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# Min-sum decoding of irregular LDPC codes with adaptive scaling based on mutual information

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## ABSTRACT

An adaptive scaling strategy is proposed for counterbalancing LLR overestimation in Min-Sum decoding. The scaling factor is a function of a single variable that can be efficiently computed online.

## MIN-SUM DECODING

Irregular (N,K) LDPC code over an AWGN channel

### NOTATIONS

$R_i$ : LLR of bit  $i$  from the observations

$E_{ji}^{(k)}$ : LLR sent at iteration  $k$  from check node  $j$  to variable node  $i$

$M_{ji}^{(k)}$ : LLR sent at iteration  $k$  from variable node  $i$  to check node  $j$

### Standard min-sum decoding

#### • Horizontal step

$$E_{ji}^{(k)} = \prod_{i' \neq i} \text{sgn}(M_{ji'}^{(k-1)}) \min_{i' \neq i} |M_{ji'}^{(k-1)}| \quad (1)$$

#### • Vertical step

$$M_{ji}^{(k)} = R_i + \sum_{j' \neq j} E_{j'i}^{(k)} \quad (2)$$

### Scaled min-sum decoding

#### KNOWN FACTS FROM THE LITERATURE:

- Min-sum decoding leads to overestimated LLR [2].
- Fixed scaling factor is sufficient for regular LDPC but not for irregular LDPC [5].
- The scaling factor should depend on the check node degree [4, 3].

#### • Vertical step (new rule)

$$M_{ji}^{(k)} = R_i + \sum_{d \in \mathcal{D}(i)} \alpha^{(k)}(d) \sum_{j' \in \mathcal{M}(d)} E_{j'i}^{(k)} \quad (3)$$

where  $\mathcal{D}(i)$  is the set of degrees and  $\mathcal{M}(d)$  is the set of indexes of parity check eq. with degree  $d$ .

Optimal choice for the scaling factor  $\Rightarrow$  depends on the *reliability* of the LLR.

## SCALING FACTOR AND MUTUAL INFORMATION

### Mutual information between extrinsics

#### NOTATIONS

$L_y$  resp.  $L_z$  rv associated with  $\{\sum_j E_{j,i}\}$  resp.  $\{\sum_j M_{j,i}\}$

$X$ : rv associated with transmitted message

The mutual information  $I(L_y, L_z)$  is defined as:

$$I(L_y, L_z) = E_{p_{L_y, L_z}(\ell_y, \ell_z)} \left[ \log_2 \left( \frac{p_{L_y, L_z}(\ell_y, \ell_z)}{p_{L_y}(\ell_y) p_{L_z}(\ell_z)} \right) \right] \quad (4)$$

The mutual information between extrinsics is related to  $I(L_y, X)$  and  $I(L_z, X)$  through [1]:

$$I(L_y, L_z) = I(L_y, X) + I(L_z, X) - I(L_y + L_z, X) \quad (5)$$

and can be evaluated with **Exit Charts**.

EXAMPLE: Irregular code of rate  $\frac{1}{2}$  and length 10032 with degree distribution

$$\rho(x) = 0.25x^4 + 0.75x^{14} \quad (6)$$

$$\lambda(x) = 0.1917x + 0.0125x^2 + 0.0417x^4 + 0.0750x^5 + 0.2040x^6 + 0.0667x^7 + 0.225x^8 + 0.0833x^9 + 0.058x^{10} + 0.0541x^{12} \quad (7)$$

Two different check-node degrees: 5 and 15  $\Rightarrow$  two scaling factors  $\alpha(5)$  and  $\alpha(15)$

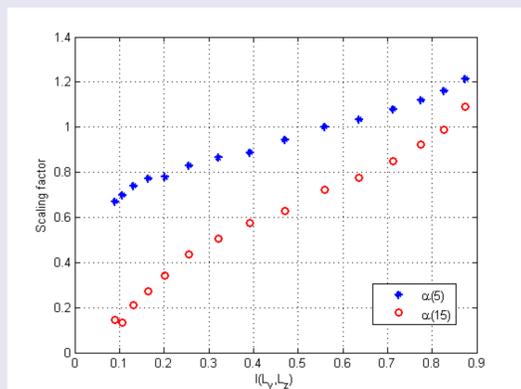


Figure 1: Scaling factors  $\alpha(5)$  and  $\alpha(15)$  vs  $I(L_y, L_z)$ .

Using polynomial fitting and with  $I = I(L_y, L_z)$ , we obtain the following rules:

$$\alpha(5) = 0.416 + 1.956I - 2.975I^2 + 1.832I^3 \quad (8)$$

$$\alpha(15) = -0.232 + 2.747I - 3.690I^2 + 2.268I^3 \quad (9)$$

## ONLINE COMPUTATION OF $I(L_y, L_z)$

Let  $\ell_{y,i}$  resp.  $\ell_{z,i}$  be a realization of random variable  $L_y$  resp.  $L_z$ . An estimator  $\tilde{I}_{yz}$  of  $I(L_y, L_z)$  is given in [1] with expression:

$$\tilde{I}_{yz} = 1 + \frac{1}{K} \sum_{k=1}^K \log_2 \left( \frac{1 + e^{\ell_{y,k} + \ell_{z,k}}}{(1 + e^{\ell_{y,k}})(1 + e^{\ell_{z,k}})} \right) \quad (10)$$

and can be implemented as

$$\tilde{I}_{yz} = 1 + \frac{1}{K \log(2)} \sum_{k=1}^K \left( f(\ell_{y,k} + \ell_{z,k}) - f(\ell_{y,k}) - f(\ell_{z,k}) \right) \quad (11)$$

with  $f(\ell) = \max(0, \ell) + \frac{\log(1 + e^{-|\ell|})}{g(\ell)}$  and pre-computed values from  $\mathcal{Q} = \{0, q, 2q, \dots, L_{max}\}$  are used for  $g(\ell)$ .

COMPLEXITY (extra cost compared to fixed scaling strategy)

- storage of pre-determined values of  $\alpha(5)$ ,  $\alpha(15)$  and  $g(\cdot)$
  - arithmetical complexity due to the computation of  $\tilde{I}_{yz}$  at each iteration.
- with LDPC in (6-7), extra cost is less than 8% for addition and less than 2% for min/max selection.

## NUMERICAL RESULTS

• CHANNEL: AWGN.

• CODE: Irregular LDPC code with rate  $1/2$ , length 10032 and degree distribution in (6)-(7).

• MODULATION: BPSK, 256-QAM.

### COMPARISON BETWEEN:

- SP: sum-product implementation
- MS,  $\alpha = 1$ : standard MS implementation with vertical step as in (2)
- MS,  $\alpha = f(IM)$ : the proposed method with vertical step as in (3) and with adaptive scaling factors in (8-9) and  $I = \tilde{I}_{yz}$ .
- MS,  $\alpha = f(EbN0_{threshold}, it)$ : MS implementation in [4] with scaling factor acquired using training LLR and with a value of  $EbN0$  corresponding to the practical threshold  $EbN0_{threshold}$  of the code.
- DNMS: MS implementation with fixed values of  $\alpha(5)$  and  $\alpha(15)$  (generalization of the normalized MS to irregular codes [2] with best scaling pair  $\alpha(5) = 0.88$  and  $\alpha(15) = 0.68$  [4]).

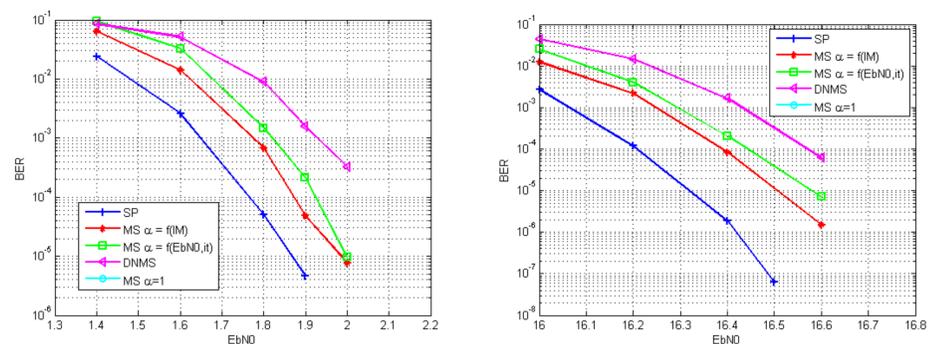


Figure 2: BER under BPSK AWGN channel (left) and 256-QAM AWGN channel (right)

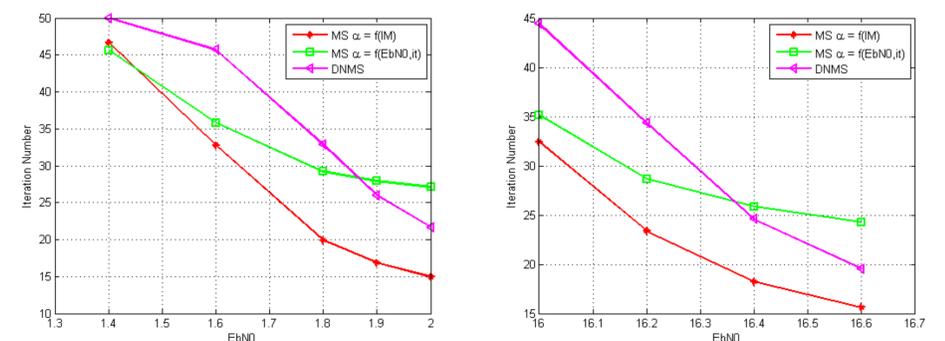


Figure 3: Iteration number under BPSK AWGN channel (left) and 256-QAM AWGN channel (right)

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