Using Mortar Element Method for Eddy Current Testing Finite Element Computations

Laurent Santandrea, Yayha Choua, Yann Le Bihan, Claude Marchand

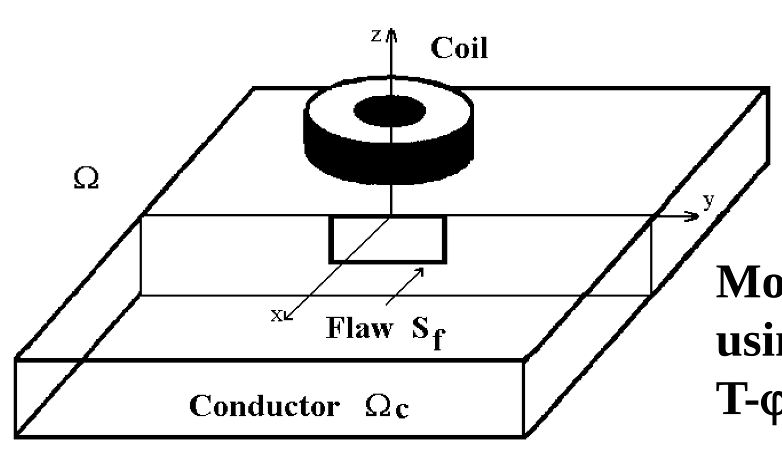
Using Mortar Element Method for Eddy Current Testing Finite Element Computations

Laurent SANTANDREA, Yayha CHOUA, Yann LE BIHAN, Claude MARCHAND
Laboratoire de Génie Électrique de Paris / SPEE Labs, CNRS UMR 8507, UPMC-P6 et Univ Paris-Sud-P11, Supelec, Plateau de Moulon 91192 - Gif sur Yvette - France
E-mail: santandrea@lgep.supelec.fr

Introduction

In this paper the use of the mortar element method (MEM) is presented to take into account the coil displacement in a three-dimensional finite element simulation of eddy current non-destructive testing (EC-NDT). The MEM is a non-overlapping, non-conforming domain decomposition method that allows for non matching grids at the sliding interface. To solve the three dimensional EC-NDT problem by finite element method (FEM) a magnetic formulation in term of combined T-φ potentials is used.

Eddy Current Testing (ECT) Problem



ECT configuration

Modelling by finite element method using combined potential formulation T-φ formulation providing the H field

Excitation
Frequency: 900 Hz
Inner radius: 6.15 mm
Outer radius: 12.4 mm
Length: 6.15 mm
Lift-off: 0.88 mm
Turns: 3790

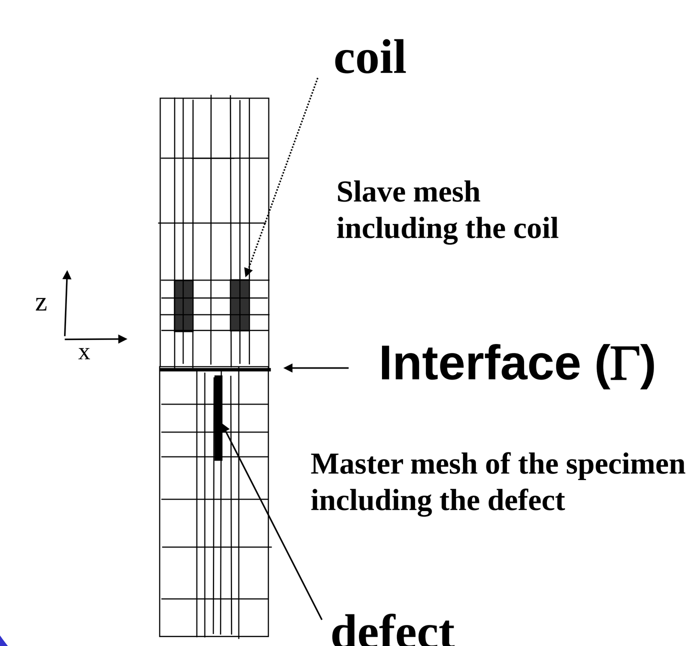
Specimen
Conductivity: 30.6 MS/m
Thickness: 12.22 mm

Crack
Orientation: ID
Length: 12.6 mm
Depth: 5.0 mm
Width: 0.28 mm

☞ **objective: reduction of the meshing process and of the matrix generation time**

Mortar Method

- **Non-overlapping, non-conforming domain decomposition method**
- **Useful for non matching grids at the sliding interface**
- **The essential transmission condition at the sliding interface is weakly imposed by means of chosen Lagrange multipliers.**
- **Geometry of the problem is decomposed in two domains**

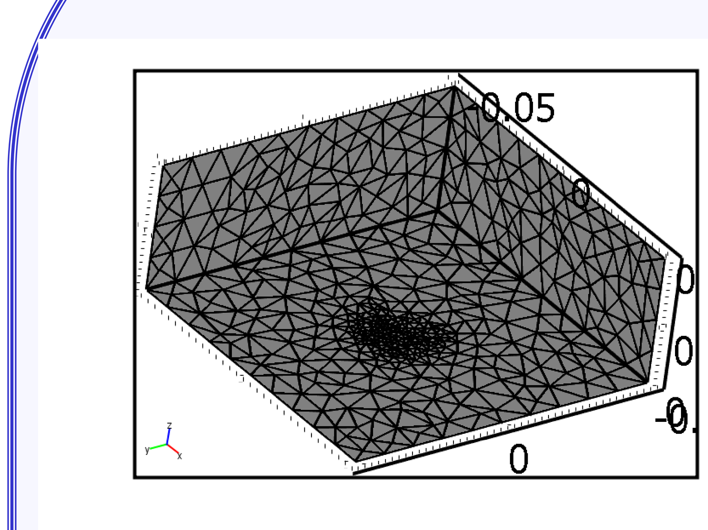


▪ At the interface between the two independent meshed domains a weak coupling condition is imposed:

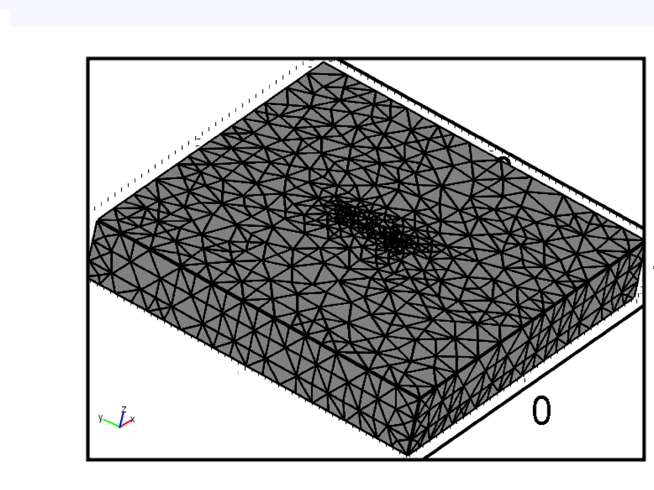
$$\int_{\Gamma} (\varphi^S - \varphi^M)\psi \, d\Gamma = 0$$

ψ is a test function, member of the Lagrange multiplier space and φ are the nodal first order 2D interpolation functions defined on the slave interface ΓS and on the master interface ΓM, respectively.

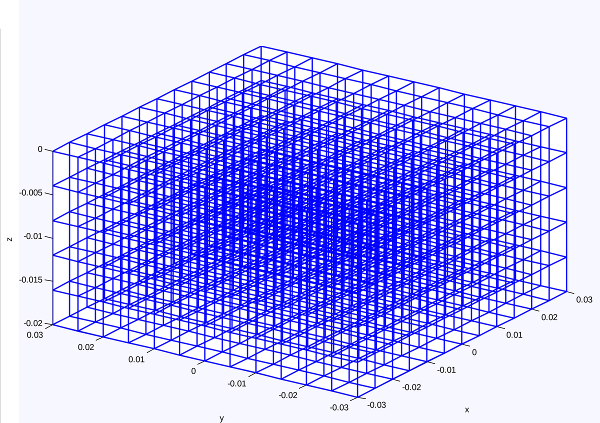
Results



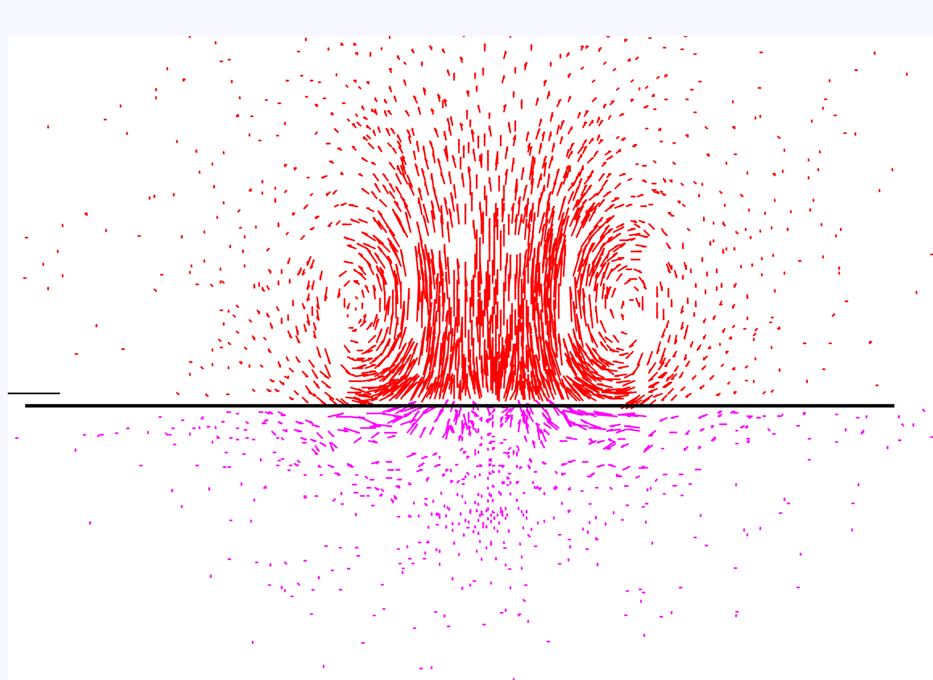
Tetraedral Mesh of the coil part



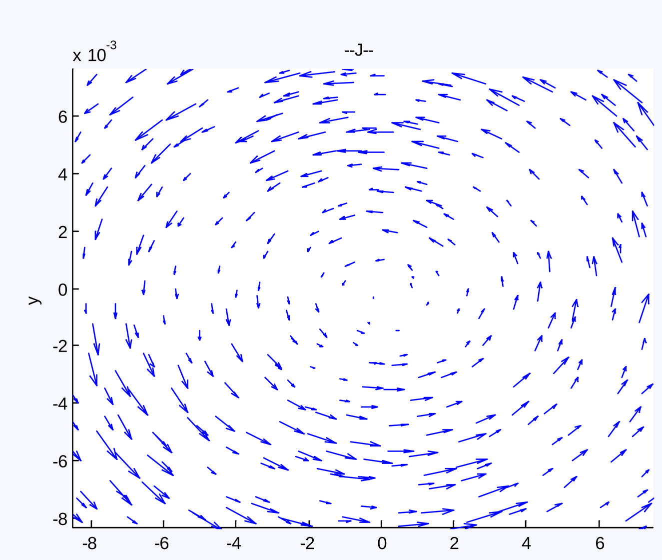
Tetraedral Mesh of the plate part



Hexaedral Mesh of the plate part



Induction magnetic field B in the cut plane y=0



Eddy current density in a cut plane in the plate

Impedance variation due to the defect

Conform mesh	ΔR=-0.14 Ω	ΔZ=2.45 Ω
Non Conform mesh	ΔR=-0.18 Ω	ΔR=2.15Ω

Matrix Form

The weak coupling condition can be written in a matrix form :

$$C\varphi_\Gamma^S = D\varphi_\Gamma^M \qquad C_{kj} = \int_\Gamma \varphi_k^S \varphi_j^S \, d\Gamma \qquad D_{ij} = \int_\Gamma \varphi_i^M \varphi_j^S \, d\Gamma$$

$$\varphi_\Gamma^S = Q\varphi_\Gamma^M$$

Q is the coupling matrix Q=C⁻¹D at the sliding interface. It is computed by using a quadrature formula on the slave side

$$(M+R)T + B\tilde{Q}\phi = F_C$$

$$\tilde{Q}^t B^t T + \tilde{Q}' K \tilde{Q}\phi = \tilde{Q}' F$$

The building of the final system leads to a sparse non-symmetric block matrix. To solve this system, a bi-conjugate gradient method is used.

Interest

- The tetrahedral elements are well adapted to mesh the inductor and air domains
- The hexahedral elements are well adapted to mesh the plate and the defect domain. In particular, they can support more distortion than the tetrahedral elements and they are more appropriate to mesh a fine defect..
- The Mortar Element Method allows to glue mesh with different element shapes.
- During the scan only the plate has to be remeshed. Structured meshing allows to have easy algorithm to execute this task which can be made directly in the C++ program. The matrix of the fixed part are not recalculated.

Conclusion

This method allows to reduce the time for the mesh generation and for the matrix computation. It is then a very useful method in a computer-aided numerical simulation environment. Investigations are in progress to increase accuracy on the interface (use of high order elements) and to have an optimized mesh. Use of more efficient conjugate gradient preconditioner are investigated to reduce the inversion matrix time.