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LabVIEW Perturbed Particle Swarm Optimization Based Approach for Model Predictive Control Tuning

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Abstract: In this paper, a new Model Predictive Controller (MPC) parameters tuning strategy is proposed using a LabVIEW-based perturbed Particle Swarm Algorithm (pPSA). This original LabVIEW implementation of this metaheuristic algorithm is firstly validated on some test functions in order to show its efficiency and validity. The optimization results are compared with the standard PSO approach. The parameters tuning problem, i.e. the weighting factors on the output error and input increments of the MPC algorithm, is then formulated and systematically solved, using the proposed LabVIEW pPSA algorithm. The case of a Magnetic Levitation (MAGLEV) system is investigated to illustrate the robustness and superiority of the proposed pPSA-based tuning MPC approach. All obtained simulation results, as well as the statistical analysis tests for the formulated control problem with and without constraints, are discussed and compared with the Genetic Algorithm Optimization (GAO)-based technique in order to improve the effectiveness of the proposed pPSA-based MPC tuning methodology.

Keywords: Model Predictive Control, parameters tuning problem, perturbed Particle Swarm Optimization, Genetic Algorithm, LabVIEW implementation, MAGLEV system.

1. INTRODUCTION

Being part of robust and optimal control techniques, the Model Predictive Control (MPC) approach is almost pervasive in various engineering applications Mayne (2014); Rawlings and Mayne (2013); Bao-Cang (2010); Rossiter (2004). The first generation of predictive control algorithms tried to solve constrained control problems manipulating several variables in the petrochemical field in the late 80s. MPC applications are present in many systems where the PID controller reaches its limits. It covers both the Single-Input Single-Output (SISO) and Multi-Inputs Multi-Outputs (MIMO) problems for linear and nonlinear systems.

However, for model predictive controller design, the relationship between the adjustment parameters and physical phenomena is shady which make tuning task tedious, error-prone and out of reach of non-expert users. Therefore, this type of tuning still presents a difficulty for manufacturers who do not have particular expertise. Well tuning MPC parameters is still challenging in control engineering field. This challenge leads the MPC tuning to be the subject of several research projects. In Lino et al. (1993), authors proposed a tuning method taking into account the robust stability based on the frequency response analysis. Rowe and Maciejowski (2000) proposed a tuning method using the $H_\infty$ loop-shaping concept. A heuristic method based on setting rules established by experts has been proposed in Drogies and Geest (1999). The methods of C. Rowe and Y. Lino require complicated calculation procedures and keep the need for qualified or rather expert user. In practice, the method presented by S. Drogies must be modified for each specific problem. Thus, none of the above mentioned methods has led to a systematic tuning procedure allowing a user to well design an MPC controller without resort to an expert as it is the case for PID controllers.

The MPC parameters tuning problem can be formulated as an optimization program. Many approaches are proposed in the literature for dealing with hard choice of MPC parameters. See Sandou and Olaru (2009); Sandou (2009). Knowing that the predictive control is treated as a black box, classic gradient-based optimization techniques will not be able to solve this problem. However, metaheuristics algorithms Yang (2010); Dréo et al. (2006); Siarry and Michalewicz (2008); Gendreau and Potvin (2010), which have reached a remarkable maturity, especially the Particle Swarm Optimization technique, present a solution to such problems. Recently, Suzuki et al. (2008) introduced an automatic tuning of predictive controller parameters
using the standard PSO technique. Newly, Xinchao (2010) introduced a new “perturbed PSO” algorithm, denoted as pPSA. This metaheuristic comes to refine the standard PSO technique, which after a premature convergence, is often dropped into local minima. Based on the principle of the possibility measure, the pPSA pursues to update the optimum position of the particle swarm noted gbest using the “possibility at gbest” strategy that seeks to model the lack of information regarding the true optimality of gbest. The pPSA is proposed so as to escape from the local optimal trap and the rapid loss of diversity during the optimization process. The new particle updating strategy aims to promote exploration capabilities in the search space. In this paper, a LabVIEW implementation of this efficient metaheuristic algorithm is proposed for using in MPC parameters tuning problem. An application to a position control of the suspended sphere in known Feedback MAGLEV system is investigated in order to show the effectiveness of our proposed MPC tuning approach.

The remainder of this paper is organized as follows. In Section 2, the MPC parameters tuning problem is introduced and formulated as an optimization program to be solved later using the proposed metaheuristic-based technique. A brief overview on MPC approach is firstly described in this paragraph. Section 3 presents the proposed perturbed PSO-based algorithm as well as its original implementation under graphical LabVIEW tool. This algorithm is validated through an optimization benchmark functions. A formulation of an optimization based tuning problem of such a problem, are presented and discussed. Section 5 concludes this paper.

2. MODEL PREDICTIVE CONTROL PARAMETERS TUNING PROBLEM

2.1 Basic Concepts of MPC Design

The MPC design approach focuses on constructing and optimizing feedback controllers at each discrete-time instant, see Rawlings and Mayne (2013); Bao-Cang (2010); Rossiter (2004), as shown in Fig. 1. Such predictive controllers can adjust the control action before a change in the output setpoint actually occurs. The ability of handling operational constraints in an explicit manner is one of the main reasons for the popularity and success of MPC strategies in various industrial applications, as shown in Sandou and Olaru (2009); Sandou (2009).

As depicted in Fig. 1, the main elements of the discrete-time model predictive control are the plant input, the controlled output and the reference trajectory which are denoted by $u \in \mathbb{R}$, $y \in \mathbb{R}$, and $r \in \mathbb{R}$, respectively. The plant model determines the predicted plant outputs on the prediction horizon, denoted $N_p$. The optimization algorithm is aimed at determining the control sequence given by \{ $u \in \mathbb{R}$, $i = 1, 2, \ldots, N_c$ \} for the control horizon, denoted $N_c$. Only the first element $u^* \in \mathbb{R}$ of the optimized control sequence is applied to the plant and the control input is updated at each sampling instant. The optimization process is repeated at the next sampling time, on the basis of the measured (or estimated from the available output $y(k)$) state $x(k)$. This sequence minimizes the specified cost function (2) in the MPC design formalism, subject to problem constraints on the input, change in input and output of the plant. The optimization algorithm assumes that $u(k - 1 + i) = u(k + N_c - 1)$ for $N_c < i \leq N_p$.

A model describing the behavior of the controlled plant is required in MPC framework. It is assumed that such a plant is represented by a discrete-time model of the form:

$$\begin{align*}
x(k + 1) &= Ax(k) + Bu(k) \\
y(k) &= Cx(k)
\end{align*}$$

(1)

This model is used to compute system predictions over a finite prediction horizon of $N_p$ samples. So, at every sampling time and for a specified prediction and control horizons, the MPC controller attempts to minimize the following cost function:

$$J(k) = \sum_{i=1}^{N_p} \tilde{e}(k + i|k)^T Q \tilde{e}(k + i|k) + \sum_{i=0}^{N_c-1} [ \Delta u^T (k + i|k) R \Delta u (k + i|k) ]$$

(2)

where $\tilde{e}(k + i|k) = \hat{y}(k + i|k) - r(k + i|k)$, $\dot{y}(k + i|k)$, $r(k + i|k)$ and $\Delta u(k + i|k)$ are the predicted plant output, the output setpoint profile and the predicted increment of change in control action at time $k + i$, given all measurements up to and including those at sampling-time $k$, respectively. $Q = Q^T > 0$ and $R = R^T > 0$ are weighting matrices on the output error and input increments.

Minimizing the cost function (2) is usually subject to the operational constraints on the control action, its rate of change and plant output signals. Thus, limitations of these entities can be defined, respectively, as given by Eqs. (3), (4) and (5):

$$u_{\text{min}} \leq u(k) \leq u_{\text{max}}$$

(3)

$$\Delta u_{\text{min}} \leq \Delta u(k) \leq \Delta u_{\text{max}}$$

(4)

$$y_{\text{min}} \leq y(k) \leq y_{\text{max}}$$

(5)

2.2 MPC Tuning Parameters Problem Formulation

A formulation of an optimization based tuning problem of parameters in MPC paradigm is given in the followings. Since prediction and control horizons, as well as operational constraints of MPC problem are fixed, weightings matrices $Q$ and $R$ of Eq. (2) are tuned in this study.
The concept of the proposed pPSA based MPC tuning procedure is depicted in Fig. 2.

In this paper, we consider the control of SISO systems so that \( Q \) and \( R \) weighting matrices become scalars to be tuned within a time-domain framework. So, the cost function, which penalizes tracking errors and to be minimized, is given as follows:

\[
J_1 (\tau, Q, R) = \sum_{\tau=0}^{\infty} [y(\tau) - r(\tau)]^2
\]

where \( \tau \) is the time expressed as number of samples from the beginning of simulation until achieving the steady-state domain.

3. PROPOSED PERTURBED PSO-BASED APPROACH FOR MPC TUNING

3.1 Perturbed PSO Algorithm

The perturbed particle swarm optimizer pPSA maintains a swarm of \( n_{PART} \) particles in the search space with dimension \( D \). Each particle in the swarm is characterized by its current position \( \mathbf{x}_i^t = (x_{i,1}^t, x_{i,2}^t, \ldots, x_{i,D}^t) \), the previous best position \( \hat{\mathbf{p}}_i^t = (\hat{p}_{i,1}^t, \hat{p}_{i,2}^t, \ldots, \hat{p}_{i,D}^t) \), and the velocity \( \mathbf{v}_i^t = (v_{i,1}^t, v_{i,2}^t, \ldots, v_{i,D}^t) \) at iteration \( t \in [1, n_{GEN}] \). Where \( n_{GEN} \) is the number of generations.

The trajectory of each particle is updated according to its own flying experience and that of the best particle in the swarm as given by the following motion equations:

\[
x_{i+1}^t = x_i^t + \mathbf{v}_i^t + w\mathbf{v}_i^t + c_1 r_{1,t}^i (\hat{\mathbf{p}}_i^t - x_i^t) + c_2 r_{2,t}^i (\hat{\mathbf{p}}_g^t - x_i^t)
\]

where \( w \) represents the inertia factor, \( c_1 \) and \( c_2 \) are the cognitive and the social scaling factors respectively, \( r_{1,t}^i \) and \( r_{2,t}^i \) are random numbers uniformly distributed in the interval \([0, 1]\), and \( \hat{\mathbf{p}}_g^t \) is the best previously obtained position of the \( i \)th particle.

In contrast to the standard PSO algorithm, the gbest \( \hat{\mathbf{p}}_g^t \) in Equation (8) of pPSA, denoted as p-gbest for "possibly at gbest", is characterized by a normal distribution as follows:

\[
\hat{\mathbf{p}}_g^t = \mathcal{N} (\hat{\mathbf{p}}_g^t, \sigma)
\]

where \( \sigma \) represents the degree of uncertainty about the optimality of the gbest \( \hat{\mathbf{p}}_g^t \) of the standard PSO algorithm as shown in Xinchao (2010).

So, for perturbed particle updating strategy, \( \sigma \) is modeled as some non-increasing function of the number of cycles \( t \), called max-min model and given by Xinchao (2010) as follows:

\[
\sigma (t) = \begin{cases} \sigma_{\text{max}}, & t < \alpha \times n_{\text{GEN}} \\ \sigma_{\text{min}}, & \text{otherwise} \end{cases}
\]

where \( \sigma_{\text{max}}, \sigma_{\text{min}} \) and \( \alpha \) are manually fixed parameters.

Two others non-increasing models (linear and random) are proposed in Xinchao (2010) and can be used instead Eq. (10). On the other hand, the function of p-gbest is to encourage the particles to explore a region beyond that defined by the search trajectory. It provides a simple and efficient exploration at the early stage when \( \alpha \) is large and encourages local fine-tuning at the latter stage when \( \alpha \) is small.

Finally, a pseudo code of this swarm algorithm is given, for a minimization problem, as follows:

- **Step 1**: Initialize a population of \( n_{PART} \) particles having random positions and velocities on \( D \) dimensions of the search space;
- **Step 2**: At every iteration \( t \) and for each particle \( \mathbf{x}_i^t \), evaluate the considered optimization fitness function on the \( D \) decision variables;
- **Step 3**: Compare particle’s fitness evaluation \( J_{1,t}^i = J_1 (\mathbf{x}_i^t) \) with its \( \text{pbest}_t^i = J_1 (\hat{\mathbf{p}}_i^t) \). If \( J_{1,t}^i \leq \text{pbest}_t^i \) then \( \text{pbest}_t^i = J_{1,t}^i \) and \( \hat{\mathbf{p}}_i^t = \mathbf{x}_i^t \);
- **Step 4**: Identify the particle in the neighborhood with the best success so far and assign its position to the global best variable noted \( gbest \);
- **Step 5**: Change the velocity and position of the particle according to the motion equations (7) and (8);
- **Step 6**: If the termination criterion is met (satisfied fitness or max of iterations), the algorithm terminates with the solution \( \mathbf{x}^* = \arg \min \{ J_1 (\mathbf{x}_i^t) \}, \forall i, t \} \). Otherwise, go to Step 2.

3.2 Implementation of the pPSO Algorithm

In order to validate the LabVIEW implementation of perturbed PSO algorithm, a benchmark optimization with 8 functions are adopted (see Appendix A.). In order to have a fair comparison between PSO and pPSA in Xinchao (2010) and our LabVIEW coded pPSA, the parameters settings are as follows: dimension of search space \( D = 30 \), number of particles \( n_{PART} = 30 \), number of generations \( n_{GEN} = 2000 \), cognitive and social coefficients \( c_1 = 0.5 \) and \( c_2 = 0.3 \), inertia weight \( w = 0.9 \) and \( \sigma_{\text{max}} = 0.15 \), \( \sigma_{\text{min}} = 0.001 \), \( \alpha = 0.5 \).

All the statistical results over 30 independent runs of the implemented algorithms, as well as those published in Xinchao (2010), are summarized in Tables 1 and 2. From these numerical optimization results, it is clear that the pPSA greatly outperforms the standard PSO algorithm. The good performance, in terms of better exploring ability for promising area and exploiting for locating the optima, is carried out.
The working excursion of $u$ is limited between -3V, corresponding to a null coil current, and +5V that defines the saturation value as given in Feedback; Santos et al. (2010). For this process, the predictive control of the suspended sphere position is investigated. The dynamics of the vertical movement of such a sphere is modeled as follows:

$$m\frac{d^2h}{dt^2} = mg_u - K\frac{i^2}{h^2}$$  \hspace{1cm} (13)

where $K$ is an electromechanical conversion gain depending on the MAGLEV system, $m$ is the mass of the sphere, $g_u$ is the acceleration of gravity, and $i$ is the coil current. According to the given sensor and current driver characteristics (11) and (12), the equation (13) can be re-written as:

$$m\frac{d^2y}{dt^2} = \gamma mg_u - K (\rho u + i_0)^2 \gamma^3$$ \hspace{1cm} (14)

Taking $x = [y \ y]^{T}$ as state vector, the following state space representation of the studied system is obtained:

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \gamma g_u - K (\rho u + i_0)^2 \gamma^3 \\
\dot{y}_k &= [1 \ 0] x_k
\end{align*}$$ \hspace{1cm} (15)

Finally, while using the values for the physical model parameters, a linear discrete-time model of the MAGLEV plant, where a zero-order-hold with 5 ms sampling period was adopted at the input of the system as in Santos et al. (2010), is given by the following state-space representation:

$$\begin{align*}
x_{k+1} &= \begin{bmatrix} 1.0108 & 0.0050 \\ 4.3185 & 1.0108 \end{bmatrix} x_k + \begin{bmatrix} -0.0142 \\ -5.6779 \end{bmatrix} u_k \\
y_k &= [1 \ 0] x_k
\end{align*}$$ \hspace{1cm} (16)

### 4.2 Simulation Results and Discussion

As described in section 2.2, the optimization is performed on a SISO system using the unconstrained control problem (6). For this design, we use the control parameters of Table 3. The simulation results for the pPSA and GA algorithms are summarized in Table 4. Recall that the GA algorithm is also implemented and executed under LabVIEW environment, see W. Golebiowski (2009) and National Instruments (2009).

**Table 3. GA and pPSA control parameters.**

<table>
<thead>
<tr>
<th>Function</th>
<th>pPSA</th>
<th>GA (WAPTIA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dim.</td>
<td>max iterations</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>c1</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>c2</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>w</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\text{max}}$</td>
<td>0.85, mutation rate</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\text{min}}$</td>
<td>0.001, min mut. rate</td>
</tr>
<tr>
<td></td>
<td>$\alpha$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The controlled position of the suspended sphere as well as the control input voltage of the MAGLEV current driver are shown in Fig. 4 and Fig. 5, respectively. The tracking performances of the controlled system are satisfactory but a high amplitude of the control action is noted and exceeds the saturation value, see Fig. 5. In an experimental point
of the proposed pPSA based MPC approach, operational constraint on upper limit of the control action signal $u_{\text{max}} = 1\text{V}$ is considered. The problem can be formulated using as a constrained optimization problem:

$$J_2 (\tau, Q, R) = \sum_{\tau = 0}^{+\infty} |y(\tau) - r(\tau)|^2$$

$$\text{s.t. } \max_{\tau \geq 0} |u(\tau)| \leq u_{\text{max}}$$

(17)

PSO being an unconstrained optimization algorithm, the objective function, to be minimized using the LabVIEW coded pSO algorithm, is augmented with the external static penalty technique, that is proportional to the degree of constraint infeasibility, and becomes as follows:

$$J_2 (\tau, Q, R) = \sum_{\tau = 0}^{+\infty} |y(\tau) - r(\tau)|^2$$

$$+ \exp \left( \frac{\max_{\tau \geq 0} |u(\tau)| - u_{\text{max}}}{u_{\text{max}}} \right)$$

(18)

The LabVIEW implementation for the constrained optimization problem (18) leads to the simulation results of Fig. 6 and Fig. 7 as well as the results in Table 5. For this design, both metaheuristic algorithms improve good performances of tuned MPC controller in terms of stability, tracking dynamic behavior and handling operational constraints. However, the superiority of our proposed LabVIEW pPSA based approach is shown mainly in terms of the fastness convergence and the simplicity of implementation.

It is important to highlight that in the MPC parameters $Q$ and $R$ optimization, only the ratio of these two coefficients is most important. The same setting is obtained if we divide by 2 these two coefficients. This is shown in the comparison of the LabVIEW implemented pPSA and GA, i.e., $0.898/0.226 = 3.97$ and $59.36/14.95 = 3.97$. 

Table 4. Performances comparison of GA and pPSA for Problem (6).

<table>
<thead>
<tr>
<th></th>
<th>pPSA</th>
<th>GA (WAPTIA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight factor $Q$</td>
<td>36.8780</td>
<td>0.6050</td>
</tr>
<tr>
<td>weight factor $R$</td>
<td>0.0004</td>
<td>7.00e-06</td>
</tr>
<tr>
<td>best value</td>
<td>0.1889</td>
<td>0.1887</td>
</tr>
<tr>
<td>$u_{\text{max}}$</td>
<td>26.3265</td>
<td>28.8861</td>
</tr>
<tr>
<td>computation time</td>
<td>12min, 22sec</td>
<td>2h, 20min, 52sec</td>
</tr>
</tbody>
</table>

Fig. 4. pPSA-based control of the MAGLEV suspended sphere position: unconstrained optimization case.

Fig. 5. pPSA-based control input voltage of the MAGLEV current driver: unconstrained optimization case.

Fig. 6. pPSA-based control of the MAGLEV suspended sphere position: constrained optimization case.

Fig. 7. pPSA-based control input voltage of the MAGLEV current driver: constrained optimization case.
As depicted in Table 5, this is the same tuning for the two proposed methods. The main observation is that these are the same two settings found by two different metaheuristic algorithms, so there is a greater probability that the obtained solution is the global optimum of the MPC tuning optimization problem. The superiority of the proposed pPSA technique is shown mainly at the remarkable fastness convergence in solving the MPC tuning problem as well as its simple software implementation and algorithm parameters choice.

5. CONCLUSION

In this paper, a new perturbed PSO based approach for MPC parameters tuning have been proposed and successfully applied to the position control of a MAGELEV system. The proposed pPSA algorithm is implemented under LabVIEW graphical environment and validated through a benchmark of test functions from the literature. All optimization results are compared with the standard PSO and statistical analysis are carried out in order to show the validity of such an implementation. The MPC tuning problem, such as the choice of weighting factors on the output error and input increments signals, is after that formulated as an optimization problem, with and without operational constraints, and solved using the GA (WAPTIA) and the proposed LabVIEW-based pPSA algorithm for the considered MAGELEV plant.

Forthcoming works deal with final implementation of the control laws on the real MAGELEV benchmark available in our laboratory. Increasing the size of the optimization-based tuning problem, i.e. while considering full weighting matrices, is also investigated.

REFERENCES


Appendix A. TEST FUNCTIONS BENCHMARK

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
<th>$f_{\min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1 = \sum_{i=1}^{n} x_i^2$</td>
<td>$[-100, 100]^n$</td>
<td>0</td>
</tr>
<tr>
<td>$f_2 = \sum_{i=1}^{n}</td>
<td>x_i</td>
<td>+ \prod_{i=1}^{n}</td>
</tr>
<tr>
<td>$f_3 = \sum_{i=1}^{n} \left( \sum_{i=1}^{n} x_i \right)^2$</td>
<td>$[-100, 100]^n$</td>
<td>0</td>
</tr>
<tr>
<td>$f_4 = \sum_{i=1}^{n} \left( 100 (x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right)$</td>
<td>$[-30, 30]^n$</td>
<td>0</td>
</tr>
<tr>
<td>$f_5 = \sum_{i=1}^{n} (x_i + 0.5)^2$</td>
<td>$[-100, 100]^n$</td>
<td>0</td>
</tr>
<tr>
<td>$f_6 = \sum_{i=1}^{n} x_i^4 + \text{random} \ [0, 1]$</td>
<td>$[-1.28, 1.28]^n$</td>
<td>0</td>
</tr>
<tr>
<td>$f_7 = \sum_{i=1}^{n} \left( x_i^2 - 10 \cos (2\pi x_i) + 10 \right)$</td>
<td>$[-5.12, 5.12]^n$</td>
<td>0</td>
</tr>
<tr>
<td>$f_8 = \frac{1}{1000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos \left( \frac{x_i}{\sqrt{n}} \right) + 1$</td>
<td>$[-600, 600]^n$</td>
<td>0</td>
</tr>
</tbody>
</table>