

LabVIEW Perturbed Particle Swarm Optimization Based Approach for Model Predictive Control Tuning

Mohamed Lotfi Derouiche* Soufiene Bouallègue**
Joseph Haggège*** Guillaume Sandou****

* *Research Laboratory in Automatic Control (LARA), National Engineering School of Tunis, BP 37, Le Belvédère, 1002 Tunis, Tunisia, e-mail:mohamedlotfi.derouiche@enit.rnu.tn*

** *e-mail: soufiene.bouallegue@issig.rnu.tn*

*** *e-mail: joseph.haggege@enit.rnu.tn*

**** *L2S, CentraleSupélec, CNRS, Université Paris Sud, Université Paris-Saclay, 3 rue Joliot Curie, 91192 Gif-sur-Yvette Cedex, France, e-mail: Guillaume.Sandou@centralesupelec.fr*

Abstract: In this paper, a new Model Predictive Controller (MPC) parameters tuning strategy is proposed using a LabVIEW-based perturbed Particle Swarm Algorithm (pPSA). This original LabVIEW implementation of this metaheuristic algorithm is firstly validated on some test functions in order to show its efficiency and validity. The optimization results are compared with the standard PSO approach. The parameters tuning problem, i.e. the weighting factors on the output error and input increments of the MPC algorithm, is then formulated and systematically solved, using the proposed LabVIEW pPSA algorithm. The case of a Magnetic Levitation (MAGLEV) system is investigated to illustrate the robustness and superiority of the proposed pPSA-based tuning MPC approach. All obtained simulation results, as well as the statistical analysis tests for the formulated control problem with and without constraints, are discussed and compared with the Genetic Algorithm Optimization (GAO)-based technique in order to improve the effectiveness of the proposed pPSA-based MPC tuning methodology.

Keywords: Model Predictive Control, parameters tuning problem, perturbed Particle Swarm Optimization, Genetic Algorithm, LabVIEW implementation, MAGLEV system.

1. INTRODUCTION

Being part of robust and optimal control techniques, the Model Predictive Control (MPC) approach is almost pervasive in various engineering applications Mayne (2014); Rawlings and Mayne (2013); Bao-Cang (2010); Rossiter (2004). The first generation of predictive control algorithms tried to solve constrained control problems manipulating several variables in the petrochemical field in the late 80s. MPC applications are present in many systems where the PID controller reaches its limits. It covers both the Single-Input Single-Output (SISO) and Multi-Inputs Multi-Outputs (MIMO) problems for linear and nonlinear systems.

However, for model predictive controller design, the relationship between the adjustment parameters and physical phenomena is shady which make tuning task tedious, error-prone and out of reach of non-expert users. Therefore, this type of tuning still presents a difficulty for manufacturers who do not have particular expertise. Well tuning MPC parameters is still challenging in control engineering field. This challenge leads the MPC tuning to be the subject of several research projects. In Lino et al. (1993), authors proposed a tuning method taking into account the robust stability based on the frequency

response analysis. Rowe and Maciejowski (2000) proposed a tuning method using the \mathcal{H}_∞ loop-shaping concept. A heuristic method based on setting rules established by experts has been proposed in Drogies and Geest (1999). The methods of C. Rowe and Y. Lino require complicated calculation procedures and keep the need for qualified or rather expert user. In practice, the method presented by S. Drogies must be modified for each specific problem. Thus, none of the above mentioned methods has led to a systematic tuning procedure allowing a user to well design an MPC controller without resort to an expert as it is the case for PID controllers.

The MPC parameters tuning problem can be formulated as an optimization program. Many approaches are proposed in the literature for dealing with hard choice of MPC parameters. See Sandou and Olaru (2009); Sandou (2009). Knowing that the predictive control is treated as a black box, classic gradient-based optimization techniques will not be able to solve this problem. However, metaheuristics algorithms Yang (2010); Dréo et al. (2006); Siarry and Michalewicz (2008); Gendreau and Potvin (2010), which have reached a remarkable maturity, especially the Particle Swarm Optimization technique, present a solution to such problems. Recently, Suzuki et al. (2008) introduced an automatic tuning of predictive controller parameters

using the standard PSO technique. Newly, Xinchao (2010) introduced a new “perturbed PSO” algorithm, denoted as pPSA. This metaheuristic comes to refine the standard PSO technique, which after a premature convergence, is often dropped into local minima. Based on the principle of the possibility measure, the pPSA pursues to update the optimum position of the particle swarm noted gbest using the “possibility at gbest” strategy that seeks to model the lack of information regarding the true optimality of gbest. The pPSA is proposed so as to escape from the local optimal trap and the rapid loss of diversity during the optimization process. The new particle updating strategy aims to promote exploration capabilities in the search space. In this paper, a LabVIEW implementation of this efficient metaheuristic algorithm is proposed for using in MPC parameters tuning problem. An application to a position control of the suspended sphere in known Feedback MAGLEV system is investigated in order to show the effectiveness of our proposed MPC tuning approach.

The remainder of this paper is organized as follows. In Section 2, the MPC parameters tuning problem is introduced and formulated as an optimization program to be solved later using the proposed metaheuristic-based technique. A brief overview on MPC approach is firstly described in this paragraph. Section 3 presents the proposed perturbed PSO-based algorithm as well as its original implementation under graphical LabVIEW tool. This algorithm is validated through an optimization benchmark functions from the literature. An application to the MPC tuning parameters for a MAGLEV system using GA and the proposed pPSA is investigated in Section 4. All simulation results, obtained for the unconstrained and constrained formulations of such a problem, are presented and discussed. Section 5 concludes this paper.

2. MODEL PREDICTIVE CONTROL PARAMETERS TUNING PROBLEM

2.1 Basic Concepts of MPC Design

The MPC design approach focuses on constructing and optimizing feedback controllers at each discrete-time instant, see Rawlings and Mayne (2013); Bao-Cang (2010); Rossiter (2004), as shown in Fig. 1. Such predictive controllers can adjust the control action before a change in the output setpoint actually occurs. The ability of handling operational constraints in an explicit manner is one of the main reasons for the popularity and success of MPC strategies in various industrial applications, as shown in Sandou and Olaru (2009); Sandou (2009).

As depicted in Fig. 1, the main elements of the discrete-time model predictive control are the plant input, the controlled output and the reference trajectory which are denoted by $u \in \mathbb{R}$, $y \in \mathbb{R}$, and $r \in \mathbb{R}$, respectively. The plant model determines the predicted plant outputs on the prediction horizon, denoted N_p . The optimization algorithm is aimed at determining the control sequence given by $\{u(k-1+i), i = 1, 2, \dots, N_c\}$ for the control horizon, denoted N_c . Only the first element $u^*(k)$ of the optimized control sequence is applied to the plant and the control input is updated at each sampling instant. The optimization process is repeated at the next sampling time, on the basis

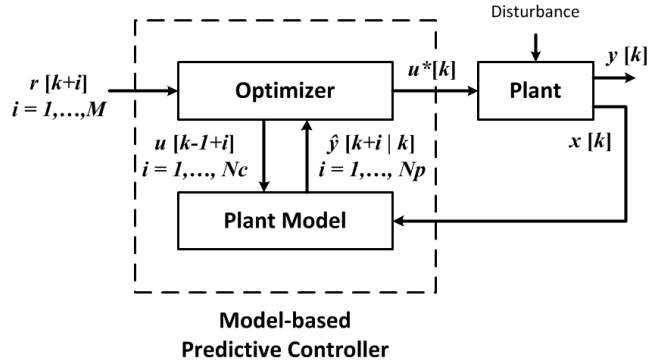


Fig. 1. Model Predictive control structure.

of the measured (or estimated from the available output $y(k)$) state $x(k)$. This sequence minimizes the specified cost function (2) in the MPC design formalism, subject to problem constraints on the input, change in input and output of the plant. The optimization algorithm assumes that $u(k-1+i) = u(k+N_c-1)$ for $N_c < i \leq N_p$.

A model describing the behavior of the controlled plant is required in MPC framework. It is assumed that such a plant is represented by a discrete-time model of the form:

$$\begin{cases} x(k+1) = \mathbf{A}x(k) + \mathbf{B}u(k) \\ y(k) = \mathbf{C}x(k) \end{cases} \quad (1)$$

This model is used to compute system predictions over a finite prediction horizon of N_p samples. So, at every sampling time and for a specified prediction and control horizons, the MPC controller attempts to minimize the following cost function:

$$J(k) = \sum_{i=1}^{N_p} \hat{e}(k+i|k)^T \mathbf{Q} \hat{e}(k+i|k) + \sum_{i=0}^{N_c-1} [\Delta u^T(k+i|k) \mathbf{R} \Delta u(k+i|k)] \quad (2)$$

where $\hat{e}(k+i|k) = \hat{y}(k+i|k) - r(k+i|k)$, $\hat{y}(k+i|k)$, $r(k+i|k)$ and $\Delta u(k+i|k)$ are the predicted plant output, the output setpoint profile and the predicted increment of change in control action at time $k+i$, given all measurements up to and including those at sampling-time k , respectively. $\mathbf{Q} = \mathbf{Q}^T > 0$ and $\mathbf{R} = \mathbf{R}^T > 0$ are weighting matrices on the output error and input increments.

Minimizing the cost function (2) is usually subject to the operational constraints on the control action, its rate of change and plant output signals. Thus, limitations of these entities can be defined, respectively, as given by Eqs. (3), (4) and (5):

$$u_{\min} \leq u(k) \leq u_{\max} \quad (3)$$

$$\Delta u_{\min} \leq \Delta u(k) \leq \Delta u_{\max} \quad (4)$$

$$y_{\min} \leq y(k) \leq y_{\max} \quad (5)$$

2.2 MPC Tuning Parameters Problem Formulation

A formulation of an optimization based tuning problem of parameters in MPC paradigm is given in the followings. Since prediction and control horizons, as well as operational constraints of MPC problem are fixed, weightings matrices \mathbf{Q} and \mathbf{R} of Eq. (2) are tuned in this study.

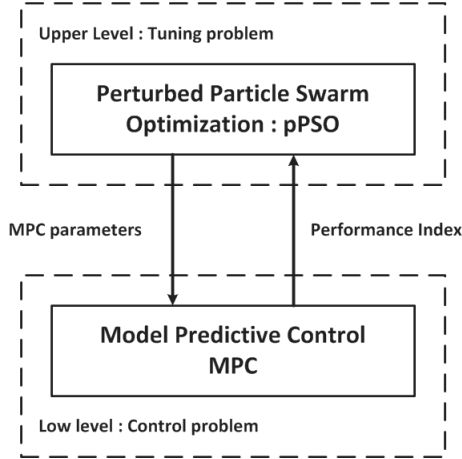


Fig. 2. Proposed pPSA based MPC tuning approach.

The concept of the proposed pPSA based MPC tuning procedure is depicted in Fig. 2.

In this paper, we consider the control of SISO systems so that \mathbf{Q} and \mathbf{R} weighting matrices become scalars to be tuned within a time-domain framework. So, the cost function, which penalizes tracking errors and to be minimized, is given as follows:

$$J_1(\tau, \mathbf{Q}, \mathbf{R}) = \sum_{\tau=0}^{+\infty} [y(\tau) - r(\tau)]^2 \quad (6)$$

where τ is the time expressed as number of samples from the beginning of simulation until achieving the steady-state domain.

3. PROPOSED PERTURBED PSO-BASED APPROACH FOR MPC TUNING

3.1 Perturbed PSO Algorithm

The perturbed particle swarm optimizer pPSA maintains a swarm of n_{PART} particles in the search space with dimension D . Each particle in the swarm is characterized by its current position $\mathbf{x}_t^i = (x_t^{i,1}, x_t^{i,2}, \dots, x_t^{i,D})$, the previous best position $\mathbf{p}_t^i = (p_t^{i,1}, p_t^{i,2}, \dots, p_t^{i,D})$, and the velocity $\mathbf{v}_t^i = (v_t^{i,1}, v_t^{i,2}, \dots, v_t^{i,D})$ at iteration $t \in \llbracket 1, n_{GEN} \rrbracket$. Where n_{GEN} is the number of generations.

The trajectory of each particle is updated according to its own flying experience and that of the best particle in the swarm as given by the followings motion equations:

$$\mathbf{x}_{t+1}^i = \mathbf{x}_t^i + \mathbf{v}_{t+1}^i \quad (7)$$

$$\mathbf{v}_{t+1}^i = w\mathbf{v}_t^i + c_1 r_{1,t}^i (\mathbf{p}_t^i - \mathbf{x}_t^i) + c_2 r_{2,t}^i (\hat{\mathbf{p}}_t^g - \mathbf{x}_t^i) \quad (8)$$

where w represents the inertia factor, c_1 and c_2 are the cognitive and the social scaling factors respectively, $r_{1,t}^i$ and $r_{2,t}^i$ are random numbers uniformly distributed in the interval $\llbracket 0, 1 \rrbracket$, and $\hat{\mathbf{p}}_t^g$ is the best previously obtained position of the i^{th} particle.

In contrast to the standard PSO algorithm, the gbest $\hat{\mathbf{p}}_t^g$ in Equation (8) of pPSA, denoted as p-gbest for "possibly at gbest", is characterized by a normal distribution as follows:

$$\hat{\mathbf{p}}_t^g = \mathcal{N}(\mathbf{p}_t^g, \sigma) \quad (9)$$

where σ represents the degree of uncertainty about the optimality of the gbest \mathbf{p}_t^g of the standard PSO algorithm as shown in Xinchao (2010).

So, for perturbed particle updating strategy, σ is modeled as some non-increasing function of the number of cycles t , called max-min model and given by Xinchao (2010) as follows:

$$\sigma(t) = \begin{cases} \sigma_{max}, & t < \alpha \times n_{GEN} \\ \sigma_{min}, & \text{otherwise} \end{cases} \quad (10)$$

where σ_{max} , σ_{min} and α are manually fixed parameters.

Two others non-increasing models (linear and random) are proposed in Xinchao (2010) and can be used instead Eq. (10). On the other hand, the function of p-gbest is to encourage the particles to explore a region beyond that defined by the search trajectory. It provides a simple and efficient exploration at the early stage when α is large and encourages local fine-tuning at the latter stage when α is small.

Finally, a pseudo code of this swarm algorithm is given, for a minimization problem, as follows:

- **Step 1:** Initialize a population of n_{PART} particles having random positions and velocities on D dimensions of the search space;
- **Step 2:** At every iteration t and for each particle \mathbf{x}_t^i , evaluate the considered optimization fitness function on the D decision variables;
- **Step 3:** Compare particle's fitness evaluation $J_{1,t}^i = J_1(\mathbf{x}_t^i)$ with its $pbest_t^i = J_1(\mathbf{p}_t^i)$. If $J_{1,t}^i \leq pbest_t^i$ then $pbest_t^i = J_{1,t}^i$ and $\mathbf{p}_t^i = \mathbf{x}_t^i$;
- **Step 4:** Identify the particle in the neighborhood with the best success so far and assign its position to the global best variable noted $gbest$;
- **Step 5:** Change the velocity and position of the particle according to the motion equations (7) and (8);
- **Step 6:** If the termination criterion is met (satisfied fitness or max of iterations), the algorithm terminates with the solution $\mathbf{x}^* = \arg \min_{\mathbf{x}_t^i} \{J_1(\mathbf{x}_t^i), \forall i, t\}$. Otherwise, go to Step 2.

3.2 Implementation of the pPSO Algorithm

In order to validate the LabVIEW implementation of perturbed PSO algorithm, a benchmark optimization with 8 functions are adopted (see Appendix A.). In order to have a fair comparison between PSO and pPSA in Xinchao (2010) and our LabVIEW coded pPSA, the parameters settings are as follows: dimension of search space $D = 30$, number of particles $n_{PART} = 30$, number of generations $n_{GEN} = 2000$, cognitive and social coefficients $c_1 = 0.5$ and $c_2 = 0.3$, inertia weight $w = 0.9$ and $\sigma_{max} = 0.15$, $\sigma_{min} = 0.001$, $\alpha = 0.5$.

All the statistical results over 30 independent runs of the implemented algorithms, as well as those published in Xinchao (2010), are summarized in Tables 1 and 2. From these numerical optimization results, it is clear that the pPSA greatly outperforms the standard PSO algorithm. The good performance, in terms of better exploring ability for promising area and exploiting for locating the optima, is carried out.



Fig. 3. Feedback Magnetic Levitation system.

4. APPLICATION TO THE MAGLEV SYSTEM CONTROL

4.1 Plant Description and Modeling

The Feedback Magnetic Levitation system (MAGLEV 33-006) of Fig. 3 is used as a process example to validate the pPSA-based approach for MPC tuning parameters.

In required range of operation, the distance h of the suspended sphere is given by the infrared photo-sensor voltage y as follows :

$$y = \gamma h + y_0 \quad (11)$$

where γ is a positive gain depending on the position sensor, and y_0 is the offset voltage such that $y \in [-2V, +2V]$.

The coil current is regulated by an inner control loop within the driver block. Its characteristic is linearly related to the input voltage u as follows, neglecting its high frequency dynamics:

$$i = \rho u + i_0 \quad (12)$$

where $\rho > 0$ is the coil resistor and $i_0 > 0$ is the offset value of current.

Table 1. Numerical optimization results over 30 runs of standard PSO from Xinchao (2010). Where Best, Mean, Median and STD mean the best, average, median result and the standard deviation over 30 runs.

Function	Best	Mean	Median	STD
f1	4.39	42.37	27.02	49.76
f2	2.03	19.95	15.37	18.04
f3	1.23e+03	6.77e+03	4.88e+03	5.37e+03
f4	1.47e+02	2.81e+03	9.98e+02	5.23e+03
f5	1.67e+02	5.62e+02	5.04e+02	2.94e+02
f6	3.81e-01	8.20e-01	7.2e-01	3.79e-01
f7	39.50	84.85	82.76	22.71
f8	1.03	1.55	1.27	7.58e-01

Table 2. Numerical optimization results over 30 runs of pPSA under LabVIEW.

Function	Best	Mean	Median	STD
f1	5.72e-06	0.00	7.32e-06	8.15e-07
f2	1.19e-02	2.4e-01	5.62e-02	5.56e-01
f3	1.66e-03	1.90	6.55e-01	2.79
f4	1.73e+01	62.40	28.10	57.10
f5	0.00	24.10	12.00	56.10
f6	1.93e-11	0.00	6.42e-11	1.88e-11
f7	53.72	81.22	82.58	15.56
f8	3.24e-07	0.00	7.40e-03	2.33e-02

The working excursion of u is limited between $-3V$, corresponding to a null coil current, and $+5V$ that defines the saturation value as given in Feedback; Santos et al. (2010). For this process, the predictive control of the suspended sphere position is investigated. The dynamics of the vertical movement of such a sphere is modeled as follows:

$$m \frac{d^2 h}{dt^2} = mg_a - K \frac{i^2}{h^2} \quad (13)$$

where K is an electromechanical conversion gain depending on the MAGLEV system, m is the mass of the sphere, g_a is the acceleration of gravity, and i is the coil current.

According to the given sensor and current driver characteristics (11) and (12), the equation (13) can be re-written as:

$$m \frac{d^2 y}{dt^2} = \gamma m g_a - K \frac{(\rho u + i_0)^2 \gamma^3}{(y - y_0)^2} \quad (14)$$

Taking $x = [y \dot{y}]^T$ as state vector, the following state space representation of the studied system is obtained:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \gamma g_a - K \frac{(\rho u + i_0)^2 \gamma^3}{m(x_1 - y_0)^2} \end{cases} \quad (15)$$

Finally, while using the values for the physical model parameters, a linear discrete-time model of the MAGLEV plant, where a zero-order-hold with 5 ms sampling period was adopted at the input of the system as in Santos et al. (2010), is given by the following state-space representation:

$$\begin{cases} x_{k+1} = \begin{bmatrix} 1.0108 & 0.0050 \\ 4.3185 & 1.0108 \end{bmatrix} x_k + \begin{bmatrix} -0.0142 \\ -5.6779 \end{bmatrix} u_k \\ y_k = [1 \ 0] x_k \end{cases} \quad (16)$$

4.2 Simulation Results and Discussion

As described in section 2.2, the optimization is performed on a SISO system using the unconstrained control problem (6). For this design, we use the control parameters of Table 3. The simulation results for the pPSA and GA algorithms are summarized in Table 4. Recall that the GA algorithm is also implemented and executed under LabVIEW environment, see W. Golebiowski (2009) and National Instruments (2009).

Table 3. GA and pPSA control parameters.

	pPSA	GA (WAPTIA)
dimension	2	2
max iterations	100	100
population size	40	40
other parameters	$c_1 = 0.5, c_2 = 0.3,$ $w = 0.9, \sigma_{\max} =$ $0.15, \sigma_{\min} = 0.001$ and $\alpha = 0.5.$	crossover prob. 0.85, mutation rate 0.005, min mut. rate 0.0005, max mut. rate 0.25 and relative fitness 1

The controlled position of the suspended sphere as well as the control input voltage of the MAGLEV current driver are shown in Fig. 4 and Fig. 5, respectively. The tracking performances of the controlled system are satisfactory but a high amplitude of the control action is noted and exceeds the saturation value, see Fig. 5. In an experimental point

Setpoint and Plant Output

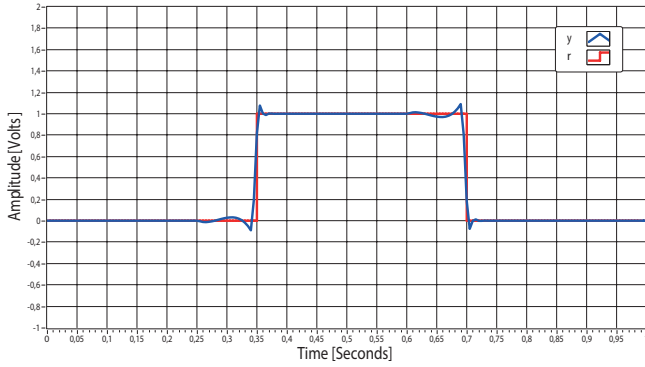


Fig. 4. pPSA-based control of the MAGLEV suspended sphere position: unconstrained optimization case.

Control Action

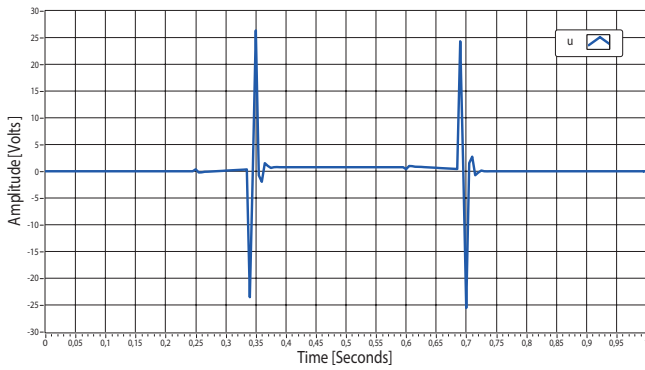


Fig. 5. pPSA-based control input voltage of the MAGLEV current driver: unconstrained optimization case.

of view, this control voltage amplitude can not be applied to the real MAGLEV system which have an upper limit of +5V. This then requires the reformulation of the control problem to incorporate this kind of operational constraints in the MPC synthesis procedure.

In order to control the real MAGLEV system and to show the effectiveness of the proposed pPSA based MPC approach, operational constraint on upper limit of the control action signal $u_{\max} = 1V$ is considered. The problem can be formulated using as a constrained optimization problem:

$$\begin{cases} J_2(\tau, \mathbf{Q}, \mathbf{R}) = \sum_{\tau=0}^{+\infty} [y(\tau) - r(\tau)]^2 \\ s.t : \max_{\tau \geq 0} |u(\tau)| \leq u_{\max} \end{cases} \quad (17)$$

PSO being an unconstrained optimization algorithm, the objective function, to be minimized using the LabVIEW

Table 4. Performances comparison of GA and pPSA for Problem (6).

	pPSA	GA (WAPTIA)
weight factor \mathbf{Q}	36.8780	0.6050
weight factor \mathbf{R}	0.0004	7.00e-06
gbest value	0.1889	0.1887
u_{\max}	26.3265	25.9861
computation time	12min, 22sec	2h, 29min, 52sec

Setpoint and Plant Output

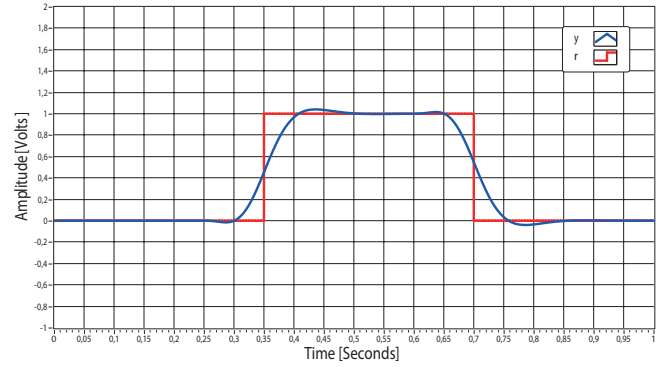


Fig. 6. pPSA-based control of the MAGLEV suspended sphere position: constrained optimization case.

Control Action

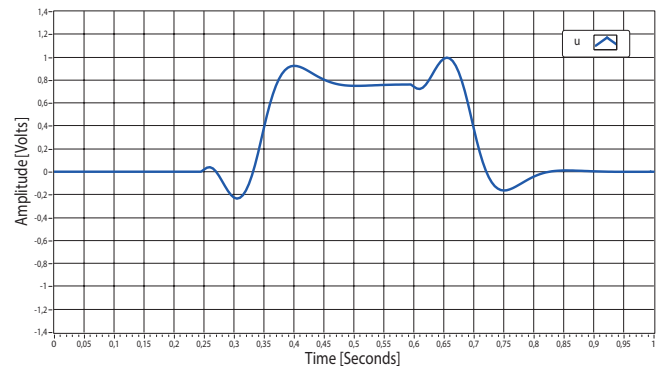


Fig. 7. pPSA-based control input voltage of the MAGLEV current driver: constrained optimization case.

coded pPSO algorithm, is augmented with the external static penalty technique, that is proportional to the degree of constraint infeasibility, and becomes as follows:

$$J_2(\tau, \mathbf{Q}, \mathbf{R}) = \sum_{\tau=0}^{+\infty} [y(\tau) - r(\tau)]^2 + \exp\left(\frac{\max_{\tau \geq 0} |u(\tau)| - u_{\max}}{u_{\max}}\right) \quad (18)$$

The LabVIEW implementation for the constrained optimization problem (18) leads to the simulation results of Fig. 6 and Fig. 7 as well as the results in Table 5. For this design, both metaheuristic algorithms improve good performances of tuned MPC controller in terms of stability, tracking dynamic behavior and handling operational constraints. However, the superiority of our proposed LabVIEW pPSA based approach is shown mainly in terms of the fastness convergence and the simplicity of implementation.

It is important to highlight that in the MPC parameters \mathbf{Q} and \mathbf{R} optimization, only the ratio of these two coefficients is most important. The same setting is obtained if we divide by 2 these two coefficients. This is shown in the comparison of the LabVIEW implemented pPSA and GA, i.e., $0.898/0.226 = 3.97$ and $59.36/14.95 = 3.97$.

Table 5. Performances comparison of GA and pPSA for Problem (18).

	pPSA	GA (WAPTIA)
weight factor \mathbf{Q}	14.9501	0.226115
weight factor \mathbf{R}	59.3676	0.898016
gbest value	2.85314	2.85314
u_{max}	0.995273	0.995252
computation time	18min, 47sec	2h, 21min, 23sec

As depicted in Table 5, this is the same tuning for the two proposed methods. The main observation is that these are the same two settings found by two different metaheuristic algorithms, so there is a greater probability that the obtained solution is the global optimum of the MPC tuning optimization problem. The superiority of the proposed pPSA technique is shown mainly at the remarkable fastness convergence in solving the MPC tuning problem as well as its simple software implementation and algorithm parameters choice.

5. CONCLUSION

In this paper, a new perturbed PSO based approach for MPC parameters tuning have been proposed and successfully applied to the position control of a MAGLEV system. The proposed pPSA algorithm is implemented under LabVIEW graphical environment and validated through a benchmark of test functions from the literature. All optimization results are compared with the standard PSO and statistical analysis are carried out in order to show the validity of such an implementation. The MPC tuning problem, such as the choice of weighting factors on the output error and input increments signals, is after that formulated as an optimization problem, with and without operational constraints, and solved using the GA (WAPTIA) and the proposed LabVIEW-based pPSA algorithm for the considered MAGLEV plant.

Forthcoming works deal with final implementation of the control laws on the real MAGLEV benchmark available in our laboratory. Increasing the size of the optimization-based tuning problem, i.e. while considering full weighting matrices, is also investigated.

REFERENCES

- Bao-Cang, D. (2010). *Modern Predictive Control*. CRC Press, Taylor & Francis Group, USA.
- Dréo, J., Pétrowski, A., Siarry, P., and Taillard, E. (2006). *Metaheuristics for Hard Optimization: Methods and Case Studies*. Springer Verlag, Berlin.
- Drogies, S. and Geest, D.D. (1999). Predictive control: propositions for the design methodology. In *Proceedings of the 1999 American Control Conference*, 647–651. San Diego CA.
- Feedback (2013). *Magnetic Levitation Control Experiments 33-942S*. www.feedback-instruments.com.
- Gendreau, M. and Potvin, J.Y. (2010). *Handbook of Metaheuristics*. Springer Verlag, New York.
- Lino, Y., Tomida, K., Fujiwara, H., and Takagi, Y. (1993). A new input-output constrained model-predictive control with frequency-domain tuning technique and its application to an ethylene plant. In *International Conference on Industrial Electronics, Control, and Instrumentation*, 457–462. Maui HI.

- Mayne, D. (2014). Model predictive control: Recent developments and future promise. *Automatica*, 50, 2967–2986.
- National Instruments (2009). *LabVIEW Control Design User Manual, 371057G-01*. www.ni.com.
- Rawlings, J. and Mayne, D. (2013). *Model Predictive Control: Theory and Design*. Nob Hill Publishing, Madison, USA.
- Rossiter, J. (2004). *Model Based Predictive Control: A practical Approach*. Control Series, CRC Press, New York.
- Rowe, C. and Maciejowski, J. (2000). Tuning mpc using h-infinity loop shaping. In *Proceedings of the 2000 American Control Conference*, 1332–1336. Chicago IL.
- Sandou, G. (2009). Metaheuristic strategy for the hierarchical predictive control of large scale energy networks. *Journal of Control Engineering and Applied Informatics*, 11, 32–40.
- Sandou, G. and Oлару, S. (2009). *Nonlinear Model Predictive Control*, chapter Particle Swarm Optimization Based NMPC: An Application to District Heating Networks. Springer Verlag, Berlin Heidelberg.
- Santos, M., Galvao, R., and Yoneyama, T. (2010). Robust model predictive control for a magnetic levitation system employing linear matrix inequalities. *ABCMSymposium Series in Mechatronics*, 4, 147–155.
- Siarry, P. and Michalewicz, Z. (2008). *Advances in Metaheuristics for Hard Optimization*. Springer Verlag, Berlin.
- Suzuki, R., Kawai, F., Nakazawa, C., Matsui, T., and Aiyoshi, E. (2008). Model predictive control using PSO. In *SICE Annual Conference*, 1981–1988. Tokyo.
- W. Golebiowski (2009). *Waptia: Genetic optimization algorithm, LabVIEW Advanced Virtual Architects*. https://lavag.org.
- Xinchao, Z. (2010). A perturbed particle swarm algorithm for numerical optimization. *Applied Soft Computing*, 10, 119–124.
- Yang, X. (2010). *Engineering optimization: an introduction with metaheuristic applications*. John Wiley & Sons, New Jersey, USA.

Appendix A. TEST FUNCTIONS BENCHMARK

Function	Domain	f_{min}
$f_1 = \sum_{i=1}^n x_i^2$	$[-100, 100]^n$	0
$f_2 = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $	$[-10, 10]^n$	0
$f_3 = \sum_{i=1}^n \left(\sum_{j=1}^i x_j \right)^2$	$[-100, 100]^n$	0
$f_4 = \sum_{i=1}^n \left[100 (x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right]$	$[-30, 30]^n$	0
$f_5 = \sum_{i=1}^n (x_i + 0, 5)^2$	$[-100, 100]^n$	0
$f_6 = \sum_{i=1}^n ix_i^4 + random [0, 1]$	$[-1.28, 1.28]^n$	0
$f_7 = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	$[-5.12, 5.12]^n$	0
$f_8 = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$[-600, 600]^n$	0