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Geometrical Properties of Optimal Volterra Filters for Signal Detection

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Abstract

Volterra filters are a particular class of nonlinear filters defined by an extension of the concept of impulse response to the nonlinear case. Linear-quadratic filters are a special example of Volterra filters limited to the second order. In the first part of this paper it is shown that all the results recently published, valid in the linear-quadratic case, can be extended with the appropriate notations to Volterra filters of arbitrary order. In particular, the optimum Volterra filter giving the maximum of the deflection for the detection of a signal in noise is wholly calculated. In the second part several geometrical properties of optimal Volterra filters are investigated by introducing appropriate scalar products. In particular the concept of space orthogonal to the signal and the noise alone reference (NAR) property are introduced allowing a decomposition of the optimal filter that exhibits a relation between detection and estimation. Extensions to the infinite case and relations with the likelihood ratio are also investigated.

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I. Introduction

THE CONCEPT of an optimal linear-quadratic system for detection and estimation was introduced in a previous paper [l]. It was also noted that linear-quadratic systems can be described by a Volterra expansion limited to the second order term. Volterra filters (VF) have been introduced into many aspects of nonlinear filtering problems, and we shall not mention here all the papers discussing their properties. For an introduction to this field applied to problems similar to those discussed hereafter, the reader should consult [21-[51.

It is thus natural to extend the results presented in [l] to VF of arbitrary order, and this is the first purpose of this paper. At first glance this extension is quite obvious [6], and to avoid excessive length, we will present the main ideas and fundamental results. Our principal effort will be to introduce simplified notations in such a way that the analogy between the second order case and the general case appears evident. In fact, the input-output relationship of a VF is extremely tedious to write out, and without our simplified notation the paper would be full of complex equations contributing little to the understanding of the physical meaning of the results. It should be clear, however, that the exact analytical solution of our condensed equations would require extensive calculations.

The extension of a second order to an arbitrary order VF exhibits new problems which were not analyzed in the linear-quadratic case. Among these problems we are mainly interested in the relation between optimal VF and the likelihood ratio (LR) filter that is the absolute optimal system for the test, or the detection, between two simple hypotheses, say the null hypothesis H_0 corresponding to the noise only situation, and the hypothesis H_1 corresponding to the presence of a signal. For this purpose filters are considered as vectors in appropriate Hilbert spaces. In particular it is shown that the filters belonging to the space orthogonal to the LR have the NAR property. This is the basis of a decomposition of the optimal VF in two terms that exhibits relations between detection and estimation. Some extensions to the infinite case are also discussed.

II. Optimal Volterra Filters

A. Principles of Volterra Filtering

A Volterra filter of order v (v positive integer) is a nonlinear system the input-output relationship of which is described by a finite Volterra expansion. To simplify the presentation we will consider the discrete time case only, and we call $x[k]$ and $y[k]$ the input and output, respectively. The output is expressed by

$$y[k] = h_0 + \sum_{m=1}^{v} \sum_{\{i_n^m\}} h_m[i_1^m, i_2^m, \ldots, i_m^m] x[k - i_1^m] x[k - i_2^m] \ldots x[k - i_m^m]. \tag{1}$$

In this expression k and i_n^m are arbitrary integers and the symbol $\{i_n^m\}$ means that the sum is taken on all the i_n^ms. The term h_0 corresponding to $m = 0$ is a constant and is sometimes omitted. In this latter case, and if $v = 1$, we obtain the classical convolution characterizing linear filters. By an obvious extension of the terminology used in the linear case, we will say that the Volterra filter has a finite impulse response if the integers i_n^m are taken in a finite interval, as, for example, $1 \leq i_n^m \leq$ N. We make this assumption henceforth to avoid convergence problems. In this case the input at time k can be considered as a vector $\mathbf{x}$ with components $x[i]$, $1 \leq i \leq$, and the output can be written as

$$y[k] = h_0 + \sum_{m=1}^{v} \sum_{\{i_n^m\}} h_m[i_1^m, i_2^m, \ldots, i_m^m] x[i_1^m] x[i_2^m] \ldots x[i_m^m] \tag{2}$$

where the i_n^ms satisfy $1 \leq i_n^m \leq N$.

For the discussion that follows it is necessary to simplify the notations a little more, and to this end we first write

$$h_m[i_1^m, i_2^m, \ldots, i_m^m] = h_m[\mathbf{i}_m]. \tag{3}$$

Furthermore we associate with the v kernels h_m the quantity

$$|h, v> = \{h_1[\mathbf{i}_1], h_2[\mathbf{i}_2], \ldots, h_v[\mathbf{i}_v]\}. \tag{4}$$

The quantity $\lambda|h, v>$ is obtained by replacing the hs by λh and the sum $|h, v> + |h', v>$ is obtained by replacing the hs by $h + h'$. With these rules the quantity$|h, v>$ becomes a vector of a linear space called $\mathbb{R}^N[v]$. For $v = 1$ and without a constant term we find the classical vector space $\mathbb{R}^N$.

As in any linear vector space, we can introduce linear operators such as

$$A|h, v> = |h, v'> \tag{5}$$

and all the concepts associated with these operators are valid in $\mathbb{R}^N[v]$.

More important is the concept of a scalar product of vectors of $\mathbb{R}^N[v]$. This scalar product of two vectors $|h, v>$ and $|h, v'>$ is defined by

$$< h, v|h', v> = \sum_{m=1}^{v} \sum_{i_m} h[\mathbf{i}_m] h'[\mathbf{i}_m] \tag{6}$$

which is a direct extension of that used in $\mathbb{R}^N[v]$. This allows the introduction of the transpose of an operator A defined by

$$< g, v|A|h, v> = < h, v|A^T|g, v> . \tag{7}$$

Finally, an operator A is said to be symmetric if $A = A^T$ and positive definite if for any $|h, v>$ we have

$$< h, v|A|h, v> \geq 0. \tag{8}$$

Let us now define the input vector $|X,v>$. The input signal in (2) is a real N-dimensional vector $\mathbf{x}$ with components $x[i]$. To this vector of $\mathbb{R}^N$ we associate the vector of $\mathbb{R}^N[v]$ defined by

$$|X,v>\stackrel{\triangle}{=}\{x[i_1^1],x[i_1^2]x[i_2^2],\ldots,x[i_1^v]x[i_2^v]\ldots x[i_v^v]\} \tag{9}$$

It is the v order input vector associated with the input x. With this notation (2) can be written as

$$y=h_0+<h,v|X,v>, \tag{10}$$

which greatly simplifies the notations. Of course the exact calculation of y requires the return to (2). It is clear that if $v=1$ or $v=2$ we again find the linear and linear-quadratic filters. Finally, to simplify a little more, we will omit the letter v when no ambiguity is possible in the order of the filter. Thus in the following $|h>$ means $|h,v>$, whenever the order v of the filter is well specified.

In detection and estimation problems the input $\mathbf{x}$ is usually random. When this is the case, the vector $|X>$ also becomes random, as does the output y given by (10). The randomness of the input introduces two points that must be briefly discussed. First, and most important, the output y can be undefined, in the sense that it is not a second order random variable. To avoid this situation we must assume that the input vector $\mathbf{x}$ has finite moments up to order $2v$. This ensures that y has a finite second order moment, or is a second order random variable. Secondly, y does not in general have a zero-mean value. To define this value let us introduce

$$|m>\stackrel{\triangle}{=}E(|X>) \tag{11}$$

which is a vector of $\mathbb{R}^N[v]$ obtained by taking the expectation value of the terms of (9). Then

$$E(y)=h_0+<h|m>, \tag{12}$$

and if $h_0=-<h|m>$, y becomes, of course, zero-mean. For any arbitrary $|h>$ it is then possible to use the constant term h_0 in the Volterra expansion (2) in such a way that y becomes zero-mean. This procedure is equivalent to using the vector $|X_0>$ defined by

$$|X_0>=|X>-|m>, \tag{13}$$

which is, of course, zero-mean and to writing (10) with $h_0=0$. This is done in what follows. This was already the case in [l] where linear-quadratic systems were written as $y=\mathbf{h}^T\mathbf{x}+\mathbf{x}^T\mathbf{M}\mathbf{x}-\mathrm{Tr}(\mathbf{CM})$, where $\mathbf{x}$ has zero-mean value and covariance matrix $\mathbf{C}$.

B. Optimal Volterra Filter for Detection

As in [1], our purpose is to find the optimal filter giving the maximum of the deflection of the output y defined by

$$D(y) = N/D, \tag{14}$$

$$N = [E_1(y) - E_0(y)]^2, \tag{15}$$

$$D = V_0(y), \tag{16}$$

where E_0 and E_1 mean-expectation value under hypotheses H_0 and H_1 respectively and V_0 denotes the variance under H_0.

To calculate the numerator we introduce the notation

$$E_{1-0}(y) \stackrel{\triangle}{=} E_1(y) - E_0(y), \tag{17}$$

and we define the signal vector by

$$|s> \stackrel{\triangle}{=} E_{1-0}(|X>) = E_{1-0}(|X_0>). \tag{18}$$

With this notation we get

$$N =< h|s>^2 . \tag{19}$$

As y is zero-mean under H_0, its variance is $E_0(y^2)$, or

$$V_0(y) = E_0[< h|X_0>< X_0|h>], \tag{20}$$

where $|X_0>$ is defined by (13). Note that as the expectation values in (11), (13), and (20) are taken under H_0 y is zero-mean under H_0 but not under H_1, which appears in (15) and (17). Finally the variance can be written as

$$V_0(y) =< h|K|h>, \tag{21}$$

with

$$K \stackrel{\triangle}{=} E_0[|X_0>< X_0|]. \tag{22}$$

This is a positive definite symmetric operator. With these notations and by using the Schwarz inequality we find that the maximum value of the deflection obtained with a Volterra filter of order v is

$$d_v^2 =< s, v|K^{-1}|s, v>, \tag{23}$$

and the corresponding optimal filter is

$$T_{o,v}(\mathbf{x}) = \alpha < s, v|K^{-1}|X_0, v> . \tag{24}$$

We will assume henceforth that $\alpha = 1$. This result gives the classical matched filter [71 for L$v = 1$, and the optimal linear-quadratic filter for $v = 2$. The exact calculation of this filter requires the solution of the equation

$$K|\bar{s}>=|s>, \tag{25}$$

which can, of course, be a tedious job for high values of v. As a result, $T_{o,v}(\mathbf{x})$ takes the form

$$T_{o,v}(\mathbf{x}) =< \bar{s}|X_0> . \tag{26}$$

III. Geometrical Concepts for Finite-Order Volterra Filters

A. The Space $\mathbb{R}^N[v, M]$

Consider an arbitrary symmetric positive definite operator M and two vectors $|a>$ and $|b>$ of $\mathbb{R}^N[v]$. The quantity $< a|M|b>$ satisfies all the requirements necessary to define a scalar product, written as

$$< a|b>_M=< a|M|b> . \tag{27}$$

The space $\mathbb{R}^N[v, M]$ is the space deduced from $\mathbb{R}^N[v]$ in which the scalar product of vectors is defined by (27). We obviously have $\mathbb{R}^N[v, I] = \mathbb{R}^N[v]$, where I is the identity operator. In this space, orthogonal vectors are called orthogonal $(\perp, M)$ and denoted by

$$< a|b>_M= 0 \Leftrightarrow< a|M|b>= 0 \Leftrightarrow |a>\perp (M)|b> . \tag{28}$$

As the projection is a concept deduced from orthogonality, we will use the term projection $(\perp, M)$. In other words the projection $(\perp, M)$ onto a subspace S is written $\mathrm{proj}(\perp, \mathrm{M})[|\mathrm{a}> |\mathrm{M}]$ and defined by the orthogonality principle, or

$$\{|a> -\mathrm{proj}(\perp, M)[|a> |S]\} \perp (M)|b> \tag{29}$$

for any vector $|b>$ of S.

B. The Space H_v

The output $y(x)$ of a VF considered in Section I1 is a zero-mean second-order random variable written as $y(x) =< h, v|X_0, v >$. This output can then be considered as a vector of the space L_2 of scalar second order random variables. In this space the scalar product between two vectors u and v noted $(u, v) = E_0[uv]$, and the orthogonality as well as the projection is written with the symbol $\perp\!\!\!\perp$, to avoid any confusion with the concepts introduced before. Note that the expectation value is taken under the hypothesis H_0 as defined after (16). The space H_v. is the subspace of L_2 containing all the outputs of VF with zero-mean value, or

$$H_v = \{F(\mathbf{x})|F(\mathbf{x}) =< h, v|X_0, v>\}, F(\mathbf{x}) \in L_2\}. \tag{30}$$

C. The Signal Subspace and Its Complement

Consider the vector $|s>$ of the space $\mathbb{R}^N[v,M]$ defined by (18) and the subspace $\mathbb{R}^N_\perp[v,M;s]$ of vectors of $\mathbb{R}^N[v,M]$ that are orthogonal $(\perp, M)$ to $|s>$. By using the projection theorem, any vector $|a>$ of $\mathbb{R}^N[v,M]$ can be written as

$$|a>= a_s|s> +|a, s_\perp>, \tag{31}$$

where

$$|a, s_\perp>= \mathrm{proj}(\perp, M)|a> \mid \mathbb{R}^N_\perp[v,M;s]. \tag{32}$$

Furthermore, the component a_s given by

$$a_s =< s|M|s>^{-1}< s|M|a> . \tag{33}$$

It is obvious that

$$|s>\perp (M)|a, s_\perp> . \tag{34}$$

D. The Signal Decomposition of the Optimal Filter

The optimal filter (26) can also be written as

$$T_o(\mathbf{x}) =< \sigma|M|X_0>=< \sigma|X_0>_M, \tag{35}$$

where

$$|\sigma>= M^{-1}K^{-1}|s>= M^{-1}|\bar{s}> . \tag{36}$$

Applying the decomposition (31) to the vectors $|\sigma>$ and $|X_0>$ of (35) and taking into account the various orthogonality $(\perp, M)$ properties, we get

$$T_o(\mathbf{x}) = T_s(\mathbf{x}) + T_{s,\perp}(\mathbf{x}), \tag{37}$$

with

$$T_o(\mathbf{x}) = \frac{< s|M|\sigma>}{< s|M|s>} < s|M|X_0>= \frac{< s|K^{-1}|s>}{< s|M|s>} < s|M|X_0>= d_v^2 \frac{< s|M|X_0>}{< s|M|s>} . \tag{38}$$

The term $T_{s,\perp}(\mathbf{x})$ is deduced from (37) with the values (35) and (38) for $T_o(\mathbf{x})$ and $T_s(\mathbf{x})$, respectively. Let us investigate a little more precisely, in a particular case, the meaning of the term $T_s(\mathbf{x})$. Suppose that $M = I$, which is the more common situation. We deduce from (38)

$$T_s(\mathbf{x}) = d_v^2 s^{-2} < s|X_0>, \tag{39}$$

where d_v^2 is given by (23) and $s^2 =< s|s >$. Suppose now, for simplification, that $v = 2$, which corresponds to linear-quadratic systems studied in [1 1 . In this case, we obtain easily

$$T_s(\mathbf{x}) = d_v^2 s^{-2}[\mathbf{s}^T\mathbf{x} + \mathbf{x}^T K\mathbf{x} - \mathrm{tr}(CK)], \tag{40}$$

where $\mathbf{s} = E_{1-0}[\mathbf{x}]$ and $C = E_0[\mathbf{x}\mathbf{x}^T]$. The first term in the bracket (40) is the output of the matched filter in a white noise, sometimes also called correlation receiver, because it performs a correlation between the input observation and the given vector $\mathbf{s}$. The second term plays a similar role, but for a quadratic receiver. In conclusion, the term $T_s(\mathbf{x})$ uses directly the primary data on the hypotheses $(\mathbf{s}, K, C)$ without any particular processing. This processing is performed through the term $T_{s,\perp}(\mathbf{x})$.

IV. Geometry in L_2 for the Optimal Volterra Filter

A. Basic Relations and the Deflection Property

Let us assume that the observation vector $\mathbf{x}(\omega)$ of $\mathbb{R}^N$ is a random variable with probability densities $p_0(\mathbf{x})$ and $p_1(\mathbf{x})$ under H_0 and H_1, respectively. The LR appearing in the statistical decision theory is defined by

$$L(\mathbf{x}) \triangleq p_1(\mathbf{x})/p_0(\mathbf{x}). \tag{41}$$

We assume in what follows that $L((\mathbf{x}))\_nL_2$ which means that

$$(L, L) = E_0(L^2) \triangleq l^2 < \infty. \tag{42}$$

There are some probability densities for which this is not satisfied. Such situations require a different approach from the following and are thus outside the scope of our discussion.

In detection problems it is interesting to introduce the displaced LR, $R(\mathbf{x})$, defined by [8]

$$R(\mathbf{x}) \triangleq L(\mathbf{x}) - 1. \tag{43}$$

It is clear that $E_0(R) = 0$ and

$$(R, R) = l^2 - 1 \triangleq d^2. \tag{44}$$

Property 4.1 : Any function $F(\mathbf{x})$ of L^2 satisfies the “deflection property” characterized by

$$(R, F) = E_{1-0}[F(\mathbf{x})]. \tag{45}$$

Proof : It is a direct consequence of the definition of the scalar product by $E_0 R(\mathbf{x}R(\mathbf{x})$ where $R(\mathbf{x}$ is replaced by (43).

Comments : The term “deflection” is due to the fact that (45) gives the numerator of the deflection criterion, as seen in (15).

As all the components of $|X, v\!>$ defined by (9) are vectors of L_2, we deduce immediately from (18) that $|s\!>$ may be written as

$$|s\!>= (R, |X\!>) = (R, |X_0\!>). \tag{46}$$

We can apply that to (35) and (38), which gives

$$E_{1-0}[T_o(\mathbf{x})] = E_{1-0}[T_s(\mathbf{x})] = d_v^2, \tag{47}$$

where d_v^2 is defined by (23).

B. The NAR Subspace

Let us introduce the subspace of L_2 defined by

$$H_{\perp\!\!\perp} = \{F(\mathbf{x}) | (F, R) = 0\}, \tag{48}$$

which is then the subspace of functions of the observation $\mathbf{x}$ orthogonal ($\perp\!\!\perp$) to R. It is clear from (45) that any vector of this subspace satisfies

$$E_1(F) = E_0(F), \tag{49}$$

which is called the NAR (noise alone reference) property [71, [9]. This means that the mean value of F is the same under H_0 and H_1, and as H_0 is the noise only hypothesis, while H_1 corresponds to the signal-plus-noise, the mean value of F has a property of the noise alone. Furthermore, it is clear from (15) that any function of this space gives a null deflection.

It is important to note that the NAR property is only valid in the mean, as seen on (49). It is sometimes possible to introduce an almost surely NAR property, which is especially the case for $v = 1$ and for the detection of a deterministic signal in noise. In fact the observation vector is $\mathbf{n}(\omega)$ under H_0 and $\mathbf{n}(\omega) + \mathbf{s}$ under H_1. If we use (31) in $\mathbb{R}^N = \mathbb{R}(1, I)$, we obtain

$$\mathbf{n}(\omega) = n_s(\omega) + \mathbf{n}_{s,\perp}(\omega), \tag{50}$$

where $n_s(\omega)$ is the projection of the noise vector $n_s(\omega)$ onto the subspace orthogonal to $\mathbf{s}$. From this it is clear that $\mathbf{n}_{s,\perp}(\omega)$ is independent of the presence or absence of a signal.

Property 4.2 : The function $T_{s,\perp}(\mathbf{x})$ introduced in (37) belongs to the NAR subspace.

Proof : From (37), we get $T_{s,\perp}(\mathbf{x}) = T_o(\mathbf{x}) - T_s(\mathbf{x})$. As a result of (47), we deduce that $E_{1-0}[T_{s,\perp}(\mathbf{x})] = 0$ and this gives with (45) $(R, T_{s,\perp}(\mathbf{x})) = 0$, which characterizes a vector of the NAR subspace.

C. Optimal VF and the Likelihood Ratio

Property 4.3 : The projection $\perp\!\!\!\perp$ of the function $R(\mathbf{x})$ given by (43) onto the subspace H_v defined by (35) is the optimal filter (401, or

$$\text{proj}(\perp\!\!\!\perp)[R(\mathbf{x})|H_v] = T_{o,v}(\mathbf{x}). \tag{51}$$

Proof : We have to show that for any element $G_v(\mathbf{x})$ of H_vI defined by

$$G_v(\mathbf{x}) =< g,v|X_0,v> \tag{52}$$

we have

$$(G_v, R - T_{o,v}) = 0. \tag{53}$$

This last equation can be written

$$E_0\{< g,v|X_0,v> [R(\mathbf{x})- < X_0,v|K^{-1}|s,v>]\} = 0. \tag{54}$$

By using (46), this equation becomes

$$< g,v|[|s,v> -KK^{-1}|s,v>] = 0, \tag{55}$$

which gives the result.

Comments : This property establishes the relation between the likelihood ratio $L(\mathbf{x})$, or its displaced version $R(\mathbf{x})$, and the optimal Volterra filtering. It is a consequence of the fact that the filter with zero-mean value that maximizes the deflection without any constraint is $R(\mathbf{x})$, [8]. As the deflection can be interpreted as a distance [8], it is natural that the optimal Volterra filter is the projection of $R(\mathbf{x})$, onto the Volterra subspace.

Furthermore note that in the linear case ($v = 1$) it is possible to use either $R(\mathbf{x})$, or $L(\mathbf{x})$, if $E_0(\mathbf{x}) = \mathbf{0}$. In fact the orthogonality relation (54) easily yields

$$\Gamma\mathbf{h} = \mathbf{c}, \tag{56}$$

where $\Gamma = E_0(\mathbf{x}\mathbf{x}^T)$. This is the equation defining the classical linear matched filter. Of course, as $E_0(\mathbf{x} == 0$, we have $E_0[\mathbf{x}L(\mathbf{x})] = E_0[\mathbf{x}R(\mathbf{x})]$, and the matched filter is the projection of the LR, or its displaced version, onto the subspace of linear filters. Various other estimation problems can also be solved by this geometrical description of Volterra filtering and will be analyzed in other publications.

REFERENCES

[1] B. Picinbono and P. Duvaut, "Optimal linear-quadratic systems for detection and estimation," *IEEE Trans. Inform. Theory*, IT-34, no. 2, pp. 304-311, Mar. 1988.

[2] P. Alper, "A consideration of discrete Volterra series," *IEEE Trans. Automat. Contr.*, vol. AC-IO, pp. 322-327, July 1965.

[3] S. Y. Fakhoury, "Identification of the Volterra kernels of nonlinear systems," *IEE Proc.*, vol. 127, pt. D, Nov. 1980, pp. 296-304.

[4] M. Schetzen, "INonlinear system modelling based on the Wiener theory," *Proc. IEEE*, vol. 69, Dec. 1981, pp. 1557-1573.

[5] P. Eykoff, "Some fundamental aspects of process-parameter estimation," *IEEE Trans. Automat. Contr.*, vol. AC-8, pp. 347-357, Oct. 1963.

[6] P. Duvaut and B. Picinbono, "Détection, estimation et filtres de Volterra," *Onzième colloque GRETSI,*, Nice, France, pp. 139-142, June 1987.

[7] L. A. Wainstein and V. D. Zubakov, *Extraction of Signal from Noise*. Englewood Cliffs, NJ: Prentice-Hall, 1962.

[8] A. Papoulis, *Signal Analysis*. New York: McGraw-Hill, 1977.

[9] T. Kailath, *Lectures on Linear Least-Squares Estimation*, Wien, Austria, Springer Verlag, Int. Center for Mechanical Sciences, Courses and Lectures, 140, Udine, 1976.

[10] A. Blanc-Lapierre and B. Picinbono, *Fonctions Aléatoires.* Paris, France: Masson, 1981.

[11] D. Sakrison, *Communication Theory; Transmission of Waveforms and Digital Information.* New York: Wiley, 1968.

[12] G. Favier, "Etude comparative de prédicteurs auto-ajustables,' *Colloque GRETSI*, 1983, p. 525-533.

[13] K. J. Astrom, *Introduction to Stochastic Control Theory.* New York: Academic Press, 1970.

[14] D. Kazakos, "Optimal constrained representation and filtering of signals," *Signal Processing*, vol. 5, pp. 347-353, 1983.

[15] A. G. Journel, "Kriging in terms of projections," *Math. Geol.*, vol. 9, pp. 563-586,1977.

[16] W. D. Stirling, "Least squares subject to linear constraints," *J. Roy. Sot. Appl. Stat.*, vol. 30, pp. 204-212, 1981

[17] B. Picinbono, "A geometrical interpretation of signal detection and estimation," *IEEE Trans. Inform. Theory*, IT-26, pp. 493-497, 1980.

[18] H. El Ayadi and B. Picinbono, "NAR AGC adaptive detection of nonoverlapping signals in noise with fluctuating power," *IEEE Trans. Acoust., Speech, Sig. Proc.*, ASSP-29, pp. 952-963, 1981.