,\gamma} \;-\; \mathcal{P}_{\nu\, \overset{e}{M}\gamma\, \overset{a}{L}\,\gamma'}]\right) \mathrm{d}\nu \\ &=\; \int_0^\infty \left(\sum_J \sum_{\gamma'} \mathcal{P}_{\nu\, \overset{e}{J}\gamma'\, \overset{a}{M}\,\gamma}\,[1 \;-\; \frac{I^\circ_\nu(T_m)}{I^\circ_\nu(T_j)}] \;+\; \sum_L [\mathcal{P}_{\nu\, \overset{e}{L}\gamma'\, \overset{a}{M}\,\gamma}\,[1 \;-\; \frac{I^\circ_\nu(T_m)}{I^\circ_\nu(T_l)}]\right) \mathrm{d}\nu\end{aligned} \tag{66}$$

where M is the cell of center of coordinates X_m, J and L are current cells belonging to the volume of the porous medium and to its boundaries, respectively. Note that $\mathcal{P}^R_\gamma(X_m)$ is independent of the ratio of the fractions per unit volume of the phases, in the third member of Eq.66.

4.2.3 Case of a homogenized phase associated to opaque interfaces (Fourier and GRTE)

In the case of a virtual homogenized phase i associated with opaque interface elements, the radiative power $\mathcal{P}^R_i(X_m)$ is determined within the volume of the propagation phase. In fact, it physically corresponds to a flux imposed to the opaque interfaces:

- In the general case, this flux is applied to the opaque interfaces when their temperature field $T_i(X_m)$ differs from the temperature fields of the two real phases $T_\gamma(X_m)$ and $T_{\gamma'}(X_m)$.

$$\varphi^R_i(X_m) \;=\; \frac{\mathcal{P}^R_i(X_m)}{A\,V_m} \tag{67}$$

where A is the specific area of the porous medium and V_m the volume of the cell M.

- Commonly, the interfaces are practically at the temperature of the solid phase S (Biot number much smaller than 1). The associated power per unit volume $P^R_S(X_m)$ is then imposed to this solid phase and not to the propagation fluid phase f

$$P^R_S(X_m) \;=\; \frac{\mathcal{P}^R_f(X_m)}{(1 \;-\; \Pi)V_m} \tag{68}$$

where Π is the medium porosity and $\mathcal{P}^R_f(X_m)$ is given by Eq.66.

A Appendix: Perturbation method applied to the ST2 case

We introduce the following functionals

$$\mathcal{F}_{\nu\,11}\Big[Y(\mathbf{u}_1)\Big](\mathbf{u}) = \left(\frac{\sigma_{\nu\,1}}{4\pi}\int_{4\pi} p_{\nu\,1}(\mu_{sc})\,Y(\mathbf{u}_1)\,\mathrm{d}\Omega_1 + \frac{\Sigma_{\nu\,11}(\mathbf{u})}{4\pi}\int_{4\pi} p_{\nu\,11}(\mathbf{u}_1,\mathbf{u})\,Y(\mathbf{u}_1)\,\mathrm{d}\Omega_1\right) [\beta_{\nu\,1} + \Sigma_{\nu\,11}(\mathbf{u}) + \Sigma_{\nu\,12}(\mathbf{u})]^{-1} \tag{69}$$

$$\mathcal{F}_{\nu\,21}\Big[Y(\mathbf{u}_2)\Big](\mathbf{u}) = \left((\frac{\Pi_2}{\Pi_1})\,\frac{\Sigma_{\nu\,21}(\mathbf{u})}{4\pi}\int_{4\pi} p_{\nu\,21}(\mathbf{u}_2,\mathbf{u})\,Y(\mathbf{u}_2)\,\mathrm{d}\Omega_2\right) [\beta_{\nu\,2} + \Sigma_{\nu\,21}(\mathbf{u}) + \Sigma_{\nu\,22}(\mathbf{u})]^{-1}, \tag{70}$$

and by obvious analogy $\mathcal{F}_{\nu\,22}\Big[Y(\mathbf{u}_2)\Big](\mathbf{u})$ and $\mathcal{F}_{\nu\,12}\Big[Y(\mathbf{u}_1)\Big](\mathbf{u})$.

As in the OST case, the solutions $I^{(0)}_{\nu\,1}$ and $I^{(0)}_{\nu\,2}$ are incremented by adding the last contributions of the previous scattering source terms. It leads to an algorithm based on a recurrence. If k is the order of recurrence, $I^{(0)\,k}_{\nu\,1}$ and $I^{(0)\,k}_{\nu\,2}$ are given by

$$\begin{aligned}
I^{(0),\,0}_{\nu\,1} &= \delta I^{(0),\,0}_{\nu\,1} = n^2_{\nu\,1}\,\kappa_{\nu\,1}\,I^{\circ}_{\nu}[T_1(\mathbf{X})],\\
I^{(0),\,0}_{\nu\,2} &= \delta I^{(0),\,0}_{\nu\,2} = n^2_{\nu\,2}\,\kappa_{\nu\,2}\,I^{\circ}_{\nu}[T_2(\mathbf{X})]\\
I^{(0),\,k}_{\nu\,1} &= I^{(0),\,k-1}_{\nu\,1} + \delta I^{(0),\,k}_{\nu\,11} + \delta I^{(0),\,k}_{\nu\,21},\\
I^{(0),\,k}_{\nu\,2} &= I^{(0),\,k-1}_{\nu\,2} + \delta I^{(0),\,k}_{\nu\,12} + \delta I^{(0),\,k}_{\nu\,22}
\end{aligned} \tag{71}$$

where $\delta I^{(0),\,k}_{\nu\,11}$ and $\delta I^{(0),\,k}_{\nu\,21}$ represent the contributions of the media 1 and 2 respectively to $I^{(0),\,k}_{\nu\,1} - I^{(0),\,k-1}_{\nu\,1}$, etc.

$$\begin{aligned}
\delta I^{(0),\,k}_{\nu\,11} &= \mathcal{F}_{\nu\,11}[\delta I^{(0),\,k-1}_{\nu\,11}]\\
\delta I^{(0),\,k}_{\nu\,21} &= \mathcal{F}_{\nu\,21}[\delta I^{(0),\,k-1}_{\nu\,21}]\\
\delta I^{(0),\,k}_{\nu\,22} &= \mathcal{F}_{\nu\,22}[\delta I^{(0),\,k-1}_{\nu\,22}]\\
\delta I^{(0),\,k}_{\nu\,12} &= \mathcal{F}_{\nu\,12}[\delta I^{(0),\,k-1}_{\nu\,12}]
\end{aligned} \tag{72}$$

up to convergence.

The same method is applied for $I^{(1)}_{\nu\,1}$ and $I^{(1)}_{\nu\,1}$. The only difference is that in Eqs.71 $n^2_{\nu\,1}\,\kappa_{\nu\,1}\,I^{\circ}_{\nu}[T_1(\mathbf{X})]$ becomes $-\frac{\mathrm{d}I^{(0)}_{\nu\,1}}{\mathrm{d}s}(\mathbf{u}_1,\mathbf{X})$, etc.

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