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***Abstract*—The magneto-mechanical behaviour of magnetic materials is the result of intricate mechanisms at different scales. These mechanisms have been described with satisfying accuracy from micro-mechanical approaches. But the corresponding constitutive laws are usually too complex to be easily implemented in structural analysis tools for the design of electromagnetic devices. In this paper, a simplified approach for the modelling of multiaxial magneto-elastic behaviour is proposed. This approach includes hysteresis effects and their dependence to stress. The corresponding very low computational time makes it suitable for an implementation into numerical tools for structural analysis.**

***Index Terms*—hysteresis loops, multiaxial stress, magneto-elasticity, magnetostriction.**

I. Introduction

The description of hysteresis in magnetic materials is key to the design of electromagnetic devices. Among the available constitutive models for magnetic behaviour, Preisach [1] and Jiles-Atherton [2] models stand as the most popular for the implementation into numerical analysis tools. In their initial version, these approaches did not consider the effect of stress on magnetic behaviour. This effect however is very significant [3] and extensions of Preisach and Jiles-Atherton models in the context of coupled magneto-mechanical loadings have been proposed [4]–[8]. They usually consider uniaxial stress configurations (pure tension or compression applied in a direction parallel to the magnetic field). But, in most practical devices, stress is multiaxial and its orientation with respect to the magnetic field can also be variable. In order to describe the complex interactions between stress and magnetic field, multiscale approaches to multiaxial magneto-elastic behaviour have been proposed [9]–[12]. They rely on an energetic description of the magneto-mechanical equilibrium. However their implementation into numerical design tools would lead to prohibitive computation times. Recently some authors proposed multiaxial constitutive laws for magneto-elastic behaviour together with a practical implementation into a finite element formulation. Fonteyn *et al.* [13] proposed a thermodynamic approach based on a magneto-mechanical definition of the Helmholtz free energy, Bernard *et al.* [14] used a simplified version of a previous multiscale model [12] allowing its use in numerical analysis and Zeze *et al.* [15] used a phenomenological approach derived from the so-called E&S model [16]. Only the latter includes hysteresis effects, the first two being anhysteretic models. In this paper we propose to include hysteresis effects in the simplified approach proposed in [14] by taking benefit of a recent extension of the multiscale approach to hysteretic behaviour [17]. In a first part, this hysteretic magneto-mechanical constitutive law is detailed. The identification of the material parameters is then discussed. Modelling results are finally compared to experimental measurements obtained from a non oriented iron-silicon steel.

II. Constitutive law

The proposed simplified approach to define the constitutive magneto-mechanical behaviour of magnetic materials is derived in two steps. The anhysteretic behaviour is calculated first from a simplification of a previous multiscale model [11], [12]. This simplified approach was proposed initially for 2D problems [14] and then extended to 3D problems [18]. The 3D version is used here. In a second step, the hysteresis effects are added by introducing a supplementary magnetic field according to the recent extension [17] of the multiscale model. The definition of this constitutive law is detailed hereafter.

The material is described as a collection of magnetic domains randomly oriented. The material is hence considered as a single crystal with specific properties identified from the macroscopic behaviour. The reversible (anhysteretic) part of the magneto-mechanical behaviour is calculated first [18]. The local free energy of the material (1) is written at the domain scale (α) and defined as the sum of three contributions. The magneto-static (Zeeman) energy (2) tends to align the local magnetisation $\mathbf{M}_\alpha$ with the applied field $\mathbf{H}$. μ_0 is the vacuum permeability. The magneto-elastic energy (3) describes the effect of the applied stress $\boldsymbol{\sigma}$ on the behaviour. It introduces the local magnetostriction strain $\boldsymbol{\varepsilon}^\mu_\alpha$. An anisotropy energy (4) can be added to describe macroscopic anisotropy effects for example resulting from the combination of crystalline anisotropy and crystallographic texture. it is given here for an uniaxial anisotropy along direction $\boldsymbol{\beta}$, J being a constant to be identified. If we assume macroscopic isotropy, this term vanishes.

$$W_\alpha = W_\alpha^{mag} + W_\alpha^{el} + W_\alpha^{an} \tag{1}$$

$$W_\alpha^{mag} = -\mu_0\,\mathbf{H}.\mathbf{M}_\alpha \tag{2}$$

$$W_\alpha^{el} = -\boldsymbol{\sigma} : \boldsymbol{\varepsilon}^\mu_\alpha \tag{3}$$

$$W_\alpha^{an} = J(\boldsymbol{\alpha}.\boldsymbol{\beta})^2 \tag{4}$$

The local magnetisation $\mathbf{M}_\alpha$ (5) is given by its direction $\boldsymbol{\alpha}$ (unit vector), and its norm is the saturation magnetisation M_s

of the material. Assuming an isotropic and isochoric magnetostriction behaviour, the magnetostriction strain ε^μ_α is given by (6). λ_s is the saturation magnetostriction of the material. The anisotropy of the macroscopic magnetostriction can be considered to the price of additional material parameters (see for instance [14]).

$$\mathbf{M}_\alpha = M_s\, \boldsymbol{\alpha} \tag{5}$$

$$\varepsilon^\mu_\alpha = \lambda_s \left(\frac{3}{2} \boldsymbol{\alpha} \otimes \boldsymbol{\alpha} - \frac{1}{2} \mathbf{I} \right) \tag{6}$$

The volume fraction f_α of domains with orientation $\boldsymbol{\alpha}$ (7) is then defined as an internal variable. It is defined with a Boltzmann probability function [9]. A_s (8) is a material parameter linked to the initial anhysteretic susceptibility χ^o [11].

$$f_\alpha = \frac{\exp(-A_s W_\alpha)}{\displaystyle\int_\alpha \exp(-A_s W_\alpha)} \tag{7}$$

$$A_s = \frac{3\,\chi^o}{\mu_0\, M_s^2} \tag{8}$$

Once the volume fraction f_α is defined, the macroscopic magnetisation $\mathbf{M}$ (9) and magnetostriction ε^μ (10) are obtained thanks to averaging operations over all possible directions for the magnetisation direction $\boldsymbol{\alpha}$:

$$\mathbf{M} = \langle \mathbf{M}_\alpha \rangle = \int_\alpha f_\alpha\, \mathbf{M}_\alpha \tag{9}$$

$$\varepsilon^\mu = \langle \varepsilon^\mu_\alpha \rangle = \int_\alpha f_\alpha\, \varepsilon^\mu_\alpha \tag{10}$$

This integration step can be performed numerically using a discretisation of a unit sphere for the possible orientations $\boldsymbol{\alpha}$ [12].

In order to describe the effect of stress on the initial domain structure, a configuration field $\mathbf{H}^\sigma$ (11) is added to the anhysteretic field $\mathbf{H}$ [17]:

$$\mathbf{H}^\sigma = \eta \left(N^\sigma - \frac{1}{3} \right) \mathbf{M} \tag{11}$$

η is a material parameter. N^σ (12) belongs to the interval [0 1] and is $1/3$ when no stress is applied. K (13) is defined as a function of A_s and λ_s. σ^{eq} is the equivalent stress for $\boldsymbol{\sigma}$ as defined by Daniel and Hubert [19], namely the projection along the magnetic field direction of the deviatoric part of $\boldsymbol{\sigma}$. $\mathbf{h}$ (unit vector) is the direction of the magnetic field $\mathbf{H}$.

$$N^\sigma = \frac{1}{1 + 2\, \exp\left(-K\, \sigma^{eq}\right)} \tag{12}$$

$$K = \frac{3}{2} A_s\, \lambda_s \tag{13}$$

$$\sigma^{eq} = \frac{3}{2} \mathbf{h} \,.\, \left(\boldsymbol{\sigma} - \frac{1}{3} \mathrm{tr}(\boldsymbol{\sigma})\, \mathbf{I} \right) .\, \mathbf{h} \tag{14}$$

This configuration field notably allows the description of the non monotonic effect of stress on the magnetic permeability observed in some materials.

Hysteresis effects are introduced in the model by adding an irreversible contribution $\mathbf{H}_{\text{irr}}$ to the anhysteretic magnetic field. The definition of $\mathbf{H}_{\text{irr}}$ is based on the works by Hauser [20], extended to magneto-mechanical loadings. $\mathbf{H}_{\text{irr}}$ is assumed to be parallel to $\mathbf{H}$ and its norm is given by (15).

$$||\mathbf{H}_{\text{irr}}|| = \delta \left(\frac{k_r}{\mu_0 M_s} + c_r ||\mathbf{H}|| \right) \left[1 - \kappa\, \exp\left(-\frac{k_a}{\kappa} ||\mathbf{M} - \mathbf{M}^{reb}|| \right) \right] \tag{15}$$

δ is equal to ± 1, depending on whether the material is being loaded or unloaded. The sign of δ starts as positive and is then changed each time there is an inversion in the loading direction. k_r, c_r, k_a and κ are material parameters. The value of κ changes each time there is an inversion in the loading direction [20]. The new value κ is calculated from the previous value κ^o according to (16). The initial value κ^{ini} of κ is a material constant. $\mathbf{M}^{reb}$ is the value of $\mathbf{M}$ at the previous inversion of the loading direction.

$$\kappa = 2 - \kappa^o\, \exp\left(-\frac{k_a}{\kappa^o} ||\mathbf{M} - \mathbf{M}^{reb}|| \right) \tag{16}$$

In order to account for the dependence of the coercive field to the applied stress, the parameter k_r (17), defining the coercive field, is assumed to show a dependence to stress similar to the stress configuration effect, k_r^0 being a material constant [17].

$$k_r = k_r^0 \left(\frac{4}{3} - N^\sigma \right) \tag{17}$$

Once the macroscopic magnetisation $\mathbf{M}$ and magnetostriction ε^μ are calculated as a function of the anhysteretic field $\mathbf{H}$ and stress tensor $\boldsymbol{\sigma}$, the effective magnetic field $\tilde{\mathbf{H}}$ (18) is calculated by adding the configuration field $\mathbf{H}^\sigma$ and the hysteresis contribution $\mathbf{H}_{\text{irr}}$ to the anhysteretic field:

$$\tilde{\mathbf{H}} = \mathbf{H} + \mathbf{H}^\sigma + \mathbf{H}_{\text{irr}} \tag{18}$$

This simplified approach allows to reduce the computational cost by a factor higher than 1000 compared to the full multiscale approach [17].

III. Identification of Model Parameters

The modelling parameters can be separated into two sets: anhysteretic and hysteretic material parameters. Four parameters are used to describe the anhysteretic behaviour. M_s is the saturation magnetisation and λ_s is the maximum magnetostriction strain. They are standard physical parameters and can be identified from a macroscopic measurement in the absence of applied stress. The parameter A_s is proportional to the initial slope of the unstressed anhysteretic magnetisation curve (8). It can then be identified from a low-field anhysteretic measurement under no applied stress. η describes the non-monotonic effect of stress on the magnetic behaviour. It can be identified from a susceptibility measurement under stress. The sensitivity to stress being much higher at low field, it is suggested to perform the identification of η from low-field anhysteretic measurements under uniaxial stress. Tension configurations should usually be sufficient for the purpose of this identification. The anhysteretic parameters are summarised in table I indicating the values used in this paper. Four parameters are also required to describe the hysteretic part of the behaviour according to Hauser's approach [20]. They can be chosen so as to adjust the description of a major magnetisation loop under no applied stress starting from a demagnetised state. k_r^0 controls the coercive field amplitude, c_r the first magnetisation behaviour, and k_a and κ^{ini} the width and inclination of the hysteresis cycle. The values used in this paper are summarised in table II.

Parameter	M_s	λ_s	A_s	η
Value	1.45 10^6	12	3.5 10^{-3}	2 10^{-4}
Unit	A/m	10^{-6}	m^3/J	-

TABLE I
MATERIAL PARAMETERS: ANHYSTERETIC PART

Parameter	k_r^0	c_r	k_a	κ^{ini}
Value	150	0.1	19 10^{-6}	1
Unit	J/m^3	-	m/A	-

TABLE II
MATERIAL PARAMETERS: HYSTERETIC PART

As a summary, the material parameters can be obtained from one anhysteretic magnetisation and magnetostriction measurement at high field, from anhyteretic measurements at low field under uniaxial stress (tension) and from a major hysteresis loop under no applied stress. The model can then be used to predict the material response under any magneto-elastic loading, including multiaxial configurations. Comparison to experimental results are proposed in next section.

IV. HYSTERESIS LOOPS AND HYSTERESIS LOSSES

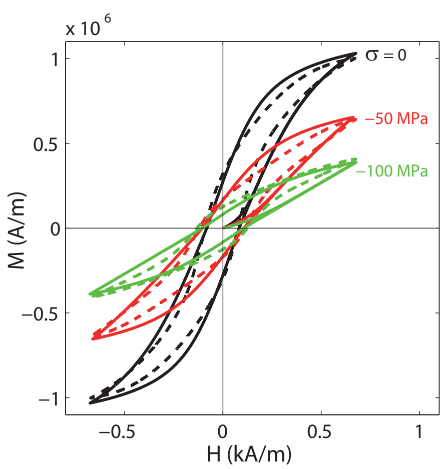

Fig. 1. Magnetisation curves for f=5Hz under uniaxial stress. Experimental results (dashed lines) and numerical results (plain lines).

The proposed simplified model enables a very fast evaluation of hysteresis loops under stress. Fig.1 gives an example of magnetisation curves for a non oriented iron-silicon alloy subjected to an uniaxial compression applied in the direction parallel to the magnetic field. The experimental results are part of an experimental characterisation campaign presented in [21]. The model does not reproduce exactly the shape of the experimental hysteresis loops, but the levels of magnetisation are correct and the effect of uniaxial compression is well described.

The model can also be used to predict the behaviour of the material under multiaxial configuration. As an example the evolution of the coercive field under biaxial loading is shown in Fig.2. The stress is a static biaxial stress (σ_1,σ_2) and the magnetic field is applied along direction 1 with a maximum amplitude of 650 A/m. It must be noticed, that hysteresis loops in this case are not major loops. The corresponding experimental results [21] are shown in Fig.3.

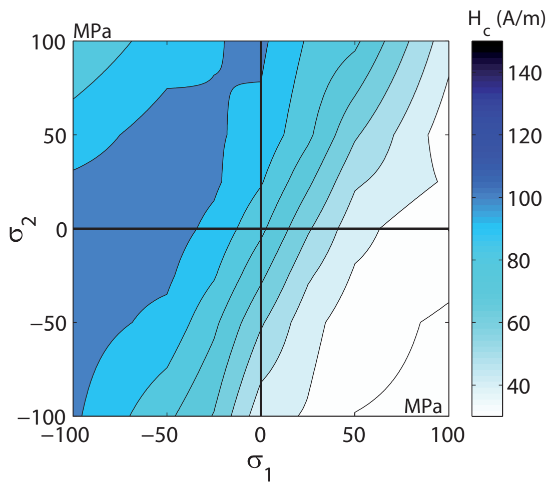

Fig. 2. Coercive field under biaxial stress. Modelling results under a maximum magnetic field H_{max}=650 A/m applied along direction 1.

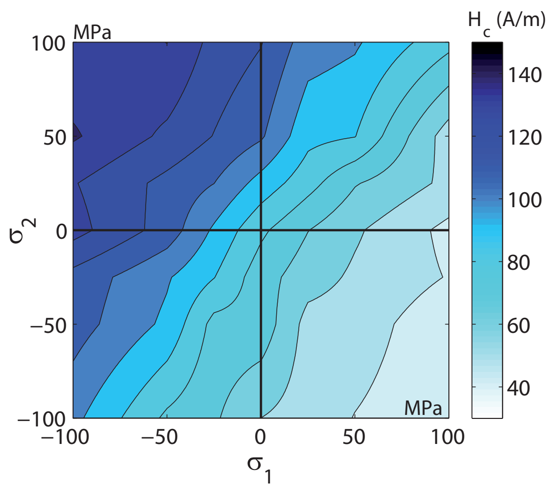

Fig. 3. Coercive field under biaxial stress. Experimental results (f=5Hz) under a maximum magnetic field H_{max}=650 A/m applied along direction 1 [21].

The modelling results show similar trends as the experimental measurements. The coercive field is decreasing with the component of the stress tensor σ_1 aligned with the magnetic field, and increasing with the perpendicular component σ_2. The order of magnitude for the coercive field is correct although the effect of stress is underestimated in the second quadrant (σ_1<0, σ_2>0), and overestimated in the fourth quadrant (σ_1>0, σ_2<0). The isovalues for the coercive field are approximately parallel lines with a slightly higher slope obtained from the model compared to the modelling. In the second quadrant, for high level of stress, the model predicts an inversion of the variation for the coercive field that is not observed experimentally for this material.

Hysteresis losses as a function of stress can also be estimated. Fig.4 shows the losses obtained under biaxial loading. For this figure the losses have been calculated on major loops, so that saturation is reached and the maximum applied field is much higher than 650 A/m. The results obtained under uniaxial tension (axis σ_2=0) are consistent with previous measurements [22] showing an increase of losses under compression and a decrease with moderate magnitude in tension.

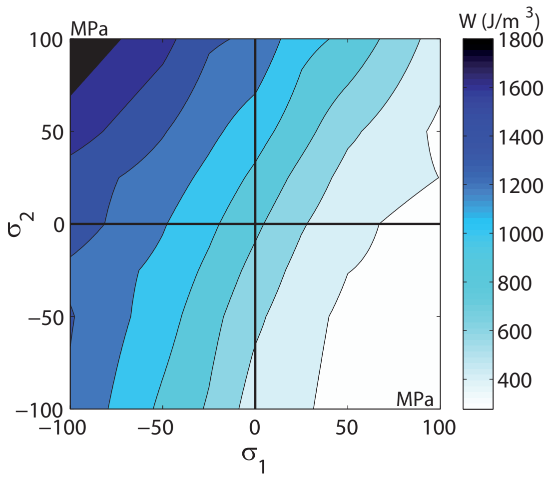

Fig. 4. Hysteresis losses under biaxial stress : modelling results (major loops).

Due to the strong simplifications made, the proposed model is expected to be less accurate than the full multiscale approach but the comparison with experimental results obtained under biaxial magneto-mechanical loading shows satisfying agreement. The approach gives a reasonable estimate of the effect of stress on both magnetisation and hysteresis losses in magnetic materials subjected to multiaxial magneto-mechanical loadings. The computation cost is low enough to allow and implementation into a numerical analysis software.

V. Conclusion

A model for the behaviour of magnetic materials under multiaxial stress is proposed. This model includes hysteresis effects. It is derived from the simplification of a multiscale approach based on an energetic description at the magnetic domain scale [17]. The compactness of this simplified approach allows its practical implementation into finite element tools for the design of electromagnetic devices.

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