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# Optimal fronthaul capacity allocation and training for joint uplink receiver in C-RAN

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**Abstract**—In Cloud Radio Access Networks (C-RAN) one of the main challenges is accurate data transfer on limited capacity fronthaul links between Remote Radio Heads (RRHs) and the central processing unit. Particularly in large-band and multi-antenna transmissions, fronthaul capacity can be a limiting factor. In this paper we study the impact of non-ideal fronthaul on uplink received signal, channel estimates and consequently on the achievable rates. We consider both the fronthaul capacity and the length of training sequences variable and find optimal values allowing high-rate transmission taking into account the costs of over-the-air transmission and fronthaul usage, thus, our approach can be used in dynamic system design and configuration given technical and economical factors.

## I. INTRODUCTION

With mobile data traffic growth, the deployment of new network architectures such as Cloud Radio Access Network (C-RAN) is planned. In C-RAN we have several single- or multi-antenna Remote Radio Heads (RRHs) located in a distributed manner, implementing only a part of Base Band (BB) processing going from only Radio Functions (RF) to Media Access Control Layer or more. The Remote Radio Heads are connected to the so-called Base Band Unit Pool (BBU-Pool) through limited capacity fronthaul links and the remaining part of the BB processing takes place there. In a fully centralized C-RAN setup, we transfer digital I/Q symbols between the RRHs and the BBU-Pool, which requires very high fronthaul data rate.

Various technologies are susceptible to be used for fronthaul connections, the most common remains optical fiber connection either in point-to-point configuration or through a switched network. Since high over-the-air bandwidth and multiple antennas at the RRH would increase fronthaul data rate, system design remains challenging.

We demonstrate the benefit of optimizing the fronthaul capacity usage by taking into account on one hand the impact of data quantization on channel estimation accuracy and on the other hand the cost of wireless spectrum and fronthaul infrastructures. The joint optimization of the length of training sequences and allocated fronthaul bandwidth allows a more efficient exploitation of available resources. The main limitation of C-RAN is related to fronthaul data-transfer, its accurate design enables good dimensioning of fronthaul links, consequently significant cost savings.

In this work, a single wireless channel is considered, the whole range of available bandwidth being contiguously filled by channels at different frequencies and all frequencies are reused between the different cells. For the sake of simplicity, we study a system with two single-antenna RRHs and two users, one in each cell, similarly to the model of the interference channel defined in [1]. In addition, receivers cooperate in the BBU-Pool to which each received digital signal is forwarded. Physical layer processing of each cell remains separated despite they take place in the same processing unit, but they benefit from centralization by exploiting channel estimates and received data of both cells in the Minimum Mean Square Error (MMSE) detection [2]. This system model guarantees scalability so that results can be extended to larger systems and to each channel in the available spectrum. The coordination of the multi-cell processing is realized in the BBU-Pool, thus it can be applied to any cell connected to the same pool.

Theoretical limits of multi-cell uplink cooperation under practical constraints has been studied in several research works. Optimal training length in uplink network MIMO systems was derived in [3] assuming large-scale cooperation and considering fixed fronthaul capacity constraint. Multi-cell cooperation with limited communication links between two Base Stations (BSs) without central processing unit was studied in [4] and the impact of channel estimation error was introduced. However, authors did not consider capacity optimization, furthermore the asymmetric configuration studied is less scalable than C-RAN which is inherently designed for multi-cell cooperation. Distributed compression scheme proposed in [5] aims reduced fronthaul usage while maintaining accurate transmission.

One of the proposed methods to increase spectral efficiency of future mobile networks is to use a large number of antennas at the BSs [6]. The promising results of multi-user reception using Massive MIMO stand under the assumption of uncorrelated channels, correlation can significantly affect system performance [7]. Spatially distributed antennas, as considered in this work can overcome this problem.

The C-RAN-based architecture proposed in [2] enables to realize joint processing for critical users with an affordable fronthaul infrastructure. The case of efficient fronthaul usage

in a C-RAN system with multi-antenna RRHs using Spatial Compression and Forward strategy was explored in [8]. Authors in [8] also point out that a system with distributed RRHs and constrained fronthaul overperforms single-RRH massive MIMO configuration.

The remainder of the paper is organized as follows: after presenting the system model in Section II, in Section III we compute the achievable rates taking into account channel estimation error and limited fronthaul capacity, in Section IV we formulate a cost-gain problem using the obtained expressions. Finally we present numerical evaluation of obtained results with various system parameters in Section V and we provide concluding remarks and future steps in Section VI.

## II. SYSTEM MODEL

The system that we study, introduced above, is depicted in Figure 1. We assume that each transmitter and each receiver has only one antenna. Users broadcast a signal  $x_i$ , with  $i \in \{1, 2\}$  (rallied in the variable  $\mathbf{x} = (x_1, x_2)^T$ ) with powers, respectively,  $p_1$  and  $p_2$ , which form  $\mathbf{P}_\mathbf{x}$ . Both receivers get the sum of the signals transported by the wireless channels represented by the channel coefficients  $h_{ij}$ , with  $i, j \in \{1, 2\}$  which form the channel matrix  $\mathbf{H} = \begin{pmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{pmatrix}$ .

The received signal by each RRH is

$$y_j = h_{1j}x_1 + h_{2j}x_2 + z_j \text{ with } j \in \{1, 2\} \quad (1)$$

which is included in the matrix formulation

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} \quad (2)$$

The added white Gaussian noise  $z_j$  has variance  $\sigma_z^2$ . The RRHs compress the received signal and forward it to the central processing unit through the fronthaul link with capacity  $c_j$ . The BBU-Pool receives  $\hat{\mathbf{y}} = (\hat{y}_1, \hat{y}_2)^T$ , then estimates the channel matrix and gets the estimate  $\hat{\mathbf{H}}$ . The actual channel matrix is  $\mathbf{H} = \hat{\mathbf{H}} + \tilde{\mathbf{H}}$  where  $\tilde{\mathbf{H}}$  is the channel estimation error matrix with independent entries of  $\mathcal{N}(0, \sigma_e^2)$ .

We use the following notational conventions: for random variables, upper case letters, e.g.,  $X$ , for scalars, bold and non-italic fonts, e.g.,  $\mathbf{V}$ , for vectors, and bold and sans serif fonts, e.g.,  $\mathbf{M}$ , for matrices. Deterministic quantities are denoted with italic letters, e.g., a scalar  $x$ , lowercase bold for a vector  $\mathbf{v}$ , and uppercase bold letters for a matrix  $\mathbf{M}$ . Logarithms are in base 2 and superscript  $(\cdot)^H$  denotes the conjugate transpose of a vector or a matrix.

## III. ACHIEVABLE TRANSMISSION RATES UNDER LIMITED FRONTHAUL CONSTRAINT

In the C-RAN model with RRHs implementing only RF, all digital signal processing is located in the BBU-Pool. On the uplink (UL), RRHs forward fixed-sized packets containing the I/Q symbols and this transmission introduces distortion on the received signal. Afterward, the channel estimation realized in the BBU-Pool using the received signal depends not only on the training length, but also on the compression. In this section, we compute the achievable transmission rates taking into account these constraints.

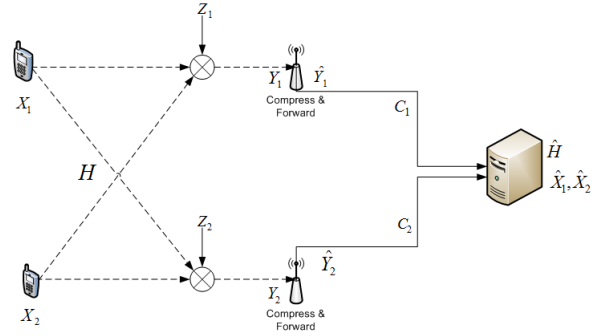


Fig. 1: Two users - two antennas interference channel with centralized processing

### A. Fronthaul quantization

The RRHs receive analog radio signals containing users' messages that they have to convert into digital signals in order to be able to transmit to the centralized processing unit. For practical reasons, even if we have high capacity fronthaul links, we need to transmit I/Q symbols in finite-size packets, consequently we have to deal with the loss of information due to quantization. In the following, we show how the statistics of the channel estimation error is related to the fronthaul capacity.

We define the distortion  $d_j$  between the signal  $y_j$  received by the RRH and the compressed signal  $\hat{y}_j$  received by the BBU-Pool as the squared-error distortion between  $y_j$  and  $\hat{y}_j$ .

$$d_j = D(y_j, \hat{y}_j) := \mathbb{E}[|y_j - \hat{y}_j|^2 | \hat{\mathbf{H}}] \leq \sigma_{d_j}^2 \quad (3)$$

The minimum achievable fronthaul rate for both of the links with capacities  $c_j$ ,  $j \in \{1, 2\}$  is given - according to the rate-distortion theorem [9] - by the mutual information between  $\mathbf{Y}$  and  $\hat{\mathbf{Y}}$  given the channel estimate  $\hat{\mathbf{H}}$ .

$$\begin{aligned} r_j &\leq c_j \\ r_j &\geq \min_{p_{\hat{y}_j|y_j}: D \leq \sigma_{d_j}^2} I(Y_j, \hat{Y}_j | \hat{\mathbf{H}}) \end{aligned} \quad (4)$$

If we choose  $\hat{y}_j$  adequately, the resulting distortion allows to quantize  $y_j$  accurately at the optimal rate  $r_j = c_j$ . We get the following bound on  $\sigma_{d_j}^2$

$$\sigma_{d_j}^2 \leq \sigma_{y_j}^2 |\hat{\mathbf{H}}|^{2-c_j} \quad (5)$$

We define  $\alpha_j = \frac{\sigma_{y_j}^2 |\hat{\mathbf{H}}|^{2-c_j}}{\sigma_{y_j}^2 |\hat{\mathbf{H}}|}$   $\forall j \in \{1, 2\}$  and in the following computations we use it in the matrix  $\boldsymbol{\alpha} = \text{diag}(\alpha_j)_{j=\{1,2\}}$ . The scaling factor  $\alpha_j$  allows to compute the upper bound of the variance of the distortion, which can be then assimilated to additive Gaussian noise with variance  $\sigma_{d_j}^2$ . The rates computed below are then valid for any distortion model.

$$\frac{\sigma_{d_j}^2}{\alpha_j} = \frac{\sigma_{d_j}^2 \sigma_{y_j}^2 |\hat{\mathbf{H}}|}{\sigma_{y_j}^2 |\hat{\mathbf{H}}| - \sigma_{d_j}^2} \leq \frac{\sigma_{y_j}^2 |\hat{\mathbf{H}}|^{2-c_j}}{1 - 2^{-c_j}} \quad \forall j \in \{1, 2\} \quad (6)$$

### B. Imperfect channel estimation

In the BBU pool we estimate the complex channel coefficient from each user to both RRHs using pilot symbols known on both sides. The error between estimates and the actual channel realization is characterized by an additive estimation error matrix  $\hat{\mathbf{H}}$ .

From a practical point of view, on one hand increasing training length would decrease channel estimation error. On the other hand, the useful sum-rate would also decrease, since less useful transmission can be realized during channel coherence time. In an optimal design we need to find a tradeoff between the pilot length and the resulting  $\sigma_e^2$  to achieve maximal useful rates.

### C. Achievable rates

The rate region of the channel defined in Section II is given by the closure of the set of the achievable rate pairs. We assume that the receivers can use as side information the estimation of the channel matrix  $\hat{\mathbf{H}}$ , the statistics of channel estimation error and of the channel noise. Received signals from both RRHs are also shared in the BBU-Pool. The achievable rates are then given by:

$$\begin{aligned} r_1 &\leq I(X_1; \hat{\mathbf{Y}} | \hat{\mathbf{H}}, x_2) \\ r_2 &\leq I(X_2; \hat{\mathbf{Y}} | \hat{\mathbf{H}}, x_1) \\ r_1 + r_2 &\leq I(\mathbf{X}; \hat{\mathbf{Y}} | \hat{\mathbf{H}}) \end{aligned} \quad (7)$$

The mutual information terms can be written as

$$I(X_1; \hat{\mathbf{Y}} | \hat{\mathbf{H}}, x_2) = h(X_1) - h(X_1 | \hat{\mathbf{y}}, x_2, \hat{\mathbf{H}}) \quad (8a)$$

$$I(X_2; \hat{\mathbf{Y}} | \hat{\mathbf{H}}, x_1) = h(X_2) - h(X_2 | \hat{\mathbf{y}}, x_1, \hat{\mathbf{H}}) \quad (8b)$$

$$I(\mathbf{X}; \hat{\mathbf{Y}} | \hat{\mathbf{H}}) = h(\mathbf{X}) - h(\mathbf{X} | \hat{\mathbf{y}}, \hat{\mathbf{H}}) \quad (8c)$$

The first entropy terms in equations (8a)-(8b) represent the amount of information that was actually sent, while the second terms the degradation through the channel, the fronthaul quantization and the receiver. These ones can be obtained from the covariance of the detection error when the received signal includes the channel estimation error. Consequently, in order to get the achievable rates, we have to find upper bounds of the second entropy terms, since the first ones depend only on transmission powers  $p_1$  and  $p_2$ . In the following equations the vectors  $\mathbf{h}_i$  denote the columns of the matrix  $\mathbf{H}$  and respectively  $\hat{\mathbf{h}}_i$  and  $\tilde{\mathbf{h}}_i$  for the columns of the matrices  $\hat{\mathbf{H}}$  and  $\tilde{\mathbf{H}}$ . Using this notation the received signal can be written

$$\hat{\mathbf{y}} = \alpha \left( (\hat{\mathbf{h}}_1 + \tilde{\mathbf{h}}_1)x_1 + (\hat{\mathbf{h}}_2 + \tilde{\mathbf{h}}_2)x_2 + \mathbf{z} + \mathbf{d} \right) \quad (9)$$

where  $\mathbf{z}$  is the additive Gaussian channel noise vector and  $\mathbf{d}$  the equivalent noise representing distortion. Since both are assumed Gaussian we denote  $\mathbf{n} = \mathbf{z} + \mathbf{d}$  and its covariance matrix  $\Sigma_N = \text{diag} \{ \sigma_{N_i}^2 \}$  with  $\sigma_{N_i}^2 = \sigma_z^2 + \frac{\sigma_{d_i}^2}{\alpha_i}$ .

Second terms of (8a) and (8b) can be computed similarly, hence following computations stand for both with appropriate indexes.

We denote  $\hat{\mathbf{y}}' = \hat{\mathbf{y}} - \alpha \hat{\mathbf{h}}_2 x_2$  and consider the linear MMSE estimate of  $x_1$  from  $\hat{\mathbf{y}}'$  the conditional entropy in (8a). By applying results in [10] to our system model, we get

$$h(X_1 | \hat{\mathbf{y}}', x_2, \hat{\mathbf{H}}) \leq \mathbb{E}_{\hat{\mathbf{H}}_1} [\log(2\pi e k_1)] \quad (10)$$

where  $k_1$  denotes the variance of the error between  $x_1$  and its LMMSE estimate  $\hat{x}_1$  given  $x_2$  and  $\hat{\mathbf{H}}$ .

Using the definition of  $\hat{\mathbf{y}}'$ , we get

$$k_1 = p_1 (1 - \hat{\mathbf{H}}_1^H (\hat{\mathbf{H}}_1 \hat{\mathbf{H}}_1^H p_1 + \Sigma_{e1})^{-1} \hat{\mathbf{H}}_1 p_1) \quad (11)$$

with  $\Sigma_{e1} = \sigma_e^2 (p_1 + |\mathbf{X}_2|^2) \mathbf{I}_2 + \Sigma_N$ .

We get the following bound on the mutual information

$$I(X_1; \hat{\mathbf{Y}} | \hat{\mathbf{H}}, x_2) \geq \mathbb{E}_{\hat{\mathbf{H}}_1, \mathbf{X}_2} \left[ \log \left( \frac{p_1}{k_1} \right) \right] \quad (12)$$

We have similar expression for the inequality (8b).

To find the achievable sum rate expression (equation (8c)) we use the array of received signals from both RRHs  $\hat{\mathbf{y}}$ .

$$h(\mathbf{X} | \hat{\mathbf{y}}, \hat{\mathbf{H}}) \leq \mathbb{E}_{\hat{\mathbf{H}}} [\log(\det(2\pi e \mathbf{K}))] \quad (13)$$

with  $\mathbf{K}$  the covariance matrix of the error between  $\mathbf{x}$  and its LMMSE estimate  $\hat{\mathbf{x}}$ .

By computing this covariance we get

$$\mathbf{K} = \mathbf{P}_X (\mathbf{I}_2 - \hat{\mathbf{H}}^H (\hat{\mathbf{H}} \mathbf{P}_X \hat{\mathbf{H}}^H + \Sigma_e)^{-1} \hat{\mathbf{H}} \mathbf{P}_X) \quad (14)$$

with  $\Sigma_e = \sigma_e^2 (p_1 + p_2) \mathbf{I}_2 + \Sigma_N$ .

Using this result we get the following bound on mutual information

$$I(\mathbf{X}, \hat{\mathbf{Y}} | \hat{\mathbf{H}}) \geq \mathbb{E}_{\hat{\mathbf{H}}} \left[ \log \frac{p_1 p_2}{\det(\mathbf{K})} \right] \quad (15)$$

From the above expressions we get the following result.

*Theorem 1:* An achievable rate-region of the system described in Section II is given by

$$r_1 \leq \mathbb{E}_{\hat{\mathbf{H}}_1, \mathbf{X}_2} \left[ \log \left( 1 + p_1 \hat{\mathbf{H}}_1^H \Sigma_{e1}^{-1} \hat{\mathbf{H}}_1 \right) \right] \quad (16a)$$

$$r_2 \leq \mathbb{E}_{\hat{\mathbf{H}}_2, \mathbf{X}_1} \left[ \log \left( 1 + p_2 \hat{\mathbf{H}}_2^H \Sigma_{e2}^{-1} \hat{\mathbf{H}}_2 \right) \right] \quad (16b)$$

$$r_1 + r_2 \leq \mathbb{E}_{\hat{\mathbf{H}}} \left[ \log \det \left( \mathbf{I}_2 + \hat{\mathbf{H}}^H \Sigma_e^{-1} \hat{\mathbf{H}} \mathbf{P}_X \right) \right] \quad (16c)$$

By developing the expression of  $\Sigma_e$  we can see that the sum-rate is an increasing function of fronthaul capacities.

$$\Sigma_e = \begin{pmatrix} \sigma_e^2 (p_1 + p_2) + & 0 \\ \sigma_z^2 + \frac{\sigma_{y1}^2 \hat{\mathbf{H}}^{2-c_1}}{1-2^{-c_1}} & \\ & \sigma_e^2 (p_1 + p_2) + \\ 0 & \sigma_z^2 + \frac{\sigma_{y2}^2 \hat{\mathbf{H}}^{2-c_2}}{1-2^{-c_2}} \end{pmatrix} \quad (17)$$

*Conjecture 1:* For an extension of our system model to  $n$  channel users and  $n$  RRHs connected to the BBU-pool through links with capacity  $c_i$  with  $i \in \{1, 2, \dots, n\}$  the following sum-rate can be derived from (16c).

$$\sum_{i=1}^n r_i \leq \mathbb{E}_{\hat{\mathbf{H}}} \left[ \log \det \left( \mathbf{I}_n + \hat{\mathbf{H}}^H \Sigma_{e,n}^{-1} \hat{\mathbf{H}} \mathbf{P}_X \right) \right] \quad (18)$$

with  $\Sigma_{e,n} = (\sigma_e^2 (\sum_{i=1}^n p_i) + \sigma_z^2) \mathbf{I}_n + \text{diag} \left( \frac{\sigma_{y_i}^2 \hat{\mathbf{H}}^{2-c_i}}{1-2^{-c_i}} \right)$

#### IV. FRONTHAUL CAPACITY AND CHANNEL TRAINING OPTIMIZATION

In the optimization of radio front-end parameters we adopt a cost-gain approach by taking into account the cost of fronthaul capacity used and the effective useful transmission. In fact, in a C-RAN architecture, the cost of the fronthaul infrastructure is an important factor to take into account, since it can happen that it is higher than the benefit from effective useful transmission.

We focus on the optimization of the sum-rate (16c) following two factors: the fronthaul capacity allocated to each user and the training length which is inversely proportional to the channel estimation error  $\sigma_e^2$ .

##### A. Effective benefit of transmissions

In the expression of the optimization problem using (16c) we have several parameters:

- The proportion of pilot symbols w.r.t. the total sequence length denoted by  $\eta$
- The input power allocated for each user that we assume equal to both of them  $p = p_1 = p_2$ .
- The parameter which allows us to model  $\sigma_e^2$  using the transmit power and  $\eta$ :  $\beta$
- The variance of the channel estimation error  $\sigma_e^2 = \frac{\beta}{\eta p + \beta}$
- The Gaussian channel noise variance  $\sigma_z^2$
- The capacity of each fronthaul link on which we forward digital base-band I/Q symbols from the base-station to the processing unit (we assume also  $c = c_1 = c_2$ )
- The cost  $\lambda$  of the fronthaul capacity allocation

The cost factor  $\lambda$  can be considered either as a price to pay for effective transmission over the fiber link, or for example a penalty depending on network conditions: allocating capacity on an already saturated link impacts other users so has a high cost.

Normalizing the estimated channel gain  $\hat{\mathbf{H}}$  makes clear the impact of training length on the channel estimation error ensures proportionality between the useful rate and the fronthaul cost. In the following equations we use the normalized channel estimate matrix  $\hat{\mathbf{H}}_N = (1 - \sigma_e^2)\hat{\mathbf{H}}$ . In order to keep the optimization results general, we use the expectancy over many occurrences of the normalized channel estimate matrix  $\hat{\mathbf{H}}_N$ .

We can formulate the following objective function

$$g(\sigma_e^2, \sigma_z^2, p, \lambda, \eta, c) = (1 - \eta)\mathbb{E}_{\hat{\mathbf{H}}_N} \left[ \log \det (\mathbf{I}_2 + p(1 - 2^{-c})(1 - \sigma_e^2)\hat{\mathbf{H}}_N^H ((2\sigma_e^2 p + \sigma_z^2)\mathbf{I}_2 + 2^{-c}p(1 - \sigma_e^2)\hat{\mathbf{H}}_N^M)^{-1} \hat{\mathbf{H}}_N) \right] - \lambda c \quad (19)$$

$$\text{with } \hat{\mathbf{H}}_N^M = \begin{pmatrix} |\hat{\mathbf{H}}_{1N}|^2 & 0 \\ 0 & |\hat{\mathbf{H}}_{2N}|^2 \end{pmatrix}$$

The value of this function for a given set of parameters gives the effective gain of the transmission.

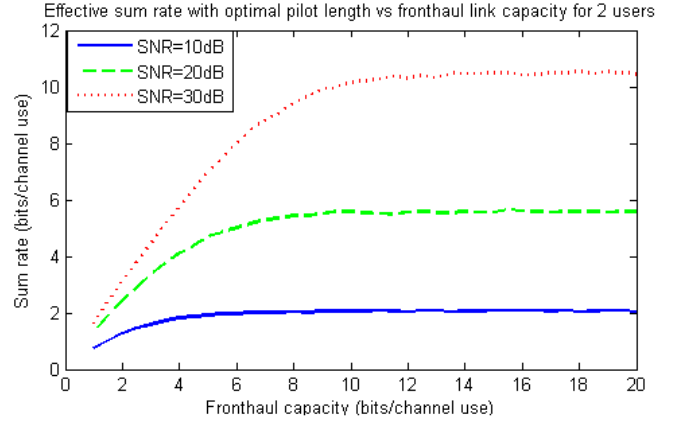
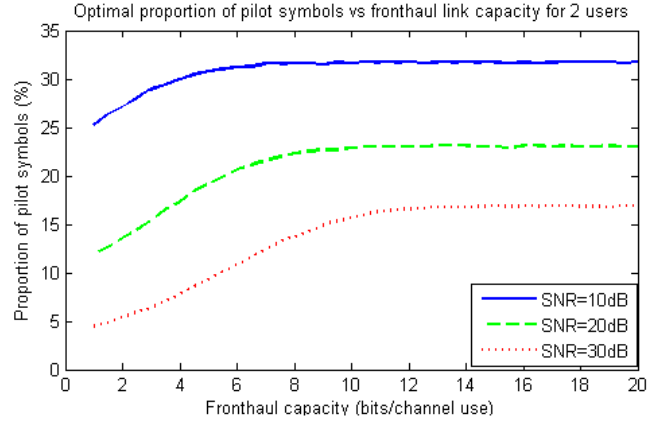


Fig. 2: Optimal pilot proportion and useful sum-rate vs fixed fronthaul capacity for several SNR values

##### B. Capacity and training length optimization

The optimization problem using function (19) is written as follows.

$$\begin{aligned} &\text{Given the parameters } \mathcal{P} = (\sigma_e^2, \sigma_z^2, p), \lambda, \beta \\ &\text{We search for } \underset{c, \eta}{\operatorname{argmax}} g(\mathcal{P}, \lambda, \eta, c) \\ &\text{subject to } \sigma_e^2 = \frac{\beta}{\eta p + \beta} \end{aligned} \quad (20)$$

The optimal  $(\eta, c)$  pair allows us to maximize the effective gain of the transmission. In order to find the optimal values of  $\eta$  and  $c$ , we compute the partial derivatives of the function  $g$  w.r.t.  $\eta$  and  $c$ .

The partial derivative w.r.t.  $\eta$  is

$$\frac{\partial g(\mathcal{P}, \lambda, \eta, c)}{\partial \eta} = (1 - \eta)\mathbb{E}_{\hat{\mathbf{H}}_N} \left[ \frac{\partial \log \det a(\sigma_e^2)}{\partial \sigma_e^2} \right] - \mathbb{E}_{\hat{\mathbf{H}}_N} [\log \det a(\sigma_e^2)] \quad (21)$$

$$\text{with } a(\sigma_e^2) = \mathbf{I}_2 + p(1 - 2^{-c})(1 - \sigma_e^2)\hat{\mathbf{H}}_N^H \times ((2\sigma_e^2 p + \sigma_z^2)\mathbf{I}_2 + 2^{-c}p(1 - \sigma_e^2)\hat{\mathbf{H}}_N^M)^{-1} \hat{\mathbf{H}}_N$$

and the partial derivative w.r.t. the capacity  $c$  is

$$\frac{\partial g(\mathcal{P}, \lambda, \eta, c)}{\partial c} = (1 - \eta) \mathbb{E}_{\hat{\mathbf{H}}_N} \left[ \frac{\partial \log \det a(c)}{\partial c} \right] - \lambda$$

with  $a(c) = \mathbf{I}_2 + p(1 - 2^{-c})(1 - \sigma_e^2) \hat{\mathbf{H}}_N^H \times$  (22)

$$((2\sigma_e^2 p + \sigma_z^2) \mathbf{I}_2 + 2^{-c} p(1 - \sigma_e^2) \hat{\mathbf{H}}_N^{M-1} \hat{\mathbf{H}}_N)$$

## V. NUMERICAL EXAMPLES

We provide in this section numerical evaluation of the results developed above. Since the achievable sum-rate depends on the actual channel realization, we need to use the expectancy w.r.t. the channel estimate value to get the optimal  $c$  and  $\eta$  in a general case, not for a particular channel realization. Despite dynamic optimization and periodical update of these parameters, computing them for each channel realization would require too high computational effort and they could only be applied for the following transmission where channel estimates are already out-of-date. We have evaluated the difference between the optimal pilot proportion computed using expectancy and the one using each channel realization and it turns out to be lower than 2% of the optimal value. One can observe that this difference is also independent of the fronthaul capacity. Thus, we can find the optimal values by numerical search so that expectancy over many values could be computed and update transmission parameters less frequently.

### A. Optimal pilot allocation

With a fixed amount of fronthaul capacity allocated and without taking into account the cost of the fronthaul usage we can find the optimal proportion of pilot symbols resulting the best achievable sum-rate for the given fronthaul capacity. More precisely, we evaluate the optimal pilot proportion  $\eta$  as formulated in the optimization problem formulated in (20) with cost factor  $\lambda = 0$  and for several fixed capacity value.

On Figure 2 we have plotted the optimal proportion of pilot symbols to be allocated depending on the available fronthaul capacity (upper part) and the achieved sum-rate values when the optimal pilot proportion is applied (lower part) for several SNR values. We notice that for capacities higher than  $\log(1 + \text{SNR})$  the sum-rate becomes constant, longer training sequence cannot increase it more because of the fronthaul capacity limitation. Also, for low SNR, the useful transmission rate is very low due to large pilot overhead.

### B. Joint optimization of the fronthaul capacity and the pilot length

We can solve also numerically the problem defined in (20) so that we get the pair  $(\eta, c)$  that gives the maximal sum-rate for a given SNR. Moreover, we consider, according to (19), a cost to get fronthaul capacity, hence increasing the capacity too much would not increase the overall gain. On Figure 3 we show jointly optimized pilot proportion and the capacity for different cost values. One can observe that while fronthaul cost increases, the optimal pilot length to be allocated decreases,

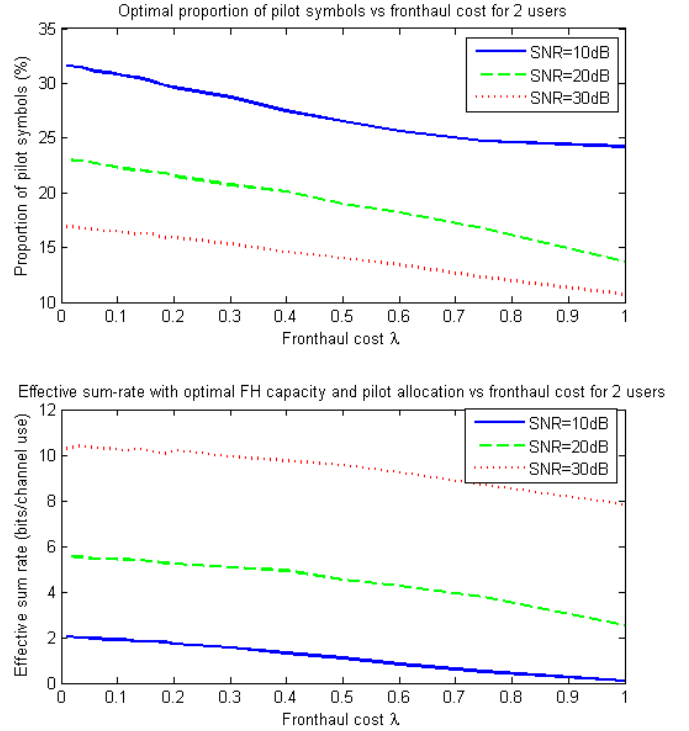


Fig. 3: Optimal pilot proportion and effective sum-rate vs fronthaul cost for several SNR values

since sum-rate is limited by the lower optimal capacity, so shorter training sequences are enough to achieve this lower sum-rate.

On Figure 4 we can see how efficiently the allocated fronthaul capacity is used if cost varies. The ratio of useful sum-rate and optimal capacity shows that fronthaul is used more efficiently with higher cost values and also for higher SNR. On one hand, with higher SNR values we need to allocate less pilots to get optimal sum-rate. On the other hand, if cost is higher, it is optimal to allocate less capacity and pilots than the ones achieving maximal sum-rate with unlimited fronthaul, but still allowing reliable transmission, so that the allocated fronthaul is used more efficiently.

## VI. CONCLUSION

In this paper we have studied uplink transmissions in a C-RAN system with limited capacity fronthaul links. We have computed the achievable rate region which depends on the fronthaul capacity and channel estimation error statistics. Using this rate region and considering the cost of fronthaul usage we have shown the benefit of jointly optimizing the length of training sequences and the allocated fronthaul capacity.

Since our system model allows scalability, these results can be applied to higher dimensional systems regarding both number of antennas and users, until the number of co-channel users is not higher than the total number of antennas over all RRHs. A next step of this work can be the creation of

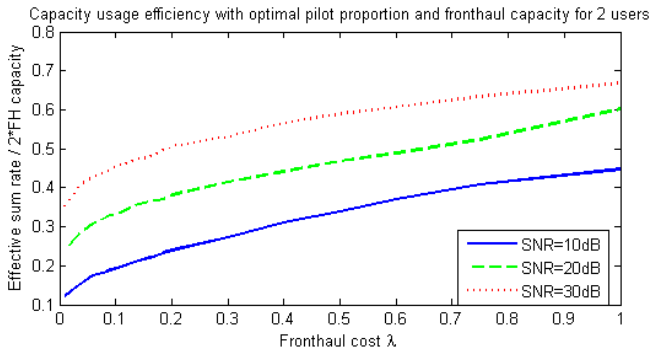


Fig. 4: Efficiency of capacity usage w.r.t. fronthaul cost

user groups transmitting on the same channel in an extension of our model where fronthaul capacity needs to be shared between transmissions on several channels. User grouping using the metric developed in this paper allows to maximize transmission rate for the whole set of channels under the practical constraints that we have cited. In another extension of the present work, correlation between antennas in various configurations in terms of distribution and centralization can be studied. Provided results can also be generalized by considering different estimation errors for each user e.g. if they use different transmission modes. This would allow to compute optimal pilot length for each of them. Finally, as a next step, we plan to implement optimization algorithms based on these

results in a prototype platform that can allow performance evaluation in real network conditions.

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