Wavelet-OFDM vs. OFDM: Performance Comparison

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***Abstract*—Wavelet-OFDM based on the discrete wavelet transform, has received a considerable attention in the scientific community, because of certain promising characteristics. In this paper, we compare the performance of Wavelet-OFDM based on Meyer wavelet, and OFDM in terms of peak-to-average power ratio (PAPR), bit error rate (BER) for different channels and different equalizers, complexity of implementation, and power spectral density. The simulation results show that, without decreasing the bandwidth efficiency, the proposed scheme based on Meyer Wavelet-OFDM, outperforms OFDM in terms of PAPR by up to 4.5 dB, and in terms of BER by up to 6.5 dB of signal to noise ratio when using the minimum mean-square error equalizer without channel coding, at the cost of a computational complexity increase.**

Keywords*—Wavelet-OFDM, Discrete Wavelet Transform (DWT), Orthogonal Frequency Division Multiplexing (OFDM), Meyer Wavelet, Haar Wavelet.

I. Introduction

Several wireline and wireless communication standards use orthogonal frequency division multiplexing (OFDM) system as a modulation technique for data transmission. OFDM has proved its worth due to its robustness against frequency selective channels compared with single carrier modulation, and also to its high spectral efficiency due to its use of orthogonal waveforms. However, OFDM suffers from some drawbacks such as high peak-to-average power ratio (PAPR), and synchronisation problems. To counter these disadvantages and to improve its performance in other respects, new multi-carrier modulation (MCM) systems have been designed, including Wavelet-OFDM [1], [2]. Instead of using the fast Fourier transform (FFT) as in OFDM, Wavelet-OFDM is based on the discrete wavelet transform (DWT).

In this paper, the performance of Wavelet-OFDM is compared with OFDM, in terms of PAPR, bit error rate (BER) on an additive white gaussian noise (AWGN) channel, a flat fading channel and a frequency selective channel. In addition, the complexity of implementation and the power spectral density (PSD) are also analysed and compared. We show that the Wavelet-OFDM system performance depends on the number of scales considered in the wavelet transform. We also apply the frequency domain equalization to the Wavelet-OFDM system to improve its BER performance. Moreover, based on the study of the PSD, we show that the selection of the wavelet has a great impact on the bandwidth efficiency of the system. As a conclusion of our study, we propose the Wavelet-OFDM based on the Discrete Meyer (Dmey) wavelet as an alternative to OFDM, since it outperforms OFDM in terms of PAPR and BER at the cost of increased complexity.

The remainder of this paper is organized as follows. In Section II, the Wavelet-OFDM system is described and its different variants with the corresponding implementation are presented. The PAPR for different variants, the BER under different channel conditions, the implementation complexity as well as the PSD are investigated in Section III, IV, V, and VI respectively. Finally, Section VII concludes the paper and gives the perspectives of the work.

II. Introduction to the Wavelet-OFDM system

A. Wavelet Basis

Let ψ and ϕ be two functions $\in L^2(\mathbb{R})$ of a finite support [1] $[0, T_0]$. such that:

$$\begin{aligned} \|\psi\|^2 = \|\phi\|^2 &= 1, && (1)\\ \text{and} \quad \int_{-\infty}^{+\infty} \psi(t) &= 0. && (2) \end{aligned}$$

$L^2(\mathbb{R})$ is the space of square integrable functions. The norm $\|.\|$ of a function $g \in L^2(\mathbb{R})$ is defined as: $\|g\|^2 = \int_{-\infty}^{+\infty} |g(t)|^2 \, dt$. ψ and ϕ can be named the mother wavelet function and the mother scaling function respectively.
Let J_0 be the first scale selected; we define contracted versions ψ_j and ϕ_j of the functions ψ and ϕ, for $j \geq J_0$,

$$\begin{aligned} \psi_j(t) &= 2^{j/2}\psi(2^j t) && (3)\\ \phi_j(t) &= 2^{j/2}\phi(2^j t). && (4) \end{aligned}$$

The contracted functions ψ_j and ϕ_j have a support of $[0, \frac{T_0}{2^j}]$. For every scale j, we define translated versions $\psi_{j,k}$ and $\phi_{j,k}$ of the functions ψ_j and ϕ_j as follows:

$$\begin{aligned} \psi_{j,k}(t) &= \psi_j(t - 2^{-j}kT_0) \\ &= 2^{j/2}\psi(2^j t - kT_0), && (5)\\ \phi_{j,k}(t) &= \phi_j(t - 2^{-j}kT_0) \\ &= 2^{j/2}\phi(2^j t - kT_0). && (6) \end{aligned}$$

The contracted translated functions $\psi_{j,k}$ and $\phi_{j,k}$ have a support of $[\frac{kT_0}{2^j}, \frac{(k+1)T_0}{2^j}]$. For $j \in [\![J_0, J-1]\!]$ and $k \in [\![0, 2^j - 1]\!]$, we define the wavelet basis as:

$$\{\phi_{J_0,k}\}_{k=0}^{k=2^{J_0}-1} \quad \bigcup_{j=J_0}^{J-1} \quad \{\psi_{j,k}\}_{k=0}^{k=2^j-1}, \qquad (7)$$

[1]The support of a function means here the interval outside which the function is equal to zero

where J is the last scale considered.

B. Expression of the transmitted Wavelet-OFDM signal

Wavelets have been used in several wireless communication applications such as data compression, source and channel coding, signal denoising and channel modeling. Moreover, wavelets have been proposed as a modulation basis for multicarrier modulation systems. The resulting system is named Wavelet-OFDM [1], [2] or also known as orthogonal wavelet division multiplexing (OWDM) [3]. In fact, the functions of the wavelet basis defined in (7), can be considered as multi-carrier waveforms for data transmission [4], in the same way as the Fourier basis is considered in a conventional OFDM system. Wavelet-OFDM is the MCM system based on the wavelet basis. Instead of carrying the input symbols upon the exponential functions of Fourier system, the carriers are represented by the wavelet functions $(\psi_{j,k})_{j\in[\![J_0,J-1]\!],k\in[\![0,2^j-1]\!]}$ and the scaling functions $(\phi_{J_0,k})_{k\in[\![0,2^{J_0}-1]\!]}$ of the first scale. The transmitted Wavelet-OFDM signal can thus be expressed as follows [2]:

$$\begin{aligned} x(t) &= \sum_{n}\sum_{j=J_0}^{J-1}\sum_{k=0}^{2^j-1} w_{j,k}\psi_{j,k}(t-nT_0) \\ &\quad +\sum_{n}\sum_{k=0}^{2^{J_0}-1} a_{J_0,k}\phi_{J_0,k}(t-nT_0). \end{aligned} \tag{8}$$

- $w_{j,k}$: wavelet coefficients located at k-th position from scale j,
- $a_{J_0,k}$: approximation coefficients located at k-th position from the first scale J_0,
- $\psi_{j,k}$ and $\phi_{J_0,k}$ are the wavelet and the scaling functions defined in (5) and (6) respectively.

The number of carriers M is equal to 2^J. The mother wavelet and the mother scaling function correspond to $\psi_{0,0}$ and $\phi_{0,0}$ respectively. For each scale j there are 2^j corresponding translated wavelet functions. From one scale to the next, the number of wavelet functions is then multiplied by two. For the first scale J_0, there are 2^{J_0} scaling functions.

Several variants of Wavelet-OFDM can be constructed, depending on the first scale J_0 considered. Fig. 1 depicts the different variants of Wavelet-OFDM defined for different values of J_0, for a total number of scales $J=3$. The position of the functions in Fig. 1 refers to the time localization and the frequency localization characteristics of the functions. We can observe that these properties change from one variant to the other, and the selection of the variant depends on the application. In general, the wavelets of the same scale are translated in time and occupy the same bandwidth. They have the same time and frequency localization properties. From one scale to the next, the time localization is divided by two (contracted wavelets) and the frequency localization is multiplied by a factor of two.

C. Implementation

In order to implement the Wavelet-OFDM system expressed in (8), we apply the Mallat algorithm [6]. For a Wavelet-OFDM signal based on the wavelets of $L=J-J_0$ scales and the scaling functions of the scale J_0, the inverse

[2]In [5], we include some material from this Section to introduce the Wavelet-OFDM system in a general way, it appears also here for completeness.

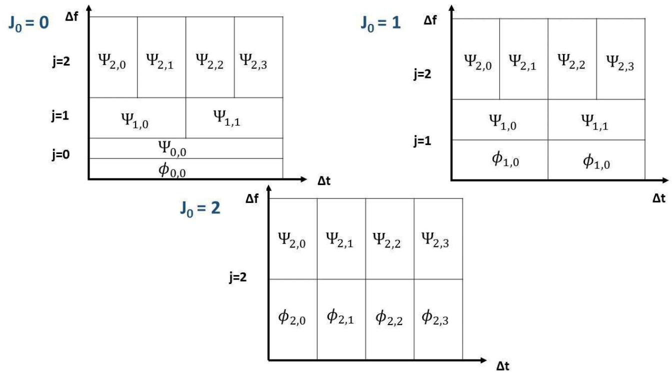

Figure 1: Variants of Wavelet-OFDM.

discrete wavelet transform (IDWT) should be performed L times. L can also be interpreted as the number of decomposition levels. Let $\boldsymbol{C_n}$ be a vector of M input complex symbols $C_{m,n}$. The 2^{J_0} first $C_{m,n}$ symbols correspond to the 2^{J_0} scaling coefficients $(a_{J_0,k})_{k\in[\![0,2^{J_0}-1]\!]}$. The second 2^{J_0} complex symbols correspond to the wavelet coefficients $(w_{J_0,k})_{k\in[\![0,2^{J_0}-1]\!]}$ of the first scale J_0. First, one IDWT is performed, which gives in its output 2^{J_0+1} scaling coefficients. After that, the next 2^{J_0+1} coefficients from the vector $\boldsymbol{C_n}$ are extracted and considered as wavelet coefficients, and the second IDWT is performed. The next symbols are processed in the same way until the last scale $j=J-1$ is reached. For example, let $\boldsymbol{C_n}$ be a vector of $M=8$ input symbols, and let us consider two cases: the first case:

- $J_0=0$, which means that we select the maximum number of decomposition levels $L=3$.
  The vector $\boldsymbol{C_n}$ of input symbols for the n^{th} period T_0 is expressed as:
  $$\begin{aligned} \boldsymbol{C_n} &= \{C_1,C_2,C_3,C_4,C_5,C_6,C_7,C_8\} \\ &= \{a_{0,0},w_{0,0},w_{1,0},w_{1,1},w_{2,0},w_{2,1},w_{2,2},w_{2,3}.\} \end{aligned}$$
- $J_0=1$, which means that we consider $L=2$ decomposition levels.
  The vector $\boldsymbol{C_n}$ of input symbols for the n^{th} period T_0 is expressed as:
  $$\begin{aligned} \boldsymbol{C_n} &= \{C_1,C_2,C_3,C_4,C_5,C_6,C_7,C_8\} \\ &= \{a_{1,0},a_{1,1},w_{1,0},w_{1,1},w_{2,0},w_{2,1},w_{2,2},w_{2,3}.\} \end{aligned}$$

As depicted in Fig. 2, the IDWT consists of upsampling by a factor of two and filtering the approximation coefficients (scaling coefficients) and the detail coefficients (wavelet coefficients) respectively by a low-pass and a high-pass filter, whose responses are derived from the wavelet considered.

D. Dmey Wavelet

The Meyer wavelet is a frequency band limited orthogonal wavelet, which has been proposed by Yves Meyer in 1985 [7], [8]. Meyer wavelets are indefinitely differentiable orthonormal wavelets, which are well localized and decay from their central peak faster than any inverse polynomial. Dmey is a discrete format approximation of the Meyer wavelet, and it

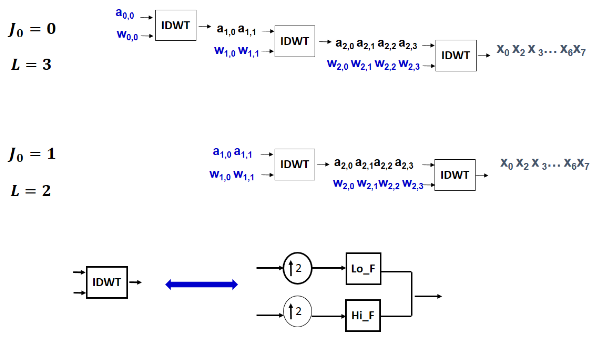

Figure 2: Wavelet-OFDM implementation for $J_0 = 0$ and $J_0 = 1$.

can approximate the Meyer wavelet based on a finite impulse response (FIR) filter as depicted in Fig. 3[3].

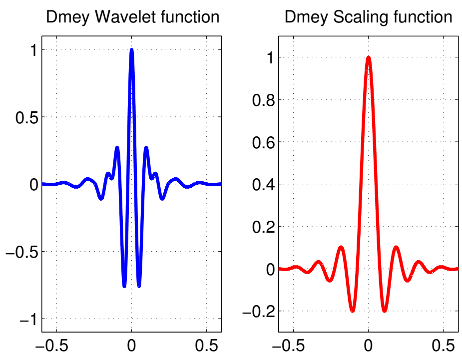

Figure 3: Dmey wavelet and scaling functions.

III. PAPR Performance

In this section, we compare the PAPR performance of different variants of Dmey Wavelet-OFDM with that of OFDM. To evaluate the PAPR performance, the complementary cumulative distribution function (CCDF) of the PAPR is simulated, which is the probability that the PAPR exceeds a defined value γ. The simulations are performed for the 4-QAM (quadrature amplitude modulation) and the 16-QAM constellations, number of carriers $M = 128$ and for 10^6 iterations. The oversampling factor is fixed to 4. The first conclusion drawn from Fig. 4 is that the PAPR performance of the Dmey depends on the number of scales L of the wavelet transform. When L is high, the CCDF is shifted to the right and the PAPR performance is therefore degraded. This can be interpreted by the consideration that in every decomposition level, the multi-carrier system can be seen as a single carrier system, because the waveforms of the same scale j have the same bandwidth and are only shifted in time. As the single carrier system does not suffer from increased PAPR, the smaller L gets, the less the effect of the PAPR is observed. Moreover, the PAPR for the different variants of the Dmey Wavelet-OFDM

[3]Generated using the MATLAB wavefun('dmey') command

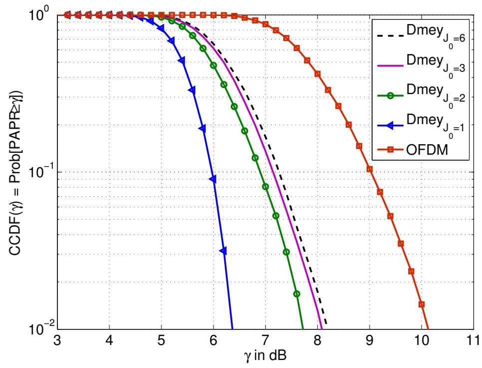

Figure 4: PAPR performance for different variants of Dmey wavelet.

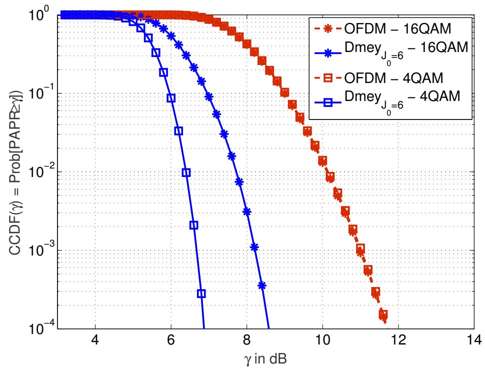

Figure 5: PAPR performance comparison for the 4-QAM and the 16-QAM constellations.

outperforms that of OFDM. The variant $J_0 = J - 1$ achieves the best PAPR performance. The PAPR performance of this variant is compared with OFDM for different constellations in Fig. 5. As depicted in this figure, given that CCDF $= 10^{-3}$, for 4-QAM (16-QAM respectively), the PAPR of the Dmey is less by 4.5 dB (3 dB respectively) than OFDM. The Dmey outperforms OFDM due to its well localized waveforms in the time domain, since Wavelet-OFDM has short time wavelets, especially for the small scales (j is high). We can also notice that, unlike OFDM, the PAPR performance of Dmey depends on the constellation used.

IV. BER Performance

The BER performance of the Wavelet-OFDM and the OFDM systems is evaluated using the parameters illustrated in Table I. The 4-QAM and the 16-QAM constellations are used in the simulations of this section.

A. AWGN and Flat Fading Channels

In this part, a comparison between the BER of OFDM and the Dmey Wavelet for AWGN and flat fading channels, is presented in Fig. 6. Since these are all orthonormal waveform sets, it is only to be expected that the performance of all of

Table II: Channel delay and power profile.

Discrete delay (ns)	0	50	120	200	230	500	1600	2300	5000
Average path gains (dB)	-1.0	–1.0	-1.0	0.0	0.0	0.0	-3.0	-5.0	-7.0

Table I: Simulation parameters.

Parameters	Definition	Values
M	Number of carriers	128
S	Number of frames	100
SNR	Signal to Noise Ratio in dB	$0:5:25$ and $0:5:35$
n_{loop}	Number of iteration loops	100000
ΔF	Intercarrier spacing	15 KHz
J_0	First scale considered	$J-1=6$

them will be the same. The simulations confirm that the Dmey is as good as OFDM in terms of BER performance.

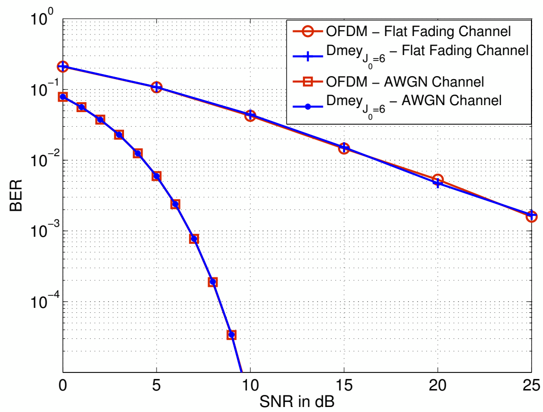

Figure 6: BER performance comparison in AWGN and flat fading channels.

B. Frequency Selective Channel

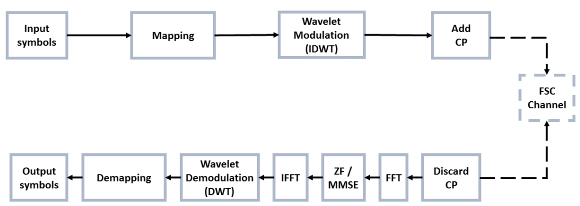

Figure 7: Wavelet transmission chain in a frequency selective channel.

In a frequency selective fading channel, a cyclic prefix is added to the Wavelet-OFDM transmitted signal in the time domain, and a frequency domain equalization is performed in the receiver side using either the zero forcing (ZF) equalizer or the minimum mean-square error (MMSE) equalizer as presented in Fig. 7.

The extended typical urban (ETU) model for LTE multipath channel standard [9], defined by the channel delay and power profile in Table II, is used in this section. As is shown in Fig. 8, OFDM outperforms the Dmey in terms of BER under frequency selective channel conditions when using ZF equalizer. However, when using MMSE equalizer as depicted in Fig. 9, the Dmey reaches a gain of 6.5 dB in terms of SNR for a BER of 10^{-3} and the 4-QAM constellation, compared with OFDM. For higher constellations (16-QAM) and for low SNR values, the performance of OFDM is comparable to that of the Dmey. Starting from SNR $=25$ dB, the Dmey outperforms OFDM. Notice that, unlike in Wavelet-OFDM, the MMSE receiver does not change the unbiased SNR in the case of OFDM, and hence does not change the BER of the OFDM system.

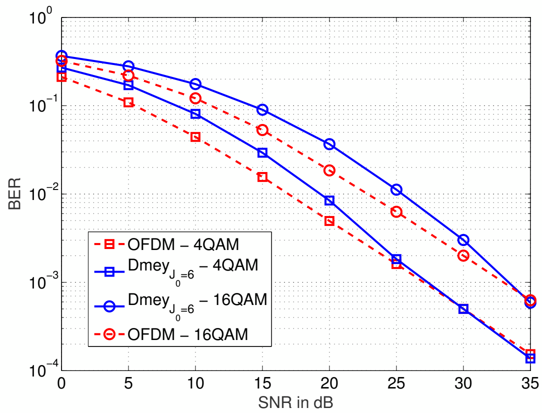

Figure 8: BER performance comparison for ZF equalizer in a frequency selective channel.

It is important to highlight that when using channel coding techniques, the gain in terms of SNR will be less significant and will depend also on the efficiency of the coding technique implemented. We have chosen not to use coding to evaluate only the effect of the modulation scheme on BER performance.

V. Complexity of Implementation

Since Wavelet-OFDM needs two more blocks (IDWT,DWT) compared with OFDM as presented in Fig. 7, its complexity is therefore higher than OFDM. Let us compute the complexity of the wavelet modulation (IDWT) and the wavelet demodulation (DWT) blocks. According to the Mallat Algorithm [6], IDWT consists of up-sampling by a factor of two and filtering the approximation coefficients $a_{j,k}$ (scaling coefficients) and the detail coefficients $w_{j,k}$ (wavelet coefficients) respectively by a low-pass filter g and

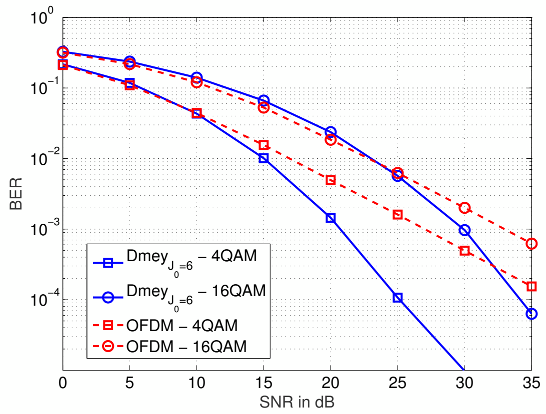

Figure 9: BER performance comparison for MMSE equalizer in a frequency selective channel.

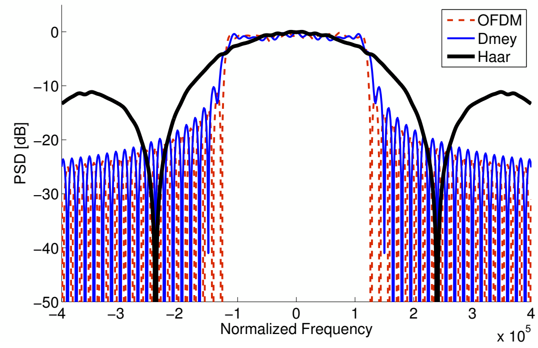

Figure 10: PSD comparison.

a high-pass filter h. Let K be the length of the filters h and g (K non-zero coefficients). The wavelet modulation is calculated with

$$\sum_{j=J_0+1}^{J} 2^j K \quad \leq \quad \sum_{j=1}^{J} 2^j K = 2MK. \tag{9}$$

The complexity order in terms of the number of additions and multiplications is therefore $\mathcal{O}(MK)$. Knowing that the complexity order of the FFT or the IFFT is $\mathcal{O}(Mlog_2(M))$, the complexity increase order is about $\mathcal{O}(\frac{K}{log_2(M)})$, which is affordable since K is bounded, and the number of carriers M is usually large.

VI. PSD Performance

The PSD is an interesting characteristic of the system, since it gives a measure of the bandwidth efficiency, the side lobe rejection and the adjacent channel interference. We believe that it may be the most important constraint in many applications. In order to compare OFDM and the Dmey wavelet, we simulate in Fig. 10 their corresponding PSD. The simulation is performed using Matlab, and the PSD is estimated via the periodogram method with a rectangular window. Before applying the wavelet modulation based on IDWT, zero padding by a factor of 8 is performed on the input signal in the frequency domain. Based on the observation of Fig. 10, the width of the main lobe of the PSD of the Dmey is comparable to that of OFDM. In fact, among the commonly used wavelets, Dmey is the most frequency compacted wavelet after Shannon, while this latter is not compatible with the discrete transform. Dmey achieves therefore the best spectrum characteristics, unlike the Haar wavelet for example, which is characterized by a PSD of main lobe larger by around 40% than that of OFDM as presented in Fig. 10. The Dmey wavelet does not have this downside and represents then an alternative waveform which provides good spectral characteristics.

VII. Conclusion

The Wavelet-OFDM system based on the Dmey wavelet for different variants has been investigated in this paper and compared with OFDM. We have shown that the Dmey wavelet is at least as good as OFDM in terms of BER performance for AWGN channel and flat fading channel. In addition, the selected variant of Dmey outperforms OFDM in frequency selective channel by up to 6.5 dB, at BER of 10^{-3}, for MMSE equalizer. Furthermore, the Dmey reaches a gain up to 4.5 dB in terms of PAPR compared with OFDM. However, the Dmey Wavelet-OFDM requires more implementation complexity.

Our future work is to study a suitable channel coding for wavelets to compare its performance with Coded-OFDM under frequency selective channel.

Acknowledegments

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