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Abstract: This paper proposes a stability criterion for the study of distributed generators equipped with local reactive power regulations. Existing formal methods propose a memory-consuming analysis with some practical issues while dealing with large-scale systems. To cope with this, a novel analytic stability criterion is established in this work. Firstly, a necessary condition for system stability is demonstrated for a feeder hosting a single generator. The approach is illustrated on a real medium voltage feeder. Then, a conjecture is proposed to study the stability of feeders hosting several generators equipped with reactive power regulations. Conjecture validity is proved over several theoretical grids hosting up to four distributed generators thanks to simulations.

Keywords: Voltage stability, Hybrid systems, Piecewise linear systems, Electric power systems, Reactive power.

1. INTRODUCTION

During the past decades, the share of generators connected to the distribution grid – the distributed generators (DGs) – has severely increased thereby strongly modifying distribution grids behavior (Azmy and Erlich, 2005). As one of the consequences of this change, the voltage along distribution feeders hosting generation has increased (Dai and Baghzouz, 2003). To cope with this, distribution grid operators (DSOs) have imagined many solutions to maintain the voltage within acceptable limits. The classic solution consists in reinforcing the network thus mitigating voltage issues but with a subsequent cost. In France, the main DSO (ERDF) estimates the cost of photovoltaic power connection to be up to 300 k€/MW in 2030. In order to try to avoid – or at least to postpone – such investments, numerous alternatives to network reinforcement have been investigated in the literature (Dutrieux et al., 2015). Among these, local control laws of DGs reactive power (Q) with respect to their voltage (U) have particularly drawn attention such as in Unger et al. (2013) or Turitsyn et al. (2011). Indeed, such a control law offers to mitigate voltage issues without limiting the DG's injection of active power. Indeed, the DG measures the voltage at its connection point and adapts, in real time, its reactive power set point according to a given lookup table. The shape of $Q(U)$ regulation adopted by ERDF the French DSO (Witkowski et al., 2013) is shown in Fig. 1.

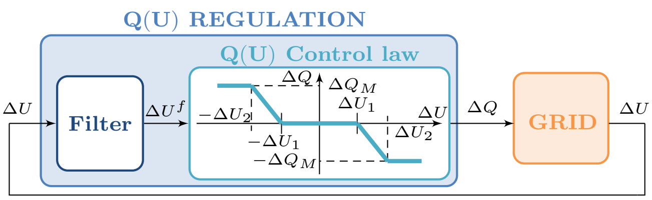

Fig. 1. General structure of the system under study

As it can be seen in Fig. 1, this is a closed-loop regulation and so may endanger voltage stability. Even if there exists a lot of work on voltage control by reactive power management with inverters, very few have studied the stability of a grid hosting many DGs equipped with $Q(U)$ regulations. This can be explained as such studies raise considerable challenges due to the non-linearity of the $Q(U)$ control law. For example, the control law considered in this work is piecewise affine with five operating modes (Fig. 1). In order to assess the stability of a grid hosting one or several

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DGs equipped with $Q(U)$ regulations, simulations and experimentations have been carried out.

In empirical studies presented by Beauné et al. (2014) and Esslinger and Witzmann (2013), $Q(U)$ regulations do not cause any system instability. However, other works have drawn contradictory conclusions. Stetz (2014) presents a simulation work evidencing unstable operating points for a grid hosting one DG equipped with a $Q(U)$ regulation. Without a formal stability study of these regulations, no general conclusion can be drawn. Recently, Andren et al. (2015) has proposed an analytic study of the stability of a grid hosting several $Q(U)$ regulations functioning in one linear operating mode. This linearity assumption allows the authors to assess stability by computing the eigenvalues of the system with some limitations on the validity domain. Preliminary work (Cosson et al., 2015a) develops a new stability study method coping with the non-linearity of the $Q(U)$ regulation. This method is based on the computation of a discrete abstraction of the system and its refinement thanks to bisimulation calculations. This method performs a formal stability analysis of a piecewise affine hybrid system but at a high computational cost.

The present paper formulates a stability criterion for a distribution grid hosting several $Q(U)$ regulations and so allows us to conclude on system stability while keeping a reduced computational load. This expression lies in the study of the possible commutations between several linear operating modes. An intensive investigation leads to an analytic criterion for the stability of one DG expressing stability limits with respect to regulation and grid parameters. Then, an extrapolation of this result to the grids hosting several DGs is presented.

Section 2 details the studied system and the proposed model. Then, the analytic expression of the stability criterion for one DG is elaborated in Section 3. The next section presents the extrapolation of the stability criterion to grids hosting several DGs and simulation results validating the criteron for realistic case-studies. Lastly, the conclusions of this work are developed in Section 5.

2. SYSTEM MODELING

A medium-voltage feeder, connecting several consumers and n generators is considered. All n DGs are supposed to be equipped with the same $Q(U)$ regulation. As presented in Fig. 1, these regulations measure and filter the voltage magnitude at the DGs buses $\mathbf{U}(k) \in \mathbb{R}^n$. The measurement filter is considered to be a discrete-time first-order low-pass filter with a sample time $T_s = 1s$ and a unit gain. Then, the filtered voltage magnitude $\mathbf{U^f}(k) \in \mathbb{R}^n$ is converted into a reactive power set point $\mathbf{Q}(k) \in \mathbb{R}^n$ through a piecewise affine $Q(U)$ law with five operating modes. The purpose of this work is to study the possible voltage oscillations caused by the $Q(U)$ regulations. If such oscillations exist, they would have a period larger than the sampling time T_s of the regulation, so approximately of a few seconds. Thus, in order to study these phenomena, the appropriate model is an electromechanical one (Kundur et al., 1994). All electromagnetic phenomena will be modeled in steady-state.

As network lines transients are electromagnetic phenomena (Kundur et al., 1994), they are modeled in steady-state. Thus, the grid behavior can be represented by the power flow equations (Bolognani and Zampieri, 2016). The model of the grid should express the voltage magnitude variation explicitly at the DGs buses $\mathbf{\Delta U}(k) \in \mathcal{U}^n \subset \mathbb{R}^n$ with respect to reactive power changes $\mathbf{\Delta Q}(k) \in \mathcal{Q}^n \subset \mathbb{R}^n$. To do so, a linear approximation of the power flow equations is computed around the operation point defined by $\mathbf{Q} = \mathbf{0}$

$$\mathbf{\Delta U}(k) = \mathbf{K_Q} \mathbf{\Delta Q}(k) + \mathbf{K_d} \mathbf{\Delta U_d}(k) \tag{1}$$

where $\mathbf{\Delta U_d}(k) = [\Delta U_{d_1}(k), \cdots, \Delta U_{d_m}(k)]^T$ is a vector of m disturbance variables. In this model, all non-controlled variables are considered as disturbances such as variations in active power, voltage at the primary substation, consumed power, etc. The matrices $\mathbf{K_Q} \in \mathbb{R}^{n\times n}$ and $\mathbf{K_d} \in \mathbb{R}^{n\times m}$ can be calculated as the sensitivity matrices of the bus voltage magnitude to the variations of the injected reactive power and disturbances.

The DGs are supposed to be connected to the grid through power electronics as it is generally the case (Machowski et al., 2011). The control of reactive power with power electronics converters, such as inverters, can be modeled as a first order low-pass filter with a unit gain and a time constant of a few milliseconds (Machowski et al., 2011). Thus, in this work, the DGs power electronics are modeled in steady-state. Lastly, the DGs behavior is modeled by their $Q(U)$ regulations composed of a measurement filter and a $Q(U)$ control law. The discrete-time first-order low-pass filter is modeled by its state equation with a unity gain and $a \in [0,\ 1[$ its rapidity.

$$\mathbf{\Delta U^f}(k+1) = a\, \mathbf{\Delta U^f}(k) + (1-a)\, \mathbf{\Delta U}(k) \tag{2}$$

where $\mathbf{\Delta U^f}(k) \in \mathcal{U}^f \subset \mathbb{R}^n$ is the vector of filtered voltage magnitudes at the DGs buses at time $t = kT_s$. Afterwards, the variation of the reactive power set point $\mathbf{\Delta Q}(k)$ is computed through a piecewise affine function.

$$\mathbf{\Delta Q}(k) = \mathbf{G}(\mathbf{I}(k))\, \mathbf{\Delta U^f}(k) \ + \ \mathbf{F}(\mathbf{I}(k)) \tag{3}$$

where $\mathbf{I} \in \mathcal{I}^n = \{1, \ldots, 5\}^n$ is the vector of the operating modes of each DG at time kT_s such as:

$$\mathbf{\Delta U}^-(\mathbf{I}(k)) \leq \mathbf{\Delta U^f}(k) \leq \mathbf{\Delta U}^+(\mathbf{I}(k)) \Leftrightarrow \mathbf{\Delta U^f}(k) \in \mathcal{D}_{\mathbf{I}(k)} \tag{4}$$

where $\mathcal{D}_i$ denotes, for every $i \in \mathcal{I}$, the polyhedron defined by the set of points $u \in \mathbf{U}^f$ such as $\Delta U^-(i) \leq u \leq \Delta U^+(i)$.

The matrix $\mathbf{G}(\mathbf{I}(k)) \in \mathbb{R}^{n\times n}$ is a diagonal matrix with $g_{jj}(i_j(k))$ being the slope of the $Q(U)$ law of the j-th DG which is in the $i_j(k)$-th operating mode. Each component $f_j(i_j(k))$ of the vector $\mathbf{F}(\mathbf{I}(k)) \in \mathbb{R}^n$ corresponds to the intercept of the j-th DG $Q(U)$ law in mode $i_j(k)$ as indicated in Table 1.

i_j	1	2	3	4	5
g_{jj}	0	$\sigma = \frac{-\Delta Q_M}{\Delta U_2 - \Delta U_1}$	0	σ	0
f_j	ΔQ_M	$\sigma \Delta U_1$	0	$-\sigma \Delta U_1$	$-\Delta Q_M$
ΔU_j^-	$-\Delta U_M$	$-\Delta U_2$	$-\Delta U_1$	ΔU_1	ΔU_2
ΔU_j^+	$-\Delta U_2$	$-\Delta U_1$	ΔU_1	ΔU_2	$+\Delta U_M$

Table 1. Parameters of the $Q(U)$ law of the j-th DG in each operating mode i_j

To simplify the study, only one disturbance term, combining the influence of all non-controlled variables, will be considered in the following $\Delta U_d(k) \in \mathbb{R}$. Furthermore, the stability study of $Q(U)$ regulations focuses on a few seconds. Over this time span, the disturbance term will be considered as constant $\Delta U_d(k) = \Delta U_d$.

To conclude, the system composed of a medium-voltage grid hosting n DGs equipped with a $Q(U)$ regulation can be modeled as shown in Fig. 2.

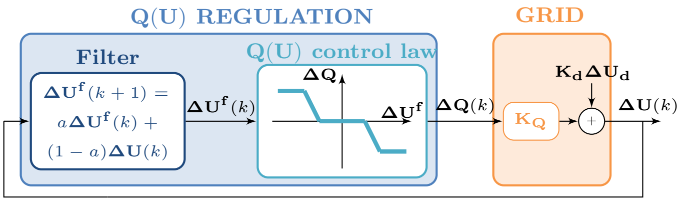

Fig. 2. Proposed model of a distribution grid hosting several DGs in order to study its stability

Let $\mathbf{A}(.)$ and $\mathbf{b}(.)$ be defined as:

$$\begin{array}{rll} \mathbf{A} : \mathcal{I}^n & \rightarrow & \mathbb{R}^{n\times n} \\ \mathbf{I} & \mapsto & a\,\mathbf{I_n} + (1-a)\mathbf{K_Q}\mathbf{G}(\mathbf{I}) \\ \mathbf{b} : \mathcal{I}^n & \rightarrow & \mathbb{R}^n \\ \mathbf{I} & \mapsto & (1-a)(\mathbf{K_d}\Delta U_d + \mathbf{K_Q}\mathbf{F}(\mathbf{I})) \end{array} \tag{5}$$

Then, from equations (1)–(5), the following closed-loop equation can be written:

$$\mathbf{\Delta U^f}(k+1) = \mathbf{A}(\mathbf{I}(k))\mathbf{\Delta U^f}(k) + \mathbf{b}(\mathbf{I}(k)) \tag{6}$$

Unstable behavior of such a piecewise affine hybrid system has been studied by Cosson et al. (2015a). The proposed method enables us to assess the stability of a distribution feeder hosting several DGs. It has evidenced the risk of voltage oscillations caused by the switching between two operating modes of the $Q(U)$ law. This method gives detailed results but its complexity grows exponentially with the number of DGs, thus limiting its application to a small number of DGs. In this paper, a stability criterion to ease the study of a large system is proposed. Let us first detail the methodology to obtain a formal expression of a stability criterion for a grid hosting a single DG equipped with a $Q(U)$ regulation.

3. STABILITY CRITERION FOR A SINGLE DISTRIBUTED GENERATOR

3.1 Methodology

The goal of this work is to express a set of sufficient conditions on system parameters that ensures system stability. To the best of our knowledge, no direct method has been found. The proposed methodology is based on deriving necessary conditions to system instability to then prove system stability. The system is said to be unstable when the operating mode of the $Q(U)$ control law oscillates. This behavior is referred to as a cycle and leads to reactive power and voltage periodic oscillations. All possible cycles must be envisaged. To do so, their existence is assumed leading to a set of necessary conditions on system parameters. The contrary of this set of necessary conditions to instability is a set of sufficient conditions to stability.

In order to be able to apply this reasoning, an exhaustive list of cycles must be established. As the number of possible cycles grows with the number of DGs, the methodology is applied here only for grids hosting a single DG. Nonetheless, an infinite number of cycles can exist even for a single DG. To cope with this, only cycles in between only two different operating modes are considered as possible as no other cycle has been evidenced in the previous work (Cosson et al., 2015a). In the following, these cycles are referred to as *simple cycles*. As five operating modes exist, ten simple cycles are considered. Table 2 presents the list of these cycles.

1↔2	1↔3	1↔4	1↔5
	2↔3	2↔4	2↔5
		3↔4	3↔5
			4↔5

Table 2. List of possible simple cycles for a single DG

3.2 System description

A grid hosting a single DG ($n = 1$) is studied here. The closed-loop dynamics equation can be expressed as follows:

$$\Delta U^f(k+1) = A(i(k))\Delta U^f(k) + b(i(k)) \tag{7}$$

where $i(k) \in \mathcal{I}$ is the operating mode at time kT_s such as $\Delta U^f(k) \in \mathcal{D}_i \subset \mathbb{R}$. Table 3 presents the dynamic equation coefficient expression for each operating mode.

i	$A(i)$	$b(i)$	$\mathcal{D}_i$
1	a	$(1-a)(K_d\Delta U_d + K_Q\Delta Q_M)$	$[-\Delta U_M,\ -\Delta U_2]$
2	$a+(1-a)\sigma$	$(1-a)(K_d\Delta U_d + K_Q\sigma\Delta U_1)$	$[-\Delta U_2,\ -\Delta U_1]$
3	a	$(1-a)K_d\Delta U_d$	$[-\Delta U_1,\ \Delta U_1]$
4	$a+(1-a)\sigma$	$(1-a)(K_d\Delta U_d - K_Q\sigma\Delta U_1)$	$[\Delta U_1,\ \Delta U_2]$
5	a	$(1-a)(K_d\Delta U_d - K_Q\Delta Q_M)$	$[\Delta U_2,\ \Delta U_M]$

Table 3. Parameters of the closed-loop equation in the i-th operating mode

3.3 Expression of the stability criterion

The stability criterion is expressed thanks to a two-step approach. Assuming the existence of a cycle between operating modes i and j implies that:

(1) it exists at least one point of $\mathcal{D}_j$ that can be reached from $\mathcal{D}_i$ and vice-versa (referred to as the predecessors study);
(2) it exists at least one point of $\mathcal{D}_i$ (respectively $\mathcal{D}_j$) involved in the cycle between operating modes i and j (referred to as the reachability study).

Please note that the existence (respectively non-existence) of a cycle between two different operating modes i and j in $\mathcal{I}^2$ will be denoted C_{ij} (resp. $\overline{C_{ij}}$).

Firstly, the predecessors study is detailed. Let us define the predecessor of a domain $\mathcal{D}$ in a domain $\mathcal{D}'$ such as

$$Pred(\mathcal{D},\mathcal{D}') = \{x \in \mathcal{D}' \subset \mathcal{D}_i \mid A(i)x + b(i) \in \mathcal{D}\} \tag{8}$$

To study the existence of a cycle between modes i and j, it can be noticed that

$$C_{ij} \Rightarrow \begin{cases} Pred(\mathcal{D}_j,\mathcal{D}_i) \neq \emptyset \\ Pred(\mathcal{D}_i,\mathcal{D}_j) \neq \emptyset \end{cases} \tag{9}$$

From these equations, a set of inequalities on system parameters can be obtained. The method is illustrated on cycle C_{35}. Its existence is assumed which implies that $Pred(\mathcal{D}_5, \mathcal{D}_3)$ is not an empty set.

$$\exists u \in \mathcal{U}^f \begin{cases} -\Delta U_1 \leq u \leq \Delta U_1 \\ \Delta U_2 \leq au + (1-a)K_d \Delta U_d \end{cases} \tag{10}$$

This is used to bound the admissible disturbance term.

$$\begin{gathered} Pred(\mathcal{D}_5, \mathcal{D}_3) \neq \emptyset \Rightarrow \\ \Delta U_2 - a\Delta U_1 \leq (1-a)K_d \Delta U_d \end{gathered} \tag{11}$$

Similarly, a necessary condition for the existence of a predecessor of $\mathcal{D}_3$ in $\mathcal{D}_5$ is obtained. Combining these conditions, it can be proved that:

$$C_{35} \Rightarrow a \leq \frac{-1 - K_Q\sigma}{1 - K_Q\sigma} \tag{12}$$

Thus, if the set of system parameters is chosen such as (12) is not satisfied then, it can be ensured that no cycle can exist in between modes 3 and 5.

$$a > \frac{-1 - K_Q\sigma}{1 - K_Q\sigma} \Rightarrow \overline{C_{35}} \tag{13}$$

This reasoning is applied to every cycle of the list (Table 2) and necessary conditions to cycle existence are obtained. Typical data for French medium-voltage grids lead to consider network lines to be shorter than 40 km with a reactance per distance unit of a few 0.1 Ω/km. The DG maximum reactive power set point is considered up to 40% of the active power which is supposed to be up to 10 MW on a medium voltage feeder. Considering these orders of magnitude, it can be proved that, for any regulation or grid parameters, necessary conditions for C_{14}, C_{15}, C_{24} and C_{25} cannot be met.

$$\begin{array}{rcl} a > \frac{-1-K_Q\sigma}{1-K_Q\sigma} & \Rightarrow & \overline{C_{13}}, \overline{C_{35}} \\ a > \frac{-K_Q\sigma}{1-K_Q\sigma} & \Rightarrow & \overline{C_{12}}, \overline{C_{23}}, \overline{C_{34}}, \overline{C_{45}} \\ \forall a \in [0,\ 1] & \Rightarrow & \overline{C_{14}}, \overline{C_{15}}, \overline{C_{24}}, \overline{C_{25}} \end{array} \tag{14}$$

To conclude, the predecessors study has restrained the list of possible cycles to six out of the ten initially considered (illustrated in Table 4). Moreover, it has been proved that none of these cycles can exist if $a > \frac{-K_Q\sigma}{1-K_Q\sigma}$ (see the green interval in Fig. 3).

1↔2	1↔3		
	2↔3		
		3↔4	3↔5
			4↔5

Table 4. List of possible simple cycles for a single DG after the predecessors study

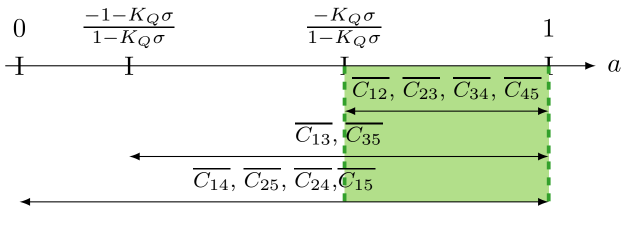

Fig. 3. Region of proved stability (green) with respect to filter rapidity a after the predecessors study

The conditions established thanks to the predecessors study are sufficient conditions to system stability. They restrain admissible values of filter rapidity in order to ensure system stability. As they are only sufficient conditions, the system might be stable for a value of a lower than the limit and thus this method might be too restrictive. It means that the range of admissible rapidity settings is narrower than the range of stable settings. To push back this limit, a reachability study is performed. It consists in verifying that, for each possible cycle, it exists at least one unstable operating point and thus detecting cycles that cannot exist. This has been done in two steps. Firstly, it has been demonstrated there is at most one point involved in a cycle C_{ij}. Lastly, this point location has been studied according to system parameters to conclude on cycle existence.

The set $\mathcal{S}_{ij} \subset \mathcal{D}_i \subset \mathbb{R}$ is defined as the set of points from $\mathcal{D}_i$ involved in the cycle C_{ij}. It corresponds to the points belonging to $\mathcal{D}_i$ which lead, in one time-step, to an operating point in $\mathcal{D}_j$ leading itself back to $\mathcal{S}_{ij}$ in the following time-step. Thus, the sets $\mathcal{S}_{ij}$ and $\mathcal{S}_{ji}$ can be defined as follows :

$$\begin{cases} \mathcal{S}_{ij} = \{x \in \mathcal{D}_i \ : \ A(i)x + b(i) \in \mathcal{S}_{ji}\} \\ \mathcal{S}_{ji} = \{x \in \mathcal{D}_j \ : \ A(j)x + b(j) \in \mathcal{S}_{ij}\} \end{cases} \tag{15}$$

Assuming that C_{ij} exists implies that the sets $\mathcal{S}_{ij}$ and $\mathcal{S}_{ji}$ are not empty sets. From now on, the existence of C_{ij} is assumed. The reachability study tries to characterize the operating points belonging to $\mathcal{S}_{ij}$ and $\mathcal{S}_{ji}$.

These two sets are assumed to be polyhedrons such as $\mathcal{D}_i$ and $\mathcal{D}_j$, it can be written that:

$$\begin{array}{lccc} \exists (K_{ij},\ L_{ij}) \in (\mathbb{R}^{n_S})^2 : & x \in \mathcal{S}_{ij} \subset \mathbb{R} & \Leftrightarrow & K_{ij}x \leq L_{ij} \\ \exists (K_{ji},\ L_{ji}) \in (\mathbb{R}^{n_S})^2 : & x \in \mathcal{S}_{ji} \subset \mathbb{R} & \Leftrightarrow & K_{ji}x \leq L_{ji} \end{array} \tag{16}$$

Combining (15) and (16), it can be said that for all $x \in \mathcal{S}_{ij}$,

$$\begin{cases} K_{ij} \quad x \leq L_{ij} \\ K_{ji}A(i)\ x \leq L_{ji} - K_{ji}b(i) \end{cases} \tag{17}$$

As this is satisfied for every point in $\mathcal{S}_{ij}$, the two polyhedrons defined by (17) must be equivalent, which implies

$$\exists\ \lambda \in \mathbb{R} : \quad \begin{cases} K_{ij} = \lambda\ K_{ji}A(i) \\ L_{ij} = \lambda\ (L_{ji} - K_{ji}b(i)) \end{cases} \tag{18}$$

Similarly, it can be established that

$$\exists\ \mu \in \mathbb{R} : \quad \begin{cases} K_{ji} = \mu\ K_{ij}A(j) \\ L_{ji} = \mu\ (L_{ij} - K_{ij}b(j)) \end{cases} \tag{19}$$

Under the assumption that $A(i)A(j)$ is not equal to zero nor one, L_{ij} can be expressed as the product of a scalar and K_{ij}.

$$L_{ij} = \frac{b(j) + A(j)b(i)}{1 - A(i)A(j)} K_{ij} \tag{20}$$

Taking this into account, the definition (16) of the polyhedron $\mathcal{S}_{ij}$ can be rewritten with $k_h \in \mathbb{R}$ defined as a component of the vector $K_{ij} = (k_h)_{h \in \{1,\ldots,n_S\}}$.

$$\forall x \in \mathcal{S}_{ij} \Leftrightarrow \forall h \in \{1, \ldots, n_S\},\ k_h x \leq \frac{b(j) + A(j)b(i)}{1 - A(i)A(j)} k_h \tag{21}$$

It can be noted that as $\mathcal{S}_{ij}$ is necessarily a closed interval of $\mathbb{R}$, then the system (21) of n_S equations must define a lower bound and an upper bound for $x \in \mathcal{S}_{ij}$. This implies that it exists at least one positive component k_h of K_{ij} (defining the upper bound) and another $k_{h'}$ which is negative (defining the lower bound).

From this statement, it can be deduced that (21) implies that:

$$\forall x \in \mathcal{S}_{ij} \quad \Leftrightarrow \quad x = \frac{b(j) + A(j)b(i)}{1 - A(i)A(j)} \tag{22}$$

The operating point of $\mathcal{D}_i$ involved in C_{ij} is now denoted $x^\star_{ij}$. Now let us express at which conditions this point actually belongs to $\mathcal{D}_i$. This method is illustrated on cycle C_{34}.

$$x^\star_{34} = \frac{1 + a + (1-a)K_Q\sigma}{1 + 1 - aK_Q\sigma} K_d \Delta U_d - \frac{K_Q\sigma}{1 + 1 - aK_Q\sigma} \Delta U_1 \tag{23}$$

Let us try to locate $x^\star_{34}$ in $\mathcal{D}_3$. It can be written that

$$\begin{cases} -\Delta U_1 \leq & x^\star_{34} \leq & \Delta U_1 \\ \Delta U_1 & \leq A(3)\, x^\star_{34} + b(3) & \leq \quad \Delta U_2 \end{cases} \tag{24}$$

From (23), the disturbance term ΔU_d can be expressed as a function of $x^\star_{34}$. Injecting this in (24), the following inequalities are obtained

$$\begin{cases} x^\star_{34} & \leq & \Delta U_1 \\ \frac{\Delta U_1}{1+a+(1-a)K_Q\sigma} & \leq & \frac{x^\star_{34}}{1+a+(1-a)K_Q\sigma} \end{cases} \tag{25}$$

On one hand, it can be seen that if $1+a+(1-a)K_Q\sigma < 0$ then there exist operating points satisfying (25) thus cycle C_{34} can exist. On the other hand, if $1+a+(1-a)K_Q\sigma > 0$, the conditions of (25) are equivalent to $x^\star_{34} = \Delta U_1$ thus the only point in C_{34} is actually the border of $\mathcal{D}_3$. It can be shown that:

$$x^\star_{43} = x^\star_{34} = \Delta U_1 = \mathcal{D}_3 \cap \mathcal{D}_4 \tag{26}$$

This means that the trajectory of C_{34} is actually a fixed point belonging both to $\mathcal{D}_3$ and $\mathcal{D}_4$ as it is their common border. To conclude, this cannot be considered as an unstable trajectory.

$$a > \frac{-1 - K_Q\sigma}{1 - K_Q\sigma} \quad \Rightarrow \quad \overline{C_{34}} \tag{27}$$

Similar calculations have been conducted to study sufficient non-existence conditions of cycles C_{12}, C_{23} and C_{45}. Finally, it has been proved that (27) implies the absence of cycles C_{12}, C_{23} and C_{45}. The reachability study has led to the expression of a larger stability region as shown in Fig. 4. Please note that there is no proof of system instability for a filter rapidity $a < a_{lim}$.

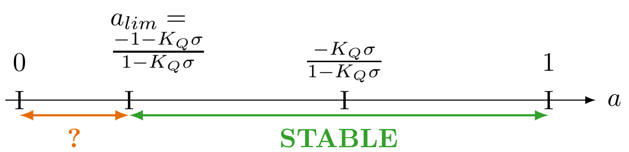

Fig. 4. Region of proved stability (green) with respect to filter rapidity a for simple cycles

Let us illustrate these results on a real case study.

3.4 Illustration on a real case-study

To illustrate the proposed method, the stability criterion of a real medium-voltage feeder is calculated. The modeled grid is a real 124 nodes distribution feeder of the ERDF network. This feeder hosts about 300 kW of consumption and a single DG: a wind farm of 6 MW located about 20 km away from the primary substation. In its present condition, this distribution feeder is experiencing over-voltage issues. In order to try to mitigate them, the DG is considered to be equipped with the $Q(U)$ regulation presented in Fig. 1.

The influence of reactive power variations at the DG connection point is modeled by a gain K_Q calculated using a sensitivity analysis of DG voltage with respect to reactive power variations.

$$K_Q = 0.13\ V/kVAr \tag{28}$$

The $Q(U)$ regulation parameters are set such as:

$$\begin{array}{ccc} \Delta Q_M = 2.4\ MVar & \text{and} & \sigma = -9.6\ \text{kVar/V} \\ \Delta U_1 = 750\ \text{V} & \text{and} & \Delta U_2 = 1000\ \text{V} \end{array} \tag{29}$$

The proposed stability criteria is computed.

$$a_{lim} = 0.11 \tag{30}$$

If the filter rapidity is set to be higher than a_{lim}, then no simple cycle can exist and the system is stable if no complex cycle arises. On the other hand, nothing can be predicted for system stability if the filter is faster than a_{lim}.

The behavior of the system in response to a disturbance step (at time equals 1 second) is simulated for four values of filter rapidity a. As it can be seen in Fig. 5, for $a > a_{lim}$, the system is stable as planned by the stability criterion. Please note that for settings closed to the limit case, even if the system is stable, however, it endures large voltage oscillations and its settling time is important. On the other hand, as a increases toward one, voltage oscillations amplitude tend to be reduced but the system dynamics is slowed down. A trade-off has to be made between stability and rapidity.

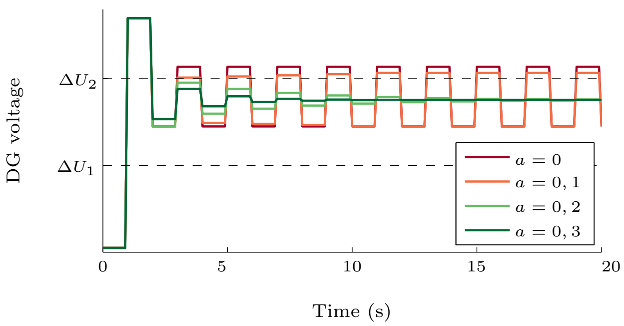

Fig. 5. Dynamical simulations of the voltage profile for a disturbance step with several rapidity filter settings

It can be seen on Fig. 5 that when $a < a_{lim}$ then this system is unstable. This result cannot be generalized to any system but it highlights the fact that the proposed stability criterion is close to the reality.

In this section, an explicit stability criterion has been proposed for grids hosting a single DG equipped with a $Q(U)$ regulation. The method has been illustrated on the $Q(U)$ regulation presented in Fig. 1 and it can be applied to any piecewise affine DG power regulation as a function of the voltage. The validity and interest of such a criterion have been illustrated on a real case-study. It demonstrates the necessity to carefully set system parameters in order to ensure system stability. Let us now discuss the stability of a grid hosting several DGs equipped with $Q(U)$ regulations.

4. STABILITY CRITERION FOR SEVERAL DISTRIBUTED GENERATORS

4.1 Methodology

The methodology detailed in the previous section proposes to study the stability of a single DG by enumerating all possible cycles. For the study of n DGs, considering only simple cycles, two among n cycles must be considered. It can be easily understood that the methodology for a single DG is not adapted to the study of the interactions in between n DGs. To overcome this issue, it can be noticed that the proposed stability criterion for a single DG can be seen as the local stability of each operating mode.

Indeed, let us consider a single DG in operating mode $i \in \mathcal{I}$. The dynamics equation of the system (7) in the operating mode i is stable if and only if the eigenvalues $\lambda(i)$ of the $A(i)$ have a module lower than one. In the considered case, $A(i)$ is a scalar and so:

$$\begin{cases} \lambda(1) = \lambda(3) = \lambda(5) = a \\ \lambda(2) = \lambda(4) = a + (1-a)K_Q\sigma \end{cases} \tag{31}$$

Dynamics in modes 1, 3 and 5 are always stable as the filter rapidity is positive and smaller than one. Modes 2 and 4 are stable if and only if:

$$-1 < a+(1-a)K_Q\sigma < 1 \quad \Leftrightarrow \quad \frac{-1-K_Q\sigma}{1-K_Q\sigma} < a < 1 \tag{32}$$

Thus, it can be noted that the stability of all operating modes dynamics is equivalent to the stability criterion shown on Fig. 4. To conclude, for a single DG, it can be said that:

$$\forall i \in \mathcal{I}, \mid \lambda(i) \mid \leq 1 \Leftrightarrow a_{lim} < a < 1 \Rightarrow \text{No simple cycle} \tag{33}$$

In this paper, it is conjectured that this result can be expanded to n DGs. The proposed stability criterion consists in computing the eigenvalues of the evolution matrix of the dynamic system of every possible operating mode. If all eigenvalues have a module smaller than one, the system is supposed to be stable.

$$\text{If } |\lambda| < 1,\ \forall \lambda \in eig(\mathbf{A}(\mathbf{I})),\ \forall \mathbf{I} \in \mathcal{I}^n \Rightarrow \text{No simple cycle} \tag{34}$$

As for now, this conjecture has not been proved to be right. In order to assess its validity, a series of numerical experiments have been done to compare, for a grid hosting n DGs, the stability conjecture to the stability computed with the formal tool presented by Cosson et al. (2015b).

4.2 Statistical validation

In order to evaluate the validity of the proposed conjecture (34), a large number of simulations is done. For every case, the formal stability of the system is computed thanks to bisimulation calculation (Cosson et al., 2015b). The result is then compared with the conjecture.

For this paper, a 20-node theoretical medium-voltage grid with lines up to 16 km is modeled. This grid hosts up to four DGs equipped with a dead-band $Q(U)$ regulation law (see Fig. 1). The maximum number of DGs has been limited to four due to the available memory of the computer [1]. For each scenario, the number of DGs is randomly chosen as well as their point of connection to the grid and their active power. The sum of active powers has been limited to 10 MW to take into account realistic data. Figure 6 presents as an example scenario number 17 which corresponds to a grid hosting 3 DGs.

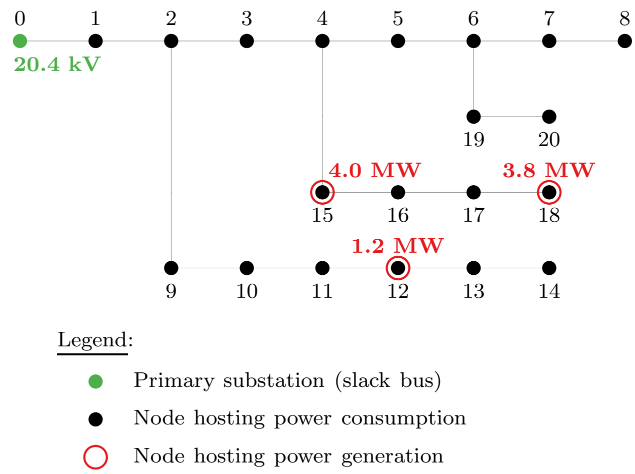

Fig. 6. Scheme of scenario 17 feeder

In this work, 60 scenarios – corresponding to different network configurations – have been randomly picked. Figure 7 presents the distribution of the number of DGs hosted by the feeder for all scenarios and Figure 8 the distribution of the cumulated active power over the feeder for all scenarios. For each scenario, all DGs filters have the same rapidity setting which is randomly chosen among $a \in \{0; 0.1; \ldots; 0.5\}$. Thus, 360 systems are under study.

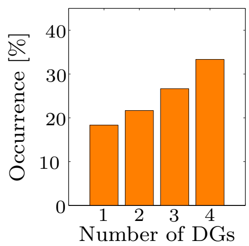

Fig. 7. Distribution of the number of DGs

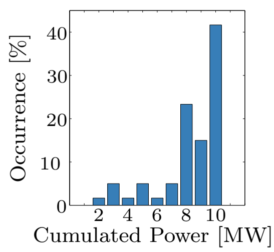

Fig. 8. Distribution of the cumulated power

Firstly, their stability is computed formally with the bisimulation technique. Then, the stability of each operating mode is analyzed to assess system stability thanks to the proposed conjecture. Results of both methods are compared in order to validate the conjectured stability criterion. With the formal stability study, 39.3% of the cases are found to be unstable. All of the unstable cases are also detected by the conjecture whereas stability of the system is not always detected by the conjecture (Table 5).

Indeed, the stability of the local linear dynamics implies the global stability of the system (34) but is not equivalent to it. For example, in scenario number 16 with a filter rapidity set to zero, the bisimulation calculation finds the system to be stable whereas the operating mode $I = [2\ 2\ 2]^T$ is unstable. This instability does not affect the

[1] These simulations have been done with a computer with 16 Go of memory.

When ... / then ...	Criterion is stable	Criterion is unstable
Bisimulation is stable	100%	12.6%
Bisimulation is unstable	0%	87.4%

Table 5. Quantitative comparison of stability analysis using the proposed criterion knowing the result of bisimulation calculation (Cosson et al., 2015b)

global stability as none of the unstable operating modes is actually reachable. For instance, electrical distance in between DGs is such as all DGs cannot be in operating mode 2 at the same time.

To conclude, the conjecture of (34) is satisfied for all the tested cases. The statistical study has highlighted the validity of the conjecture at least for systems up to three DGs. The proposed stability criterion for n DGs seems to be a sufficient condition to system stability but not a necessary condition.

5. CONCLUSION

This paper proposes a stability study of n DGs equipped with dead-band $Q(U)$ regulations based on previous observations of unstable behavior (Cosson et al., 2015a). The presented work consists in the establishment of an analytic stability criterion which pushes back the maximum number of DGs that can be considered as the numerical complexity is reduced compared to bisimulation calculation.

Firstly, the existence as well as the explicit expression of a stability criterion have been demonstrated for grids hosting a single DG and considering only simple cycles. Then, its extension to several DGs is conjectured. The proposed stability criterion is: the local stability of each linear operating mode implies the global stability of the system (34). To validate it, a stability study over 360 systems is conducted. The comparison with formal stability study confirms the validity of the conjecture in all tested cases.

Please note that due to numerical complexity of the bisimulation calculation, the validation study was limited to feeders hosting up to four DGs. Even though, the bisimulation calculation over the 360 studied cases took more than 87 hours with up to 102 iterations before being able to conclude. Further work will include the improvement of the bisimulation calculation convergence in order to extend stability study to more complex cases. Indeed, a broader validation study is needed to confirm conjecture validity for feeders hosting more than four DGs.

To conclude, this paper focuses on the stability of dead-band $Q(U)$ regulations. Further work will generalize these results to the stability study of various DG power regulations.

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