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Abstract: A robust alternative of the delayed resonator is proposed by spectral approach. By double root assignment at the excitation frequency, which is projected to widening the stop-band in the active absorber frequency response, the performance sensitivity to the mismatch between the design and true excitation frequency is considerably decreased. Additionally, the overall scheme is supplemented by a control loop which improves the stability margin. The design is validated by simulations and the results are compared with classical delayed resonator.

Keywords: Double root assignment, active vibration control, robust control, acceleration feedback, spectral method, delayed resonator

1. INTRODUCTION

The delayed resonator (DR) concept proposed in 1990s by N. Olgac and his co-workers Olgac and Holm-Hansen (1994); Olgac (1995), has become an established tool for vibration suppression and one of the typical examples of benefits of time delay in the feedback loop. The time-delay feedback is applied to modify the active absorber properties resulting in entire suppression of undesirable oscillations. The absorber is tuned to an ideal resonator, marginally stable, where the DR acts as a perfect resonator and absorbs the vibrations entirely. The resonator feedback can be implemented using position, velocity or acceleration measurements, depending on the type of sensor selected for a particular vibration control application at hand. In this paper, the acceleration feedback is considered as it allows closing the feedback directly from the accelerometer sensors, which are easy to apply and of relatively low cost.

Modifications of the resonator concept include a single-mass dual frequency absorber Olgac (1996), relative position feedback absorber in Olgac and Hosek (1997), a torsional vibrational absorber Hosek et al. (1997), to name a few. An auto-tuning algorithm enhancing robustness against uncertainties in the absorber parameters was presented in Hosek and Olgac (2002). An algorithm for multi-degree of freedom (MDOF) mechanical structures with multiple delayed resonators is described in Jalili and Olgac (1999), where multiple absorbers are used to suppress different harmonics. Stability of the vibration absorber using acceleration and displacement feedback was derived in Alujević et al. (2012) utilizing the Nyquist criterion. In Rivaz and Rohling (2007) a delay free PI alternative of the resonator has been proposed for the acceleration feedback. However, due to risky noise integration phenomenon, the feedback needs to be supplemented with high-pass filters, which makes the overall feedback more complex compared to the time delay case.

Recently, a complete dynamics analysis of a DR with acceleration feedback was performed in Vyhlídal et al. (2014) revealing unfavorable neutral character of spectral properties of the DR, which is also transferred to the overall set-up. In order to mitigate this undesirable effect, an alternative distributed delay resonator (DDR) was proposed and analyzed in Pilbauer et al. (2016) resulting in retarded spectral properties, which are more convenient. The second benefit of this novel resonator scheme is that the distributed delay provides filtering of the measured noise. By applying the method of *Cluster Treatment of Characteristic Roots* Olgac and Sipahi (2005), it was shown in Vyhlídal et al. (2014) and Pilbauer et al. (2016) that for both the DR and DDR, their operable frequency range is limited. From below, it is limited by the stability boundary of the DR, while the delay implementation aspects limit the range from the above - due to the exponential decay of the delay length with respect to growing frequency. A methodology for further extension

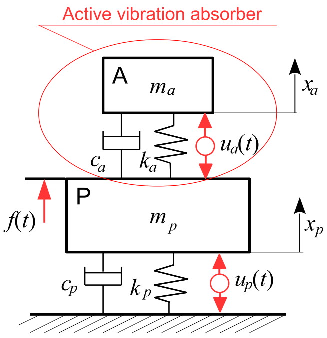

Fig. 1. Primary Structure (P), with an active vibration absorber (A) to suppress displacement x_p induced by harmonic disturbance force $f(t)$

of the operable frequency range was recently proposed in Kučera et al. (2017). It is based on extending the feedback by a delay free factor virtually modifying the mass of the absorber and thus its natural frequency. Besides, implementation aspects of the resulting, so-called *extended delay resonators* have been discussed in Kučera et al. (2017).

As it results from the frequency domain analysis performed in application sections of Pilbauer et al. (2016); Kučera et al. (2017), the entire vibration suppression takes place only if the resonant frequency adjusted by the resonator feedback is equivalent with the true excitation frequency. The performance of the resonator decays considerably even for very small differences in these frequencies. The main objective of this paper is to propose a robust alternative to the classical delayed resonator, which will widen the frequency stop-band of the resonator. Note that in this aspect, the paper is analogous to the recent work Pilbauer et al. (2017), where the optimization approach is applied directly on shaping the frequency characteristic of the DR. In this paper, we propose a more straightforward and easy to apply approach, which is based on double root assignment.

A fundamental feature of the resonator concept is that the parameters of the primary structure do not play a role in the resonator tuning. The purpose of the design is only to suppress the vibrations, while stability aspects of the overall set-up are left aside. As discussed in Vyhlídal et al. (2014); Pilbauer et al. (2016), in fact, the resonator may decrease the stability margin or even destabilize the overall set-up. Thus, the second objective of the presented work is to handle this stability aspect by including a stabilizing controller, similarly as it was done in Fenzi et al. (2017).

The paper is organized as follows. In Section 2, the concept of vibration absorber and functioning of the overall set-up including a stabilizing controller is outlined. Section 3 then provides the main result with introducing the robust delayed resonator concept and providing two analytic rules for its parametrizations. Next, design of the additional finite order controller is adressed together with its parametrization by minimizing the spectral abscissa. The validation of the proposed concept is done in section 4, where it is also compared with the classical delayed resonator concept. In section 5, brief conclusions are given.

2. PRELIMINARIES AND PROBLEM STATEMENT

The dynamic model of the absorber, depicted in Fig. 1, with the physical parameters m_a, c_a, k_a denoting the mass, the damping and the stiffness coefficients, can be described by

$$m_a\ddot{x}_a(\vartheta) + c_a\dot{x}_a(\vartheta) + k_a x_a(\vartheta) = \bar{u}_a(\vartheta) \tag{1}$$

with x_a being the displacement of the absorber and u_a a (scaled) external input. The damping ratio and the natural frequency of the absorber are given by $\zeta = \frac{c_a}{2\sqrt{m_a k_a}}$ and $\Omega = \sqrt{\frac{k_a}{m_a}}$, respectively. The objective of the absorber is to suppress vibrations of the primary structure being excited by a harmonic force $f(t)$ at the frequency $\bar{\omega}_f$.

The absorber equation can be turned to

$$\ddot{x}_a(\vartheta) + 2\zeta\Omega\dot{x}_a(\vartheta) + \Omega^2 x_a(\vartheta) = \bar{u}_a(\vartheta). \tag{2}$$

Scaling the time ϑ with respect to the frequency Ω, i.e. by introducing dimensionless time $t = \Omega\vartheta$, we obtain an absorber form

$$\ddot{x}_a(t) + 2\zeta\dot{x}_a(t) + x_a(t) = u_a(t). \tag{3}$$

with the only parameter ζ, assuming $u_a = \frac{1}{m_a\Omega^2}\bar{u}_a$. Note that scaled excitation frequency is given by $\omega_f = \frac{\bar{\omega}_f}{\Omega}$.

Considering the scaled parameters with respect to m_a and Ω, the overal set-up in Fig. 1 is given by

$$\ddot{x}_a(t) + 2\zeta\dot{x}_a(t) + x_a(t) - 2\zeta\dot{x}_p(t) - x_p(t) = u_a(t), \tag{4}$$

$$m_p\ddot{x}_p(t) + (2\zeta + c_p)\dot{x}_p(t) + (1 + k_p)x_p(t) - 2\zeta\dot{x}_a(t) - x_a(t) = -u_a(t) + u_p(t) + f(t), \tag{5}$$

where $x_p(t)$ is the position, $u_p(t)$ is the control input for positioning the primary structure and m_p, c_p, k_p are the scaled mass, the damping and the stiffness parameters of the primary structure.

The absorber input u_a is to be used for enhancing the absorber parameters so that the vibrations at the frequency ω_f are entirely suppressed. Due to problem generalization purpose, we consider the feedback in the form

$$U_a(s) = P(s)X_a(s), \tag{6}$$

where $P(s)$ is the feedback transfer function.

Compared to the classical delayed resonator concept, where (6) is the only active feedback at the set-up, we assume that also the primary structure is equipped with an active feedback

$$U_p(s) = K(s)X_p(s), \tag{7}$$

with controller $K(s)$, which can be used for platform positioning or just for improving the overall dynamical properties in vibration suppression.

For the overall set-up (4)-(7), the transfer function between the excitation force f and the position of the platform with feedback (6) is given by

$$G_{x_p,f}(s) = \frac{X_p(s)}{F(s)} = \frac{R(s) - P(s)}{(R(s) - P(s))(V(s) - K(s)) + (P(s) - Q(s))Q(s)} \tag{8}$$

with

$$R(s) = s^2 + 2\zeta s + 1 \tag{9}$$

being the characteristic function of the absorber (1), and

$$\begin{aligned} V(s) &= m_p s^2 + (2\zeta + c_p)\, s + (1 + k_p)\,, \\ Q(s) &= 2\zeta s + 1. \end{aligned} \tag{10}$$

If the transfer function $P(s)$ is parameterized so that the characteristic equation of the resonator composed of the absorber (1) and the feedback (6), given by

$$M(s) = R(s) - P(s) = 0, \tag{11}$$

has a root couple $s_{1,2} = \pm j\omega_f$, composing a pole couple of the resonator, then

$$G_{x_p,f}(j\omega_f) = 0, \tag{12}$$

indicating that no vibrations at the given frequency $\omega = \omega_f$ are transferred in this f to x_p channel and the vibrations are ideally suppressed.

Note however, that including the resonator feedback affects the dynamical properties of the entire system, which is given by the roots of the characteristic equation

$$(R(s) - P(s))\,(V(s) - K(s)) + (P(s) - Q(s))\, Q(s) = 0 \tag{13}$$

Assuming presence of the delay terms in $P(s)$, the equation (13) has infinitely many roots. For the stability implications, all of them need to be located safely in the left half of the complex plane.

3. ROBUST RESONATOR FEEDBACK

The resonator feedback is considered in the following form

$$u(\vartheta) = -h\ddot{x}_a(\vartheta) + \int_0^{\vartheta} \bar{p}(\theta) x_a(\vartheta - \theta) d\theta, \tag{14}$$

where $\bar{p}(\vartheta)$ is the weighting function (impulse response) corresponding to the resonator transfer function $\bar{P}(\bar{s})$, which is to be transformed to $P(s)$ in the dimensionless form. Motivated by Kučera et al. (2017), the term with h is included in (14) to modify virtually the mass of the absorber so that the natural frequency of the absorber $\Omega = \sqrt{\frac{k_a}{m_a + h}}$ is identical with the excitation frequency $\bar{\omega}_f$, i.e. $\omega_f = 1$, which results to

$$h(\bar{\omega}_f) = \frac{k_a}{\bar{\omega}_f^2} - m_a. \tag{15}$$

Note that if $h \neq 0$, for the design purposes, the absorber (1) as well as the primary structure needs to be scaled with respect to $m+h$, i.e. with respect to the virtually modified mass.

Coupling the absorber (1) with the feedback (14), and scaling the time with respect to $\Omega = \omega_f$, the delayed resonator is given in the form

$$\ddot{x}_a(t) + 2\zeta \dot{x}_a(t) + x_a(t) = \int_0^t p(\theta) x_a(t - \theta) d\theta. \tag{16}$$

where $p(\theta)$ is the scaled impulse response $\bar{p}(\theta)$.

The transfer function is considered in the form

$$P(s) = \mathcal{L}\{p(t)\} = \frac{X_a(s)}{U_a(s)} = \frac{\sum_{i=0}^{2} a_i e^{-s\tau_i}}{s} s^2, \tag{17}$$

i.e. of (effectively) integral character with delays $\tau_0 = 0, \tau_2 > \tau_1 > \tau_0$ and parameters a_0, a_1, a_2 to be tuned. Note that the term s^2 corresponds to the acceleration measurements $\ddot{x}_a$. When considering the measured output as $y_a = \ddot{x}_a$, the transfer function turns to the mentioned integral form $\frac{Y_a(s)}{U_a(s)} = \frac{\sum_{i=0}^{2} a_i e^{-s\tau_i}}{s}$.

As will be shown later, selection of this specific form of the transfer function (17) is advantageous due to easy parametrization of the resonator. However, on the other hand, it should be noted that from the practical point of view, the integral character of the transfer function can be risky when the measurements in the feedback path are noisy Kučera et al. (2017). As has been shown in Rivaz and Rohling (2007), this problem can be solved by supplying the feedback path by a high-pass filter. Another option is to impose the finite impulse response of $P(s)$, which can be achieved by satisfying the condition

$$\sum_{i=0}^{2} a_i = 0. \tag{18}$$

3.1 DR parametrization by assigning a double root

The objective in the robust resonator design is widening the frequency stop-band of $|G_{x_p,f}(j\omega)|$, where $G_{x_p,f}(s)$ is given by (8), in the vicinity of $\omega_f = 1$. This can be done by assigning a double root $s_{1,2} = j$ to the characteristic equation of the delayed resonator (11), which appears in the numerator of (8). To perform the double root assignment, the root needs to be assigned not only to (11) given by

$$M(s) = s^2 + \left(2\zeta - \sum_{i=0}^{2} a_i e^{-s\tau_i}\right) s + 1 = 0, \tag{19}$$

but also to its first derivative

$$\frac{dM(s)}{ds} = 2s + 2\zeta - \sum_{i=0}^{2} a_i e^{-s\tau_i}(1 - \tau_i s) = 0. \tag{20}$$

This will naturally lead to widening the range where $|M(j\omega)|$ is *close to zero* in the vicinity of $\omega_f = 1$, due to its locally parabolic shape. This property is transferred to $|G_{x_p,f}(j\omega)|$ supposing that the poles of $G_{x_p,f}(s)$ are not located close to the zeros at j, which will be imposed by the stability requirement and will be targeted in the subsequent step.

Assigning the complex root $s_1 = j$ to (20) and splitting the equation to real and imaginary parts, we obtain

$$\Re(M(s)|_{s=j}) = -\sum_{i=0}^{2} a_i \sin\tau_i = 0, \tag{21}$$

$$\Im(M(s)|_{s=j}) = 2\zeta - \sum_{i=0}^{2} a_i \cos\tau_i = 0. \tag{22}$$

Analogously, assigning the root to (20), we obtain the following two equations

$$\Re\left(\frac{dM(s)}{ds}|_{s=j}\right) = 2\zeta - \sum_{i=0}^{2} a_i(\cos\tau_i - \tau_i \sin\tau_i) = 0, \tag{23}$$

$$\Im\left(\frac{dM(s)}{ds}|_{s=j}\right) = 2 + \sum_{i=0}^{2} a_i(\sin\tau_i + \tau_i \cos\tau_i) = 0. \tag{24}$$

Taking into account the equality (22), the equation (23) reduces to

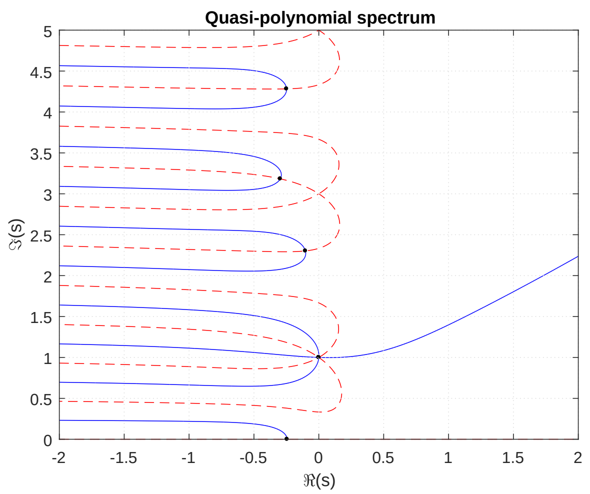

Fig. 2. Rightmost spectrum of the double root Delayed Resonator characteristic equation (29), parametrized by Lemma 1.

$$\sum_{i=0}^{2} a_i \tau_i \sin \tau_i = 0. \tag{25}$$

In the same line, taking into account (21), the eqation (24) reduces to

$$2 + \sum_{i=0}^{2} a_i \tau_i \cos \tau_i = 0. \tag{26}$$

Thus, we have four nonlinear equations (21), (22), (25) and (26), to determine the parameter set $a_i, i = 0, 1, 2$, τ_1, τ_2 (recall that the first delay was preselected as $\tau_0 = 0$). The nonlinearity of the problem may be removed by preselecting also the other delay values. Equations (21) and (25) are satisfied independently of the gain values a_i if the delays are selected as $\tau_1 = \pi$ and $\tau_2 = 2\pi$ (and possibly its $2k+1$ multiples, which are not considered here). Then, (22) and (26) form the linear set

$$2\zeta - a_0 + a_1 - a_2 = 0, \tag{27}$$

$$2 - \pi a_1 + 2\pi a_2 = 0. \tag{28}$$

Still the set of two equations with three variables have infinitely many solutions. We provide two reasonable options to solve the parametrization task.

Lemma 1: Consider the preselected delay values $\tau_0 = 0, \tau_1 = \pi$ and $\tau_2 = 2\pi$, then a *universal* parametrization of the resonator feedback (17) is given by

$$a_0 = 2\zeta, a_1 = a_2 = -\frac{\pi}{2}.$$

Proof: By pre-selecting $a_0 = 2\zeta$, the other two parameters a_1 and a_2 result directly from (27) and (28). The universality lies in the independence of the resulting characteristic equation

$$M(s) = s^2 + \frac{\pi}{2}\left(e^{-s\pi} + e^{-s2\pi}\right) s + 1 = 0. \tag{29}$$

on the resonator parameters. □

Applying the QPmR algorithm Vyhlídal and Zítek (2014), the rightmost spectrum of the retarded characteristic equation (29) is shown in Fig. 2. Next to the spectrum values denoted as black dots, the iso-lines $\Re(M(s)) = 0$ and $\Im(M(s)) = 0$ are shown. The root multiplicity two at $s = j$ is confirmed by that two couples of $\Re(M(s)) = 0$ and $\Im(M(s)) = 0$ isolines intersect at this point, see Vyhlídal and Zítek (2014). As can be seen from the figure, the given root $s_{1,2}$ is the rightmost root.

From the physical and univerality points of view, this setting may be considered as a reasonable option. However, a drawback of this setting is that as the equation (18) is not satisfied, the impulse response of $P(s)$ is not of finite length, which can have undesirable consequences for the control loop performance, taking into account accumulation of the measurement noise due to the integral character of $\frac{P(s)}{s^2}$. This problem can be overcome by the setting given in the following Lemma obtained directly by solving the set of equations (18), (25) and (26).

Lemma 2: Consider the preselected delay values $\tau_0 = 0, \tau_1 = \pi$ and $\tau_2 = 2\pi$, then parametrization of the resonator feedback (17) for which the impulse response is of finite length is given

$$a_0 = \frac{3}{2}\zeta + \frac{1}{\pi}, a_1 = -\zeta, a_2 = -\frac{1}{2}\zeta - \frac{1}{\pi}.$$

3.2 Stabilizing the overall set-up

The absorber and primary structure coupled together as given in (4) and (5) can be described as

$$\begin{cases} \dot{x}(t) &= Ax(t) + B_1 u_a(t) + B_2 u_p(t) + B_3 f(t), \\ y_a(t) &= C_a \dot{x}(t) \\ &= C_a A x(t) + C_a B_1 u_a(t) + C_a B_2 u_p(t) + C_a B_3 f(t), \\ y_p(t) &= C_p x(t) \end{cases} \tag{30}$$

where the output y_a is the measured acceleration $y_a = \ddot{x}_a$, and matrices A, B_1, B_2, B_3 correspond to a linear model of a mechanical system (4)–(5) translated into state space representation with a state vector $x = [x_a\ \dot{x}_a\ x_p\ \dot{x}_p]^T$. Matrix $C_a = [0\ 1\ 0\ 0]$ defines the measured acceleration of the absorber, and $C_p = [0\ 0\ 1\ 0]$ determines the control system output $y_p = x_p$, i.e. the position of the primary structure.

Concerning the resonator feedback implementation, which in Laplace form is given as

$$U_a(s) = \frac{\sum_{i=0}^{N} a_i e^{-s\tau_i}}{s} Y_a(s), \tag{31}$$

it can be turned to

$$\dot{u}_a(t) = \sum_{i=0}^{N} a_i y_a(t - \tau_i). \tag{32}$$

It should be mentioned that this transformation results in additional dynamics, characterized by the introduction of two eigenvalues at zero Pilbauer et al. (2015).

In order to stabilize and optimize the spectrum of the resulting infinite dimensional system, following the methodology proposed in Michiels (2011) we consider a fixed order dynamic feedback controller in the form

$$K : \begin{cases} \dot{x}_K(t) = A_K x_K(t) + B_K y_p(t), \\ u_p(t) = C_K x_K(t) + D_K y_p(t), \end{cases} \tag{33}$$

with k denoting order of the controller.

The system (30) and the feedback (32) create together System of Delay Differential Algebraic Equations (DDAE)

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1 u_a(t) + B_2 u_p(t) + B_3 f(t), \\ \dot{u}_a(t) = \sum_{i=0}^{2} a_i y_a(t-\tau_i), \\ y_a(t) = C_a A x(t) + C_a B_1 u_a(t) + C_a B_2 u_p(t) + C B_3 f(t), \\ y_p(t) = C_p x(t) \end{cases} \tag{34}$$

which can be rewritten in a compact form of DDAEs Michiels and Niculescu (2014) as

$$\mathbf{E}\dot{\xi}(t) = \mathbf{A_0}\xi(t) + \sum_{i=1}^{2} \mathbf{A}_i \xi(t-\tau_i) + \mathbf{B}_0 f(t), \tag{35}$$

where $\xi = [x \quad u_a \quad y_a \quad y_p]^T$ is a new state vector.

For the resulting system, it is required that the spectrum of (35) lies safely in the left half of the complex plain. Defining the spectral abscissa of the system as

$$\alpha(K) := \sup(\Re(s) : s \in \Sigma), \tag{36}$$

with

$$\Sigma := \left\{ s \in C : \det\left(s\mathbf{E} - \mathbf{A}_0 - \sum_{i=1}^{2} \mathbf{A}_i e^{-s\tau_i} \right) = 0 \right\}, \tag{37}$$

the requirement on the stability is given as $\alpha < 0$. Note that if α is very close to the imaginary axis, next to the undesirably small stability margin, it may result in undesirably long transients due to the *low damping* or *slow modes* brought by the rightmost roots. Therefore, it will be required so that the spectral abscissa is located *sufficiently far* to the left of the imaginary axis. This motivates a controller design based on minimizing the *spectral abscissa* over the free parameters, the elements of matrices (A_K, B_K, C_K, D_K). For this task we use the algorithm and software described in Michiels (2011).

It is important to note that the function $K \mapsto \alpha(K)$ is in general not everywhere differentiable, even not everywhere Lipschitz continuous, even though the dependence of the system matrices of the closed-loop system on the controller parameters is smooth. To tackle these challenges, the core optimization tasks in the method of Michiels (2011) are done using the software HANSO (Hybrid Algorithm for Nonsmooth Optimization) Overton (2009).

4. CASE STUDY EXAMPLE

Let's consider coupled absorber with the platform as shown in Fig. 1 with following parameters $m_a = 0.223\,\mathrm{kg}$, $c_a = 1.273\,\mathrm{kgs^{-1}}$, $k_a = 350\,\mathrm{Nm^{-1}}$ on the absorber's side and $\bar{m}_p = 1.52\,\mathrm{kg}$, $\bar{c}_p = 10.11\,\mathrm{kgs^{-1}}$, $\bar{k}_p = 1960\,\mathrm{Nm^{-1}}$ on the primary structure. These parameters were also considered in Kučera et al. (2017).

To describe this set-up by (4)-(5) we use normalized parameters. Absorber has natural frequency $\Omega = 39.627\,\mathrm{s^{-1}}$ and damping ratio $\zeta = 0.072$. For simplicity, we assume that the excitation frequency $\bar{\omega}_f = \Omega$, which implies $h = 0$ in (14). Then, the primary structure has normalized weight $m_p = 6.816$, damping $c_p = 1.144$ and stiffness $k_p = 5.6$. System is excited by the external harmonic force with amplitude $\Delta\bar{f} = 10\,\mathrm{N}$, which implies the normalized amplitude $\Delta f = 0.029$.

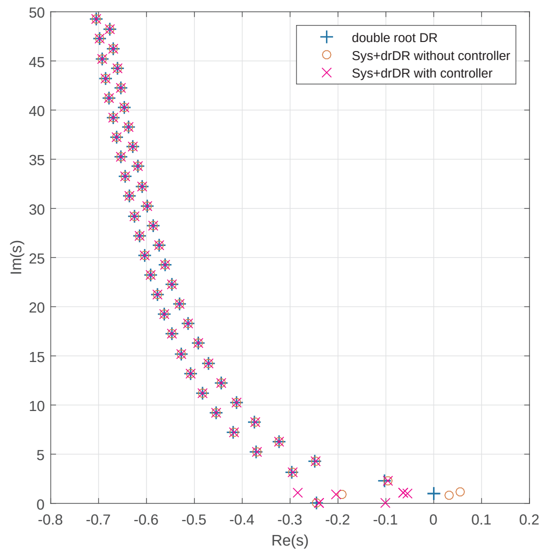

Fig. 3. Spectra of the drDR (16)-(17) alone, drDR - system interconnection (34) without controller (unstable), and with controller (33) (stable).

For parametrization of the robust resonator, we apply Lemma 1. In Fig. 3 the spectrum of the *double root* delayed resonator (drDR) (16)-(17) as well as the spectrum of its coupling with the primary structure (34) is shown. As can be seen, the spectrum of the interconnected system is unstable, as two root couples are located to the right of the stability boundary. Thus, the controller (33) of the order $k = 3$ is tuned by minimizing the spectral abscissa, as described above. After spectral abscissa optimization, defining the controller matrices

$$A_K = \begin{bmatrix} -1.0897 & 0.0147 & 0.4740 \\ 1.4890 & 0.8037 & -3.0751 \\ 0.2966 & 0.6138 & -0.8099 \end{bmatrix}, \; B_K = \begin{bmatrix} 1.3048 \\ 0.5531 \\ 0.9221 \end{bmatrix}, \\ C_K = [6.6243 \; -0.1113 \; -0.3087], \; D_K = [-4.2371], \tag{38}$$

the drDR with the primary structure is stable (as shown in Figure 3).

In the next step, the performance validation is done via simulations. Next to the nominal excitation frequency $\omega = 1$ case, two cases with 3% mismatch between the design and true excitation frequency ($\omega = 0.97$ and $\omega = 1.03$) are considered to study the robustness. For comparison results by the *classical* delayed resonator with lumped delay feedback

$$u_a(t) = g_{cDR}\ddot{x}_a(t - \tau_{cDR}) \tag{39}$$

are shown, with gain $g_{cDR} = 0.1441$ and time delay $\tau_{cDR} = 1.5708$, parametrized by assigning a single root $s_1 = j$ to the delayed resonator Olgac and Holm-Hansen (1994), Olgac (1995). From the results shown in Figure 4 it can be seen that for nominal frequency $\omega = 1$ both DRs performs well. However, for the cases with the 3% frequency mismatch, the drDR performs considerably better with approximately ten times smaller residual vibration amplitudes.

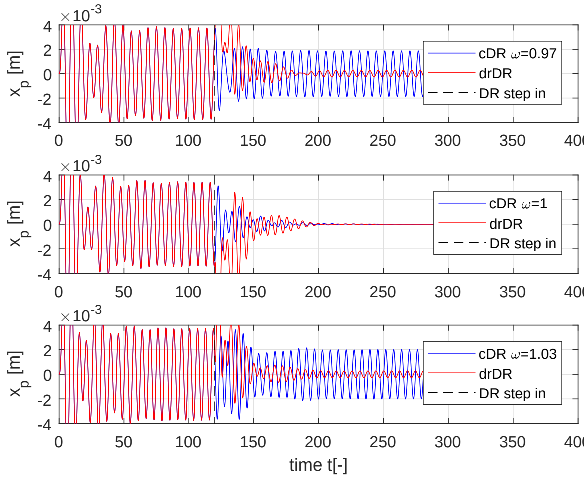

Fig. 4. Simulation results of delayed resonator performance at three excitation frequencies i) $\omega = 0.97$, ii) the nominal $\omega = 1$ and iii) $\omega = 1.03$. Next to the results of the scheme with robust drDR (16)-(17) and the stabilizing controller (33) (red line), the results by the classical scheme with cDR with feedback (39) (blue line) are shown.

Finally, the enhancement of the robustness is studied via the frequency responses. Next to the magnitude $|G_{x_p,f}(j\omega)|$ of the transfer function (8) between the excitation force f and the position of the primary x_p, also the *normalized transmissibility function*

$$T(\omega) = \left| \frac{R(j\omega) - P(j\omega)}{Q(j\omega)} \right| \tag{40}$$

in the vicinity of $\omega = 1$ is studied. Notice that this measure, which was introduced in Pilbauer et al. (2017), is fully determined by the absorber parameters in $R(s)$ and $Q(s)$ defined by (9) and (10) and by the parameters of active feedback $P(s)$. The denominator $Q(s)$ normalizes the function (40) in order to achieve $\lim_{\omega \to 0} T(\omega) = 1$. Note that by coupling equation (4) with the resonator feedback, the transmissibility can be interpreted as $T(\omega) = \frac{1}{|G_{x_a,x_p}(j\omega)|}$, where $G_{x_a,x_p}(s) = \frac{X_a(s)}{X_p(s)}$.

If the cDR (39) is applied, then $T(\omega)$ goes to zero for the nominal frequency $\omega = 1$ as shown in Fig. 5. For the frequencies in the vicinity of $\omega = 1$, this function tends to rapidly increase and therefore, if there is an uncertainty in the excitation frequency, the quality of the vibration suppression is likely to decay. As can be seen in Fig. 5, drDR transmissibility has parabolic shape in the vicinity of $\omega = 1$ with considerably smaller increase in its neighborhood, compared to the cDR. This broadening of the effective frequency stop-band is transformed to $|G_{f,x_p}(j\omega)|$ shown in the Fig. 6. Another positive aspect is considerably lower H_∞ norm (maximum of the frequency response) compared to the case with cDR, caused by applying the platform position controller K, which is not coupled for the cDR case.

5. CONCLUSION

A new type of delayed resonator with a double root assigned at nominal frequency was presented as the main result. For the parametrization of the proposed resonator feedback with time delays, two fully analytic formulas have been proposed. The overall design procedure gains from the fact that it was proposed for the dimensionless models, normalized with respect to the absorber mass and natural frequency. An important aspect is also application of delay free feedback part in the resonator feedback (14), by which the resonant frequency of the absorber is unified with the excitation frequency. This step makes the successive parametrization much easier, as already revealed in Kučera et al. (2017). It was also demonstrated that application of the additional controller of the platform position is beneficial from both the stability and robustness points of view.

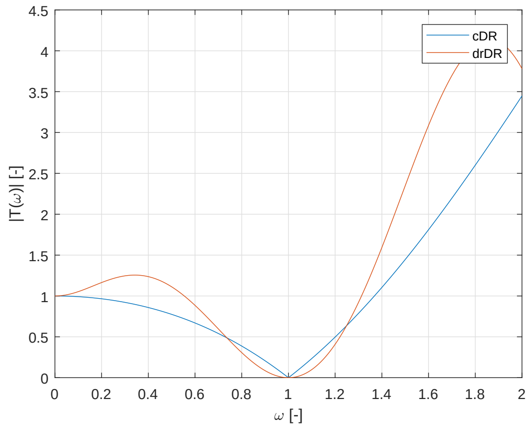

Fig. 5. Transmisibility of the drDR (16)-(17) and classical DR with the feedback (39).

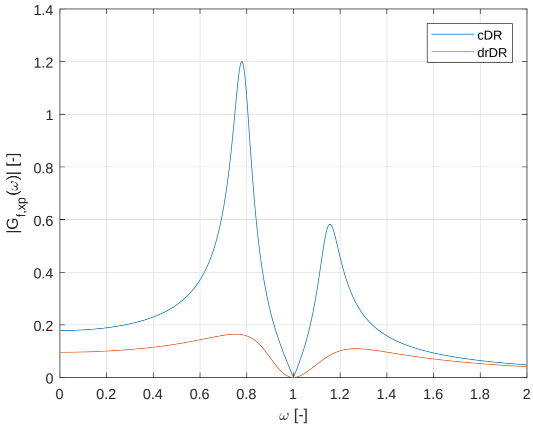

Fig. 6. Magnitude of the transfer function $G_{x_p,f}(j\omega)$ given by (8): i) with robust drDR (16)-(17) and the stabilizing controller (33), ii) with a classical cDR with feedback (39)

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