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Said Lazreg, Olivier Hubert

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S. Lazreg, O. Hubert
LMT-Cachan
61, avenue du Président Wilson- 94235 CACHAN Cedex- FRANCE
Téléphone : 01 47 40 77 66 - 22 24
Télécopie : 01 46 40 22 40
Adresse(s) électronique(s) : lazreg@lmt.ens-cachan.fr, hubert@lmt.ens-cachan.fr

Mots clés : Kinematic hardeneing, Internal stress, Magnetic behavior, Magnetostrictive behavior, Multidomain modeling

1 INTRODUCTION

Magnetic non destructive methods are widely used in the industries of steel since magnetic behavior demonstrates a good sensitivity to the microstructural and/or mechanical changes. It is well known that plastic deformations lead to a sharp degradation of the magnetic properties of ferromagnetic materials (strong decrease of susceptibility, increase of hysteresis losses) especially for weak plastic strain levels [1] . Therefore, Magnetic method can be potentially used to evaluate the plastic state of a material. This evaluation requires an accurate model as principal part of the non destructive procedure. This model has to predict the magnetic behavior of the material involved. It must on the other hand demonstrate low computation time to allow "on line" inverse identification of the mechanical and metallurgical state. Such magneto-mechanical modeling is proposed. It is based on the so-called multidomain model. Plasticity is introduced through internal stress characterizing heterogenous biphasic structure. It is shown that kinematic hardening can be estimated by magnetic measurements.

2 MECHANICAL MODELING

This modeling must respect time constraints inherent with the calculation specifications for NDE. Thus, the micro-macro approach commonly used in mechanics should be avoided. The modeling proposed requires first the calculation of residual stress fields considering the material as a two phased material as initially proposed by Mughrabi [2]. The effect of plastic deformation on the macroscopic magnetic behavior is supposed to correspond to an average effect of the residual stresses on the magnetic behavior of each phase.

2.1 Composite model

At the macroscopic scale, we consider a representative volume element (RVE) consisting of two phases: a soft phase s and a hard phase h, meaning that s phase exhibits a lower yield stress and strengthening than the h phase. f_s and f_h indicate the volume fraction of s and h phases. The RVE is submitted to an elastoplastic stress tensor $\mathbf{\Sigma}$. $\mathbf{E}^e$, $\mathbf{E}^p$ and $\mathbf{E}$ denote the elastic, plastic and total deformation tensors respectively so that :

$$\mathbf{E} = \mathbf{E}^e + \mathbf{E}^p = \mathbb{C}^{-1}\mathbf{\Sigma} + \mathbf{E}^p \tag{1}$$

where $\mathbb{C}$ indicates the stiffness tensor of the medium supposed isotropic and homogeneous whatever the phase. The same decomposition can be made for each phase:

$$\boldsymbol{\epsilon}_h = \boldsymbol{\epsilon}_h^e + \boldsymbol{\epsilon}_h^p = \mathbb{C}^{-1}\boldsymbol{\sigma}_h + \boldsymbol{\epsilon}_h^p \qquad \boldsymbol{\epsilon}_s = \boldsymbol{\epsilon}_s^e + \boldsymbol{\epsilon}_s^p = \mathbb{C}^{-1}\boldsymbol{\sigma}_s + \boldsymbol{\epsilon}_s^p \tag{2}$$

Macroscopic stress and deformation $\mathbf{\Sigma}$ and $\mathbf{E}$ are given by:

$$\mathbf{\Sigma} = f_h\boldsymbol{\sigma}_h + f_s\boldsymbol{\sigma}_s \qquad \mathbf{E} = f_h\boldsymbol{\epsilon}_h + f_s\boldsymbol{\epsilon}_s \tag{3}$$

The local stress is given on the other hand by the Hill's relationship [3] so that:

$$\boldsymbol{\sigma}_s = \boldsymbol{\Sigma} + \mathbb{C}^*(\mathbf{E} - \boldsymbol{\epsilon}_s) \qquad \boldsymbol{\sigma}_h = \boldsymbol{\Sigma} + \mathbb{C}^*(\mathbf{E} - \boldsymbol{\epsilon}_h) \tag{4}$$

where $\mathbb{C}^*$ indicates the Hill's constraint tensor.
Because of isotropic elasticity, the plastic deformation tensors verify:

$$\mathbf{E}^p = f_s \boldsymbol{\epsilon}_s^p + f_h \boldsymbol{\epsilon}_h^p \tag{5}$$

so that it is possible to define two residual stress tensors $\mathbf{D}_s$ and $\mathbf{D}_h$ satisfying:

$$\boldsymbol{\sigma}_s = \boldsymbol{\Sigma} + \mathbf{D}_s \qquad \boldsymbol{\sigma}_h = \boldsymbol{\Sigma} + \mathbf{D}_h \tag{6}$$

$$f_s \mathbf{D}_s + f_h \mathbf{D}_h = 0 \tag{7}$$

2.2 Correlation with kinematic hardening

On the other hand the plastic straining of a material is suitably described by the yield function f which can be expressed as function of the macroscopic deviatoric stress tensor $\mathbf{S}$[1], yield stress Σ_y, isotropic R and kinematic $\mathbf{X}$ hardening components [4]:

$$f(\boldsymbol{\Sigma}) = \sqrt{\frac{3}{2}(\mathbf{S} - \mathbf{X}) : (\mathbf{S} - \mathbf{X})} - \Sigma_y - R \tag{8}$$

assuming that the strengthening is suitably described by a von Mises criterion. $\mathbf{X}$ tensor is a non linear function of the plastic strain tensor $\dot{\mathbf{E}}^p$ (9), related to the position of the yield function ($f = 0$) in the stress space and representative of heterogeneous and multiaxial residual stress field within the material.

$$\dot{\mathbf{X}} = g(\dot{\mathbf{E}}^p) \tag{9}$$

Coming back to the previous decomposition in soft and hard phases, it can be shown that the kinematic hardening is directly associated to the residual stress within the soft phase, so that:

$$\mathbf{D}_s = -\frac{3}{2}\mathbf{X} \tag{10}$$

Assuming that the volume fraction of hard and soft phases are known, an experimental estimation of the quantity $\mathbf{X}$ allows to define the stress field within the two phases.

3 MAGNETIC MODELING [5]

The multidomain modeling is a two-scales (domain α and grain g) reversible modeling allowing the prediction of the magneto-mechanical behavior of isotropic polycrystals. Based on an energetic minimization, this model can predict the magneto-mechanical behavior of single crystal submitted to a magnetic field $\vec{H}$ and/or uniaxial stress $\boldsymbol{\sigma}$ applied along a same direction $\vec{n}_c$ defined by angles ϕ_c and θ_c of the spherical frame. This direction is restricted to the standard triangle defined by crystallographical directions $< 100 >$, $< 110 >$ and $< 111 >$: cubic symmetry means that at any loading direction, there is a corresponding direction in this triangle. Uniform strain and field hypotheses are used over the crystal and domain walls contribution to the total energy is neglected [6]. Thanks to an analytical minimization of the total energy of each domain α, it is possible to express constitutive laws for the evolution of magnetization angles of each domain and of the volumetric fraction f_α, as function of magnetic field, stress, and loading direction parameters (see [5] for more details). Average magnetization and magnetostriction (11) are projected along the loading axis $\vec{n}_c$ leading to the behavior of the single crystal $M(H,\sigma)$ and $\epsilon^{\mu}_{//}(H,\sigma)$ (12). We assume on the other hand that because all possible loading directions are restricted to the standard triangle, the behavior of an isotropic polycrystal is necessarily given by a loading along a specific direction inside the triangle. Since behaviors are not linear, this direction is not the average direction and is theoretically changing with stress or magnetic field level. We consequently make the assumption that this change is small enough to be neglected. To simulate the magnetic behavior of a magnetic material, parameters to be identified are

1. with $\mathbf{S} = \boldsymbol{\Sigma} - \frac{1}{3}tr(\boldsymbol{\Sigma})\mathbf{I}$; $\mathbf{I}$: identity tensor.

ϕ_c, θ_c of the specific direction and A_s (as an adjustment parameter).

$$\vec{M} = \sum_\alpha f_\alpha \, \vec{M}_\alpha \qquad \boldsymbol{\epsilon}^\mu = \sum_\alpha f_\alpha \, \boldsymbol{\epsilon}^\mu_\alpha \tag{11}$$

$$M = \vec{M}.\vec{n}_c \qquad \epsilon^\mu_{//} = {}^t\vec{n}_c.\boldsymbol{\epsilon}^\mu.\vec{n}_c \tag{12}$$

We consider now a plastified material composed of hard (h) and soft (s) phases. We suppose on the other hand that macroscopic stress $\mathbf{\Sigma}$, kinematic hardening $\mathbf{X}$ and volume fraction of s and h phases are known. The stress field within the two phases is consequently defined ($\boldsymbol{\sigma}_s$,$\boldsymbol{\sigma}_h$). Considering finally homogeneous magnetic field condition and assuming that the magnetic behavior of each phase is known, a mixing law allows the estimation of the magneto-mechanical behavior of the whole material:

$$\vec{M}(\vec{H},\mathbf{\Sigma}) = f_s\vec{M}_s(\vec{H},\boldsymbol{\sigma}_s) + f_h\vec{M}_h(\vec{H},\boldsymbol{\sigma}_h) \tag{13}$$

$$\boldsymbol{\epsilon}^\mu(\vec{H},\mathbf{\Sigma}) = f_s\boldsymbol{\epsilon}^\mu_s(\vec{H},\boldsymbol{\sigma}_s) + f_h\boldsymbol{\epsilon}^\mu_h(\vec{H},\boldsymbol{\sigma}_h) \tag{14}$$

Magnetization and magnetostriction of the s and h phases can be modeled separately thanks to the multidomain modeling.
A first step is to get the parameters of each phase (loading axis and constant A_s). A second step is to change the multiaxial stresses $\boldsymbol{\sigma}_s$ and $\boldsymbol{\sigma}_h$ into uniaxial magneto-mechanical equivalent stresses according to the direction of the magnetic loading. We use for that purpose the simplified equivalent stress recently defined in [7]:

$$\sigma_i^{eq} = \frac{3}{2}\, {}^t\vec{n}\mathbf{S}_i\vec{n} \tag{15}$$

Index i indicates s or h phase. $\mathbf{S}_i$ is the deviatoric tensor associated to $\boldsymbol{\sigma}_s$ and $\boldsymbol{\sigma}_h$ respectively. $\vec{n}$ indicates the direction of the magnetic loading.

4 EXPERIMENTAL VALIDATION: TENSILE STRENGTHENING

4.1 Experimental protocol

A dual phase steel is used for the study. Its microstructure consists of about 30%vol of hard martensite islands dispersed in a soft and ductile ferritic matrix. As a first approximation, the martensite does not play any significant role in the magnetic behavior. The ferritic matrix can be considered as pure iron. The stress-strain $\Sigma(E)$ behavior of the material is carried out and unloading/reloading tests permit to estimate the kinematic X and isotropic R hardenings as function of the plastic strain E^p thanks to a Cottrell's method. An experimental protocol permits to inspect the magnetic state as function of plastic straining.The prestrained samples are submitted to an increasing level of tensile stress. Magnetic and magnetostrictive measurements are performed. Figures 1.a et 1.b show respectively the evolution of the magnetic and magnetostrictive behaviors of the sample prestrained at 3% and reloaded at various stress levels indicated in the figure. We observe that a critical stress Σ_c^X (resp. Σ_c^μ) allows to recover the initial magnetic (resp. magnetostrictive) behavior of the material.

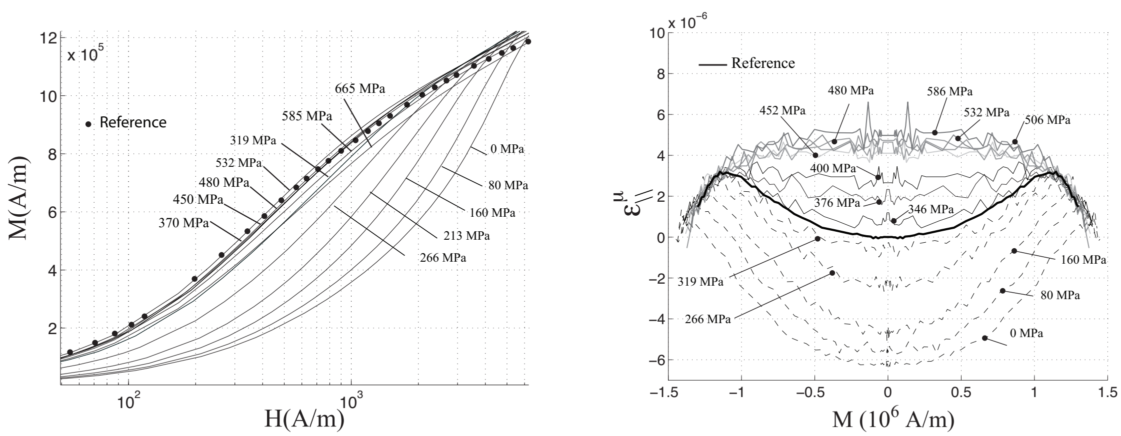

FIG. 1 – *Anhysteretic magnetic (a) and parallel magnetostrictive (b) behaviors of the sample prestrained at* 3% *and reloaded at different stress levels*

4.2 Comparison between experiments and modeling

Let consider a tensile loading of axis $\vec{x}$ leading to an axial plastic deformation E^p. The material can be reloaded along the same direction. The macroscopic plastic strain tensor is constant, diagonal and deviatoric, as well as the kinematic hardening. The stress tensors are:

$$\mathbf{\Sigma} = \begin{pmatrix} \Sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{(\vec{x},\vec{y},\vec{z})} ; \boldsymbol{\sigma}_s = \begin{pmatrix} \Sigma + \frac{3}{2}X & 0 & 0 \\ 0 & -\frac{3}{4}X & 0 \\ 0 & 0 & -\frac{3}{4}X \end{pmatrix} ; \boldsymbol{\sigma}_h = \begin{pmatrix} \Sigma - \frac{f_s}{f_h}\frac{3}{2}X & 0 & 0 \\ 0 & \frac{f_s}{f_h}\frac{3}{4}X & 0 \\ 0 & 0 & \frac{f_s}{f_h}\frac{3}{4}X \end{pmatrix} \quad (16)$$

Considering on the other hand a magnetic loading along $\vec{x}$ axis, the magneto-mechanical equivalent stresses in s is $\sigma_s^{eq} = \Sigma - \frac{9}{4}X$ and in h is $\sigma_h^{eq} = \Sigma + \frac{f_s}{f_h}\frac{9}{4}X$.

The following points can be highligthed:

- At zero applied stress, we observe that the s phase is submitted to compression, the h phase to traction. This result joins the hypotheses of Cullity [1] in order to interpret the results carried out on a plastic strained iron-silicon alloy (the h phase was actually corresponding to the grain boundaries of the material).
- In order to annul the equivalent stress in the s phase, a tensile stress must be superimposed $\Sigma_0 = \frac{9}{4}X$. The equivalent stress in the h phase is non zero: $\sigma_h^{eq} = \frac{\Sigma_0}{f_h} \neq 0$.

This result joins the experimental observations of Iordache [8] observed on Fe-3%Si. The h phase corresponding to the grain boundaries of the material does not participate to the magnetic behavior although the stress in the h phase is high. The reason is that the volume fraction of grain boundaries is negligible compared to the volume fraction of matrix.

E^p	Σ_c^{χ} (MPa)	Σ_c^{μ} (MPa)	Σ_0 (MPa)
0.001	180	80	225
0.01	300	260	405
0.03	380	340	540

TAB. 1 – *Recovery stresses - experimentaly estimated (Σ_c^{χ},Σ_c^{μ}) and foreseen by the model (Σ_0).*

Table 1 includes the recovery stresses as experimentally estimated (Σ_c^{χ},Σ_c^{μ}) and predicted by the model (Σ_0). Values of Σ_c^{χ} and Σ_c^{μ} are in accordance. The recovery stress predicted by the model is excessive. Another interpretation is that kinematic hardening measured thanks to Cortell's method is not accurate. Magnetic method leads to another evaluation (e.g. $\epsilon^p = 0{,}03$, $X = \frac{4}{9}(\Sigma_c^{\chi} + \Sigma_c^{\mu})\frac{1}{2} = 160MPa \pm 18$).

5 CONCLUSION

The experimental procedure presented in this study allows a qualitative validation of a multidomain modeling applied to the plastic straining thanks to a composite model. The role of martensite in the magnetic behavior should be taken in account for a better result. Nevertheless this model is not able to predict the influence of plastic deformation on coercive field or hysteresis losses since the pinning effect due to the metallurgical defects is not considered. Magnetic measurement appears finally as an interesting way to evaluate the kinematic hardening in magnetic materials.

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