# Hierarchical Control of a Wind Farm for Wake Interaction Minimization

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**Abstract:** The problem of controlling a wind farm for power optimization by minimizing the wake interaction among wind turbines is addressed. We aim to evaluate the real gain in farm power production when the dynamics of the controlled turbines are taken into account. The proposed local control enables the turbines to track the required power references in the whole operating envelope. Simulations are carried out based on a wind farm of 600 $kW$ turbines and they show the actual benefit of considering the wake effect in the optimization algorithm.

*Keywords:* Wind turbines, power tracking, feedback linearization, predictive control.

## 1. INTRODUCTION

Technology and control methods for wind energy production through the use of wind turbines have nowadays reached a mature level of the state of the art. Variable-speed-variable-pitch wind turbines allow a good degree of adaptability to a wide range of wind conditions for the maximum power capture. Nonetheless, new challenges, as the ones posed by Europe 2020, and opportunities, thanks to developments in the field of control and optimization, have pushed the target further ahead towards a better exploitation of the wind source. For this reason, in recent years, we have witnessed a relevant increase in the installation of wind farms composed of several wind turbines, e.g. more than one hundred for some offshore wind farms. This in turns suggested to take in consideration aerodynamic interaction among the turbines when the power maximization of large wind farms is concerned. Indeed, when extracting kinetic energy from the wind, a wind turbine causes a reduction of the wind speed in the downstream wake. As a result a turbine, standing in the wake of an upstream one, experiences a reduction of available wind power (see Gebraad et al. (2013)). Intuitively as the number of wind turbines of a wind farm increases, the wake effect becomes more important, so that considering it when optimizing the wind production proves potential gain with respect to classic individual turbine maximum power point tracking (MPPT), (e.g. Park and Law (2015)). This mainly justifies a growing interest in *cooperative* methods to control wind turbines belonging to large wind farms.

One of the difficulties when dealing with wind farm power maximization subject to wake interaction lies in the solution of the optimization problem itself. This is mainly due to the lack of reliable models of the involved aerodynamic phenomena and, even when the latter are available, simplification is needed to let practical implementation feasible. Indeed, since a new optimization needs to be run each time that wind conditions change, one important feature of the optimization algorithm is fast convergence. As a result two main approaches to the optimization problem have been explored. On the one hand, model-free, data-driven methods have been employed by Marden et al. (2013), Gebraad et al. (2013), and Park and Law (2016), inspired by game theory and decentralized approaches. On the other hand, model-based optimization has been proposed, as in (Tian et al. (2014)), and (Park and Law (2015)), where a trade-off between model complexity and speed of convergence to a solution has to be taken into account. However, as stated by Marden et al. (2013), the aforementioned optimization algorithms assume the existence of local control strategies for individual wind turbines that can stabilize around any feasible optimal set point, solution of the wind farm optimization problem. Even if proving a potential benefit in the amount of extracted wind power, these assumptions are not necessarily realistic as either the dynamics of the optimization variables and the performance of the local controllers play an important role in the actual gain. In this paper we address the problem of maximizing a wind farm power production, based on a static optimization at high level for optimal set points generation and local control at low level to stabilize the wind turbines around them. Nonetheless we mainly focus on the local controller performance to demonstrate what can actually be achieved by means of the proposed hierarchical architecture. In literature few works have analyzed the effective gain of wind farm optimization under wake effect when the dynamics of the controlled turbines are considered. In (Heer et al. (2014)) a local controller based on system linear approximation is employed and it shows a 1% energy gain with respect to classic *greed* control, where farm optimization would not be performed.

The central aspect that motivated this work is that, when cooperative optimization is employed, the optimal set points delivered to each turbine in the wind farm can deviate from the classic power references typically used in greed control. As it is well known, the former consist in the MPPT algorithm at low wind speed, and in stabilizing the power at its nominal value at high wind speed (see Ackermann (2005)). To do so, according to the current value of wind speed, references for the turbine rotor angular speed and for the pitch angle are obtained via the static aerodynamic relation between the mentioned variables and the aerodynamic power. However, when the desired aerodynamic power is lower than its optimal value, different set points for the rotor angular speed and the pitch angle must be provided. This is the case when considering the wake effect, as, to optimize the global power production, upstream turbines would degrade their *own* power production in order to increase the one of downstream turbines. Even though different strategies have been proposed in the literature for the choice of the *local* set points (e.g. Yingcheng and Nengling (2011), Žertek et al. (2012)), in the most cases the control architecture is based on standard linear controllers such as PID (e.g. Ramtharan et al. (2007)) and gain scheduling approaches (e.g. Wang and Seiler (2014)). To the extent of our knowledge, nonlinear techniques applied to the turbine control as in (Thomsen (2006)), and (Boukhezzar and Siguerdidjane (2011)) are conceived for well-defined operating modes, again either MPPT or power limiting at high wind speed. As a consequence their application for the entire turbine operating envelope as well as their extension to the more general task of tracking non conventional power references is not trivial. In this paper we employ a nonlinear control for power tracking, based on a combination of feedback linearization (FL) technique and model predictive control (MPC). The controller is applied to 600 $kW$ turbines, and they are not confined to work in a specific region, i.e. no assumptions were made concerning the wind speed.

The rest of the paper is organized as follows. In Section 2 the wake and the wind turbine model are provided. The main control problem and its objectives are stated in Section 3. In Section 4 we present the proposed local control architecture. We carry out simulations to test performance at the wind farm scale in Section 5. The paper ends with conclusions and future perspectives in Section 6.

## 2. WIND FARM MODELING

This section is composed of two main parts. In the first one we consider an analytic representation of the wake model and the global wind farm power function. The reason is twofold. Firstly, this model will serve for the computation of the optimal set points via a gradient-based optimization. Secondly, we aim to use it for a dynamic simulation of the wind farm. In the second part we derive the wind turbine model. The composition of the two models let us describe the global wind farm functioning.

### *2.1 Wake model and global power function*

The wake model describes the aerodynamic coupling among the wind turbines of a wind farm. In other words, it is a mathematical representation of the phenomenon according to which the wind blowing on the rotor disk of a given turbine is influenced by the free stream wind $u_\infty$ blowing on the wind farm and by the operating conditions of all the upstream turbines. For the sake of simplicity we assume the wind speed $u_\infty$ uniform and its direction $\vartheta^W$ constant. In addition, without loss of generality, we consider the wind turbines oriented in the direction of the free stream wind. This simplification is allowed mainly because the slow yaw angle $\gamma$ dynamics is decoupled from the other turbine variables. It can be argued, though, that the optimal choice of $\gamma$ values in a wind farm lead to a power production improvement, as shown by Park and Law (2015). Nonetheless, consideration of $\gamma$ for wind farm optimization goes beyond the scope of this paper, which is to evaluate the actual power gain when the system *dynamics* is taken into account. For the same reason we do not aim to evaluate the effectiveness of the proposed approach with respect to less restrictive assumptions on the wind source. If the latter are to considered the higher level optimization should be complexified, and this is subject of future near work. As a consequence, for the stated purpose of this work, the turbine operating conditions having an impact on the wake effect can be represented via the induction factor $\alpha \triangleq (u_\infty - u_R)/u_\infty$, where $u_R$ is the wind speed right behind the rotor disk of a turbine. Variable $\alpha$ serves as an indicator of the extracted power from $u_\infty$. Indeed, the latter has a theoretical value of

$$P \triangleq \frac{1}{2}\rho\pi R^2 u_\infty^3 4\alpha(1-\alpha)^2\eta \tag{1}$$

where $\rho$ is the air density, $R$ the radius of the rotor disk, $\eta \in (0,1)$ the efficiency, and $C_p \triangleq 4\alpha(1-\alpha)^2$ the theoretical power coefficient. From the latter one can easily find the Betz limit $C_{p,Betz} = 0.59$ corresponding to a value of $\alpha_{Betz} = 1/3$. Note that operating a turbine at $\alpha_{Betz}$ corresponds to extracting the maximum power, i.e. MPPT. However, in real applications, $C_p$ is typically provided in turbine specifications as a look-up-table, so that $C_{p,real} \cong C_p\eta$. In the sequel we make use of a continuous wake model presented by Park and Law (2015). Here we employ a simplified version of the latter, since for the choice we made, $\gamma$ does not intervene in the model equations. The reader may refer to the aforementioned reference for a more complete description. According to the model, a turbine $i$, in a wind farm of N turbines, experiences a wind deficit with respect to $u_\infty$ such that $u_i = u_\infty(1-\delta\bar{u}_i)$, where $\delta\bar{u}_i$ is the result of multiple wakes due to all the upstream turbines $j$ with respect to turbine $i$. A widely used method to take into account multiple wakes is the conservation of kinetic energy (see Katic et al. (1986)) given by

$$\delta\bar{u}_i = \sqrt{\sum_{j=1|j\neq i}^{N} \delta\bar{u}_{ij}^2 \varphi(i,j,\vartheta^W)}$$

where $\varphi(i,j,\vartheta^W) \triangleq \frac{1}{2}(1+sign(y_i'-y_j'))abs(sign(y_i'-y_j'))$ is a simple way to determine whether a turbine $j$ is upstream with respect to the turbine $i$, $(j \to i)$, given a wind direction $\vartheta^W$, and $y_i' \triangleq x_i sin(\vartheta^W) + y_i cos(\vartheta^W)$ is the rotated coordinate with respect to the original system of coordinates shown in Fig. 1. The wind speed deficit due to the single wake of $j \to i$ is a function of the wind direction, wind farm geometry and induction factor $\alpha_j$ of turbine $j$,

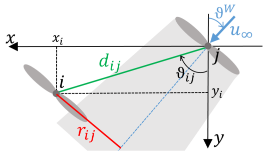


Fig. 1. Single wake and wind farm coordinates.

according to

$$\delta \bar{u}_{ij} = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R \delta u_{ij}(d_{ij}, r_{ij}, r', \theta', \alpha_j) r'\, dr' d\theta'$$
$$\text{where} \quad \delta u_{ij} \triangleq 2\alpha_j \left(\frac{R}{R+\kappa d_{ij}}\right)^2 \exp\left(-\frac{r}{R+\kappa d_{ij}}\right)^2$$

and $r = \sqrt{(r_{ij} - r'cos\theta')^2 + (r'sin\theta')^2}$, being $(r', \theta')$ the local polar coordinates on the rotor disk of turbine $i$, and $\kappa$ the wake expansion rate depending on the surface roughness of the site. In addition $d_{ij}$, $r_{ij}$ are uniquely determined via the wind farm geometry and wind direction as show in Fig. 1. Finally, if we define $\boldsymbol{\alpha} \triangleq (\alpha_1, \dots, \alpha_N)$, the global wind farm mechanical power can be expressed as the summation of individual powers, as

$$P_{tot} = \sum_{i=1}^{N} \frac{1}{2}\rho\pi R^2 u_i^3(\boldsymbol{\alpha}, u_\infty, \vartheta^W) C_p(\alpha_i)\eta \tag{2}$$

### *2.2 Wind turbine model*

The wind turbine model describes the conversion from wind power to electric power. The wind kinetic energy captured by the turbine is turned into mechanical energy of the turbine rotor, turning at an angular speed $\omega_r$ and subject to a torque $T_r$. In terms of extracted power, it can be described by the nonlinear function

$$P_r = \omega_r T_r = \frac{1}{2}\rho\pi R^2 v^3 C_{p,real}(\lambda, \vartheta) \tag{3}$$

where $\vartheta$ is the pitch angle, $v$ is the equivalent wind speed representing the wind field impact on the turbine, obtained by filtering the time series of wind data as described by Petru and Thiringer (2002), $\lambda$ is the tip speed ratio given by $\lambda = \frac{\omega_r R}{v}$. The main difference with respect to (1) is concerned with $C_p$. If for wind farm modeling it is convenient to describe the latter as a function of $\alpha$, in order to synthesize a local controller a relation between $C_p$ and the controlled variables $\omega_r$, and $\vartheta$ is needed. It is then clear that $(\lambda, \vartheta)$ and $\alpha$ are related, so that by acting on them we are able to control the value of $\alpha$. In this work we make use of the CART (Controls Advanced Research Turbine) power coefficient, shown in (Boukhezzar and Siguerdidjane (2011)), so that $C_{p,real} \equiv C_{p,CART}$. A drive train turns the slow rotor speed into high speed on the generator side, $\omega_g$. As in (Boukhezzar and Siguerdidjane (2011)), and (Thomsen (2006)), we use a two-mass model represented in Fig. 2, where $J_r$ is the rotor inertia, $K_s$ is the spring constant, $D_s$ is the damping coefficient, $n_g$ the gear ratio and $J_g$ the generator inertia. If we neglect the generator loss, then the electric power delivered to the grid is $P_e = T_g\omega_g$, where $T_g$ is the torque applied to the generator. The implicit dynamic model is then obtained by applying the Newton's law. It follows the same system of differential equations as in (Thomsen (2006))

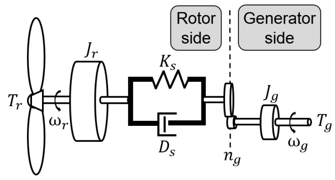


Fig. 2. Two-mass model of the turbine mechanics.

$$\begin{bmatrix} \dot{\omega}_r \\ \dot{\omega}_g \\ \dot{\delta} \\ \dot{\vartheta} \\ \dot{T}_g \end{bmatrix} = \begin{bmatrix} \frac{1}{J_r}\frac{P_r(\omega_r, \vartheta, v)}{\omega_r} - \frac{D_s}{J_r}\omega_r + \frac{D_s}{J_r n_g}\omega_g - \frac{K_s}{J_r}\delta \\ \frac{D_s}{J_g n_g}\omega_r - \frac{D_s}{J_g n_g^2}\omega_g + \frac{K_s}{J_g n_g}\delta - \frac{1}{J_g}T_g \\ \omega_r - \frac{1}{n_g}\omega_g \\ -\frac{1}{\tau_\vartheta}\vartheta + \frac{1}{\tau_\vartheta}\vartheta_r \\ -\frac{1}{\tau_T}T_g + \frac{1}{\tau_T}T_{g,r} \end{bmatrix} \tag{4}$$

where the state $\delta$ was introduced to describe the twist of the flexible drive train. Moreover the last two equations in (4) depict the dynamic of the system actuators: respectively the pitch angle and the generator torque actuator. Their dynamics are supposed to behave as a first order system. The controlled input of the system is $u \triangleq [\vartheta_r \quad T_{g,r}]^\top$, its state vector is $x \triangleq [\omega_r \quad \omega_g \quad \delta \quad \vartheta \quad T_g]^\top$, and we suppose to measure the vector $y \triangleq [\omega_r \quad \omega_g \quad \vartheta \quad T_g]^\top$, to which we add a zero-mean Gaussian noise to model the measurement noise. It is easy to see that the system is affine in the control, i.e. of the form

$$\begin{cases} \dot{x} = f(x, v) + g(x)u \\ P_e = h(x) \end{cases}$$

where $f(x, v)$, $g(x)$ can be identified from equation (4) and $h(x) \triangleq T_g\omega_g$. Note that $v$ acts as a disturbance and it makes the system *time-varying* for $v$ is a function of time. The CART turbine parameters and physical limitations are provided in (Boukhezzar and Siguerdidjane (2011)).

## 3. PROBLEM STATEMENT

### *3.1 Control objectives and optimization for the wind farm*

The high level control is responsible for the generation of the optimal power set points for a given value of $u_\infty$, and $\vartheta^W$. This means that the first control objective is to solve the following optimization problem.

$$\begin{aligned} \boldsymbol{\alpha}^* = \arg \max_{(\alpha_1, \dots, \alpha_N)} \quad & P_{tot}(\boldsymbol{\alpha}, u_\infty, \vartheta^W) \\ \text{subject to} \quad & \\ & 0 \le \alpha_i \le \alpha_{Betz} \quad i = 1, \dots, N \end{aligned} \tag{5}$$

Where the objective function in (5) is given by (2). We remind that a choice of $\alpha_i = \alpha_{Betz}$ corresponds to the MPPT power reference to the turbine $i$. Note that, according to the chosen wake model, the optimal global wind farm power $P^*_{tot}$ depends on parameters $(u_\infty, \vartheta^W)$, whereas the optimal value $\boldsymbol{\alpha}^*$ only depends on $\vartheta^W$. This basically means that the optimization needs to be run each time that the wind direction changes.

In this paper we do not focus the attention on the wide range of possibilities to solve the presented optimization problem, for which some relevant references are given in Section 1. Here we choose a gradient-based optimization method as in Park and Law (2015). In particular we employ available MATLAB®software based on SQP.

### *3.2 Control objectives for an individual wind turbine*

In standard conditions, when operating below the rated power, a wind turbine is controlled to extract the maximum power from the wind. For a given effective wind speed $v$, the former is given by

$$P_{MPPT,i} = \max_{(\omega_{r,i},\vartheta_i)} P_{r,i}(\omega_{r,i}, \vartheta_i, v) \tag{6}$$

However, when optimization (5) is performed, $\alpha_i^*$ is related to the amount of power that the turbine $i$ has to extract from the wind in such a way that the optimal *deloaded* power reference, for a given $v$, is

$$P_i^* = P_{MPPT,i} \frac{C_p(\alpha_i^*)\eta}{C_{p,CART}(\omega_{r,MPPT,i}, \vartheta_{MPPTi}, v)}$$

being $(\omega_{r,MPPT,i}, \vartheta_{MPPT,i})$ argument of optimization (6), related to turbine $i$. Since $P_i^*$ can assume values above the turbine nominal power $P_{e,n}$, the overall power reference for the generic turbine $i$ is

$$\forall t \geq 0 : 0 \leq P_{e,i}^*(t) \leq min(P_i^*, P_{e,n}) \tag{7}$$

In the sequel we drop the index $i$ when referring to a single turbine for ease of notation. Since according to (3), for a given $P_e^* < P_{MPPT}$, the choice of $(\omega_r^*,\vartheta^*)$ that yields $P_e^*$ is not unique, there exist different strategies to deload a wind turbine. They are typically based either on pitch control or on speed control (see Yingcheng and Nengling (2011)). The former consists in keeping $\omega_r$ at its MPPT value, and modifying the pitch angle. The latter involves operating the turbine at increased rotor speed. This second approach seems to be preferable. Indeed, if the wind turbine has to be deloaded, part of the mechanical power $P_r$ can be used to increase the rotor speed. As a result, part of the undelivered energy to the grid can be stored in the rotor kinetic energy

$$\Delta W_k = \frac{1}{2} J_r(\omega_{r,increased}^2 - \omega_{r,opt}^2) \tag{8}$$

If then, temporary additional power needs to be delivered to the grid the rotor has to slow down back to its MPPT value and the surplus of kinetic energy can be released to the grid. In this paper we make use of the strategy proposed by Žertek et al. (2012) which allows the turbine to work at an optimal operating point with respect to the amount of kinetic energy of the rotating masses. When deloading needs to be performed the set points of $(\omega_r,\vartheta)$ are calculated using

$$\begin{aligned} (\omega_r^*, \vartheta^*) = \arg\max_{\omega_r,\vartheta} \ \omega_r, &\quad \text{subject to} \\ P_e^* = P_r(\omega_r, \vartheta, v)& \\ \omega_{r,min} \leq \omega_r \leq \omega_{r,n}& \\ \vartheta_{min} \leq \vartheta \leq \vartheta_{max}& \end{aligned} \tag{9}$$

### *3.3 Problem formulation for an individual wind turbine*

Consider the system described by equation (4). Given an effective wind speed signal $v(t)$ and a time-varying reference trajectory of power $P_e^*(t)$ verifying (7) and such that it is an admissible steady state target for system (4), i.e. $\forall t \geq 0$ it always exists an admissible solution $(x_s(t), u_s(t))$ to the following set of equations.

$$\begin{cases} 0 = f(x_s(t), v(t)) + g(x_s(t))u_s(t) \\ P_e^*(t) = h(x_s(t)) \end{cases} \tag{10}$$

We can define the control problem as that of finding the input vector $u(t)$ that minimizes the distance between the system variables $(x(t), u(t))$ and the pair $(x_s(t), u_s(t))$, $\forall t \geq 0$. Note that (10) has to be solved together with the solution of (9), thus yielding a *unique* solution. In addition, in this paper, we make use of an observer to determine, among other relevant variables of the system, an estimation of the effective wind speed $v$, namely $\hat{v}$, blowing towards the turbine axial direction.

## 4. LOCAL CONTROL ARCHITECTURE

As it has been said, the proposed local controller is based on the composition of two techniques which basically divide its design in two phases: the FL and the MPC stage. While MPC allows to deal with state and inputs constraints explicitly, FL enables solving an optimal control problem with nonlinear constraints and whose underlying dynamic system is made *linear* by the FL itself, (see Nevistić and Morari (1995)).
In this section we state the main motivations that led us to the choice of this particular architecture and we provide a brief description of the proposed local controller. The reader can refer to our previous work in (Gionfra et al. (2016)) for details of its design.

### *4.1 Choice of FL+MPC architecture*

For the choice made, the question arises: why not directly employ nonlinear MPC (NMPC)? The FL+MPC architecture for the sake of wind turbine control is mainly motivated by the following:

- *Reduction of computational burden*: FL together with some approximations of the nonlinear constraints yields a quadratic program (QP). Being the system nonlinear, NMPC would lead to the employement of non linear programming (NLP) techniques, which generally imply a substantial increase of the computational burden, (see Nevistić and Morari (1995)).
- *Reduction of system approximations*: NLP techniques, such as SQP, are based on local approximations on either the objective function and the problem constraints. The chosen architecture only requires approximations on the latter in order to reduce the problem to a QP.
- *Choice of MPC ingridients*: being the system exactly linearized, FL can simplify the choice of some MPC ingridients, e.g. the terminal cost function. Also, the solution of the QP is a global optimum and this fact could help for the sake of stability analysis, as it represents a usual assumption in the classic proof of stability, (see Rawlings and Mayne (2009)).

### *4.2 Local Control Scheme*

The overall local controller is composed as follows (see Fig. 3). Being the wind speed value not directly available and the anemometer measurement poorly reliable,

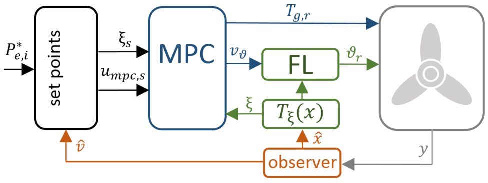


Fig. 3. FL + MPC scheme.

as previously stated, we employ a Kalman filter for the estimation of the effective wind speed and the system state. For the sake of brevity we do not provide here the filter equations. The reader may refer to (Boukhezzar and Siguerdidjane (2011)) for the basic concept and implementation in the case of wind turbine control. As far as the controller is concerned, a first FL stage is employed to linearize system (4). As in Thomsen (2006), FL is employed to target directly the system non linearity and it makes use of a change of coordinates, which defines the new system state variables

$$\xi \triangleq T_\xi(x) = [x_1 \quad L_f x_1 \quad x_2 \quad x_3 \quad x_5]^\top$$

and of the linearizing input

$$\vartheta_{r,FL} \triangleq \vartheta_r = \frac{1}{\beta(\xi,\vartheta,v)}(-\alpha(\xi,\vartheta,v,\dot{v}) + v_\vartheta)$$

where $v_\vartheta$ is left as degree of freedom as in classic FL technique, $\beta(\xi,\vartheta,v)$ is such that $\beta(\xi,\vartheta,v)\vartheta_r = L_g L_f x_1 u$, and $\alpha(\xi,\vartheta,v,\dot{v})$ collects all non linearities in $L_f^2 x_1$. The feedback linearized system is then

$$\begin{aligned}\dot{\xi} &= A\xi + B[v_\vartheta \quad T_{g,r}]^\top \\ &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ a_{2,1} & 0 & a_{2,3} & a_{2,4} & a_{2,5} \\ \dfrac{D_s}{n_g J_g} & 0 & -\dfrac{D_s}{n_g^2 J_g} & \dfrac{K_s}{n_g J_g} & -\dfrac{1}{J_g} \\ 1 & 0 & -\dfrac{1}{n_g} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\dfrac{1}{\tau_T} \end{bmatrix} \xi + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \dfrac{1}{\tau_T} \end{bmatrix} \begin{bmatrix} v_\vartheta \\ T_{g,r} \end{bmatrix}\end{aligned} \tag{11}$$

MPC is used to control (11) subject to the system constraints, which, because of the FL stage, happen to be nonlinear and state dependent. Eventually, the wind speed estimation together with the desired power reference $P_e^*$ is used to determine the controller set points via

$$\begin{cases} A\xi_s + B[v_{\vartheta,s} \quad T_{g,r,s}]^\top = 0 \\ \xi_{3,s}\xi_{5,s} = P_e^* \\ \xi_{1,s} = \omega_r^* \end{cases}$$

where $(\xi_s, v_{\vartheta,s}, T_{g,r,s})$ is the steady state solution for a given $P_e^*$ and $\omega_r^*$, (argument of (9)).

## 5. SIMULATION RESULTS

We are now ready to evaluate the wind farm performance. For this simulation we consider a 25 turbine farm as shown in Fig. 4, excited for 600 $s$ by a uniform wind speed $u_\infty$, whose direction is $\vartheta^W = 0°$ with respect to the chosen reference coordinates. $u_\infty$, together with an estimation of its effective value, are shown in Fig. 5. Such a real wind data series is purposely chosen to possibly excite the turbines in all the regions of functioning, i.e. power optimization as well as power limiting. Static optimization, via equation (5), states a theoretical gain of $\sim$ 9%

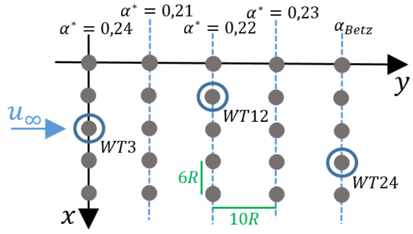


Fig. 4. Wind farm layout.

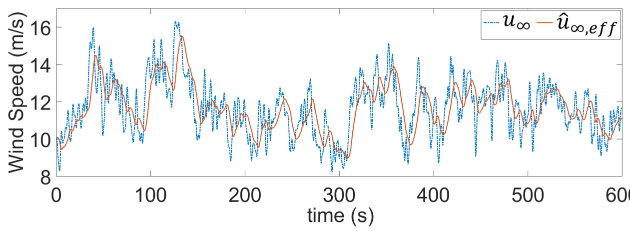


Fig. 5. Wind speed signals.

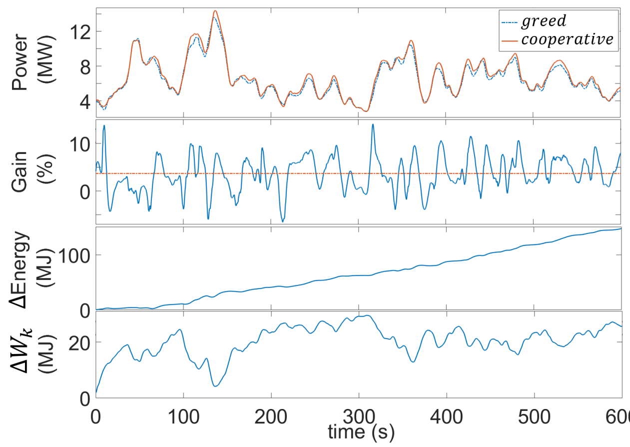


Fig. 6. Comparison of global mechanical power and energy.

with respect to the greed control, and provides the $\boldsymbol{\alpha}^*$ reference show in Fig. 4. Fig. 6 shows the system behavior with respect to the global extracted mechanical power and energy. In practice such a gain cannot be achieved at each time step, as, for wind speed fluctuations, the proposed local controllers are not able to perfectly track $\boldsymbol{\alpha}^*$, (e.g. see Fig. 7). It is clear, then, how global performance is highly influenced by the local controller one. Nonetheless good results are achieved, yielding a mean gain of $\sim$ 4%. In addition, thanks to the strategy of equation (9), a surplus of kinetic energy with respect to classic MPPT functioning is stored in the rotating masses according to equation (8), and it is shown in the last plot of Fig. 6.
Eventually, to get insight into the wake interaction phenomenon, we compared the behavior of turbines 3, 12, and 24 of Fig. 4, in both the scenarios of greed and cooperative optimization. Referring to Fig. 7, we see how in the cooperative framework, upstream turbines track other $\alpha$ values than the $\alpha_{Betz}$ one. It corresponds a deloaded power production of the individual turbines (see Fig. 8).

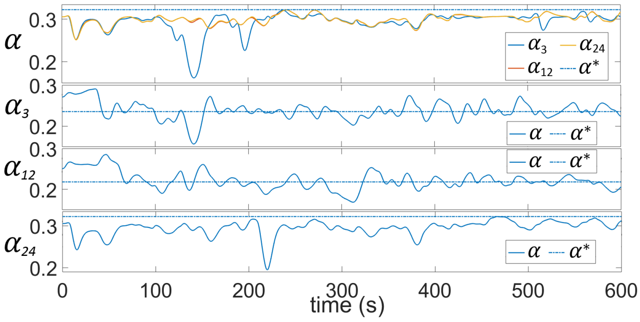

Fig. 7. Comparison of greed ($1^{st}$ *top*) and cooperative ($2^{nd}$, $3^{rd}$, *and* $4^{th}$ *from top*) induction factors of WTs 3, 12, and 24.

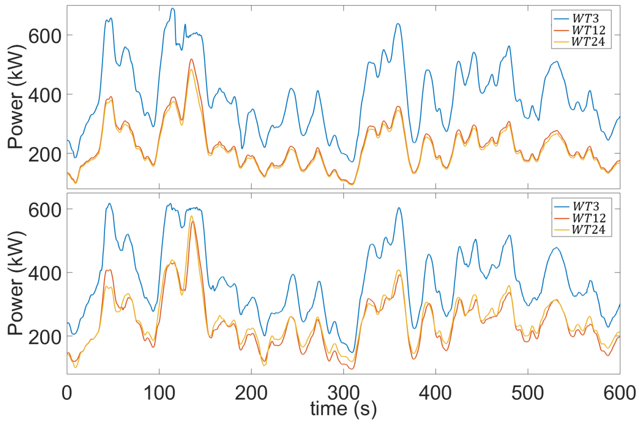

Fig. 8. Comparison of greed (*top*) and cooperative (*bottom*) powers of WTs 3, 12, and 24.

## 6. CONCLUSION

A hierarchical control for wind farm optimization under the effect of wake interaction was presented. Even if it exists a gap between theoretical static optimization and actual attainable gain when system dynamics is considered, good performance is achieved, and it consolidates the need for cooperative control of large wind farms. The overall optimization is highly influenced by the performance of the local controller, which in this paper is obtained by composition of FL and MPC techniques.
A main drawback of the presented architecture is the lack of feedback on the high level optimization. Indeed, the latter is only based on the measurement of the free stream wind and on the wake model, i.e. it does not exploit real time information of turbine state and wind inside the wind farm. As a result the system behavior is sub optimal with respect to the real achievable power production. An interesting opportunity for research would be to employ optimal distributed control in order to take the dynamics of the system into account at the control synthesis step, while eliminating the need for a supervisor controller.

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