Controlling bulk Reynolds number and bulk temperature in channel flow simulations

Yufang Zhang, Ronan Vicquelin

Controlling bulk Reynolds number and bulk temperature in channel flow simulations

Y. F. Zhang[a,1], R. Vicquelin[a,*]

[a]Laboratoire EM2C, CNRS, CentraleSupélec, Université Paris-Saclay, Grande Voie des Vignes, 92295 Chatenay-Malabry cedex, France

Abstract

Bulk Reynolds number and bulk temperature are key quantities when reporting results in channel flow simulations. There are situations are when one wishes to accurately control these parameters while changing some numerical or physical conditions. A method to control the bulk Reynolds number and the bulk temperature in channel flow simulations is detailed. An ordinary differential equation is prescribed for the additional source term in the momentum balance equation so that the transient regime of the simulation is thoroughly tuned in order to efficiently and accurately reach the target Reynolds number value. A similar treatment is applied for the additional volume heat source term in the energy balance equation. The proposed method is specifically interesting when studying complex multi-physics in channel flow configurations when non-dimensionalization of the equations is no longer practical.

Keywords: Channel flow, Control, Bulk Reynolds number, Bulk temperature

1. Introduction

Channel flow is a simple configuration that has been widely studied numerically in order to analyze turbulent boundary layers and validate models [1, 2]. For fully developed channel flows, periodic conditions are considered in the infinite spanwise direction, and also along the streamwise direction. A homogeneous source term S_i is then added to the momentum equations to compensate for viscous forces and drive the flow at a given bulk velocity. In direct numerical simulations, the momentum equation in the streamwise direction ($i = 1$) reads

$$\frac{\partial}{\partial t}(\rho u_1) + \frac{\partial}{\partial x_j}(\rho u_j u_1) = -\frac{\partial p}{\partial x_1} + \frac{\partial \tau_{1j}}{\partial x_j} + S_1 \tag{1}$$

where S_1 is the homogeneous source term in the streamwise direction while $S_2 = S_3 = 0$. The source term S_1 enforces the flow mass flow rate and then determines the obtained Reynolds number. In standard flows with constant flow properties and without multiple physical phenomena (chemistry, radiation, ...), non-dimensional equations can be written and this source term is directly related to the intended bulk or friction Reynolds number. Alternatively, the source term can be simply be updated step by step to compensate for the change of the intended mass flux [3]. However, in more complex flows featuring variable properties due to an explicit dependency on temperature for example [4], or involving multiphysics such as chemistry [5] or radiation [6, 7], equations are kept in their dimensional form and S_1 must be determined differently to reach a target bulk Reynolds number Re_b^t.

A first method to determine S_1 consists in taking a fixed constant in time S^{ref} [5, 6] that is either chosen arbitrarily or more carefully evaluated from friction coefficient formulae that depend on the Reynolds number. Usually, the considered formulae are accurate for simple flows (constant properties, no multiphysics) but can become considerably erroneous as the studied flow is more and more complex. In such a case, the final Reynolds number that is obtained is different from the intended one.

*Corresponding author
Email address: ronan.vicquelin@ecp.fr (R. Vicquelin)
[1]Current address: AVIC Commercial Aircraft Engine Co., Ltd., Shanghai, 200241, P. R. China

A second kind of method consists in dynamically adapting the source term value after each iteration so that the Reynolds number is brought towards its target value [8, 4, 5, 9]. However, none of these methods have been carefully characterized with a dedicated study. The procedures reported in [4, 9] are similar to

$$S_1^{n+1} = S^{\mathrm{ref}} + \frac{\rho_b^{\mathrm{t}}\, u_b^{\mathrm{t}} - \frac{\int_V \rho^n u^n\, dV'}{V}}{\tau^{\mathrm{ref}}}, \tag{2}$$

where S_1^{n+1} is the updated value of the uniform momentum source term at iteration $n+1$, ρ_b^{t} is the target value of bulk density and u_b^{t} is the target value of bulk velocity. Equation 2 describes the adaptation of S_1^{n+1} because of the difference between the bulk mass flux at iteration n and its target value $\rho_b^{\mathrm{t}}\, u_b^{\mathrm{t}}$. This difference is related to the difference between the simulation bulk Reynolds number and the target value Re_b^t. The relaxation time τ^{ref} used in Eq. 2 is typically expressed in terms of the channel time scale δ/u_τ. When using Eq. 2, the permanent regime is reached after a transient stage and S_1^{n+1} tends towards a constant value denoted by $\overline{S_1}$, and

$$\rho_b^{\mathrm{t}}\, u_b^{\mathrm{t}} - \frac{\int_V \rho^n u^n\, dV'}{V} = \left(\overline{S_1} - S^{\mathrm{ref}}\right)\tau^{\mathrm{ref}}. \tag{3}$$

Since the obtained stationary value $\overline{S_1}$ is different from the empirically determined S^{ref} in complex flows, a finite bias between Re_b and Re_b^t unfortunately remains in such flows. This biais is not present in the formulation from Lenormand *et al.* [8] which can be reformulated as a PI controller with constant coefficients and a time response close to the computation time step. The efficiency of the approach was established *a posteriori*.

Equation 2 was thus a first attempt to control the bulk Reynolds number. This objective is here pursued by proposing an approach which ensures that the channel flow converges exactly and efficiently to the target bulk Reynolds number *a priori* and *a posteriori*. The approach appears as a modified PI controller with time varying coefficients. Carefully tuning the time varying coefficients allows to derive a second order ordinary differential equation with constant coefficients for Re_b whose time response can then be exactly controlled. The method is described in the following section. Its efficiency is then demonstrated in several channel flow configurations. Similarly, the method is derived and applied for the equivalent control of bulk temperature when studying turbulent heat transfer. Finally, a strategy for the control of both bulk Reynolds number and bulk temperature with variable thermo-physical properties accounted for is derived and validated in the section 4.

2. Control of the bulk Reynolds number

2.1. Formulation

Integration of Eq. 1 over the whole computational domain V gives

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_V \rho u_1\, \mathrm{d}V'\right) = \int \tau_{1j} n_j\, \mathrm{d}S + S_1 V \tag{4}$$

where n_j is the outward surface normal vector. The integration of the pressure gradient and the convective terms is null because of the applied periodic boundary conditions in the X and Z directions.

The integrated term on the left side of Eq. 4 is related to the bulk Reynolds number $\mathrm{Re}_b = \frac{\rho_b u_b \delta}{\mu_b}$,

$$\int_V \rho u_1\, dV' = \rho_b\, u_b\, V = \frac{\mu_b\, \mathrm{Re}_b\, V}{\delta} \tag{5}$$

where δ is the half-width of the channel and the subscript b is related to the aforementioned bulk quantities.

The integrated wall shear stress is split into two contributions,

$$\int \tau_{1j} n_j\, \mathrm{d}S = -(\tau_{w,1} + \tau_{w,2}) S_w \tag{6}$$

as $\tau_{w,1}$ and $\tau_{w,2}$, the average wall shear stresses on the lower and upper wall respectively, can be different in the general case. S_w denotes the surface area of each wall.

Neglecting variations of the bulk dynamic viscosity and noticing that $V = 2\delta S_w$, Eq. 4 becomes

$$\frac{\mathrm{dRe}_b}{\mathrm{d}t} = -\frac{\tau_{w,1} + \tau_{w,2}}{2\,\mu_b} + \frac{\delta}{\mu_b} S_1. \tag{7}$$

Differentiating Eq. 7 to make $\frac{\mathrm{d}S_1}{\mathrm{d}t}$ appear gives

$$\frac{\mathrm{d}^2\mathrm{Re}_b}{\mathrm{d}t^2} = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\tau_{w,1} + \tau_{w,2}}{2\,\mu_b}\right) + \frac{\delta}{\mu_b}\frac{\mathrm{d}S_1}{\mathrm{d}t}. \tag{8}$$

As in many control algorithms, we choose the evolution of the homogeneous source term S_1 to be determined by the dynamics of the controlled quantity Re_b:

$$\frac{\mathrm{d}S_1}{\mathrm{d}t} = a_1(t)\,\frac{\mathrm{dRe}_b}{\mathrm{d}t} + a_2(t), \tag{9}$$

where $a_1(t)$ and $a_2(t)$ are to be determined. Equation 8 is then written as

$$\frac{\mathrm{d}^2\mathrm{Re}_b}{\mathrm{d}t^2} - \left(a_1(t)\frac{\delta}{\mu_b}\right)\frac{\mathrm{dRe}_b}{\mathrm{d}t} - a_2(t)\frac{\delta}{\mu_b} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{|\tau_{w,1}| + |\tau_{w,2}|}{2\,\mu_b}\right) = 0. \tag{10}$$

In order to obtain a controlled relaxation towards the target value Re_b^t, Eq. 10 is transformed into the following second-order ordinary differential equation with constant coefficients α and β for the difference $(\mathrm{Re}_b - \mathrm{Re}_b^t)$,

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}(\mathrm{Re}_b - \mathrm{Re}_b^t) + \alpha\frac{\mathrm{d}}{\mathrm{d}t}(\mathrm{Re}_b - \mathrm{Re}_b^t) + \beta\,(\mathrm{Re}_b - \mathrm{Re}_b^t) = 0, \tag{11}$$

by setting $a_1(t)$ and $a_2(t)$ as

$$a_1(t) = -\alpha\frac{\mu_b}{\delta}, \tag{12}$$

$$a_2(t) = \frac{\mu_b}{\delta}\left[\beta\,(\mathrm{Re}_b^t - \mathrm{Re}_b(t)) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{|\tau_{w,1}| + |\tau_{w,2}|}{2\,\mu_b}\right)\right]. \tag{13}$$

The dynamics of $\mathrm{Re}_b(t)$ is then controlled by the discriminant $\Delta = \alpha^2 - 4\beta$. Among the different possible regimes, the critically damped regime ($\Delta = 0$) gives the shortest decay time in order for the controlled bulk Reynolds number to reach the desired targeted value. It is therefore chosen, leading to $\alpha = 2\beta^{1/2}$. Defining $\tau = \frac{2}{\alpha}$ introduces the controlling decay time that can be chosen arbitrarily. Accounting for the initial conditions $\mathrm{Re}_b\big|_{t=0}$ and $\frac{\mathrm{dRe}_b}{\mathrm{d}t}\big|_{t=0}$, the temporal evolution obtained for the bulk Reynolds number is

$$\mathrm{Re}_b(t) = \mathrm{Re}_b^t + \left[\left(\mathrm{Re}_b\big|_{t=0} - \mathrm{Re}_b^t\right)\left(1 + \frac{t}{\tau}\right) + \frac{\mathrm{dRe}_b}{\mathrm{d}t}\Big|_{t=0}\, t\right] e^{-\frac{t}{\tau}}, \tag{14}$$

The final formulation for the source term is therefore:

$$\frac{\mathrm{d}S_1}{\mathrm{d}t} = -\frac{\mu_b}{\delta}\left[\frac{2}{\tau}\,\frac{\mathrm{dRe}_b}{\mathrm{d}t} + \frac{\mathrm{Re}_b(t) - \mathrm{Re}_b^t}{\tau^2} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\tau_{w,1} + \tau_{w,2}}{2\,\mu_b}\right)\right], \tag{15}$$

which appears as a modified PI controller with thoroughly tuned coefficients. Indeed, equation 15 contains time varying terms (apart from $\mathrm{Re}_b(t)$ and $\frac{\mathrm{dRe}_b}{\mathrm{d}t}$) instead of constants. Beside, their values are determined from the exact evolution law of the bulk Reynolds number rather than by linearization of the system equations around a mean state. Hence, any non-linear effects in the physical response due to a change in the source term is accounted for. The temporal evolution described by Eq. 14 is exact when bulk dynamic viscosity is constant. This is the only assumption that has been made. Even when this is not true, Eq. 15 ensures that the statistically steady state will always correspond to $\mathrm{Re}_b = \mathrm{Re}_b^t$.

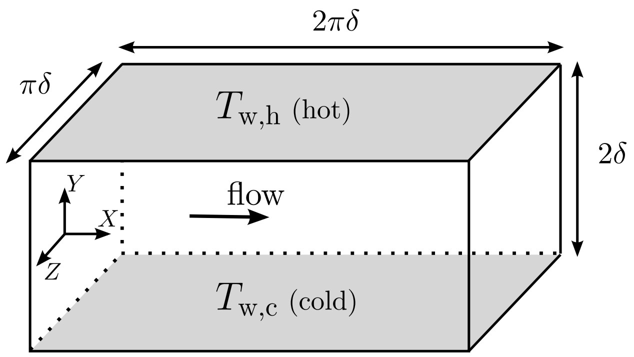


Figure 1: Channel flow configuration of half-width δ. In heat transfer simulations, both wall temperatures $T_{w,c}$ (lower wall) and $T_{w,h}$ (upper wall) are prescribed where $T_{w,h} \geq T_{w,c}$.

Initial condition $S_1(0)$ could be determined from Eq. 7 to impose an smooth departure from the initial Reynolds number by imposing $\frac{\mathrm{dRe}_b}{\mathrm{d}t}\Big|_{t=0} = 0$. Alternatively, the initial source term value is here fixed to $S^{\mathrm{ref}} = \frac{\tau_w^{\mathrm{ref}}}{\delta}$ a reference value that estimates the source term for the targeted Reynolds number. Equation 15 is coupled with the set of Navier-Stokes conservative equations. Using forward Euler time integration, the value of the momentum source term is updated at each new iteration. This approach has been applied successfully in direct numerical simulations and large-eddy simulations of channel flows with or without radiative energy transfer for identical bulk Reynolds number [7, 10].

Sometimes, the source term in the momentum equation is expressed as a body force ρS_1. In variable density flows, the source term ρS_1 is then not uniform but the proposed approach to determine S_1 can be simply adapted to this case.

2.2. Validation

The computational domain is shown in figure 1 is $2\pi\delta \times 2\delta \times \pi\delta$. The mesh resolution is the same as in case C3 reported in [7]. The set of governing equations is solved with the finite-volume solver YALES2 [11, 12] under a low Mach-number approximation. As detailed in [7], the numerical setup is composed of a centered fourth-order spatial discretization and a fourth-order time integration.

2.2.1. Isothermal case

The only assumption made in the previous derivation is neglecting the variations of the bulk dynamic viscosity. The first studied case is isothermal (T = 1000 K) and the constant viscosity assumption is correct. Three different decay times τ are considered: $0.33\delta/u_\tau^{ref}$, $0.033\delta/u_\tau^{ref}$, and $0.0033\delta/u_\tau^{ref}$. They are set as proportionate to the estimated channel time scale δ/u_τ^{ref}. The initial state of the channel flow simulation is a statistically steady turbulent flow at $\mathrm{Re}_b = 5850$. The control method is applied to obtain $\mathrm{Re}_b = \mathrm{Re}_b^t = 7000$. The mesh resolution remains adequate for this larger Reynolds number.

Figure 2 (a) shows that, whatever the imposed time scale τ, the temporal evolution of the bulk Reynolds number expressed in non-dimensional time units t/τ follows the theoretical solution given by equation 14. It is then possible to reach the target value Re_b^t at any speed. Obviously, the time scale must still be chosen in order not to penalize the stability of time integration of the full set of equations. The very small deviation from the theoretical solution of the case with the smallest time response is due to numerical errors in the time integration because of the time decay τ becoming of the same order as the simulation time step.

Figure 2 (b) shows the corresponding temporal evolution of the friction Reynolds number defined as $\mathrm{Re}_\tau = \rho_w u_\tau \delta/\mu_w$ where u_τ is the actual friction velocity averaged in homogeneous directions on both walls, ρ_w and μ_w are the density and dynamic viscosity at the wall. Re_τ is here plotted against the normalized time $t\,u_\tau^{ref}/\delta$ to highlight that, for short decay times τ, the adaptation of the flow field to the change in the bulk Reynolds number takes a given amount of time that roughly scales with u_τ/δ.

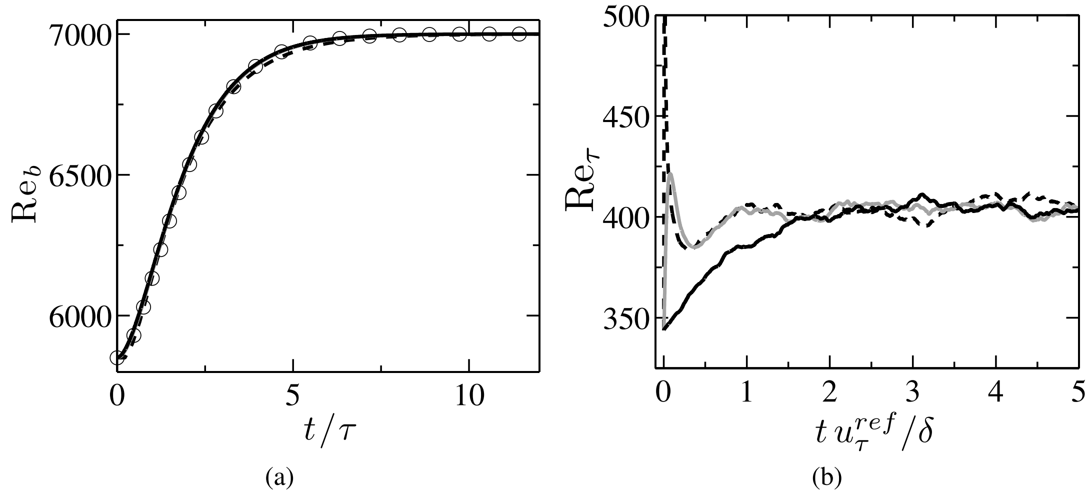


Figure 2: Temporal evolution of Re_b (a) and Re_τ (b) for an isothermal case (T=1000K). Circles: theoretical solution; Black plain line: $\tau = 0.33\delta/u_\tau^{ref}$; Gray plain line: $\tau = 0.033\delta/u_\tau^{ref}$; Black dashed line: $\tau = 0.0033\delta/u_\tau^{ref}$.

2.2.2. Case with variable flow properties

Another computational case is now considered to study the impact of the constant bulk viscosity assumption made to derive the controlling method. Wall temperatures are set to $T_{w,c} = 950$ K and $T_{w,h} = 2050$ K for the cold and hot wall, respectively. From the instantaneous turbulent field of temperature, the dynamic viscosity μ is computed from a tabulated function of temperature built from detailed computation with the CHEMKIN package [13, 14]. This time, the initial state for the control method is a turbulent channel flow at $\mathrm{Re}_b = 5850$ whose statistically steady state is reached with the modified wall temperatures $T_{w,c}$ and $T_{w,h}$. The target value for the bulk Reynolds number is set to $\mathrm{Re}_b^t = 7000$.

The equation for $\frac{dS_1}{dt}$ can be adapted to account rigorously for a variable bulk viscosity (see section 4). It was chosen here to rely instead on the robustness of the derived controller. Results are shown in figure 3 for two different decay times τ. In this case, the evolution of the bulk Reynolds number follows the ideal response with very small deviations. Then, the target value of the bulk Reynolds number is effectively reached as imposed by Eq. 15. Because of the different density and viscosity at each wall, a friction Reynolds number Re_τ is defined for each side. As observed in the isothermal case, the adaptation of the mean wall shear stress takes a given physical time different from the prescribed value of τ.

3. Bulk Temperature

The analogical issue of bulk Reynolds number control raises when studying compressible flows or turbulent heat transfer in channel flow simulations. The bulk temperature or enthalpy is then controlled by the wall heat transfer. When both walls have a different temperature or when heat is generated by viscous dissipation in high-speed flows, the bulk temperature is physically determined by the problem setup. In other configurations ([15] for example), one might want to control the bulk temperature by adding a new source term in the enthalpy or energy balance equation. The introduction of an energy source term is also required to maintain the bulk temperature T_b in periodic simulations when considering identical temperatures at both walls that are different from T_b without any physical heat source term. By adding an energy source term Ω, the enthalpy transport equation is given by

$$\frac{\partial}{\partial t}(\rho \mathrm{h}) + \frac{\partial}{\partial x_j}(\rho u_j \mathrm{h}) = -\frac{\partial q_j}{\partial x_j} + \underbrace{\frac{\partial p}{\partial t} + u_j\frac{\partial p}{\partial x_j} + \tau_{ij}\frac{\partial u_i}{\partial x_j} + \dot{Q} + S_i u_i}_{\dot{Q}^{tot}} + \Omega \tag{16}$$

where pressure effects, viscous heating, additional physical energy source term $\dot{Q}$ and the work $S_i u_i$ due to the added momentum source term are considered in the general case and gathered in $\dot{Q}^{tot}$. The previous method is now derived

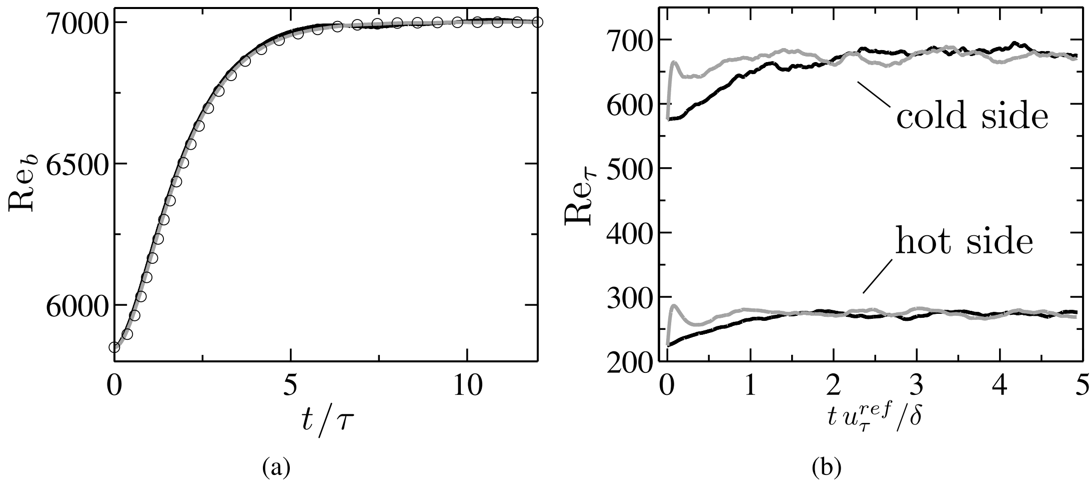


Figure 3: Temporal evolution of Re_b (a), Re_τ on the cold and hot sides (b) for a non-isothermal case ($T_{w,c}$=950K,$T_{w,h}$=2050K). Circles: theoretical solution; Black plain line: $\tau = 0.33\delta/u_\tau^{ref}$; Gray plain line: $\tau = 0.033\delta/u_\tau^{ref}$.

for the controlling source term Ω. It is first validated when temperature is considered as a passive scalar field and where the bulk Reynolds number is fixed. Then, results for simultaneous control of the bulk Reynolds number and temperature are presented.

3.1. Formulation

The control method is obtained similarly to the one developed for the bulk Reynolds number, *i.e.* it is exact for a flow with constant properties. More complex expressions are given in section 4 by accounting for effects of variables flow properties. Integrating the enthalpy balance equation gives

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_V \rho \mathrm{h} dV'\right) = -\int q_j n_j dS + \int_V \dot{Q}^{tot} dV' + \Omega V, \tag{17}$$

where a bulk enthalpy h_b can be introduced, with

$$\int_V \rho \mathrm{h} dV' = \rho_b h_b V. \tag{18}$$

When variable properties are considered, the thermal capacity at constant pressure c_p is not constant and the bulk temperature T_b can be defined by inverting the relation $h_b = h(T_b)$. Neglecting temporal variations of bulk density, equation 17 becomes

$$\rho_b V \frac{\mathrm{d}h_b}{\mathrm{d}t} = -\int q_j n_j dS + \int_V \dot{Q}^{tot} dV' + \Omega V. \tag{19}$$

The integrated wall heat flux is split into two parts,

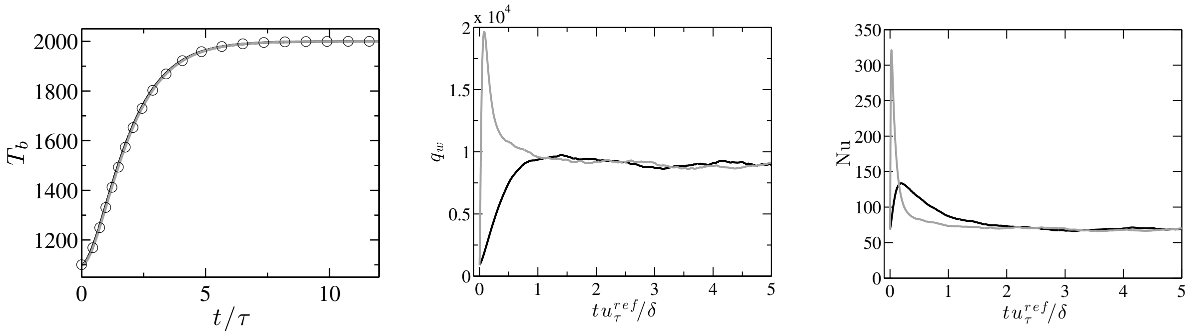
$$\int q_j n_j dS = (q_{w,1} + q_{w,2}) S_w \tag{20}$$

where $q_{w,1}$ and $q_{w,2}$ are the average wall conductive fluxes on the lower and upper wall, respectively. Neglecting variations of c_p gives, $\frac{\mathrm{d}h_b}{\mathrm{d}t} = c_{p_b}\frac{\mathrm{d}T_b}{\mathrm{d}t}$, and the temporal evolution of the bulk temperature is described by

$$\frac{\mathrm{d}T_b}{\mathrm{d}t} = -\frac{q_{w,1} + q_{w,2}}{2\delta \rho_b c_{p_b}} + \frac{1}{\rho_b c_{p_b} V}\int_V \dot{Q}^{tot} dV' + \frac{1}{\rho_b c_{p_b}}\Omega \tag{21}$$

Differentiating in respect to time gives

$$\frac{\mathrm{d}^2 T_b}{\mathrm{d}t^2} = -\frac{1}{2\delta}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{q_{w,1} + q_{w,2}}{\rho_b c_{p_b}}\right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho_b c_{p_b} V}\int_V \dot{Q}^{tot} dV'\right) + \frac{1}{\rho_b c_{p_b}}\frac{\mathrm{d}\Omega}{\mathrm{d}t} \tag{22}$$



(a) (b) (c)

Figure 4: Temporal evolution of T_b in Kelvin (a), q_w in W/m^2 (b) and Nu for $\mathrm{Re}_b = 5850$ with contant properties. Circles: theoretical solution; Black plain line: $\tau = 0.33\delta/u_\tau^{ref}$; Gray plain line: $\tau = 0.033\delta/u_\tau^{ref}$.

Taking the following form for the source term Ω,

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = b_1(t)\,\frac{\mathrm{d}T_b}{\mathrm{d}t} + b_2(t), \tag{23}$$

the time functions $b_1(t)$ and $b_2(t)$ are adapted to give a second-order ordinary differential equation with constant coefficients for $T_b - T_b^t$ whose response is set to the critically damped regime. It is then found that

$$\begin{aligned} b_1(t) &= -\frac{2\rho_b c_{p_b}}{\tau} \\ b_2(t) &= \rho_b c_{p_b}\left[\frac{T_b^t - T_b(t)}{\tau^2} + \frac{1}{2\delta}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{q_{w,1}+q_{w,2}}{\rho_b c_{p_b}}\right) - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho_b c_{p_b} V}\int_V \dot{Q}^{tot} dV'\right)\right] \end{aligned} \tag{24}$$

Different choices are possible for the initial condition $\Omega(t = 0)$. It is set to a reference value $\Omega^{ref} = \frac{q_w^{ref}}{\delta}$ which is estimated from empirical Nusselt number formula for the desired operating condition.

3.2. Validation

3.2.1. Temperature as a passive scalar

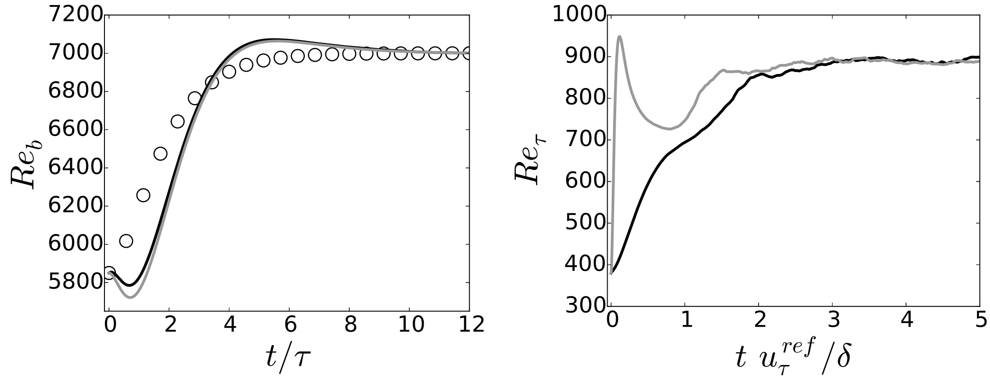
Temperature is first treated as a passive scalar in order to validate the proposed method in the case of a flow with constant thermo-physical properties. The bulk Reynolds number is then not sensitive to the variations of temperature and is set as $\mathrm{Re}_b = 5850$. The wall temperature are identical $T_{w,c} = T_{w,h} = 1000$ K and the initial instantaneous field of temperature is obtained for a channel flow simulation where $T_b = 1100$ K. The target bulk temperature is set to $T_b^t = 2000$ K. The wall heat flux averaged in homogeneous directions on both walls can be expressed as a Nusselt number Nu defined as

$$\mathrm{Nu} = \frac{4 q_w \delta}{\lambda_w (T_b - T_w)}, \tag{25}$$

where the reference length scale 4δ corresponds to the channel flow hydraulic diameter and where λ_w is the thermal conductivity at the walls. Figure 4 (a) shows for two different decay times τ that the bulk temperature follows the theoretical response curve. Similarly, to the wall shear stress, for small imposed value of τ, the average wall heat flux shown in figure 4 (b) needs an incompressible physical time to adapt to the change of T_b. As expected, the obtained steady value of the wall heat flux is larger than its initial value due to the significant change in the bulk temperature. The Nusselt number is shown in figure 4 (c). Its final value remains close to the initial one because of the weak dependency of Nu with the parameter T_b/T_w.

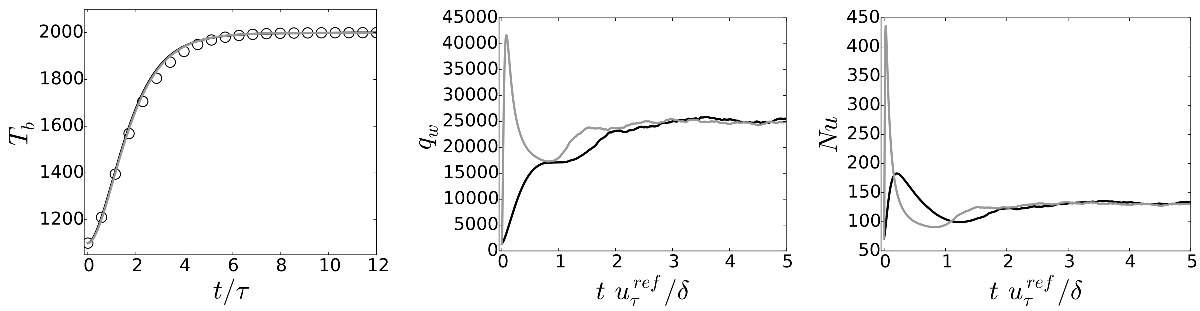
3.2.2. Simultaneous control of bulk Reynolds number and bulk temperature

The temperature field is now coupled to the velocity field by taking into account the change of properties with temperature. Applying the control method for both Re_b and T_b is necessary as controlling and varying one quantity



(a) (b)

Figure 5: Temporal evolution of Re_b (a) and Re_τ (b) for with varying T_b at the same time. Circles: theoretical solution; Black plain line: $\tau = 0.33\delta/u_\tau^{ref}$; Gray plain line: $\tau = 0.033\delta/u_\tau^{ref}$.



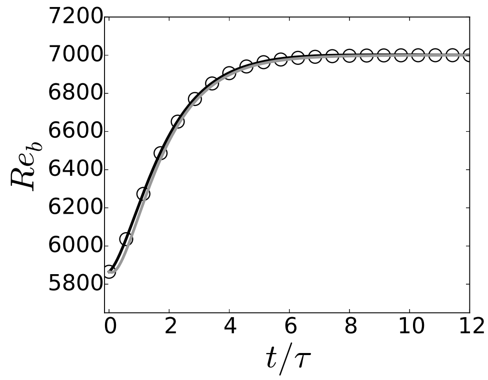
(a) (b) (c)

Figure 6: Temporal evolution of T_b in Kelvin (a), q_w in W/m^2 (b) and Nusselt number (c) for varying Re_b at the same time . Circles: theoretical solution; Black plain line: $\tau = 0.33\delta/u_\tau^{ref}$; Gray plain line: $\tau = 0.033\delta/u_\tau^{ref}$.

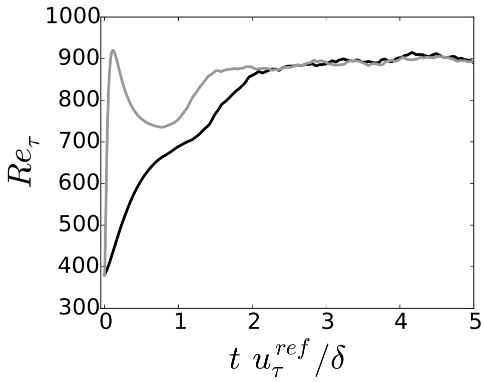
alone would change the other. The controllers derived for constant-properties flow are here considered. The initial state of the simulation corresponds to a steady turbulent channel flow accounting for variable properties with $\mathrm{Re}_b = 5850$, $T_b = 1100$ K and $T_{w,c} = T_{w,h} = 1000$ K. Target values to be reached are set to $\mathrm{Re}_b^t = 7000$ and $T_b^t = 2000$ K with two different decay times $\tau = 0.33\delta/u_\tau^{ref}$ and $\tau = 0.033\delta/u_\tau^{ref}$. The temporal evolution of bulk and friction Reynolds numbers are presented in figure 5. Deviation of the bulk Reynolds number is larger compared to results in section 2.2.2 due to the variation of the bulk temperature from 1100 K to 2000 K which induces large temporal changes in physical properties such as the bulk dynamic viscosity. Despite such strong perturbations, the controlling approach derived for constant properties flow remains robust and brings Re_b to its target within a duration close to the desired decay time.

Similarly, the bulk temperature shown in figure 6 (a) deviates slightly from the theoretical response curve before reaching a plateau at the desired target value T_b^t=2000 K. Corresponding wall heat flux and Nusselt number are given in figure 6 (b) and (c), respectively. This time, the final value taken by the Nusselt number changes because of the modified bulk Reynolds number.

This coupled case with simultaneous control of the bulk Reynolds number and the bulk temperature demonstrates the adequacy of the method even with strong deviations from the assumptions made to derive the controllers. As mentioned previously, the equations to derive the method can be modified to account for the temporal evolutions of bulk variable properties (see next section). As shown here, relying instead on the robustness of the obtained constant-properties controllers is also a good strategy.



(a)



(b)

Figure 7: Temporal evolution of Re_b (a) and Re_τ (b) for with varying T_b at the same time with the control equations.. Circles: theoretical solution; Black plain line: $\tau = 0.33\delta/u_\tau^{ref}$; Gray plain line: $\tau = 0.033\delta/u_\tau^{ref}$.

4. Accounting for variable flow properties during control of bulk Reynolds number and bulk temperature

For the sake of completeness, controllers that account for temporal variations of thermo-physical properties of the mixture are derived. The equations 7 and 21 and for the derivative of Re_b and T_b are then replaced by

$$\frac{d\mathrm{Re}_b}{dt} = -\frac{\tau_{w,1}+\tau_{w,2}}{2\mu_b} + \frac{\delta}{\mu_b}S_1 - \mathrm{Re}_b\frac{1}{\mu_b}\frac{d\mu_b}{dt} \tag{26}$$

$$\frac{dT_b}{dt} = -\frac{q_{w,1}+q_{w,2}}{2\delta\rho_b c_{p_b}} + \frac{1}{\rho_b c_{p_b} V}\int_V \dot{Q}^{tot}dV' + \frac{1}{\rho_b c_{p_b}}\Omega - \frac{h_b}{\rho_b c_{p_b}}\frac{d\rho_b}{dt} \tag{27}$$

Computing the temporal derivative of theses expressions gives modified expressions for the control equations. Hence, the momentum source term is still set by the differential equation $\frac{\mathrm{d}S_1}{\mathrm{d}t} = a_1(t)\frac{\mathrm{dRe}_b}{\mathrm{d}t} + a_2(t)$ but with modified coefficients:

$$a_1(t) = -\frac{2}{\tau}\frac{\mu_b}{\delta}, \tag{28}$$

$$a_2(t) = \frac{\mu_b}{\delta}\left[\frac{\mathrm{Re}_b^t - \mathrm{Re}_b(t)}{\tau^2} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{|\tau_{w,1}|+|\tau_{w,2}|}{2\mu_b}\right) + \frac{\delta S_1}{\mu_b^2}\frac{d\mu_b}{dt} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{Re}_b}{\mu_b}\frac{d\mu_b}{dt}\right)\right]. \tag{29}$$

Similarly, it is found that

$$b_1(t) = -\frac{2\rho_b c_{p_b}}{\tau} \tag{30}$$

$$\begin{aligned} b_2(t) = \rho_b c_{p_b}\Bigg[&\frac{T_b^t - T_b(t)}{\tau^2} + \frac{1}{2\delta}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{q_{w,1}+q_{w,2}}{\rho_b c_{p_b}}\right) - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho_b c_{p_b} V}\int_V \dot{Q}^{tot}dV'\right) \\ &-\Omega\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho_b c_{p_b}}\right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{h_b}{\rho_b c_{p_b}}\frac{d\rho_b}{dt}\right)\Bigg] \end{aligned} \tag{31}$$

with the energy source term determined by $\frac{\mathrm{d}\Omega}{\mathrm{d}t} = b_1(t)\frac{\mathrm{d}T_b}{\mathrm{d}t} + b_2(t)$.

The same simulation case as in section 3.2.2 is carried out with these new equations. The obtained temporal evolutions of bulk Reynolds number and temperature are shown in figures 7 and 8 with their corresponding profiles of Re_τ and Nusselt number. Thanks to the modified equations to take into account temporal variations of thermo-physical properties, the bulk quantities henceforth follow their prescribed behavior.

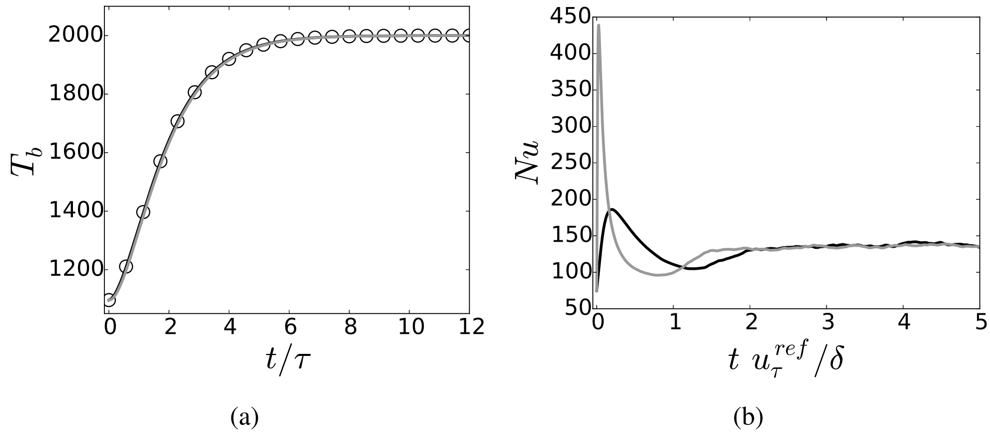


Figure 8: Temporal evolution of T_b in Kelvin (a) and Nusselt number (b) for varying Re_b at the same time with the control equations. Circles: theoretical solution; Black plain line: $\tau = 0.33\delta/u_\tau^{ref}$; Gray plain line: $\tau = 0.033\delta/u_\tau^{ref}$.

5. Conclusion

Bulk Reynolds number and bulk temperature are key quantities when reporting results in channel flow simulations. There are situations are when one wishes to control these parameters although they are actually results of the simulations when imposing a fixed source term in the momentum and energy equations. A control strategy has been developed to carefully set the desired target values for the bulk Reynolds number and bulk temperature by temporally adapting the prescribed source terms. Final equations appear as PI controllers with a thorough setting of the coefficients so that the dynamics of the quantities of interest is exactly controlled when the flow thermo-physical properties are constant or even variable. The controlling method is validated in several cases and is shown to be robust and efficient. Such a method enables to safely study the effect fo different numerical or physical description of the flow while safely maintaining identical bulk Reynolds numbers and/or bulk temperatures.

Acknowledgements

This work was granted access to the HPC resources of [CCRT/CINES/IDRIS] under the allocations 2013-2b0164 and 2015-2b0164 made by GENCI (Grand Equipement National de Calcul Intensif). Vincent Moureau and Ghislain Lartigue from the CORIA lab, and the SUCCESS scientific group are acknowledged for providing the YALES2 code.

[1] P. Moin, K. Mahesh, Direct numerical simulation: A tool in turbulence research, Annual Review of Fluid Mechanics 30 (1998) 539–578.

[2] U. Piomelli, E. Balaras, Wall-layer models for large-eddy simulations, Annual review of fluid mechanics 34 (2002) 349–374.

[3] R. Garcia-Mayoral, J. Jimenez, Hydrodynamic stability and breakdown of the viscous regime over riblets, Journal of Fluid Mechanics 678 (2011) 317–347. doi:10.1017/jfm.2011.114.

[4] S. Bocquet, P. Sagaut, J. Jouhaud, A compressible wall model for large-eddy simulation with application to prediction of aerothermal quantities, Physics of Fluids 24 (6) (2012) 065103.

[5] O. Cabrit, F. Nicoud, Direct simulations for wall modeling of multicomponent reacting compressible turbulent flows, Phys. Fluids 21 (5) (2009) 055108.

[6] S. Ghosh, R. Friedrich, M. Pfitzner, C. Stemmer, B. Cuenot, M. El Hafi, Effects of radiative heat transfer on the structure of turbulent supersonic channel flow, Journal of Fluid Mechanics 677 (2011) 417–444.

[7] Y. F. Zhang, R. Vicquelin, O. Gicquel, J. Taine, Physical study of radiation effects on the boundary layer structure in a turbulent channel flow, International Journal of Heat and Mass Transfer 61 (2013) 654–666.

[8] E. Lenormand, P. Sagaut, L. Phuoc, Large eddy simulation of subsonic and supersonic channel flow at moderate Reynolds number, International Journal for Numerical Methods in Fluids 32 (4) (2000) 369–406.

[9] O. Malaspinas, P. Sagaut, Wall model for large-eddy simulation based on the lattice boltzmann method, Journal of Computational Physics 275 (2014) 25–40.

[10] Y. F. Zhang, R. Vicquelin, O. Gicquel, J. Taine, A wall model for les accounting for radiation effects, International Journal of Heat and Mass Transfer 67 (2013) 712–723.

[11] V. Moureau, P. Domingo, L. Vervisch, Design of a massively parallel CFD code for complex geometries, Comptes Rendus Mécanique 339 (2-3) (2011) 141–148.

[12] V. Moureau, P. Domingo, L. Vervisch, From large-eddy simulation to direct numerical simulation of a lean premixed swirl flame: Filtered laminar flame-pdf modeling, Combustion and Flame 158 (7) (2011) 1340–1357.
[13] R. J. Kee, G. Dixon-lewis, J. Warnatz, M. E. Coltrin, J. A. Miller, A fortran computer code package for the evaluation of gas-phase, multicomponent transport properties, Tech. Rep. SAND89-8009, Sandia National Laboratories (1986).
[14] R. J. Kee, F. M. Rupley, J. A. Miller, CHEMKIN-II: A Fortran Chemical Kinetics Package for the Analysis of Gas-Phase Chemical Kinetics, Tech. Rep. SAND89-8009, Sandia National Laboratories (1989).
[15] G. Coleman, J. Kim, R. Moser, A numerical study of turbulent supersonic isothermal-wall channel flow, Journal of Fluid Mechanics 305 (1995) 159–183.