New state estimator for decentralized event-triggered consensus for multi-agent systems

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Abstract: This paper extends recent work of Garcia et al on event-triggered communication to reach consensus in multi-agent systems. It proposes an improved agent state estimator as well as an estimator of the state estimation uncertainty to trigger communications. Convergence to consensus is studied. Simulations show the effectiveness of the proposed estimators in presence of state perturbations.

Keywords: Multi-agent system, event-triggered, consensus, undirected communication graph.

1. INTRODUCTION

Consensus is an important problem in cooperative control: several agents have to be synchronized to the same state Reza Olfati-Saber and Murray (2007); Ren (2008); Cao and Ren (2012); Garcia et al. (2014c,b). In distributed cooperative control, consensus usually requires significant exchange of information between neighbouring agents so that each agent can properly evaluate its control law. This communication may be either permanent, as in Reza Olfati-Saber and Murray (2007); Ren (2008), or takes place at discrete time instants, which is much more practical. In the latter case, communications may occur periodically, as in Garcia et al. (2014b), may be intermittent Wen et al. (2012a,b); Guanghui Wen (2013), or may be event-triggered as in Dimarogonas and H.Johansson (2009); Jiangping et al. (2011); Dimarogonas et al. (2012); Fan et al. (2013); Garcia et al. (2014c); Zhang et al. (2015).

Event-triggered communication is the most promising approach to save communication energy, while allowing a consensus to be reached. In Dimarogonas et al. (2012), each agent performs an estimation of the state of other agents. Communications occurs when the error between the model and the actual state reaches some threshold. Nevertheless, the communication frequency increases close to consensus. Each agent is modeled as a double integrator in Seyboth et al. (2013). The triggering condition depends on a state-independent and exponentially decreasing threshold. With this method, communication frequency still increases close to consensus, but slower. For the general linear model of the dynamic of agents considered in Zhu et al. (2014); Garcia et al. (2014c,a), the error between the measurement of an agent state and its estimate of this state is used to trigger communication. Measurement errors make this approach sensitive to perturbations. Such issues have been partly addressed in Hu et al. (2014); Cheng et al. (2014), which study the influence of perturbations on the state of each agent and propose an event-triggered method with a time-varying coefficients to mitigate the influence of the noise.

This paper considers the event-triggered communication strategy developed in Garcia et al. (2014c,a), and briefly recalled in Section 2, to reach a consensus for identical agents with a fixed topology. It introduces in Section 3 a new agent state estimator to further reduce the amount of communications. The new estimator better represents the agent behaviour by accounting for the control input evaluated by each agent. To implement this strategy, each agent, instead of estimating the state of its neighbours only, will estimate the states of all agents of the network. Each agent is then able to estimate the control inputs applied by all other agents. Section 4 shows that the proposed approach allows to reduce the need for communications when the perturbations remain bounded. Finally, conclusions are drawn in Section 5.

2. PRELIMINARIES

We start introducing classical notations taken from Cortes and Martinez (2009). The Kronecker product is denoted as $\otimes$. Note $\lambda_{\min}(M)$, $\lambda_{\min>0}(M)$, $\lambda_{\max}(M)$ the smallest eigenvalue, the smallest strictly positive eigenvalue and the largest eigenvalue of a matrix M.

Consider a network of N agents with fixed undirect communication graph $\mathcal{G}$ and fixed adjacency matrix A. The set of neighbours of an Agent i is $\mathcal{N}_i = \{j \in \mathcal{N} \,|\, (i,j) \in \mathcal{E},\ i \neq j\}$. N_i is the cardinal number of $\mathcal{N}_i$.

In what follows, the state perturbation affecting Agent i is assumed additive with

$$d_i(t) = m(t) + s_i(t), \tag{1}$$

where $m(t) \in \mathbb{R}^n$ is a bounded time-varying perturbation with $\|m(t)\| \leq M_{\max}$ identical for all agents and

$s_i(t) \in \mathbb{R}^n$ is a bounded agent-specific perturbation with $\|s_i(t)\| \leq S_{\max}\ \forall t$, $i = 1, \dots, N$. The vector of all state perturbations is then

$$d(t) = \mathbf{1}_N \otimes m(t) + \left[s_1(t)^T \dots s_N(t)^T \right]^T . \qquad (2)$$

3. DECENTRALIZED CONTROL WITH NEW AGENT STATE ESTIMATOR

Here, as in Garcia et al. (2014c), undirected communication graph and fixed topology are considered. Agent dynamics is modelled as

$$\begin{aligned} \dot{x}_i(t) &= \mathbf{A}x_i(t) + \mathbf{B}u_i(t) + d_i(t) \\ u_i(t) &= c_1 \mathbf{F} \sum_{j \in \mathcal{N}_i} \left(y_i^i(t) - y_j^i(t) \right) \end{aligned} \qquad (3)$$

where $x_i \in \mathbb{R}^n$ is the state of Agent i and $u_i \in \mathbb{R}^m$ is its control input, $i = 1, \dots, N$. $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathbf{B} \in \mathbb{R}^{n \times m}$. In Garcia et al. (2014c), $c_1 = c + c_2$ with $c = 1/\lambda_2(L)$ and $c_2 \geq 0$ is a design parameter. $\mathbf{F} = -\mathbf{B}^T\mathbf{P}$ where $\mathbf{P}$ is the symmetric positive semi-definite matrix, solution of the Riccati equation

$$\mathbf{P}\mathbf{A} + \mathbf{A}^T\mathbf{P} - 2\mathbf{P}\mathbf{B}\mathbf{B}^T\mathbf{P} + 2\alpha\mathbf{P} < 0, \qquad (4)$$

with $\alpha > 0$. The esimation $y_j^i(t)$ is here expressed as

$$y_j^i\left(t_{j,k}^i\right) = x_j\left(t_{j,k}^i\right), \qquad (5)$$

$$\dot{y}_j^i(t) = \mathbf{A}y_j^i(t), \ \forall t \in \left] t_{j,k}^i, t_{j,k+1}^i \right[, \qquad (6)$$

In order to reduce the number of messages broadcast by each agent, a new estimated dynamic is built to represent the agent behaviours by accounting for the control input evaluated by each agent and his dynamic behaviour. To implement this strategy, each agent, instead of estimating the states of its neighbours only, will estimate the states of all agents of the network. Each agent is then able to estimate the control inputs applied by all other agents. If the estimators perform well, this may reduce the need for communications.

In Theorem 1, we assume that the estimation errors are perfectly known by all agents. In Section 3.2, a communication protocol and an estimator of the estimation errors are introduced to allows a practical implementation of the proposed technique.

The following section describes the resulting decentralized control algorithm.

3.1 Estimation model

In this part, each agent is assumed to perform its own estimation of the states of all other agents. Note $y^i = \left[y_1^{iT}, y_2^{iT}, \dots y_N^{iT} \right]^T$ with $y^i \in \mathbb{R}^{Nn}$ is the vector of state estimates of all agents by Agent i. Define the estimation errors $e_j^i = y_j^i - x_j$ with $y_j^i(t)$ is an estimate of the state of Agent j by Agent i and denote $e_i = e_i^i$. Define also $e = y - x = \left[e_1^T, e_2^T, \dots e_N^T \right]^T$ with $e \in \mathbb{R}^{Nn}$.

The new estimation $y_j^i(t)$ is here expressed as

$$\begin{aligned} \dot{y}_j^i(t) &= \mathbf{A}y_j^i(t) + \mathbf{B}\tilde{u}_j^i(t), \ \forall t \in \left] t_{j,k}^i, t_{j,k+1}^i \right[\\ \tilde{u}_j^i(t) &= c_1 \mathbf{F} \sum_{p \in \mathcal{N}_j} \left(y_j^i(t) - y_p^i(t) \right) \\ y_j^i\left(t_{j,k}^i\right) &= x_j\left(t_{j,k}^i\right), \end{aligned} \qquad (7)$$

with $t_{j,k}^i$ the time at which the k-th message sent by Agent j has been received by Agent i. It is assumed that there is no communication delay between agents. The time instant at which the k-th message has been sent by Agent j is denoted $t_{j,k}$ and $t_{j,k+1}$ denotes the time instant for the $(k+1)$-th message. The time of reception by Agent i of the ℓ-th message is t_ℓ^i, whatever the sending agent.

The estimate of their own state for all agents is $y = \left[\left(y_1^1\right)^T \ \dots \ \left(y_N^N\right)^T \right]^T$. Define $y_i = y_i^i$.

The problem considered consists in designing a control scheme to reach an bounded consensus, while limiting the communications between agents. For that purpose, times $t_{i,k}$ are chosen locally by Agent i using an event-triggered approach considering a threshold δ_i depending on the state estimation error e_i.

Theorem 1. Assume that $(\mathbf{A}, \mathbf{B})$ is controllable and that the communication graph is connected and undirected with a fixed topology described by Laplace matrix L. Then the agents described by the dynamics (3) achieve a bounded consensus where the difference between any two states of Agents i and j is bounded by

$$\lim_{t \to \infty} \|x_i - x_j\|^2 \leq \frac{N\eta}{\beta \lambda_{\min}(\mathbf{P})} \qquad (8)$$

with $\eta > 0$ is a design parameter, if the following condition on the perturbation bound is satisfied:

$$S_{\max} \leq \frac{N\eta}{\lambda_{\max}(\mathbf{P})\lambda_2(L)} \left\| -2\alpha\mathbf{P} + 2(1 - c_1\lambda_2(L))\mathbf{P}\mathbf{B}\mathbf{B}^T\mathbf{P} \right\| \qquad (9)$$

and if communication events are triggered when

$$\bar{\delta}_i > \sigma z_i^T \Theta z_i + \eta \qquad (10)$$

with $\Theta_i = (2c_2 - b_i N_i (c_2 - c))\mathbf{M}$ and

$$\begin{aligned} \bar{\delta}_i = c_1 \Big[& |z_i - N_i e_i|^T \mathbf{M} \textstyle\sum_{j \in \mathcal{N}_i} |\Delta_{ij}| + N_i(N-1)\left(\frac{1}{2b_i} e_i^T \mathbf{M} e_i \right) \\ & + \left(\tfrac{1}{2b_i} + b_i \right) N_i \textstyle\sum_{j \in \mathcal{N}_i} \left(\Delta_{ij}^T \mathbf{M} \Delta_{ij} \right) \Big] + 2(c_2 - c) N_i z_i^T \mathbf{M} e_i \\ & + \left[2c(N_i)^2(1 + b_i) + \tfrac{c_2 - c}{b_i} N_i + cN_i(N-1)\left(b_i + \tfrac{3}{b_i} \right) \right] e_i^T \mathbf{M} e_i \end{aligned}$$

and $\mathbf{M} = \mathbf{P}\mathbf{B}\mathbf{B}^T\mathbf{P}$, $0 < b_i < \frac{2c_2}{(c - c_2)N_i}$ if $c_2 > c$, $b_i > 0$ otherwise. $z_i = \sum_{j \in \mathcal{N}_i} \left(y_i - y_j^i \right)$. $\Delta_{ij} = y_i^j - y_i^i = y_i^j - y_i$ is the difference between the estimates of state of Agent i by Agents i and j.

If $\eta = 0$ and if there is no perturbation, the system achieves an assymptotic consensus. If $t = 0$, all agents are assumed to broadcast a message. When $t > 0$, communications are then triggered according to Theorem 1. The proof of Theorem 1 is detailed in Appendix A.

3.2 Δ_{ij} estimation and communication protocol

Theorem 1 involves the Δ_{ij}s which are in practice not accessible at all time instants. Agent i knows precisely Δ_{ij} only when it receives (possibly with some delay) a packet containing y_i^j from Agent j. To allow each agent to perform an estimate of the Δ_{ij}s, one has to consider a communication protocol in which each agent transmits its own estimate of the state of all other agents, but also his estimates of all other agents (c.f. Communication protocol).

Note first Theorem 1 only uses Δ_{ij} such as $j \in \mathcal{N}_i$: we will only create the estimates of Δ_{ij} for Agents $j \in \mathcal{N}_i$.

For $j \in \mathcal{N}_i$ and if the k-th packet has been transmitted by Agent j at time $t_{j,k}$, then

(1) Agent i has access to $y_i^j(t_{j,k})$ and is able to evaluate exactly $\Delta_{ij}(t_{j,k}) = y_i^j(t_{j,k}) - y_i(t_{j,k})$.
(2) Agent i is also able to reinitialize its estimate $y_j^i(t_{j,k})$ of the state of Agent j at time $t_{j,k}$, and thus Agent j can reset $\Delta_{ji}(t_{j,k}) = 0$.

As a consequence, the estimates $\tilde{\Delta}_{ij}^i$ performed by Agent i may be updated at $t_{j,k}$ as follows

$$\tilde{\Delta}_{ij}^i(t_{j,k}) = y_i^j(t_{j,k}) - y_i(t_{j,k}) \quad \text{if } t = t_{j,k}, \tag{11}$$

$$\tilde{\Delta}_{ij}^i(t_{i,k}) = 0_n \quad \text{if } t = t_{i,k}, \tag{12}$$

$$\tilde{\Delta}_{ij}^i(t) = \bar{\mathbf{Z}}_{ij}(t - t_{\mathrm{M}})\,\tilde{\Delta}_{ij}^i(t_{\mathrm{M}}),\ \forall t \in]t_{\mathrm{M}}, t_{\mathrm{M}+1}[\tag{13}$$

where $\bar{\mathbf{Z}}_{ij}(t) = \mathbf{E}_{ij}\exp(\mathbf{Z}t)\,\mathbf{E}_{ij}^+$ with

$\mathbf{E}_{ij} = [0_n, \ldots, 0_n, \mathbf{I}_n, 0_n, \ldots, 0_n]$ composed of $\left(N^2 - 1\right)$ matrix $0_{n\times n}$ and with $\mathbf{I}_{n\times n}$ is the $((i-1)*N+j)$ -th matrix element of $\mathbf{E}_{ij}$. Define $\mathbf{E}_{ij}^+$ the pseudo-inverse matrix of $\mathbf{E}_{ij}$ such as $\mathbf{E}_{ij}^+\mathbf{E}_{ij} = \mathbf{I}_{N^2 n}$. $t_{\mathrm{M}} = \max\{t_{j,k}, t_{i,k}\}$ and $t_{\mathrm{M}+1} = \min\{t_{i,k}, t_{i,k+1}\}$, $\mathbf{Z} = \tilde{\mathbf{A}}_c + \tilde{\mathbf{N}}$, $\tilde{\mathbf{A}}_c = \mathbf{I}_{N^2} \otimes \mathbf{A} + c_1(\mathbf{N} \otimes \mathbf{I}_N \otimes (\mathbf{BF}))$ with $\mathbf{N} = \mathrm{diag}([\,N_1\ N_2\ \ldots\ N_N\,])$, $\tilde{\mathbf{N}} = \tilde{A} \otimes (\mathbf{BF}) - \bar{A} \otimes (\mathbf{BF})$ with $\tilde{A} = \left(\left(\mathbf{I}_N \otimes \mathbf{1}_N^T\right).*\left(\mathbf{1}_N^T \otimes A\right)\right)$ and $\bar{A} = \left[\,(\mathbf{I}_N \otimes A(1,:))^T\ \ldots\ (\mathbf{I}_N \otimes A(N,:))^T\,\right]$, where $A(i,:)$ is the i-th line of the adjacency matrix A. Calculation of $\tilde{\Delta}_{ij}^i(t)$ is detailed in Appendix A. At $t = 0$, all agents are assumed to broadcast a message so $\tilde{\Delta}_{ij}(0) = 0_n$.

$\tilde{\Delta}_{ij}^i(t)$ increases with t when it received a packet containing y_i^j from Agent j: in this case, Δ_{ij} is overevaluated. However, when Agent i broadcasts a message at $t_{i,k}$, since Agent j broadcasts a message at $t_{j,k}$, 13 results in $\tilde{\Delta}_{ij}^i(t) = 0$, so Δ_{ij} is underevaluated.

Using the method described in this paper involves that all agents must estimate all other agent states even if they do not belong to their neighbourhood. When the graph is not fully-connected, an Agent i can not receive information from an Agent j if $j \notin \mathcal{N}_i$, so it can not adjust its estimation y_j^i. However, Agent i can receive the missing information from one of its neighbours k, if Agent k is a neighbour of Agent j. The communication protocol below describes how informations are relayed when an Agent broadcasts a new message: Agent i will send its vector y^i at $t_{i,k}$(instead of only y_i^i).

Communication protocol:

Assume that, according to Theorem 1 , Agent i broadcasts a message at time $t_{j,k}^i$:

- Note $Lt^i = [t_{1,k_1} \ldots t_{N,k_N}]$ the list of the last reception times corresponding to y^i.
- At $t_{i,k}$, $y_i(t_{i,k}) = x_i(t_{i,k})$ and Agent i broadcasts $y^i(t_{i,k})$ and Lt^i. Agent $j \in \mathcal{N}_i$ compares the times in Lt^i of reception of the estimated states with the ones in its own list Lt^j. The components of y^j are replaced by those of y^i if they have been received more recently. Conversely, Agent j broadcasts to Agent i the estimates of the states of other agents that have been more recently received than those of Agent i. The final step consists in resetting $\tilde{\Delta}_i$ to zero.
- As the graph is assumed to be connected, all state estimates are iteratively broadcasted to Agent i.

Transmitting the estimation y_j^i instead of the information received allows to account for its dynamical evolution between the time of reception and the time of sending the new message.

4. EXAMPLE

This section illustrates Theorem 1 considering an unstable system. Comparison with the estimator develops in Garcia et al. (2014c) is achieved when possible. Perturbations will be considered. Consider the $N = 5$ third-order agents with unstable linear dynamics given by:

$$\mathbf{A} = \begin{bmatrix} 0.48 & 0.29 & -0.3 \\ 0.13 & 0.23 & 0 \\ 0 & -1.2 & -1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ -1.5 & 1 \\ 0 & 1 \end{bmatrix},$$

$\mathbf{P}$ is obtained by solving (4)

$$\mathbf{P} = \begin{bmatrix} 4.8436 & 5.4783 & -1.1082 \\ 5.4783 & 7.0514 & -1.4299 \\ -1.1082 & -1.4299 & 0.3778 \end{bmatrix},$$

and we take $\alpha = 2.5$ for the evaluation of $S_{\max}$ in (9). $\mathbf{L_5}$ is the Laplace matrix associated to the graph with $N = 5$:

$$\mathbf{L_5} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

The following initial states are used:

$$x(0) = \begin{bmatrix} -4.72 & 0.67 & -2.61 & -1.96 & 3.03 \\ -1.14 & 0.73 & -0.99 & -0.81 & -0.63 \\ -0.10 & 0.25 & 0.01 & 0.06 & -0.05 \end{bmatrix} \tag{14}$$

Robustness with respect to initial conditions is also tested by adding a random Gaussien noise $(\mu_{\mathrm{b}}, \sigma_b^2)$ with $\mu_{\mathrm{b}} = 0$, $\sigma_b^2 = 5$ to the values of the initial states known by the agents. The total number of messages broadcasted during a simulation is denoted N_{m}. With the simulation duration $T = 5\ s$, N_{m} is upper bounded by $\overline{N}_{\mathrm{m}} = NT/dt$.

The added perturbation is $s_i = [0, s_{i2}, 0]^T$ with s_{i2} is a Gaussian noise $(0, \sigma_b^2)$ where $\sigma_b^2 = S_{\max}$, and truncated such as $|s_{i2}| < S_{\max}$ $(|s_i| \leq [0, S_{\max}, 0]^T \Leftrightarrow \|s_i\| \leq S_{\max})$, $m(t) = [0, m_2(t), 0]^T$ where $m_2(t)$ can be either constant (see e.g. Figure 2 (b)) or represent a Gaussian noise $(0, \sigma_b^2)$ (Figure 2 (a)).

To evaluate the efficiency of the proposed technique, the reduction ratio of broadcasted messages, expressed in % is approximated by $R_{\mathrm{com}} = 100 * \frac{N_{\mathrm{m}}}{\overline{N}_{\mathrm{m}}}$.

We set $\eta = 0.1$, $c = \frac{1}{\lambda_2(L)}$, and $c_2 = 0.1$. Euler integration with a step $dt = 0.01$ s is used. As system has been discretised, the minimum delay between the transmission of two messages by the same agent is set to $dt = 0.01$ s.

4.1 Results obtained with an unstable dynamics without perturbation

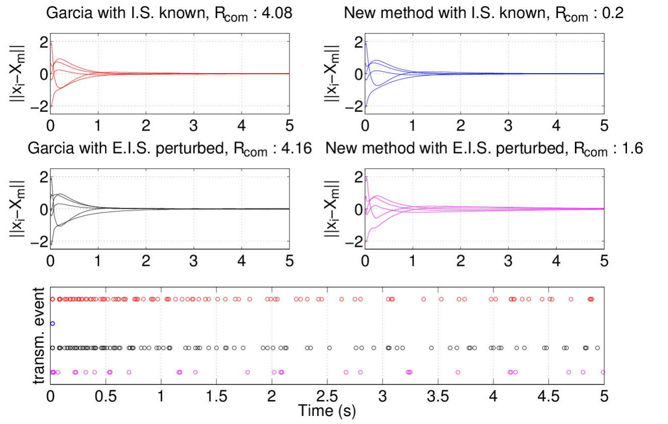

Fig. 1. Comparison between observer (6) and observer (7) with known Initial States (I.S.) and perturbed Estimation of the Initial States (E.I.S.).

Figure 1 illustrates the compared performance in terms of consensus errors and number of event-trigger communications for observer (7) and (6). When initial conditions are perfectly known and there is no perturbation, using observer (7) allows to limit the number of communications to the only initial one. When initial conditions are not perfectly known, the required number of communication for (7) increases but remains lower than the one required using (6). For both methods, the consensus error is bounded and tends to zero and the number of communication decreases with this error.

4.2 Perturbations

Figure 2 illustrates that using observer (7) lessens the number of messages required by observer (6) when perturbations are low. However, when the level of perturbations increases, the performance of observer (7) in terms of communication ratio become equivalent to observer (6) for time-varying perturbations.

5. CONCLUSION

This paper presents an event-triggered communication to reach consensus in multi-agent systems, with an improved agent state estimator to trigger communications using an estimate of the estimation errors. Convergence to consensus has been studied. Simulations have shown the effectiveness of the proposed estimators in presence of state perturbations.

An extensions of this work is the use of the new estimator to direct graph. Future works will adapt the method to time delay on the communication network and influence of the packet drops.

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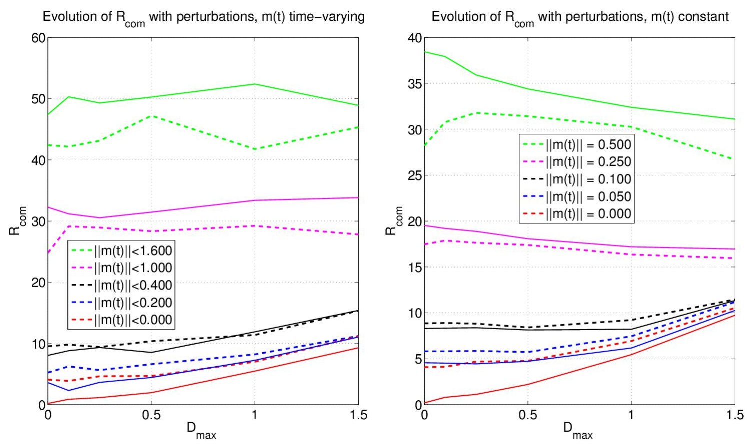

(a) with Gaussian $m(t)$

(b) with constant $m(t)$

Fig. 2. Comparison between observer (6) (Dash line) and observer (7) (Thick line).

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Appendix A.

A.1 Proof consensus convergence

Define $x = \left[x_1^T \dots x_N^T\right]^T$ the global state vector. The system gathering the dynamics of all the agents is then

$$\dot{x}(t) = \bar{\mathbf{A}}x(t) + \tilde{\mathbf{B}}\tilde{y}(t) + d(t)$$

where $\bar{\mathbf{A}} = \mathbf{1}_N \otimes \mathbf{A}$, $\tilde{\mathbf{B}} = \mathbf{T}\left(\mathbf{I}_N \otimes \overline{\mathbf{B}}_1\right)$, $\overline{\mathbf{B}}_1 = c_1 L \otimes \mathbf{BF}$, $\tilde{y} = \left[y^{1T}, y^{2T} \dots, y^{NT}\right]^T$with $\tilde{y} \in \mathbb{R}^{N^2 n}$ is the vector of states of Agents $1, \dots, N$ estimated by each agent. $\mathbf{T}$ is a matrix such as $\mathbf{T}\tilde{y} = y$, $\mathbf{T} = \left(\left(\mathbf{I}_N \otimes \mathbf{1}_N^T\right) .* \left(\mathbf{1}_N^T \otimes \mathbf{I}_N\right)\right) \otimes \mathbf{1}_n$, with $.*$ is the term-to-term matrix product. $\mathbf{T}$ satisfies $\mathbf{T}\left(\mathbf{1}_N \otimes y\right) = y$.

Define the Lyapunov function : $V = x^T \widehat{L} x$, with $\widehat{L} = L \otimes \mathbf{P}$ and L a Laplacian matrix, so $L\mathbf{1}_N = 0$. In this section, the graph is undirect so L is symmetric.

$$\dot{V} = 2\left(x^T \widehat{L}\left(\bar{\mathbf{A}}x + \tilde{\mathbf{B}}\tilde{y}\right) + d(t)\widehat{L}x\right) \tag{A.1}$$

Define $\dot{V}_1 = 2x^T \widehat{L}\left(\bar{\mathbf{A}}x + \tilde{\mathbf{B}}\tilde{y}\right)$ and $\dot{V}_2 = 2d(t)\widehat{L}x + x^T \bar{L}x$. Upper bounds on $\dot{V}_1$ and on $\dot{V}_2$ are derived in the two following sections.

Upper bounds on $\dot{V}_1$: Let $\Delta(t) = \left[\Delta_{11}^T(t), \Delta_{12}^T(t), \dots, \Delta_{N,N-1}^T(t), \Delta_{NN}^T(t)\right]^T$, $\Delta(t) \in \mathbb{R}^{N^2 n}$. Note first that $\tilde{y} = \mathbf{1}_N \otimes y + \Delta$.

$$\dot{V}_1 = 2x^T \widehat{L}\left(\bar{\mathbf{A}}x + \tilde{\mathbf{B}}\left(\mathbf{1}_N \otimes y + \Delta\right)\right) \tag{A.2}$$

Remind that $\tilde{\mathbf{B}} = \mathbf{T}\left(\mathbf{I}_N \otimes \overline{\mathbf{B}}_1\right)$, $\overline{\mathbf{B}}_1 = c_1 L \otimes (\mathbf{BF})$ as $\mathbf{T}\left(\mathbf{1}_N \otimes y\right) = y$. One obtains

$\tilde{\mathbf{B}}\left(\mathbf{1}_N \otimes y\right) = \mathbf{T}\left(\mathbf{I}_N \otimes \overline{\mathbf{B}}_1\right)\left(\mathbf{1}_N \otimes y\right) = \mathbf{T}\left(\mathbf{I}_N \otimes \left(\overline{\mathbf{B}}_1 y\right)\right) = \overline{\mathbf{B}}_1 y$:

$$\dot{V}_1 = 2x^T \widehat{L}\tilde{\mathbf{B}}\Delta + 2x^T \widehat{L}\left(\bar{\mathbf{A}}x + \overline{\mathbf{B}}_1 y\right) \tag{A.3}$$

Let us introduce the two following terms

$\dot{V}_{11} = 2x^T \widehat{L}\left(\bar{\mathbf{A}}x + \overline{\mathbf{B}}_1 y\right)$ and $\dot{V}_{12} = 2x^T \widehat{L}\tilde{\mathbf{B}}\Delta$. The expression of $\dot{V}_{11}$ is the one in Garcia et al. (2014c). From Garcia et al. (2014c) one obtains $\dot{V}_{11} = x^T \bar{L}x + \sum_{i=1}^N \left(\delta_i - z_i^T \Theta_i z_i\right)$.

Consider now $\dot{V}_{12} = 2x^T \widehat{L}\tilde{\mathbf{B}}\Delta$:

$$\dot{V}_{12} = 2\left(\widehat{L}(y - e)\right)^T \mathbf{T}\left(\mathbf{I}_N \otimes \left(c_1 L \otimes (\mathbf{BF})\right)\right)\Delta \tag{A.4}$$

as $\mathbf{T}\tilde{y} = y$, :

$$\mathbf{T}\left(\mathbf{I}_N \otimes \left(c_1 L \otimes (\mathbf{BF})\right)\right)\Delta =$$

$$\left[\, c_1 \textstyle\sum_{k \in \mathcal{N}_1} \left(\mathbf{BF}(\Delta_{11} - \Delta_{1k})\right)^T \ \dots \ c_1 \sum_{k \in \mathcal{N}_N} \left(\mathbf{BF}(\Delta_{NN} - \Delta_{Nk})\right)^T \,\right]^T$$

Remark $\Delta_{ii} = 0.\mathbf{F} = -\mathbf{B}^T\mathbf{P}$ and $\dot{V}_{12}$ expresses as

$$\begin{aligned}\dot{V}_{12} \leq c_1 \sum_{i=1}^N \Bigg[&\sum_{j \in \mathcal{N}_i} (y_i - y_j)^T \left(-\mathbf{PBB}^T\mathbf{P}\right) \sum_{k \in \mathcal{N}_i} (-\Delta_{ik}) \\ &- \sum_{j \in \mathcal{N}_i} (e_i - e_j)^T \left(-\mathbf{PBB}^T\mathbf{P}\right) \sum_{k \in \mathcal{N}_i} (-\Delta_{ik}) \Bigg]\end{aligned} \tag{A.5}$$

Remark $\Delta_{ji} = y_j^i - y_j$ so if can be shown $\sum_{j \in \mathcal{N}_i} (y_i - y_j)^T = z_i^T + \sum_{j \in \mathcal{N}_i} \Delta_{ji}^T$. Replacing this expression in(A.5) and defining $\mathbf{M} = \mathbf{PBB}^T\mathbf{P}$:

$$\begin{aligned}\dot{V}_{12} \leq c_1 \sum_{i=1}^N \Bigg[&z_i^T \mathbf{M} \sum_{k \in \mathcal{N}_i} \Delta_{ik} + \sum_{j \in \mathcal{N}_i} (\Delta_{ji})^T \mathbf{M} \sum_{k \in \mathcal{N}_i} \Delta_{ik} \\ &- N_i e_i^T \mathbf{M} \sum_{k \in \mathcal{N}_i} \Delta_{ik} + \sum_{j \in \mathcal{N}_i} e_j^T \mathbf{M} \sum_{k \in \mathcal{N}_i} \Delta_{ik} \Bigg]\end{aligned} \tag{A.6}$$

Using $|xy| \leq \frac{1}{2b_i} x^T x + \frac{b_i}{2} y^T y$, with $b_i > 0$:

$$\begin{aligned}\dot{V}_{12} \leq\ & c_1 \textstyle\sum_{i=1}^N \Big[(z_i - N_i e_i)^T \mathbf{M} \sum_{k \in \mathcal{N}_i} \Delta_{ik} + \sum_{j \in \mathcal{N}_i} N_i \left(\frac{1}{2b_i} + \frac{b_i}{2}\right) \\ & \times \Delta_{ij}^T \mathbf{M} \Delta_{ij} + N_i \textstyle\sum_{j \in \mathcal{N}_i} \left(\frac{1}{2b_i} e_j^T \mathbf{M} e_j + \frac{b_i}{2} \Delta_{ij}^T \mathbf{M} \Delta_{ij}\right) \Big] \\ \leq\ & c_1 \textstyle\sum_{i=1}^N \Big[|z_i - N_i e_i|^T \mathbf{M} \sum_{k \in \mathcal{N}_i} |\Delta_{ik}| + \frac{N_i(N-1)}{2b_i} e_i^T \mathbf{M} e_i \\ & + \textstyle\sum_{j \in \mathcal{N}_i} N_i \left(\frac{1}{2b_i} + b_i\right) \Delta_{ij}^T \mathbf{M} \Delta_{ij} \Big]\end{aligned} \tag{A.7}$$

For $\dot{V}_1$, the expression becomes

$\dot{V}_1 \leq x^T \bar{L}x + \sum_{i=1}^N \left(\bar{\delta}_i - \sigma z_i^T \Theta_i z_i\right)$ with

$$\bar{\delta}_i = c_1 \Big[|z_i - N_i e_i|^T \mathbf{M} \textstyle\sum_{k \in \mathcal{N}_i} |\Delta_{ik}| + \frac{N_i(N-1)}{2b_i} e_i^T \mathbf{M} e_i$$

$$+\left(\tfrac{1}{2b_i}+b_i\right)N_i\sum_{j\in\mathcal{N}_i}\left(\Delta_{ij}^T\mathbf{M}\Delta_{ij}\right)\Big]+\delta_i. \tag{A.8}$$

Then $\dot{V}_1 \leq 0$ if, for $i,j=1\dots N$, the events are triggered when $\bar{\delta}_i > \sigma z_i^T\Theta z_i$.

Remark 2. In order to reduce the number of broadcasted communications, a threshold η can be introduced so that $\bar{\delta}_i > \sigma z_i^T\Theta z_i + \eta$.

Upper bounding of $\dot{V}_2$

$$\dot{V}_2 = 2x^T\left(L\otimes\mathbf{P}\right)\left(\mathbf{1}_N\otimes m + s\left(t\right)\right) + x^T\bar{L}x$$
$$= 2x^T\widehat{L}s\left(t\right) + x^T\bar{L}x$$

because $\left(L\otimes\mathbf{P}\right)\left(\mathbf{1}_N\otimes m\right) = \left(\left(L\mathbf{1}_N\right)\otimes\left(\mathbf{P}m\right)\right) = 0$ and $L\mathbf{1}_N = 0$. Let $\dot{V}_3 = 2x^T\widehat{L}s\left(t\right)$ and $\dot{V}_4 = x^T\bar{L}x$.

$$\dot{V}_3 = 2\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\left(x_i-x_j\right)^T\mathbf{P}s_i$$
$$\dot{V}_3 \leq 2\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\left\|x_i-x_j\right\|\lambda_{\max}\left(\mathbf{P}\right)S_{\max} \tag{A.9}$$

Bounding $\dot{V}_4$ requires first to note that:

$$L\otimes\left(\mathbf{PA}+\mathbf{A}^T\mathbf{P}\right)+\left(LL\right)\otimes\left(-2c\mathbf{PBB}^T\mathbf{P}\right)$$
$$=\frac{1}{\lambda_2\left(L\right)}\left(\lambda_2\left(L\right)L\right)\otimes\left(\mathbf{PA}+\mathbf{A}^T\mathbf{P}\right)+\left(LL\right)\otimes\left(-2c\mathbf{M}\right)$$
$$\leq\frac{1}{\lambda_2\left(L\right)}\left(LL\right)\otimes\left[\mathbf{PA}+\mathbf{A}^T\mathbf{P}-2\mathbf{M}+2\left(1-c\lambda_2(L)\right)\mathbf{M}\right]$$
$$\leq\frac{1}{\lambda_2\left(L\right)}\left(LL\right)\otimes\left(-2\alpha\mathbf{P}+2\left(1-c\lambda_2\left(L\right)\right)\mathbf{M}\right) \tag{A.10}$$

Note: $c_1 \geq c \geq \frac{1}{\lambda_2(L)}$, so $\left(1-c_1\lambda_2\left(L\right)\right) \leq 0$. It can be shown that $\dot{V}_4 = x^T\bar{L}x$ is equal to:

$$\dot{V}_4 = x^T\left[L\otimes\left(\mathbf{PA}+\mathbf{A}^T\mathbf{P}\right)-\left(LL\right)\otimes\left(2c_1\mathbf{PBB}^T\mathbf{P}\right)\right]x$$

Using (A.10):

$$\dot{V}_4 \leq x^T\left[\frac{1}{\lambda_2\left(L\right)}\left(LL\right)\otimes\left(-2\alpha\mathbf{P}+2\left(1-c_1\lambda_2\left(L\right)\right)\mathbf{M}\right)\right]x$$

$$\leq \tfrac{1}{\lambda_2(L)}\sum_{i=1}^{N}\left[\sum_{j\in\mathcal{N}_i}(x_i-x_j)^T(-2\alpha\mathbf{P}\ +2(1-c_1\lambda_2)\mathbf{M})\sum_{j\in\mathcal{N}_i}(x_i-x_j)\right]$$
$$\leq\frac{1}{\lambda_2(L)}\sum_{i=1}^{N}\left[N\eta\left\|-2\alpha\mathbf{P}+2\left(1-c_1\lambda_2(L)\right)\mathbf{M}\right\|N\eta\right]$$

with η the positive constant threshold of Section A.1. $\dot{V}_2 \leq \dot{V}_3+\dot{V}_4$ so

$$\dot{V}_2 \leq \sum_{i=1}^{N}\Big[\sum_{j\in\mathcal{N}_i}(x_i-x_j)^T(-2\alpha\mathbf{P}+2(1-c_1\lambda_2(L))\mathbf{M})$$
$$\times\sum_{j\in\mathcal{N}_i}(x_i-x_j)+\sum_{j\in\mathcal{N}_i}\|x_i-x_j\|\lambda_{\max}(\mathbf{P})S_{\max}\Big] \tag{A.11}$$

The condition of Theorem 1 is then:

$$S_{\max} \leq \frac{N\eta}{\lambda_{\max}\left(\mathbf{P}\right)\lambda_2(L)}\left\|-2\alpha\mathbf{P}+2\left(1-c_1\lambda_2(L)\right)\mathbf{M}\right\|$$
$$2\sum_{j\in\mathcal{N}_i}\left\|x_i-x_j\right\|\lambda_{\max}\left(\mathbf{P}\right)S_{\max} \leq \sum_{j\in\mathcal{N}_i}\left\|x_i-x_j\right\|$$
$$\times\left\|-2\alpha\mathbf{P}+2\left(1-c_1\lambda_2(L)\right)\mathbf{M}\right\|\sum_{j\in\mathcal{N}_i}\left\|x_i-x_j\right\|\frac{1}{\lambda_2(L)}$$
$$2\sum_{j\in\mathcal{N}_i}\left\|x_i-x_j\right\|\lambda_{\max}\left(\mathbf{P}\right)S_{\max} \leq -\sum_{j\in\mathcal{N}_i}\left(x_i-x_j\right)$$
$$\times\left(-2\alpha\mathbf{P}+2\left(1-c_1\lambda_2(L)\right)\mathbf{M}\right)\sum_{j\in\mathcal{N}_i}\left(x_i-x_j\right)\frac{1}{\lambda_2(L)}$$

Using it in (A.11), one obtains $\dot{V}_3 \leq 0$. System converges to a bounding consensus.

Upper bound on $\dot{V}$: Assume there is no perturbation. Study now the value of $\|x_i-x_j\|$ when the conditions are satisfied. Let's first remark that $x^T\widehat{L}x \geq 0$ so $x^T\widehat{L}x \leq \lambda_{\max}\left(\widehat{L}\right)x^Tx$ and $x^T\bar{L}x \leq 0$ so $-x^T\bar{L}x \geq \lambda_{\min}\left(-\bar{L}\right)x^Tx$.

Using this, one obtains $x^T\bar{L}x \leq \frac{-\lambda_{\min}(-\bar{L})}{\lambda_{\max}\left(\widehat{L}\right)}x^T\widehat{L}x$. Define $\beta_2 = \frac{\lambda_{\min}(-\bar{L})}{\lambda_{\max}(\widehat{L})}$. With the triggering condition defined in Theorem 1, we obtain:

$$\dot{V}\left(t\right) \leq x^T\bar{L}x + \sum_{i=1}^{N}\left(\delta_i-\sigma z_i^T\Theta_i z_i\right)$$
$$\leq -\beta_2 V\left(t\right)+N\eta \tag{A.12}$$

Solving this differential equation results in $V\left(t\right) \leq V\left(0\right)e^{-\beta_2 t}+\frac{N\eta}{\beta_2}$ and when $t\to\infty$, $V\left(t\right)\leq\frac{N\eta}{\beta_2}$. We can express $V\left(t\right)=\frac{1}{2}\sum_{i=1}^{N}\left[\sum_{k\in\mathcal{N}_i}\left(x_i-x_k\right)^T\mathbf{P}\left(x_i-x_k\right)\right]$ and a bound on the difference between any two states i,j can be obtained. We can deduce $\lambda_{\min}\left(\mathbf{P}\right)\sum_{i=1}^{N}\left[\left\|x_i-x_j\right\|^2\right] \leq \frac{N\eta}{\beta_2}$ and so $\left\|x_i-x_j\right\|^2 \leq \frac{N\eta}{\lambda_{\min}(\mathbf{P})\beta_2}$.

Note that perturbation terms don't appear in terms $\bar{\delta}_i$ and Θ_i, but they will have an impact on error measurement, and then on the frequency of event triggering.

A.2 Estimation of $\tilde{\Delta}_{ik}$

It can be shown that the time derivative of the component Δ_{ik} of Δ is defined as :

$$\dot{\Delta}_{ik} = \left(\mathbf{A}+c_1N_i\mathbf{BF}\right)\Delta_{ik}+c_1\mathbf{BF}\sum_{j\in\mathcal{N}_i}\left(\Delta_{ji}-\Delta_{jk}\right)$$

using (7). Then $\dot{\Delta}$ satisfies:

$$\dot{\Delta} = \left[\mathbf{I}_{N^2}\otimes\mathbf{A}+c_1\left(\mathbf{N}\otimes\mathbf{I}_N\otimes\left(\mathbf{BF}\right)\right)\right]\Delta$$
$$+c_1\left[\tilde{A}\otimes\left(\mathbf{BF}\right)\right]\Delta-c_1\left[\bar{A}\otimes\left(\mathbf{BF}\right)\right]\Delta \tag{A.13}$$
$$\dot{\Delta} = \mathbf{Z}\Delta \tag{A.14}$$

with $\mathbf{Z} = \tilde{\mathbf{A}}_c+\tilde{\mathbf{N}}$, $\mathbf{Z}\in\mathbb{R}^{NNn}$, $\tilde{\mathbf{A}}_c = \mathbf{I}_{N^2}\otimes\mathbf{A}+c_1\left(\mathbf{N}\otimes\mathbf{I}_N\otimes\left(\mathbf{BF}\right)\right)$, $\tilde{\mathbf{N}} = \tilde{A}\otimes\left(\mathbf{BF}\right)-\bar{A}\otimes\left(\mathbf{BF}\right)$.

The solution of the differential equation $\dot{\Delta} = \mathbf{Z}\Delta$ is of the form $\Delta\left(t\right) = \exp\left(\mathbf{Z}\left(t-t_k\right)\right)\Delta\left(t_k\right)$ where t_k is the last communication time in the network $\mathcal{N}$. Remark $\Delta_{ij}\left(t\right) = \mathbf{E}_{ij}\exp\left(\mathbf{Z}\left(t-t_k^i\right)\right)\Delta\left(t_k^i\right)$ and remind $\mathbf{E}_{ij}^+$ the pseudo-inverse matrix of $\mathbf{E}_{ij}$ such as $\mathbf{E}_{ij}^+\mathbf{E}_{ij} = \mathbf{I}_{N^2n}$. So define

$$\tilde{\Delta}_{ij}^i\left(t\right) = \mathbf{E}_{ij}\exp\left(\mathbf{Z}\left(t-t_k^i\right)\right)\mathbf{E}_{ij}^+\mathbf{E}_{ij}\Delta\left(t_k^i\right).$$
$$\tilde{\Delta}_{ij}^i\left(t\right) = \bar{\mathbf{Z}}_{ij}\left(t-t_{j,k}^i\right)\Delta_{ij}\left(t_{j,k}^i\right) \tag{A.15}$$

with $\bar{\mathbf{Z}}_{ij}\left(t\right) = \mathbf{E}_{ij}\exp\left(\mathbf{Z}t\right)\mathbf{E}_{ij}^+$.