Global sensitivity analysis applied to traffic rescheduling in case of power shortage

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GLOBAL SENSITIVITY ANALYSIS APPLIED TO TRAFFIC RESCHEDULING IN CASE OF POWER SHORTAGE

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ABSTRACT

The present work addresses electric infrastructure incidents: rescheduling is needed in order to optimally share the available power among the different trains. We propose a rescheduling process based on dynamic multiphysic railway simulations that compute physical quantities such as the train speed profiles, voltage along the catenary lines, temperatures … The optimization problem to solve is characterized by a large number of continuous and discrete variables, constraints on dynamic outputs (typically voltage limits), and a high computation cost. We use sensitivity analysis in order to analyse the behaviour of this complex system and provides us more information around the adjustment operations helping to reorganize train traffic in an optimal way. Our approach is based on statistics of the output of simulations of the dynamic railway system, with predefined variation ranges of the input parameters. Two parts of analysis are considered: variance decomposition based sensitivity analysis (generalized Sobol indexes) used for prioritization and fixing factors (quantitative analysis); regional sensitivity analysis (qualitative analysis) used for factor mapping. The proposed approach has been tested on a simple case, with a nominal traffic running on a single track line. The considered incident is the loss of a feeding power substation. The variables to be adjusted are: incrementing of the train departure times and speed reduction at given positions. The results show that the incrementing of train departure times is the most influential variable and we got Pareto-optimal front that meets multi-criteria such as: minimize travelling time, train delays, traction energy …

Keywords: rescheduling, railway simulation, sensitivity analysis, Sobol indexes, Monte Carlo filtering

1 INTRODUCTION

The problem of railway traffic rescheduling in the event of an operating incident has been studied for a long time. Many research studies in this area deal with situations where a minor incident on a track affects a large number of trains due to cascade effects [1], [2], [3]. Authors also discuss the impact of major incidents [4],[5],[6] and focus on the reconstruction of schedules taking into account the availability of rolling stock and crews, but the problematic of the impact of electrical power shortage is never studied.

The present work addresses train traffic rescheduling, in the case of electric power shortage, due to an electric equipment outage. Such incidents limit the power available for train traction, and hence the transportation capacity of the railway system. We propose a method to reschedule the train traffic, and to optimally share the available power among the different trains. Our approach is based on global sensitivity analysis and Monte Carlo filtering in order to rank the influence of the decision variables while accounting for various operational constraints.

The paper is organized as follows: Section 2 describes the problem with more details; Section 3 introduces global sensitivity analysis and presents how the method is applied to traffic rescheduling; Section 4 provides first results in a simple case and Section 5 concludes.

2 PROBLEM DESCRIPTION

The electrical infrastructure of a railway network is designed and controlled so as to provide the power needed by the trains. It is a complex system, in which the main elements are the feeding sub-stations, the catenaries, the rails (return conductor) and the trains. Other devices allow to configure the electrical network's topology according to the needs. Numerous trains travel at the same time on different lines, and the analysis of the system relies on simulation.

In the present work, we use ESMERALDA NG [7], a simulator developed by the SNCF. This simulator is based on a multi-physical model of the railway network: mechanical, electrical, and thermal. The input data are the physical description of the railway network on one hand (topology, position and characteristics of all devices, including the trains) and the description of the intended traffic on the other hand (type of trains, departure and passage times at various points, and reference speed profiles along the way). The equations of train dynamics are coupled with the circuit equations of the electrical network and solved step by step over time in order to determine the position of the different trains at each time, as well as different electrical quantities such as the voltage at the pantographs, powers passing through catenaries and transformers and the resulting heating. It is a nonlinear numerical model with a large number of inputs and outputs. The computation cost is high, with a large number of variables involved and simulation times around ten minutes.

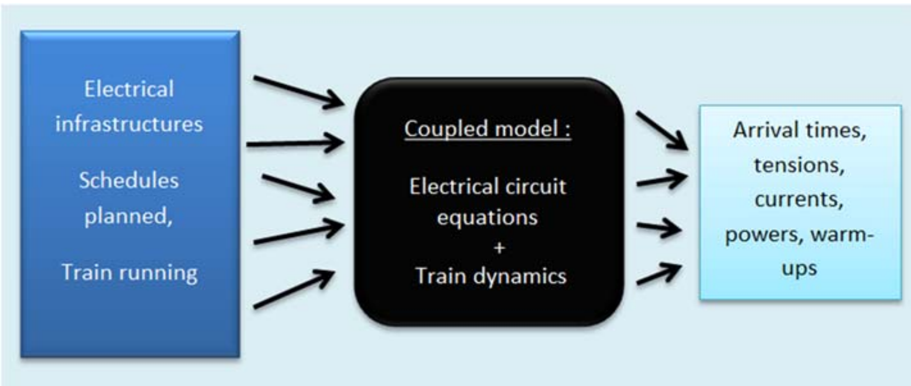

Figure 1: Structure of simulation model

The simulator calculates the actual traffic, for a given physical infrastructure and traffic instructions. For example, a train is scheduled to leave at a certain time, and travel according to a certain speed profile, but if the actual available power is too low, the actual speed profile will not be the expected one and the train will be late. From a technical point of view, the quality of power distribution is monitored through the catenary voltage: too small values indicate that the electrical network is overloaded, which is not desired and requires the traffic to be adapted.

If a component of the electrical infrastructure is unavailable, due to either a technical incident or a maintenance operation, the power available for traction is reduced. It is then necessary to check if the residual capacity allows to maintain the traffic initially planned. The criterion is the catenary voltage, which must remain within the range defined by the standards. If not, the train traffic must be rescheduled.

In the current situation, traffic rescheduling is carried out according to an iterative trial-and-errors method: on the basis of their experience, operators propose re-planning solutions and run simulation to check if the catenary voltage remains within the prescribed limits. This process is slow because the analyzed situation is complex (many trains, many lines) and numerous simulation runs are needed. Furthermore, the outcome of the process, both in terms of quality of the solution and time to reach it, fully depends on the operator experience and know-how. There is no guarantee that an optimal solution will be reached.

The goal of the presented work is to assist the operator in the rescheduling process, thanks to sensitivity analysis. Sensitivity analysis is used for a better understanding of the problem and answers questions like: which adjustment operation is the most influential? How does a given input variable affect the output? Which traffic adjustments are needed to reschedule the traffic while respecting operational constraints?

We propose a global stochastic sensitivity analysis, associated to Monte Carlo filtering, in order to efficiently explore the decision space and build a set of acceptable solutions. 'Acceptable solutions' means traffic grids that respect all the operational constraints of the system. Sensitivity indicators are calculated in order to quantify the influence of the different traffic grid parameters (decision variables) on the traffic. In addition, the set of acceptable solutions is processed in order to build a practical Pareto front according to criteria defined by the traffic operator, and guide him for the choice of the final solution.

3 SENSITIVITY ANALYSIS APPLIED TO TRAFFIC RESCHEDULING

3.1. Traffic rescheduling process

Traffic rescheduling requires to adjust the traffic grid according to the actual power feeding capacity of the railway system: less train can circulate in a given time frame on a given line, and/or their speed must be reduced. To do this, a set of decision variables must be defined, such as time and/or space intervals between trains or speed limits at particular points of the lines. Each point of the decision variables space corresponds to a particular traffic grid. The traffic operator uses simulation in order to explore the decision space and find the points which correspond to acceptable solutions, in the sense that they respect all physical and operational constraints of the system; then the operator has to choose which points are the best ones, with respect to different performance criteria.

The numerical simulator, ESMERALDA NG, is based on a dynamic nonlinear electro-mechanical model. All trains motions are computed step-by-step over time, along with other quantities such as train pantograph voltages, catenary currents, engine temperatures …). All these quantities constitute the simulator outputs, and the operator must verify that they meet the various operational constraints they are subjected to. In the rescheduling process, the simulator is used as a black box in order to find input which result in acceptable outputs, in the sense that they meet the operational constraints.

The numerical model is a highly nonlinear one, has a large number of variables (tens of trains traveling during hours), and is computationally expensive (typically around ten minutes for a real case simulation). Hence, a rational procedure is needed in order to obtain as much information as possible about the relationships between the model inputs and outputs, at the lowest computation cost. Global stochastic sensitivity analysis is a possible approach to this type of problem, and the rest of this section explains how it is applied to traffic rescheduling.

3.2. Purposes of sensitivity analysis

Sensitivity analysis is the process of varying the input parameters of a model and observing the resulting outputs variations. It is used to explore how the model outputs variations can be qualitatively and quantitatively attributed to the different inputs variations.

While local sensitivity analysis considers variability stemming from input variations around a specific point, global sensitivity analysis considers inputs variations within their entire feasibility range. In this paper we will consider global sensitivity analysis, in order to explore the whole decision space. Three so-called ‘settings’ are commonly defined in order to quantify the inputs importance [8]:

- Factor prioritization aims at ranking the inputs in terms of their relative contribution to the output variability;
- Factor fixing aims at determining which inputs, if any, do not have any contribution to the output variability;
- Factor mapping aims at determining which part of the inputs space produces outputs in a specific region, for instance above a prescribed threshold.

Factor prioritization and factor fixing are linked settings: the second one focuses on input that have so little influence that they can be fixed and dropped in order to simplify the model. Factor mapping is a more difficult concept, dealing with model inversion and useful to handle output constraints. In the present work, we use Sobol sensitivity indexes for factor prioritization and factor fixing (section 3.3). Factor mapping is handled thanks to Monte Carlo filtering and empirical cumulative density functions (section 3.4).

3.3. Sobol sensitivity indexes

Sobol’s method is a variance-based sensitivity analysis that determines the contribution of each input parameter and their interactions to the overall model output variance. This method is based on the decomposition of the model output variance into summands of variances of the input parameters in increasing dimensionality [9].

Let us denote the nonlinear model by $Y = f(X)$, where $X \in \mathbb{R}^p$ is the model input vector and $Y \in \mathbb{R}^m$ is the model output vector. The function f represents the simulator. The input and output vectors are also written as : $X = (X_i)_{i=1,p}$ and $Y = (Y_r)_{r=1,m}$

Let us first consider a single output model, i.e $m = 1$. In [10], an indicator of the influence of the parameter X_i on this single output is proposed. It is denoted S_i and is defined by:

$$S_i = \frac{V_i}{V(Y)} = \frac{V(E(Y|X_i))}{V(Y)} \tag{1}$$

where $V_i = V(E(Y|X_i))$ represents the conditional variance of Y at the parameter X and $V(Y)$ the total variance of Y. The sensitivity index S_i is between 0 and 1. The closer the index is to 1, the more influential X_i is.

The sensitivity parameters S_i are derived from the Hoeffding variance decomposition [11]. Y may be decomposed in the following way:

$$Y = f_0 + \sum_i^p f_i(X_i) + \sum_{i<j}^p f_{ij}(X_i, X_j) + \cdots + f_{1,2,..p}(X_1, \dots, X_p) \tag{2}$$

If the variables X_i are random and independent, equation (2) makes it possible to obtain the decomposition of the functional variance by orthogonality (called functional ANOVA):

$$V(Y) = \sum_{i=1}^p V_i(Y) + \sum_{i<j} V_{ij}(Y) + \sum_{i<j<p} V_{ijp}(Y) + \cdots + V_{12..k}(Y) \tag{3}$$

where :

$$V_i(Y) = V[E(Y|X_i)]$$
$$V_{ij}(Y) = V\left[E\left(Y\middle|X_i X_j\right)\right] - V_i(Y) - V_j(Y)$$

The two-factor case can be used as an illustration. For any function $Y = f(X_1, X_2)$, there exists a single orthogonal decomposition:

$$Y = f_0 + f_1(X_1) + f_2(X_2) + f_{1,2}(X_1, X_2) \tag{4}$$

$$V(Y) = V(f_1) + V(f_2) + V(f_{1,2})$$

$$1 = \frac{V(f_1)}{V(Y)} + \frac{V(f_2)}{V(Y)} + \frac{V(f_{1,2})}{V(Y)}$$

$$1 = Si_1 + Si_2 + Si_{12}$$

Si_1 and Si_2 quantify the influence of X_1 and X_2 respectively, and Si_{12} quantifies their interaction effect.

In the general case, two parameters can be defined for each input parameter (equations (5) and (6)): the first quantifies the main effect (interaction with other variables are not taken into account), and the second accounts for all the effects:

$$S_i = \frac{V_i}{V(Y)} \qquad i = 1, \ldots, p \tag{5}$$

$$S_{Ti} = \frac{V_{Ti}}{V(Y)} \qquad i = 1, \ldots, p \tag{6}$$

where $V_i = V[E(Y|X_i)]$, $V_{Ti} = V(Y) - V[E(Y|X_{-i})]$ and X_{-i} denotes the set of all inputs excluding X_i [11]. The total effects of X_i takes into account both the main effects and its interaction effects.

In general, models have several outputs and generalized Sobol indexes, defined by equation (7) were proposed in [13] is expressed by:

$$S_i = \frac{\sum_{r=1}^{m} V[E(Y_r|X_i)]}{\sum_{r=1}^{m} V(Y_r)} \tag{7}$$

The numerator is the sum of X_i first order effects on all the model outputs, and the denominator is the sum of the variances of all the model outputs.
Sobol sensitivity indexes are used for factor prioritization, and for factor fixing if some inputs are found to have very low sensitivity indexes.

3.3. Regional sensitivity analysis

As mentioned before, outputs constraints need to be accounted for. This is done through regional sensitivity analysis and factor mapping [14]. The operator is interested in determining what input values cause the model output to be in a certain region, defined by a performance indicator or a constraint. In our case, we want the catenary voltage to be above a given threshold for all trains and at all times.

A simple qualitative method is Monte Carlo filtering: Monte Carlo runs are performed and the sampled input space is partitioned into two groups, depending on whether the associated model output satisfies or not the desired condition. The so-called « behavioral group », denoted by (X_i/R), of size n, contains the elements that respect the performance indicators, while the « non-behavioral group », denoted by $(X_i/\bar{R})$, of size $\bar{n}$, contains those that do not. The sum $n + \bar{n}$ corresponds to the total number of runs.

For each input, the empirical cumulative distribution functions of both groups, respectively denoted by $F_n(X_i/R)$ and $F_{\bar{n}}(X_i/\bar{R})$, are computed and plotted (Fig. 2). Visual comparison between $F_n(X_i/R)$ and $F_{\bar{n}}(X_i/\bar{R})$ provides qualitative information about the influence of each input variable with respect to factor mapping: if $F_n(X_i/R)$ and $F_{\bar{n}}(X_i/\bar{R})$ are significantly different, then the sensitivity of the performance indicator to X_i is high.

The shape and the relative positions of the curves also contain information (Fig 3). If $F_n(X_i/R)$ lies on the left side of $F_{\bar{n}}(X_i/\bar{R})$, it means that the performance indicator is statistically more respected for smaller values of X_i : increasing X_i has a negative impact on the indicator performance criterion. Conversely, if $F_n(X_i/R)$ lies on the right side of $F_{\overline{n}}(X_i/\bar{R})$, increasing X_i has a positive impact. The shape of the curves also gives information: the steeper the slope, the larger the local X_i influence.

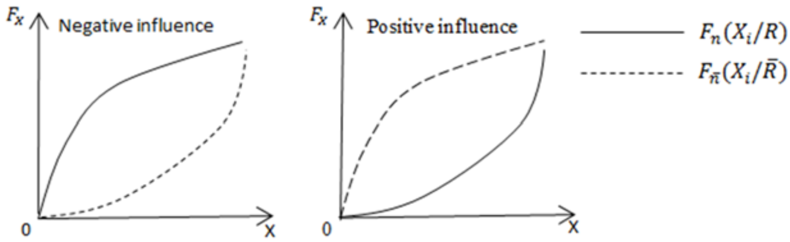

Figure 2: Relative position of $F_n(X_i/R)$ and $F_{\overline{n}}(X_i/\bar{R})$, according to the influence of X_i

3.4. Application to traffic rescheduling

The principles presented above have been applied to train traffic rescheduling, in order to better understand the influence of the rescheduling parameters on the model outputs. At the time being, we focus on the main performance criterion, which is the power feeding of the trains. The power feeding quality is monitored through the voltage pantograph, which must remain between standard limits

The rescheduling process is organized in four stages:

1. Problem specification: the decision-maker defines the traffic adjustment variables (time or distance intervals between trains, speed references…), their range of variation, output performance indicators that define the set of acceptable solutions (voltage standard in the present case), and the criteria used to generate practical Pareto-optimal fronts (travel time, delays, electric consumption…).
2. Monte Carlo runs: the simulator manager generates the sampling of the input space defined in stage 1 and runs the simulations. Quasi-random sampling should be used in order to avoid gaps and clusters in the sampled space (Fig. 3).
3. Sensitivity analysis: the post-processor calculates the generalized Sobol indexes, generates the X_i/R and $X_i/\bar{R}$ subsets, computes and plots the cumulative distribution functions, and generates practical Pareto-optimal fronts.
4. Choice of the rescheduled traffic: the decision-maker interprets the sensitivity analysis results and Pareto-optimal front in order to choose the best point in X_i/R.

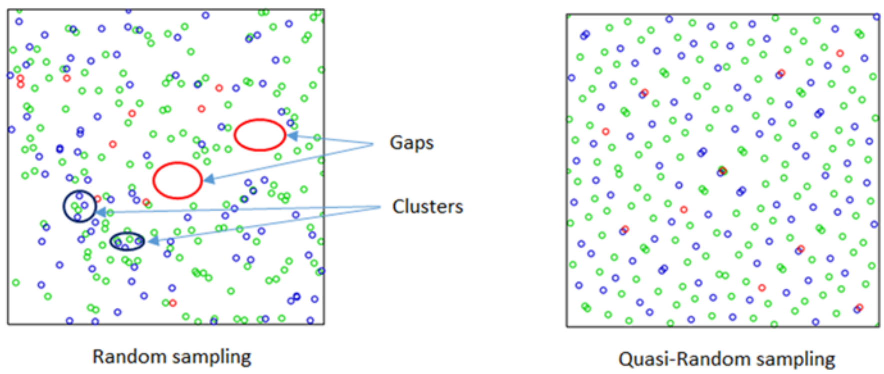

Figure 3: Comparison between random and quasi-random samplings

4. TEST CASE

The proposed approach has been tested on a very simple first example: a single-track line, fed by three substations (SST) and traveled by ten high speed trains (Fig. 4). In the nominal case (no default), trains depart every five minutes. The present analysis focuses on the respect of the pantograph voltage operational constraint, that must remain between 17.5 kV and 27.5 kV at all time in order to ensure the proper operation of traction engines. Fig 5 shows the pantograph voltage of the ten trains. Which curve corresponds to which train is not a concern: what matters is that all curves remain in the desired range, between 17.5 kV and 27.5 kV.

In the default case, the substation located in the middle of the line is assumed to go down, which limits the power available and disturbs the traffic. Fig. 6 shows that the pantograph voltage drops below the lowest allowed level. Some trains are more affected than others, but again, what matters is the global view. Operational constraints are not respected, hence the traffic cannot be maintained as it is.

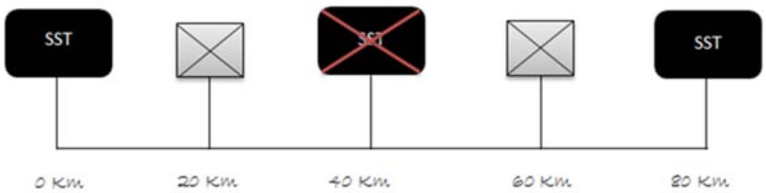

Figure 4: Position of the feeding substations (SST) along the studied line. The substation at 40 km is out of order

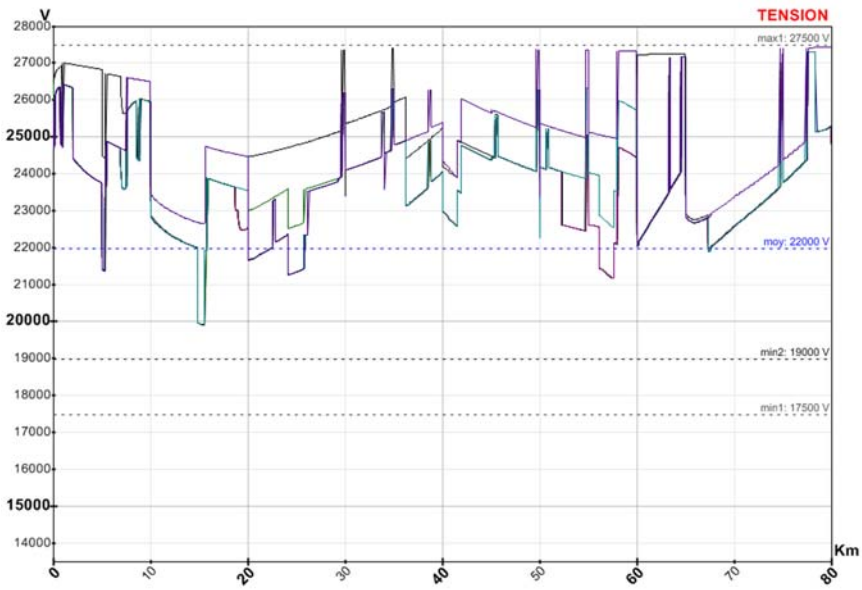

Figure 5: Trains pantographs voltages in the nominal case

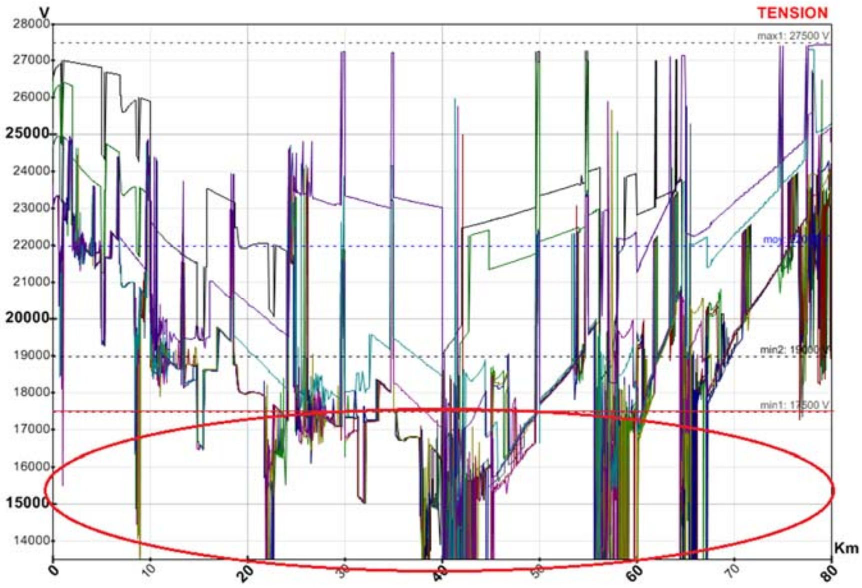

Figure 6: Trains pantographs voltages in the default case

For this first test case, two rescheduling actions are considered: increase of the time interval between two trains and speed reduction on both sides on the disabled substation. These actions are converted into two input variables: X_1 is the time interval increase, ranging between 0 and 15 minutes, and X_2 is the speed reduction, ranging between 0 and 35 km/h. The considered model outputs are the pantograph voltages at each computation time, which correspond to the data plotted in Fig. 5 and Fig. 6.

The sensitivity analysis methods presented in the previous section has been applied. Thousand quasi-random samples were generated according to a uniform distribution, and the corresponding simulations were done.

First, the generalized Sobol indexes were computed for each of the ten trains. Similar results were obtained for all of them, which is consistent with the fact that they have identical missions. Table 1 reports the generalized Sobol indexes for the third train. These indexes allow to quantify the influence of the two rescheduling actions on the pantograph voltage quality. It shows that increasing the time interval between trains has a larger influence that reducing their speed.

Table 1: Result of the generalized Sobol analysis

Input variable	Generalized Sobol indexes
X_1 : time interval increase	0.27
X_2 : speed reduction	0.09

The next step is the regional analysis, needed to account for the pantograph voltage constraint. Samples were filtered according to the voltage constraint criterion and divided into the behavioral set and non-behavioral set. The empirical cumulated density functions were computed for each adjustment variables.

Fig. 7 displays the cumulated density functions relative to X_1 (time interval increase). The blue and red curves respectively correspond to the behavioral and non- behavioral subsets. The blue curve indicates that the voltage constraint is never satisfied when X_1 is less than 7.5 minutes. Starting from this value, more and more points respect the voltage constraints as X_1 increases. The red curve lies on the left side of the blue one, indicating that increasing X_1 has a positive influence with respect to the voltage constraint satisfaction. The large space between the two curves shows that this influence is large.

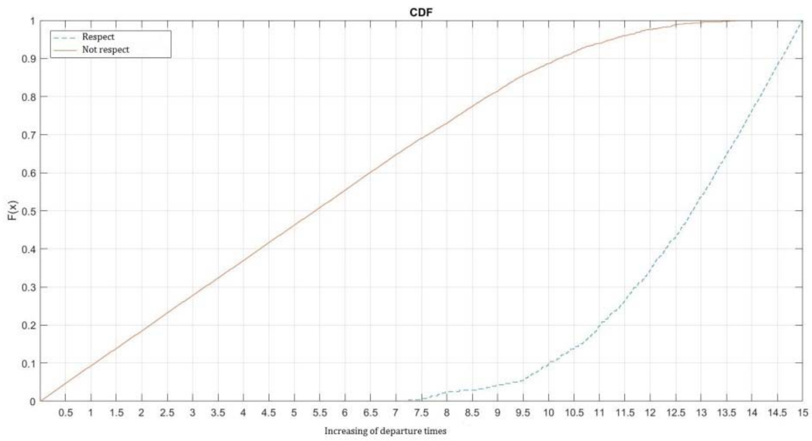

Figure 7: Cumulative density functions relative to X_1 (time interval increase)

Fig. 8 displays the cumulated density functions relative to X_2 (speed reduction). Again, the blue and red curves respectively correspond to the behavioral and non- behavioral subsets. Both curves are very close to each other, meaning that X_2 has a very small positive influence with respect to the voltage constraint satisfaction

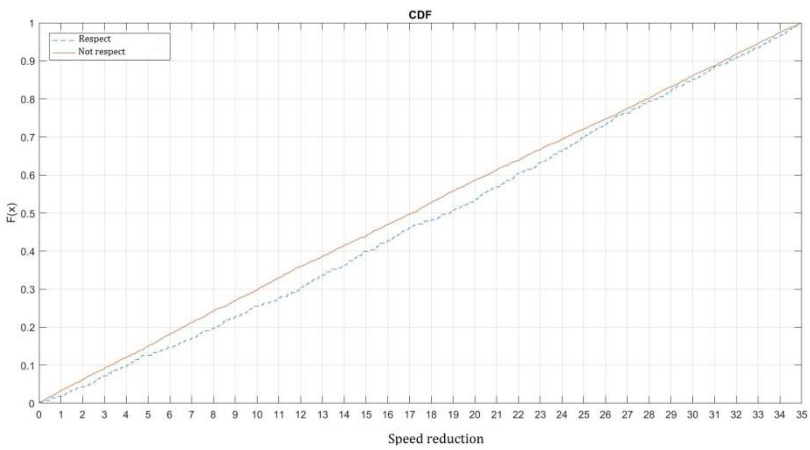

Figure 8: Cumulative density functions relative to X_2 (speed reduction)

A closer look of the behavioral subset, not detailed here, shows that it is possible to respect the voltage constraint by acting on the time interval only: with a time interval increase above ten minutes, there is no need to reduce the speed around the disabled substation to respect the voltage standards. The contrary is not true: the voltage constraint cannot be respected by the single speed reduction.

The last part of the analysis is to provide performance criteria to help the choice of the final solution. Performance criteria are defined by the decision maker and the behavioral subset points are used to build practical Pareto front. Fig. 9 shows a Pareto plot which represents the trade-off between two criteria: F1 is the average train travel time, whereas F2 is the time interval increase between trains. The red points correspond to non-dominated solutions.

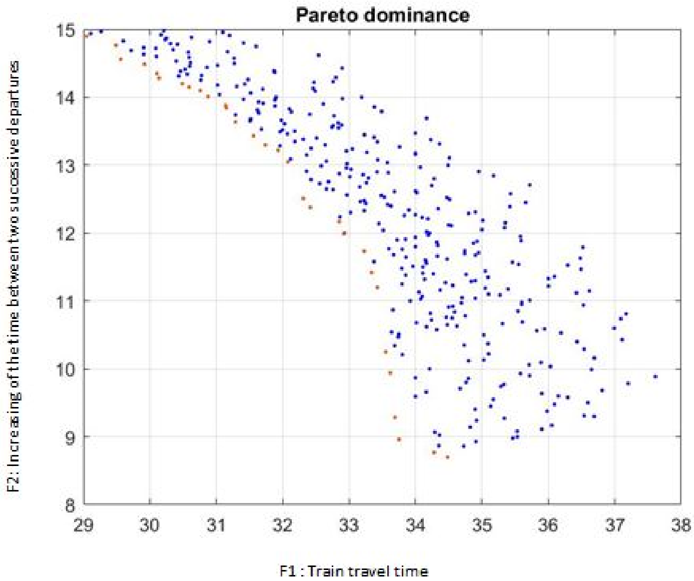

Figure 9: Pareto dominance of criteria F1and F2, built with the acceptable solutions.

The decision maker now has qualitative and quantitative information to support his final choice, according to his priorities. For example, he can choose to have fast trains with large time interval between them, or conversely prefer to have closer but slower trains. The choice is his. Fig. 10 shows the train pantograph voltage in the case where the criteria F2 is favored.

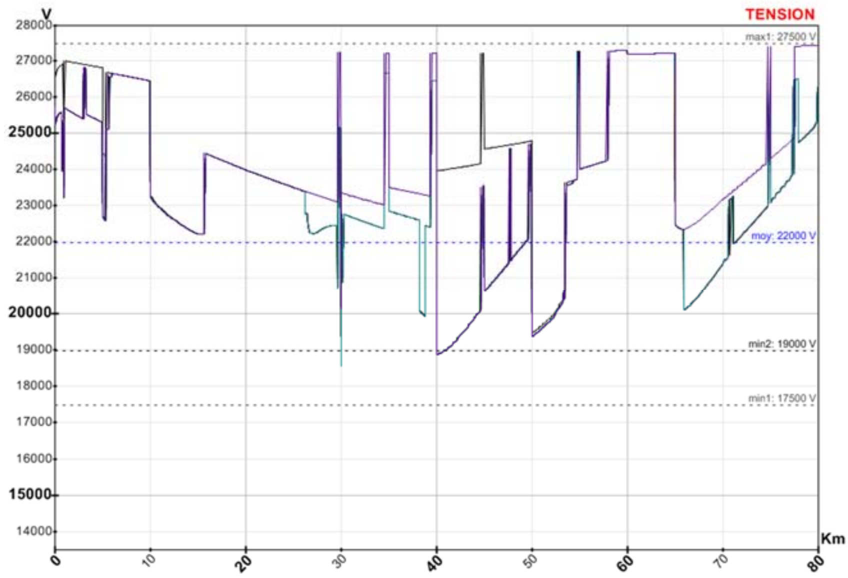

Figure 10: Pantographs voltage for the solution that minimizes F2 (trains departure delay)

5. CONCLUSION

Railway traffic rescheduling in case of electric power shortage is an important process for proper operation of the railway network. In this paper, we propose to use global and regional sensitivity analysis in order to provide qualitative and quantitative information to the decision maker. The method is applied to a simple test case and results show that the proposed approach is helpful to assist the operators in charge of railway traffic rescheduling. The influence of the different decision variables are determined, and the voltage operational constraints are accounted for. Pareto-optimal front are built and the operator can choose the final solution according to his priorities.

In the present paper, the proposed approach is tested on a very simple case. Further work will test more complex situations, but Sobol's method requires a rather fine sampling of the decision space, which will limit the size of the problem that can be efficiently dealt with. Hence future work should consider less expensive methods, as well as ways to breakdown large problems into smaller ones.

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