BMOO: a Bayesian Multi-Objective Optimization algorithm

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Ph.D. expected duration: 2014-2017

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Abstract:

We address the problem of derivative-free multi-objective optimization of real-valued functions subject to multiple inequality constraints. The problem consists in finding an approximation of the set

$$\Gamma = \{x \in \mathbb{X} : c(x) \leq 0 \text{ and } \nexists\, x' \in \mathbb{X} \text{ such that } \; f(x') \prec f(x)\} \tag{1}$$

where $\mathbb{X} \subset \mathbb{R}^d$ is the search domain, $c = (c_i)_{1 \leq i \leq q}$ is a vector of constraint functions ($c_i : \mathbb{X} \to \mathbb{R}$), $c(x) \leq 0$ means that $c_i(x) \leq 0$ for all $1 \leq i \leq q$, $f = (f_j)_{1 \leq j \leq p}$ is a vector of objective functions to be minimized ($f_j : \mathbb{X} \to \mathbb{R}$), and $\prec$ denotes the Pareto domination rule. Both the objective functions f_j and the constraint functions c_i are assumed to be continuous. The search domain $\mathbb{X}$ is assumed to be compact—typically, $\mathbb{X}$ is a hyper-rectangle defined by bound constraints. Moreover, the objective and constraint functions are regarded as black boxes and, in particular, we assume that no gradient information is available. Finally, the objective and the constraint functions are assumed to be expensive to evaluate, which arises for instance when the values $f(x)$ and $c(x)$, for a given $x \in \mathbb{X}$, correspond to the outputs of a computationally expensive computer program. In this setting, the emphasis is on building optimization algorithms that perform well under a very limited budget of evaluations (e.g., a few hundred evaluations).

We adopt a *Bayesian approach* to this optimization problem. The essence of Bayesian optimization is to choose a prior model for the expensive-to-evaluate function(s) involved in the optimization problem—usually a Gaussian process model [6, 7] for tractability—and then to select the evaluation points sequentially in order to obtain a small average error between the approximation obtained by the optimization algorithm and the optimal solution, under the selected prior.

Our contribution is twofold. First, we propose a new sampling criterion, which can handle both multiple objectives and multiple non-linear constraints. This criterion corresponds to a one-step look-ahead Bayesian strategy, using the dominated hyper-volume as a utility function (following in this respect [4]). The dominated hyper-volume is defined using an *extended domination rule*, which handles both the objectives and constraints jointly, in the spirit of [5]. This makes it possible to deal with problems where no feasible solution is known at the beginning of the optimization procedure. The sampling criterion may be written under the form of an integral:

$$\rho_n(x) = \int_{G_n} \mathbb{P}_n(\xi(x) \lhd y)\, \mathrm{d}y\,, \quad x \in \mathbb{X} \tag{2}$$

where G_n is the set of all non-dominated points, $\mathbb{P}_n$ stands for the posterior probability given n evaluations, ξ is the vector-valued random process modeling the objectives and the constraints, and $\lhd$ denotes our extended domination rule. Several criteria from the literature are recovered as special cases.

The second part of our contribution resides in the numerical methods employed to compute and optimize the sampling criterion. Indeed, this criterion takes the form of an integral over the

space of constraints and objectives, for which no analytical expression is available in the general case. Besides, it must be optimized at each iteration of the algorithm to determine the next evaluation point. In order to compute the integral, we use an algorithm similar to the *subset simulation method* [1, 3], which is well known in the field of structural reliability and rare event estimation. For the optimization of the criterion, we resort to *Sequential Monte Carlo* (SMC) techniques, following earlier work by [2] for single-objective bound-constrained problems. The resulting algorithm is called BMOO (for Bayesian multi-objective optimization).

The new algorithm is compared with reference methods on both single- and multi-objective optimization problems with very promising results.

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Acknowledgements. This research work has been carried out in the frame of the Technological Research Institute SystemX, and therefore granted with public funds within the scope of the French Program *Investissements d'Avenir*.

Short biography – I am titular of an engineering diploma from the french engineering school ISAE-Supaero (Toulouse) and of a Master of Research from the Paul Sabatier University (Toulouse). I started my PhD thesis with Supelec at the Technological Research Institute SystemX in January 2014 under the ROM project (model Reduction and Multi-physic Optimization). Industrial partners of the project are SAFRAN (Snecma), Cenaero, Airbus group, Renault and ESI group.