New criteria for iterative decoding

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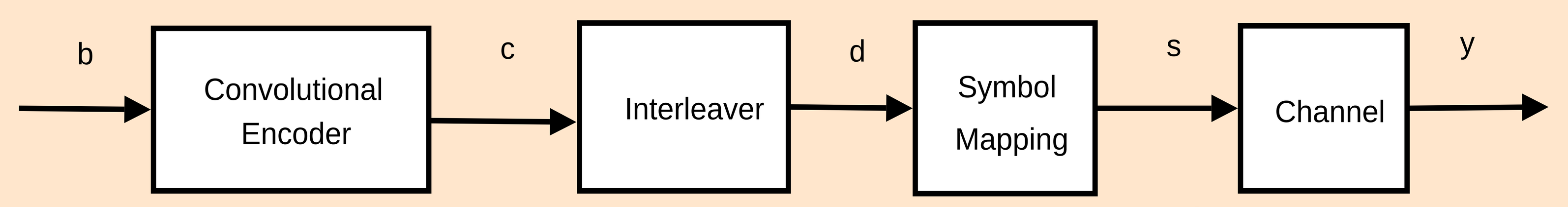
Context: Turbo-like algorithms with iterative decoding

Goal: Make the link between iterative decoding and classical optimization techniques. Improve the performance of iterative decoding.

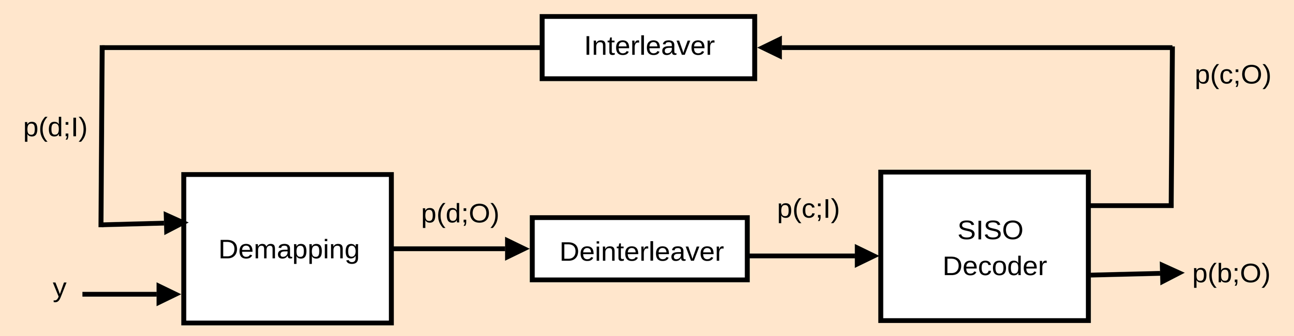
System model (Bit Interleaved Coded Modulation)(Zehavi, 1992; Li, 2002)

Transmission model



AWGN Channel $y_k = s_k + n_k$ $\mathbf{n}_k \rightsquigarrow \mathcal{N}(\mathbf{0}, \sigma_n^2)$
$s_k = Mapping(d_{km+1}, d_{km+2}, ..., d_{(k+1)m})$

BICM-ID receiver with soft-decision feedback



$$\textbf{APP (Demapping)} \quad p_{APP}(d_{km+i} = b) = K'_m \sum_{\mathbf{s}_k \in \Psi_b^i} p(\mathbf{y}_k \mid \mathbf{s}_k) \prod_j p(d_{km+j}; I)$$

Prior $\times$ Channel probability $\rightarrow$ Marginalization

$$\textbf{Extrinsic (Demapping)} \quad p(d_{km+i} = b; O) = K_m \sum_{\mathbf{s}_k \in \Psi_b^i} p(\mathbf{y}_k \mid \mathbf{s}_k) \prod_{j \neq i} p(d_{km+j}; I)$$

$$\textbf{APP (Decoder)} \quad p_{APP}(c_l = b) = K'_c \sum_{\mathbf{c} \in \mathcal{R}_b^l} \mathbf{I}_{\mathcal{C}}(\mathbf{c}) \prod_j p(c_j; I) \quad (\mathbf{I}_{\mathcal{C}} \textit{ indicator function of the code})$$

Prior $\times$ Indicator function $\rightarrow$ Marginalization

$$\textbf{Extrinsic (Decoder)} \quad p(c_l = b; O) = K_c \sum_{\mathbf{c} \in \mathcal{R}_b^l} \mathbf{I}_{\mathcal{C}}(\mathbf{c}) \prod_{j \neq l} p(c_j; I)$$

Information geometry and iterative decoding

Basic tools and Notations (Walsh, 2006)

$(\mathbf{B_i}) \in \{0,1\}^N$ **binary representation** of integer i gathered into matrix $\mathbf{B} = (\mathbf{B_0}, \mathbf{B_1}, ..., \mathbf{B_{2^N-1}})^T$.

PMF $\eta = (\mathbf{Pr}[\chi = \mathbf{B_0}], \mathbf{Pr}[\chi = \mathbf{B_1}], ..., \mathbf{Pr}[\chi = \mathbf{B_{2^N-1}}])^{\mathbf{T}}$

Log-coordinates of PMF η $(\theta_i)_{0 \leq i \leq 2^N-1} = \ln(Pr[\chi = \mathbf{B_i}]) - \ln(Pr[\chi = \mathbf{B_0}])$

Bitwise log-probability ratio $(\lambda_j)_{0 \leq j \leq N-1} = log(\frac{Pr[\chi_j=1]}{Pr[\chi_j=0]})$

For **factorisable probability measures**, the log-coordinates take the form $\theta = \mathbf{B}\lambda$.

Link with iterative decoding

- **Demapping subblock**

$$p_{APP}(d_{km+i} = b) = K'_m \textstyle\sum_{\mathbf{s}_k \in \Psi_b^i} p(\mathbf{y}_k \mid \mathbf{s}_k) \; \prod_j p(d_{km+j}; I)$$
$$\downarrow \theta_{\mathbf{m}} \qquad \downarrow_{Log-coordinates} \mathbf{B}\lambda_{\mathbf{1}}$$

$$p_{APP}(d_{km+i} = b) = K''_m \; p(d_{km+i}; I) \; p(d_{km+i}; O)$$
$$\downarrow \mathbf{B}\lambda_{\mathbf{1}} \qquad \downarrow_{Log-coordinates} \mathbf{B}\lambda_{\mathbf{2}}$$

The **demapping sub-block** solves, with respect to λ_2, $p_{\mathbf{B}(\lambda_1+\lambda_2)} = p_{\mathbf{B}\lambda_1+\theta_{\mathbf{m}}}$

- **Decoding sub-block**

$$p_{APP}(c_l = b) = K'_c \textstyle\sum_{\mathbf{c} \in \mathcal{R}_b^l} \mathbf{I}_{\mathcal{C}}(\mathbf{c}) \; \prod_j p(c_j; I)$$
$$\downarrow \theta_{\mathbf{c}} \qquad \downarrow_{Log-coordinates} \mathbf{B}\lambda_{\mathbf{2}}$$

$$p_{APP}(c_l) = K''_c \; p(c_l; I) \; p(c_l; O)$$
$$\downarrow \mathbf{B}\lambda_{\mathbf{2}} \qquad \downarrow_{Log-coordinates} \mathbf{B}\lambda_{\mathbf{1}}$$

The **decoding sub-block** solves, with respect to λ_1, $p_{\mathbf{B}(\lambda_1+\lambda_2)} = p_{\mathbf{B}\lambda_2+\theta_{\mathbf{c}}}$

- **Global criterion**
Let $D_{FD}(\mathbf{p}, \mathbf{q}) = \sum_j p_j \ln\left(\frac{p_j}{q_j}\right) + \sum_j (1-p_j) \ln\left(\frac{1-p_j}{1-q_j}\right)$ denote the **Fermi-Dirac entropy** (Kullback-Leibler distance for bit probabilities).

The demapping sub-block solves the minimization problem $\min_{\lambda_2} D_{FD}(p_{\mathbf{B}\lambda_1+\theta_{\mathbf{m}}}, p_{\mathbf{B}(\lambda_1+\lambda_2)})$.

The decoding sub-block solves the minimization problem $\min_{\lambda_1} D_{FD}(p_{\mathbf{B}\lambda_2+\theta_{\mathbf{c}}}, p_{\mathbf{B}(\lambda_1+\lambda_2)})$.

New criteria

An hybrid proximal point algorithm

Goal: Link the two (independent) criteria using **proximal point technique** (Luque, 1984).

$$\textbf{Demapping} \quad \min_{\lambda_2} J_{\theta_m}(\lambda_1, \lambda_2) = \min_{\lambda_2} D_{FD}(\mathbf{p}_{\mathbf{B}\lambda_1+\theta_{\mathbf{m}}}, \mathbf{p}_{\mathbf{B}(\lambda_1+\lambda_2)}) + \mu_m D_{FD}(\mathbf{p}_{\mathbf{B}(\lambda_1^{(\mathbf{k})}+\lambda_2^{(\mathbf{k})})}, \mathbf{p}_{\mathbf{B}(\lambda_1+\lambda_2)})$$
$$\textbf{Decoding} \quad \min_{\lambda_1} J_{\theta_c}(\lambda_1, \lambda_2) = \min_{\lambda_1} D_{FD}(\mathbf{p}_{\mathbf{B}\lambda_2+\theta_{\mathbf{c}}}, \mathbf{p}_{\mathbf{B}(\lambda_1+\lambda_2)}) + \mu_c D_{FD}(\mathbf{p}_{\mathbf{B}(\lambda_1^{(\mathbf{k})}+\lambda_2^{(\mathbf{k+1})})}, \mathbf{p}_{\mathbf{B}(\lambda_1+\lambda_2)})$$

Optimal choice of the step-size :

$$\textbf{Choose } \mu_{\mathbf{m}} \textbf{ such that } J_{\theta_m}(\lambda_1^{(\mathbf{k})}, \lambda_2^{(\mathbf{k+1})}) \leq J_{\theta_c}(\lambda_1^{(\mathbf{k})}, \lambda_2^{(\mathbf{k})})$$
$$\textbf{Choose } \mu_{\mathbf{c}} \textbf{ such that } J_{\theta_c}(\lambda_1^{(\mathbf{k+1})}, \lambda_2^{(\mathbf{k+1})}) \leq J_{\theta_m}(\lambda_1^{(\mathbf{k})}, \lambda_2^{(\mathbf{k+1})})$$

Both criteria **decrease with the iterations**. Convergence towards the same stationary point than the classical iterative decoding.

An hybrid minimum entropy algorithm

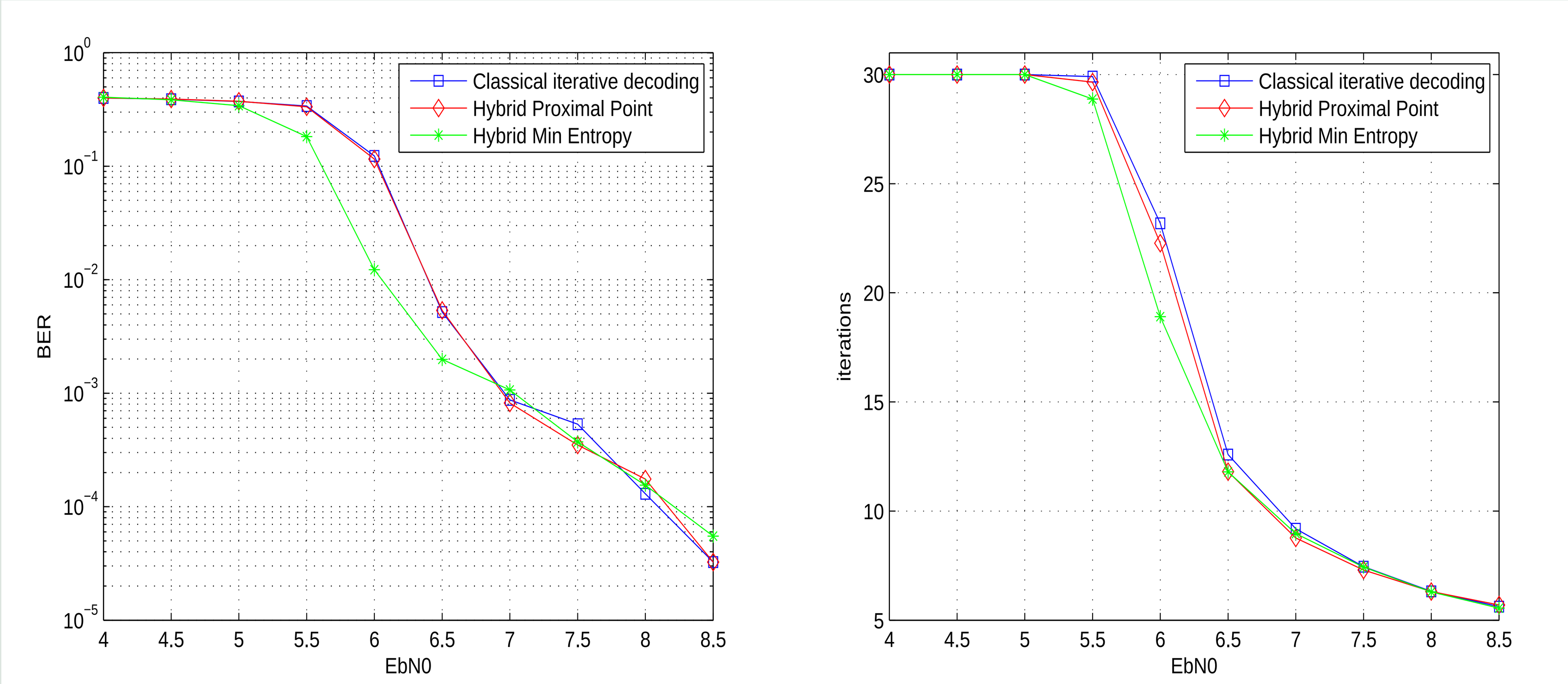
Goal: Improve the performance of the iterative decoding.

Rationale: The entropy of the APP gives a measure of the **reliability** of the decision : $E_{B(\lambda_1+\lambda_2)} \rightarrow 0$ means that the iterative decoding is **confident about its decisions** (Kocarev, 2006).

$$\textbf{Demapping} \quad \min_{\lambda_2} D_{FD}(\mathbf{p}_{\mathbf{B}\lambda_1+\theta_{\mathbf{m}}}, \mathbf{p}_{\mathbf{B}(\lambda_1+\lambda_2)}) + \eta_m E_{B(\lambda_1+\lambda_2)}$$
$$\textbf{Decoding} \quad \min_{\lambda_1} D_{FD}(\mathbf{p}_{\mathbf{B}\lambda_2+\theta_{\mathbf{c}}}, \mathbf{p}_{\mathbf{B}(\lambda_1+\lambda_2)}) + \eta_c E_{B(\lambda_1+\lambda_2)}$$

Simulation

The generator polynomial of the encoder is $g = [111; 001; 100]$. The bits are mapped using subset partitioning to a 8-PSK modulation. The length of the coded bit sequence is $L_c = 6000$. The step-sizes η_m and η_c in the HMEA are both chosen equal to 0.05.



Left: BER versus EbN0 – Right: Iteration number versus EbN0

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