A finite element modelling tool of a buried system for glass furnace wall characterization

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Abstract : This study deals with the wall characterization of a glass furnace based on an electrical resistance measurement between two buried electrodes. In this aim, the relationship between the wall geometry and the electrical resistance is established. This paper presents the modelling aspects of this inspection technique.

Keywords : Wall characterization, thermal problem, electrokinetic problem, 3D finite element modelling, mortar method.

1. Introduction

The refractory material erosion due to the melted glass flowing is a well known phenomenon in glass technology especially for gas heated furnaces. The erosion of walls have to be checked without stop of the furnace. Consequently, different non destructive investigation methods are being applied such as optical evaluation, mechanical measuring techniques, ultrasound or microwave techniques. The method presented in this paper is based on electrical resistance measurement between the two buried electrodes. A modelling approach is developed to evaluate the erosion influence on the electrical resistance and to optimise the electrical device. This numerical modelling based on the finite element method (FEM) must take into account two physical phenomena (thermal and electrical). In this aim, a FEM software has been developed in the Matlab environment.

2. Problem description

2.1 Studied structure

In this study, we consider a finite thickness wall having infinite lateral dimensions composed of several layers of materials (Figure 1). One side (Γi, temperature Thot) is in contact with the melted glass an the other (Γe, temperature Tcold) with the ambient air. Two electrodes are buried in the wall. The electrical resistance measured between the electrodes mainly depends of the temperature distribution and consequently of the wall geometry and allows to characterize it.

The two following configurations are considered

a- Evaluation of a thickness (th) variation due to an uniform erosion.

b- Analyse of a local erosion characterized by a cubic defect.

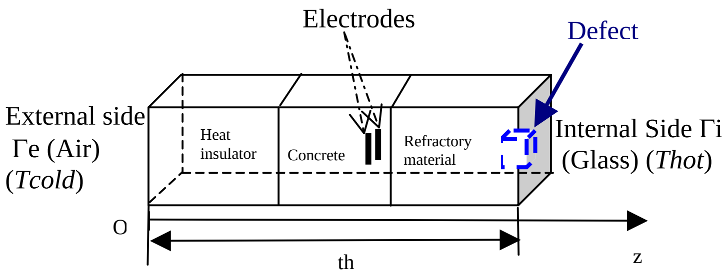

Figure 1: wall configuration example

2.2 Computational aspects

The objective is to compute the electrical resistance between the two electrodes. This computation necessitates the knowledge on the electrical conductivity distribution in the wall. This conductivity depending of the temperature T, a thermal problem have first to be solved. The temperature distribution in the wall is obtained solving a non linear problem. The cartography of the electric conductivity σ(T) in the volume is then deducted. In a second step, the distribution of the current density in the wall created by an electrical potential difference applied between the two electrodes is determined and the equivalent resistance between the electrodes is finally calculated.

3. Formulations

Two decoupled physical problems (thermal-electrical) are solved numerically using the FEM.

3.1 Thermal problem formulation

The heat equation is solved in the volume of the wall (Figure 1). On the cold side Γe two boundary conditions are possible (imposed temperature or convection condition).
On the internal side, the temperature is imposed to the temperature of the melted glass. The thermal probem is then formulated as :

$$\begin{cases} -div(k(T)grad(T))=0 \\ T(z=th)=Thot \text{ on } \Gamma i \\ -kgrad(T).n=h(T(z=0)-Tair) \text{ or } T(z=0)=Tcold \text{ on } \Gamma e \end{cases} \quad (1)$$

with T the temperature, $k(T)$ the thermal conductivity, and h the equivalent exchange coefficient. n unitary vector to the surface.

In order to solve the non-linear problem, an iterative fixed point method is used because of its robustness. The resolution is carried out in 1D for (a) configuration whereas in the (b) configuration a 3D resolution is needed.

Experimental results was used to determine an effective transfer coefficient h . An expression of h is obtained using the continuity of the heat flux on the external wall.

$$\varphi = -k\frac{dT}{dz} = -h(Tcold - Tair) \qquad (2)$$

Considering a wall constituted of a single material layer and integrating the equation along the wall thickness below

$$\int_{z=0}^{L} -k\frac{dT}{dz}dz = \int_{z=0}^{L} -h(Tcold - Tair)dz \qquad (3)$$

we obtain : $\int_{Tfroid}^{Tchaud} kdT = -h(Tcold - Tair)L \qquad (4)$

and finally

$$h = \frac{\int_{Tcold}^{Thot} kdT}{(Tcold - Tair)L} \qquad (5)$$

The non-linear expression of k (in $Wm^{-1}K^{-1}$)

Thickness 30mm		**Thickness 52mm**	
Thot(°C)	Tcold(°C)	Thot(°C)	Tcold(°C)
900	436	900	340
1200	577	1200	449
1500	735	1500	563
1600	796	1600	602

depending of T is taken as following

k(T) = A + B*(T/1000) + C*(T/1000)^2 (6)

Using these experimental data for Thot and Tcold, values of h can be calculate in function of the temperature.

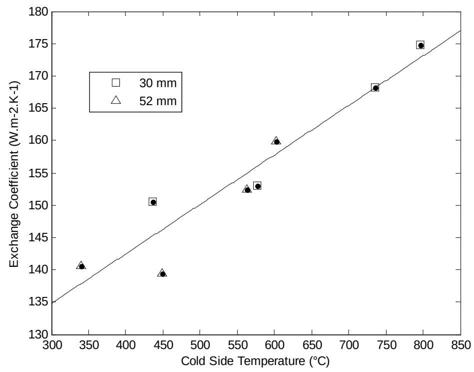

Figure 2 : least-squares fitting of the experimental results

The least-squares method with first order polynomial approximation allows to obtain a good estimate of h.

h(Tcold) = a +b Tcold (°C) with : a = 111,8 and b = 0,0766. (7)

3.2 Electrical problem formulation

To compute the resistance, the relation R= V^2/P is used where V is the electric potential difference applied between the two electrodes and P is the Joule losses in the wall which are given by:

$$P = \int_{\Omega} \sigma E^2 d\Omega \qquad (8)$$

E represents the electrokinetic field ($\boldsymbol{E}$) norm. $\boldsymbol{E}$ = -grad$V$. The electric problem can be formulated as:

$$\begin{cases} \sigma(T)\Delta V = 0 \\ V = V1 \text{ on electrode 1} \\ V = V2 \text{ on electrode 2} \end{cases} \qquad (9)$$

This equation is solved using the FEM with a first order hexahedral element grid on the domain of Figure 1. The electrical conductivity has been first computed at the centre of gravity of each element using the temperature distribution obtained in 3.1.

4. Software considerations

The objective is to develop a software tool usable by a non-specialist of the FEM. The Matlab environment is selected for implementation. In particular, it allows an easy matrix manipulation well suited in a FEM context, and it has powerful tools for visualization and for graphical user interface development. For this purpose the mesh generation is made as automatic as possible. The user needs only to introduce geometrical parameters (electrode size, electrode spacing, defect size, material layer thicknesses, etc …) and the desired mesh density.

The 3D mesh is obtained by an extrusion method. This procedure is easily implemented for the (a) configuration. In the defect case (b), it is difficult to have a general 2D mesh to extrude, taking into account simultaneously the electrode geometries and the defect geometry. The mesh generation is then decomposed into two steps (Figure 2). The first step allows to obtain the part of the mesh with the defect. In the second step, the mesh including the electrodes is generated.

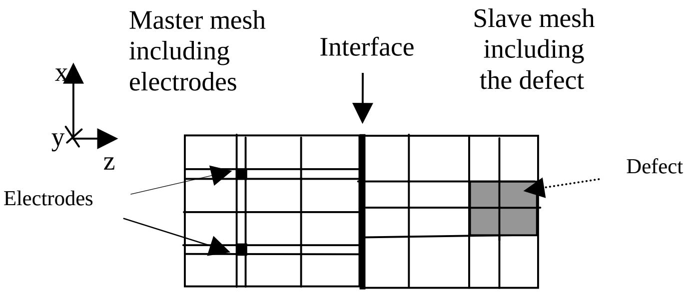

Figure 3:Cutting plane of a 3D mesh

To assemble the two non conform grids, the mortar method introduced in [1][2] is used. The specificity of the method lies in the choice of the unknowns of one interface as Lagrange multipliers to impose the continuity of the calculated quantity.
The building of the 3D FEM matrix in Matlab requires an important computational time. In order to reduce this one, compiled routine written in C language are interfaced with Matlab.
Two kinds of use of the software are possible : graphical user interface and scripting program.

5. Software validation

5.1 Thermal Problem Validation

Firstly, the thermal problem is validated. Results obtained with the developed tool are compared with the ones obtained with the PDE-toolbox of Matlab. As an example, a wall constituted of a single material is considered here. The main characteristics of the test problem are as follows: Thickness = 52 mm, Th = 1600°C. The transfer coefficient h is taken constant : h =160 $Wm^{-2}K^{-1}$.

k(T) = A + B*(T/1000) + C*(T/1000)^2 , k in W/(mK) and T in °C with : A=7,028, B=-8,161, C=5,223

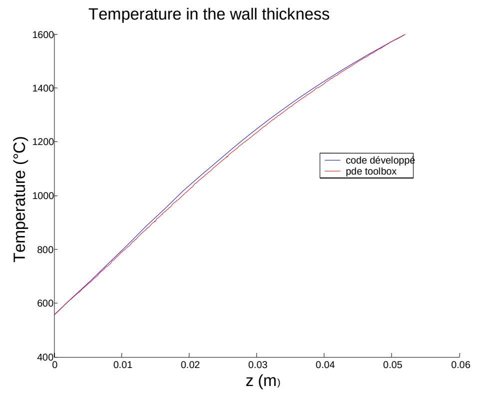

Figure 4: Example of validation of the thermal calculation: evolution of the temperature through the wall thickness.

A good agreement is observed between the results of the two codes.

5.2 Electric Problem Validation

In a second time, the electrical part is checked. Our results are compared to numerical results obtained with the commercial software ANSYS. The same finite element mesh is use for the both calculations. In first, the temperature distribution is obtained by our Matlab program without defect (1D calculation). Using experimental data for electrical conductivity in function of the temperature, the electrical conductivity for each finite element is calculated and used for resistance calculation.

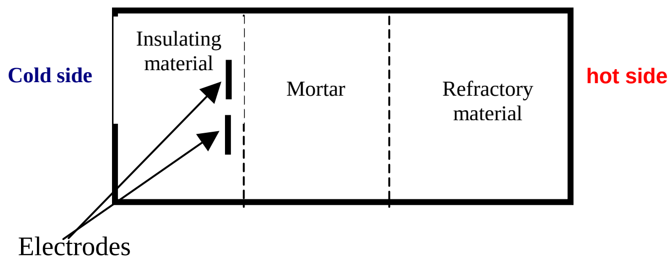

Figure 5 :typical wall furnace configuration

After computation, the maximum mean error of numerical results is less than 4%.

6. Use of the numerical tool

This numerical modelling tool is used to make a parametric study. In particular, the influence of the defect position compared to the electrodes position, the influence of the electrical conductivity of the refractory material (layer of the wall in contact with the glass), the influence of the size of the defect and the influence of the wall thickness wall are investigated.

6.1 Wall thickness estimation

The dependence of the resistance to the wall thickness is investigated for the following configuration:

Only one layer with only one material
Thickness of the material : 25 mm to 60 mm
The electrodes are buried in the insulating material

Electrodes geometry

depth	2 mm
thickness	2 mm
width	2 mm
height	46 mm
gap	10 mm

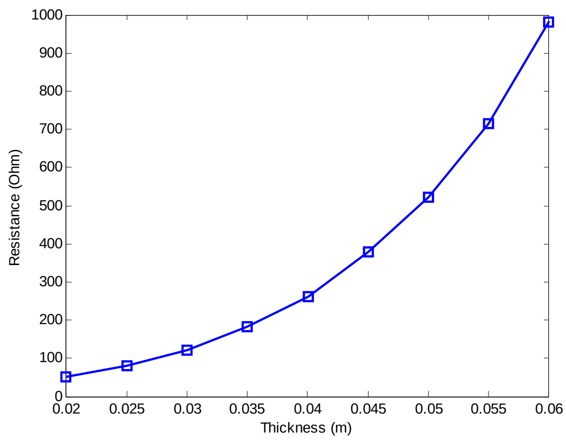

Figure 6 :Evolution of the resistance in function of the wall thickness

It appears that the resistance decreases with the thickness decreasing. This can be attributed to the increase of the temperature into the wall when it thickness decreases. Indeed, considering the materials which are used in the glass furnace realisation, the electrical conductivity is a highly increasing function of the temperature.

6.2 Electrical conductivity influence

The influence of the electrical conductivity in the refractory material is investigated. A temperature independent low electric conductivity (10^{-4} S/m) and a high conductivity (10^{4} S/m) are successively imposed in the refractory material.

Thickness of the wall

Thermal insulating material	200mm
Concretes	125 mm
Refractory material	150 mm

The electrodes are buried in the insulating material

Electrodes geometry

depth	193 mm
thickness	2 mm
width	2 mm
height	150 mm
gap	40 mm

Internal furnace temperature : 1347 °C
External furnace temperature : 600 °C

The thermal conductivity is fixed using the formula (6) with the following coefficients A,B,C.

Electrical resistivity

Refractory material		Mortar		Insulating material	
T (°C)	ρ (Ohm*m)	T (°C)	ρ(Ohm*m)	T (°C)	ρ(Ohm*m)
850	10	800	50,00	800	500
1000	4	1000	10,00	1000	100
1100	2,9	1200	4,00	1200	40
1300	1	1400	2,50	1400	25
1500	0,8	1500	2,00	1500	20
1600	0,7			1600	0,7

Thermal conductivity

Refractory material

A	B	C
5,608	-5,396	3,348

Mortar

A	B	C
5,087	-5,391	2,464

Insulating material

A	B	C
0,816	1,202	-0,589

The temperature distribution computed the electrical resistivity is deducted by fitting the experimental data below.

Remark : the glass is supposed to have a very high electrical resistivity in comparison of others materials (10^4 Ohm.m)

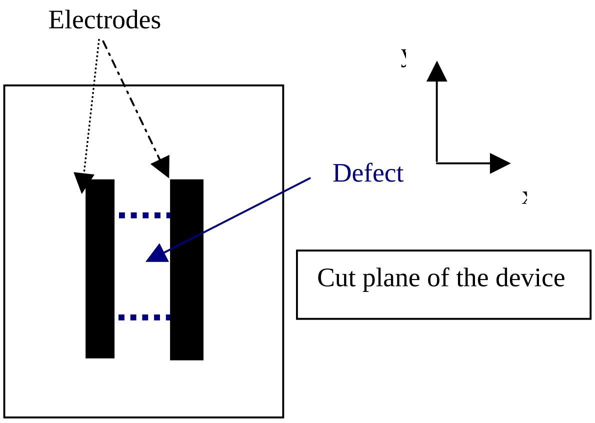

Figure 4 : defect and electrodes position

Electrical Conductivity (S/m)	10^{-4}	10^{4}
Resistance (Ohm)	313.70	312.8

These results show that the electric conductivity of the refractory has nearly no influence on the resistance. It is rather the change of the temperature distribution in the region close to the electrodes which impacts on this quantity.

We consider a case with a defect centered between the electrodes. Defect parameter : lx = 50 mm, ly = 70 mm, lz = 100 mm

Defect	No defect	defect centered (0,0)	defect no decentered $(10^{-2},10^{-2})$	Uniform Diminution of thickness Wall
Resistance (Ohm)	346.38	313.62	316.16	285.18

The difference of the resistance value between the case with defect and the case without defect can be explained by the non uniform temperature distribution obtained between the electrodes in the defect case as shown in figure 5.

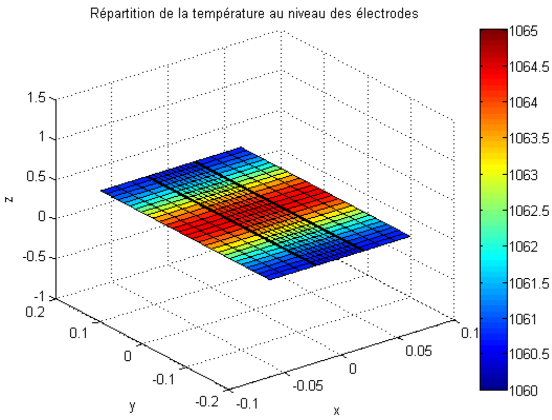

Figure 5 : temperature distribution between the electrodes

The results indicates the possibility of the investigated method to detect such kind of defects.

6.3 Electrodes gap influence

We consider a case with a defect centered between the electrodes. Defect parameters are lx = 50 mm, ly = 70 mm, lz = 80 mm

In this case, the influence of the electrodes gap is studied. Two computations are made successively varying the electrodes gap (along x axis). The first no defect is considered and the second has a defect described below. The sensibility of the device given as fellow is visualised.

S=(Rwd-Rd)/R

Rwd represents the resistance without defect and Rd represents the resistance with d.

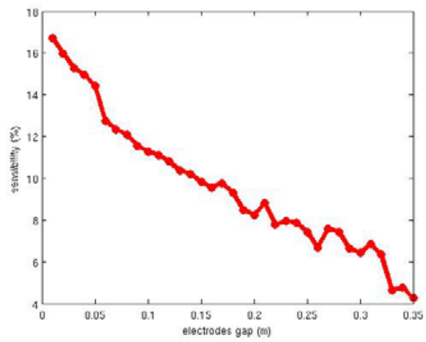

Figure 6 : sensibility of the resistance in function of the electrodes gap

7. Conclusion

In this study, a numerical software tool, based on FEM has been developed under Matlab environment. It allows to a non-specialist of the FEM to evaluate the system behaviour, to optimise the size and the position of the electrodes and finally to generate databases for wall characterization. In particular, as it has been presented, the numerical results show that it is the value of the temperature on the region close to the electrodes rather than the volume of material flowed by the current that has a significant effect on the value of the resistance.

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