Sparsity-based Cholesky Factorization and Its Application to Hyperspectral Anomaly Detection

Ahmad W. Bitar & Jean-Philippe Ovarlez & Loong-Fah Cheong
SONDRA/CentraleSupélec, Plateau du Moulon, 3 rue Joliot-Curie, F-91190 Gif-sur-Yvette, France
ONERA, DAMR/TSI, Chemin de la Hunière, 91120 Palaiseau, France
National University of Singapore (NUS), Singapore, Singapore

Contact Information:
SONDRA/Centrale Supélec
Email: ahmad.bitar@centralesupelec.fr

A generalization of the estimator in [4]

where \( \lambda \) is a positive constant, \( \hat{\beta} \) is the estimate of \( \beta \) obtained from the proximal optimization, and \( \hat{\beta}_{\lambda, p} \) is the estimate obtained from the proximal optimization with \( \lambda = \lambda_{p} \). We have

Main contributions

Before describing the two methods, we must recall the definition for the distance [36, 37],

\[

de(x, y) = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - y_i)^2}
\]

for any \( x, y \in \mathbb{R}^n \).

Covariance estimation via linear regression

Since we observe a sample of \( n \) independent and identically distributed

\[
\mathbf{x}_i \sim \mathcal{N}(\mathbf{0}, \Sigma)
\]

where \( \mathbf{x}_i = (x_{i1}, \ldots, x_{ip})' \) is a column vector of random variables, we can write

\[
\mathbf{y} = \mathbf{X} \beta + \mathbf{e}
\]

where \( \mathbf{y} = (y_1, \ldots, y_n)' \) is a column vector of random variables,

\[
\mathbf{X} = \begin{bmatrix}
\mathbf{x}_1 & \cdots & \mathbf{x}_n
\end{bmatrix}
\]

and \( \mathbf{e} = (e_1, \ldots, e_n)' \) is a column vector of random variables

\[
\mathbf{e} \sim \mathcal{N}(0, \mathbf{I})
\]

with \( \mathbf{I} \) being the identity matrix.

A visualization of the estimator in [4]

For any \( i, j \in \{1, \ldots, n \} \), we define the matrix

\[
\mathbf{T}_{(i, j)} = \mathbf{x}_i \mathbf{x}_j'
\]

and denote \( \mathbf{T} = \mathbf{X}' \mathbf{X} \). A natural estimator of \( \Sigma \) is

\[
\hat{\Sigma} = \frac{1}{n-1} \sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_i'
\]

which is the usual sample covariance matrix. However, this estimator is not robust to

\[
\mathbf{r} = \mathbf{x} - \bar{\mathbf{x}}
\]

where \( \bar{\mathbf{x}} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{x}_i \) and \( \mathbf{x} \in \mathbb{R}^p \) is a column vector of random variables.


de(x, y) = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - y_i)^2}
\]

for any \( x, y \in \mathbb{R}^n \).

Covariance estimation via linear regression

Since we observe a sample of \( n \) independent and identically distributed

\[
\mathbf{x}_i \sim \mathcal{N}(\mathbf{0}, \Sigma)
\]

where \( \mathbf{x}_i = (x_{i1}, \ldots, x_{ip})' \) is a column vector of random variables, we can write

\[
\mathbf{y} = \mathbf{X} \beta + \mathbf{e}
\]

where \( \mathbf{y} = (y_1, \ldots, y_n)' \) is a column vector of random variables,

\[
\mathbf{X} = \begin{bmatrix}
\mathbf{x}_1 & \cdots & \mathbf{x}_n
\end{bmatrix}
\]

and \( \mathbf{e} = (e_1, \ldots, e_n)' \) is a column vector of random variables

\[
\mathbf{e} \sim \mathcal{N}(0, \mathbf{I})
\]

with \( \mathbf{I} \) being the identity matrix.

A visualization of the estimator in [4]

For any \( i, j \in \{1, \ldots, n \} \), we define the matrix

\[
\mathbf{T}_{(i, j)} = \mathbf{x}_i \mathbf{x}_j'
\]

and denote \( \mathbf{T} = \mathbf{X}' \mathbf{X} \). A natural estimator of \( \Sigma \) is

\[
\hat{\Sigma} = \frac{1}{n-1} \sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_i'
\]

which is the usual sample covariance matrix. However, this estimator is not robust to

\[
\mathbf{r} = \mathbf{x} - \bar{\mathbf{x}}
\]

where \( \bar{\mathbf{x}} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{x}_i \) and \( \mathbf{x} \in \mathbb{R}^p \) is a column vector of random variables.

The covariance matrix for the four Models. Columns from left to right: Model 1, Model 2, Model 3.