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Sparsity-based Cholesky Factorization and Its Application to Hyperspectral Anomaly Detection

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Abstract
Estimating large covariance matrices has been a long-standing problem in many applications and has attracted increased attention over several decades. This paper deals with two main problems: (1) recovering the covariance matrix via its unit lower triangular matrix, and (2) imposing sparsity on the covariance matrix via its unit lower triangular matrix. The main results can be summarized as follows:

1. A new sparsity-based Cholesky factorization method is proposed to recover the covariance matrix via its unit lower triangular matrix.
2. A new algorithm is proposed to impose sparsity on the covariance matrix via its unit lower triangular matrix.
3. The performance of the proposed methods is evaluated through numerical experiments.

Introduction

Hyperspectral imagery (HSI) is a powerful tool for remote sensing applications, such as land use classification, mineral exploration, and disease detection. However, the huge size of HSI datasets poses significant challenges in terms of storage and computational cost. One way to address these challenges is to impose sparsity on the covariance matrix of the HSI data.

Main contributions

Before describing the two main results, we need to recall the definition of a sparsity model. Given a sample $\{x_i\}_{i=1}^n$, we have

$$\begin{align*}
\sum_{i=1}^n |x_i| &\leq \beta \quad \text{(0)}
\end{align*}$$

where $\beta$ is a pre-defined parameter. The main results are as follows:

1. A new sparsity-based Cholesky factorization method is proposed to recover the covariance matrix via its unit lower triangular matrix.
2. A new algorithm is proposed to impose sparsity on the covariance matrix via its unit lower triangular matrix.

Generalized sparsity-based Cholesky Factorization

For an HSI dataset, we divide the matrix-thresholding operation $T_k$ (and denote by $T_kT_k^T = C_k$) in two steps: (a) thresholding the matrix and (b) applying a specific thresholding operation $\hat{T}_k$ to $C_k$ (SMT) in each channel of the matrix $E_k = \{e_i\}_{i=1}^p$. We consider the following sparsity model:

$$\begin{align*}
\sum_{i=1}^p |e_i| &\leq \beta (\text{SMT})
\end{align*}$$

Covariance estimation via linear regression

Impulse that we observe a sample of $n$ independent and identically distributed random vectors, $(x_1, \ldots, x_n)$, follows a multivariate Gaussian distribution with mean vector and unknown covariance matrix $\Sigma \in \mathbb{R}^{p \times p}$. The main results are as follows:

1. A new sparsity-based Cholesky factorization method is proposed to recover the covariance matrix via its unit lower triangular matrix.
2. A new algorithm is proposed to impose sparsity on the covariance matrix via its unit lower triangular matrix.

A generalization of the estimator in [4]

Note that both $\bar{F}_k$ and $T_k$ are defined as follows:

$$\begin{align*}
\bar{F}_k &= \frac{1}{p} \sum_{i=1}^p T_{k,i} \\
T_k &= (1 - \lambda_k) T_{k,\text{soft}} + \lambda_k T_{k,\text{scad}}
\end{align*}$$

Table 1. AUCs for different detectors.

<table>
<thead>
<tr>
<th>Detector</th>
<th>AUC (on MUSE MHS)</th>
<th>AUC (on MUSE HS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCM</td>
<td>0.9264</td>
<td>0.9503</td>
</tr>
<tr>
<td>OLS</td>
<td>0.7879</td>
<td>0.8331</td>
</tr>
<tr>
<td>SCAD</td>
<td>0.9274</td>
<td>0.9509</td>
</tr>
<tr>
<td>Soft</td>
<td>0.9083</td>
<td>0.9541</td>
</tr>
</tbody>
</table>

Application on experimental data

The proposed methods are evaluated for galaxy detection on the Multi-View Spectroscopy Explorer (MUSE) dataset (see Fig. 3). It is a 400 x 105 image, made of 4000 bands representing imaging data from 4 metal bands in total. The MUSE HSI data was split into two parts: (a) a training set and (b) a testing set.

References