Min-sum decoding of irregular LDPC codes with adaptive scaling based on mutual information

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ABSTRACT

An adaptive scaling strategy is proposed for counterbalancing LLR overestimation in Min-Sum decoding. The scaling factor is a function of a single variable that can be efficiently computed online.

MIN-SUM DECODING

Irregular (N,K) LDPC code over an AWGN channel

NOTATIONS
R_i: LLR of bit i from the observations
$E_{ji}^{(k)}$: LLR sent at iteration k from check node j to variable node i
$M_{ji}^{(k)}$: LLR sent at iteration k from variable node i to check node j

Standard min-sum decoding

- **Horizontal step**

$$E_{ji}^{(k)} = \prod_{i' \neq i} sgn(M_{ji'}^{(k-1)}) \min_{i' \neq i} |M_{ji'}^{(k-1)}| \quad (1)$$

- **Vertical step**

$$M_{ji}^{(k)} = R_i + \sum_{j' \neq j} E_{j'i}^{(k)} \quad (2)$$

Scaled min-sum decoding

KNOWN FACTS FROM THE LITERATURE:
- Min-sum decoding leads to overestimated LLR [2].
- Fixed scaling factor is sufficient for regular LDPC but not for irregular LDPC [5].
- The scaling factor should depend on the check node degree [4, 3].

- **Vertical step** (new rule)

$$M_{ji}^{(k)} = R_i + \sum_{d \in \mathcal{D}(i)} \alpha^{(k)}(d) \sum_{\substack{j' \in \mathcal{M}(d) \\ j' \neq j}} E_{j'i}^{(k)} \quad (3)$$

where $\mathcal{D}(i)$ is the set of degrees and $\mathcal{M}(d)$ is the set of indexes of parity check eq. with degree d.

Optimal choice for the scaling factor ⇒ depends on the *reliability* of the LLR.

SCALING FACTOR AND MUTUAL INFORMATION

Mutual information between extrinsics

NOTATIONS
L_y resp. L_z rv associated with $\{\sum_j E_{j,i}\}$ resp. $\{\sum_j M_{j,i}\}$
X: rv associated with transmitted message

The mutual information $I(L_y, L_z)$ is defined as:

$$I(L_y, L_z) = E_{p_{L_y,L_z}(\ell_y,\ell_z)}\left[\log_2\left(\frac{p_{L_y,L_z}(\ell_y,\ell_z)}{p_{L_y}(\ell_y)p_{L_z}(\ell_z)}\right)\right] \quad (4)$$

The mutual information between extrinsics is related to $I(L_y, X)$ and $I(L_z, X)$ through [1]:

$$I(L_y, L_z) = I(L_y, X) + I(L_z, X) - I(L_y + L_z, X) \quad (5)$$

and can be evaluated with **Exit Charts**.

EXAMPLE: Irregular code of rate $\frac{1}{2}$ and length 10032 with degree distribution

$$\rho(x) = 0.25x^4 + 0.75x^{14} \quad (6)$$

$$\lambda(x) = 0.1917x + 0.0125x^2 + 0.0417x^4 + 0.0750x^5 + 0.2040x^6 + 0.0667x^7 + 0.225x^8 + 0.0833x^9 + 0.058x^{10} + 0.0541x^{12} \quad (7)$$

Two different check-node degrees: 5 and 15 ⇒ two scaling factors $\alpha(5)$ and $\alpha(15)$

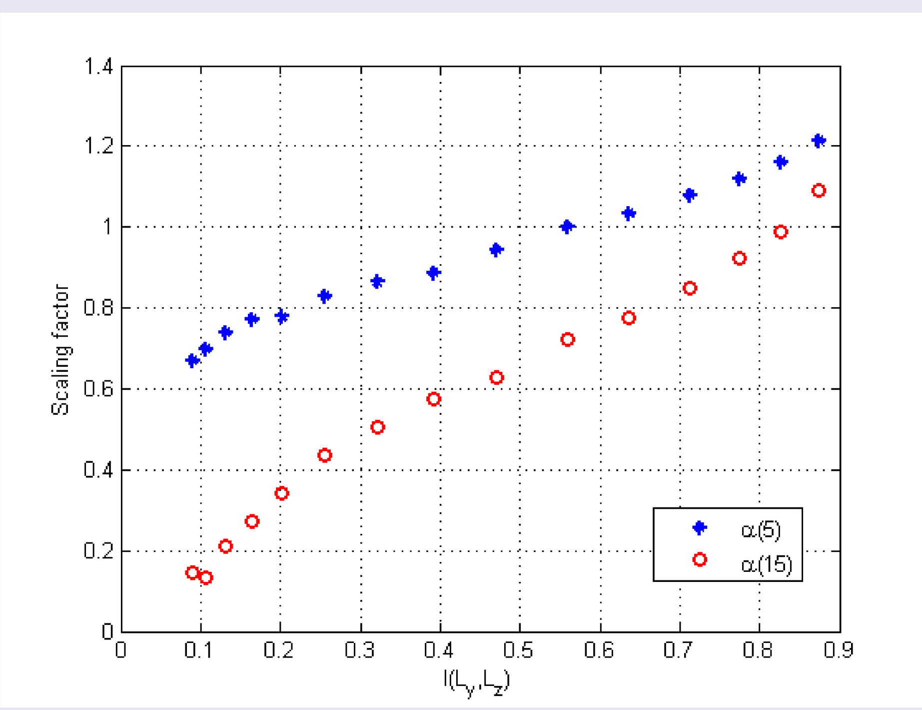

Figure 1: Scaling factors $\alpha(5)$ and $\alpha(15)$ vs $I(L_y, L_z)$.

Using polynomial fitting and with $I = I(L_y, L_z)$, we obtain the following rules:

$$\alpha(5) = 0.416 + 1.956I - 2.975I^2 + 1.832I^3 \quad (8)$$
$$\alpha(15) = -0.232 + 2.747I - 3.690I^2 + 2.268I^3 \quad (9)$$

ONLINE COMPUTATION OF $I(L_y, L_z)$

Let $\ell_{y,i}$ resp. $\ell_{z,i}$ be a realization of random variable L_y resp. L_z. An estimator $\widetilde{I}_{yz}$ of $I(L_y, L_z)$ is given in [1] with expression:

$$\widetilde{I}_{yz} = 1 + \frac{1}{K}\sum_{k=1}^{K} \log_2\left(\frac{1 + e^{\ell_{y,k}+\ell_{z,k}}}{(1+e^{\ell_{y,k}})(1+e^{\ell_{z,k}})}\right) \quad (10)$$

and can be implemented as

$$\widetilde{I}_{yz} = 1 + \frac{1}{K\log(2)}\sum_{k=1}^{K}\Big(f(\ell_{y,k}+\ell_{z,k}) - f(\ell_{y,k}) - f(\ell_{z,k})\Big) \quad (11)$$

with $f(\ell) = \max(0,\ell) + \underbrace{\log(1+e^{-|\ell|})}_{g(\ell)}$ and pre-computed values from $\mathcal{Q} = \{0, q, 2q, ..., L_{max}\}$ are used for $g(\ell)$.

COMPLEXITY (extra cost compared to fixed scaling strategy)
- storage of pre-determined values of $\alpha(5)$, $\alpha(15)$ and $g(.)$
- arithmetical complexity due to the computation of $\widetilde{I}_{yz}$ at each iteration.
  with LDPC in (6-7), extra cost is less than 8% for addition and less than 2% for min/max selection.

NUMERICAL RESULTS

- CHANNEL: AWGN.
- CODE: Irregular LDPC code with rate $1/2$, length 10032 and degree distribution in (6)-(7).
- MODULATION: BPSK, 256-QAM.

COMPARISON BETWEEN:
- SP: sum-product implementation
- MS, $\alpha = 1$: standard MS implementation with vertical step as in (2)
- MS, $\alpha = f(IM)$: the proposed method with vertical step as in (3) and with adaptive scaling factors in $(8-9)$ and $I = \widetilde{I}_{yz}$.
- MS, $\alpha = f(EbN0_{threshold}, it)$: MS implementation in [4] with scaling factor acquired using training LLR and with a value of $EbN0$ corresponding to the practical threshold $EbN0_{threshold}$ of the code.
- DNMS: MS implementation with fixed values of $\alpha(5)$ and $\alpha(15)$ (generalization of the normalized MS to irregular codes [2] with best scaling pair $\alpha(5) = 0.88$ and $\alpha(15) = 0.68$ [4]).

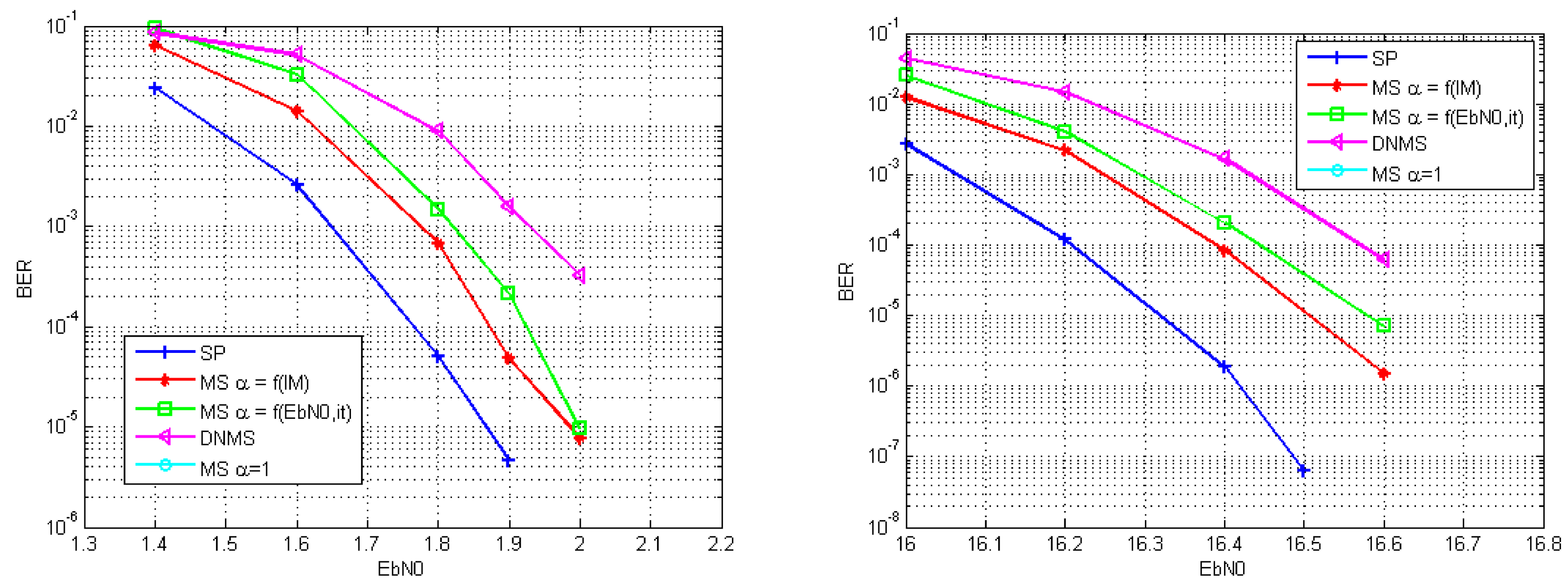

Figure 2: BER under BPSK AWGN channel (left) and 256-QAM AWGN channel (right)

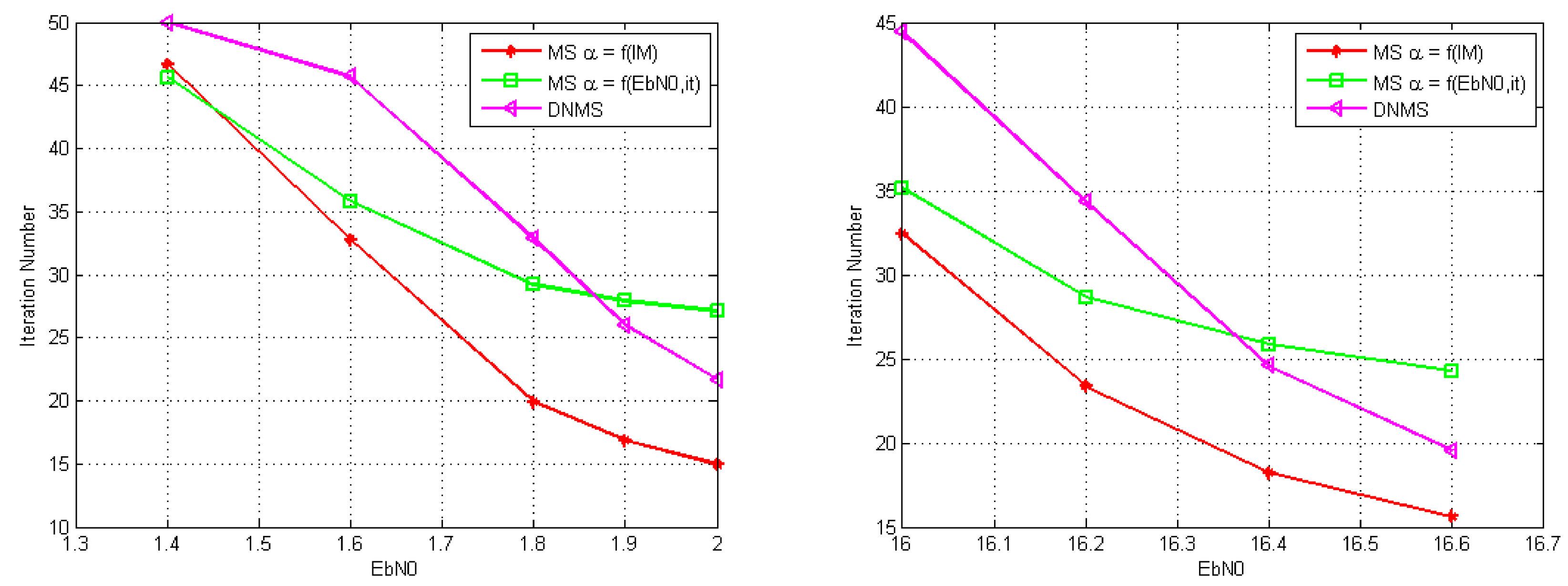

Figure 3: Iteration number under BPSK AWGN channel (left) and 256-QAM AWGN channel (right)

References

[1] F. Alberge. On Some Properties of the Mutual Information between Extrinsics with Application to Iterative Decoding. *IEEE Trans. on Commun.*, 63(5):1541–1553, May 2015.

[2] J. Chen and M.P.C. Fossorier. Density evolution for two improved BP-Based decoding algorithms of LDPC codes. *IEEE Communications Letters*, 6(5):208–210, May 2002.

[3] G. Lechner and J. Sayir. Improved Sum-Min Decoding for Irregular LDPC Codes. In *6th International ITG-Conference on Source and Channel Coding (TURBOCODING)*, pages 1–6, April 2006.

[4] Y. Xu, L. Szczecinski, B. Rong, F. Labeau, D. He, Y. Wu, and W. Zhang. Variable LLR Scaling in Min-Sum Decoding for Irregular LDPC Codes. *IEEE Trans. on Broadcasting*, 60(4):606–613, Dec 2014.

[5] J. Zhang, M. Fossorier, D. Gu, and J. Zhang. Two-dimensional correction for min-sum decoding of irregular LDPC codes. *IEEE Communications Letters*, 10(3):180–182, Mar 2006.