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Spectral design of robust delayed resonator
by double-root assignment

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Abstract: A robust alternative of the delayed resonator is proposed by spectral approach. By
double root assignment at the excitation frequency, which is projected to widening the stop-
band in the active absorber frequency response, the performance sensitivity to the mismatch
between the design and true excitation frequency is considerably decreased. Additionally, the
overall scheme is supplemented by a control loop which improves the stability margin. The
design is validated by simulations and the results are compared with classical delayed resonator.

Keywords: Double root assignment, active vibration control, robust control, acceleration
feedback, spectral method, delayed resonator

1. INTRODUCTION

The delayed resonator (DR) concept proposed in 1990s
by N. Olgac and his co-workers Olgac and Holm-Hansen
(1994); Olgac (1995), has become an established tool for
vibration suppression and one of the typical examples of
benefits of time delay in the feedback loop. The
time-delay feedback is applied to modify the active absorber
properties resulting in entire suppression of undesirable
oscillations. The absorber is tuned to an ideal resonator,
marginally stable, where the DR acts as a perfect resonator
and absorbs the vibrations entirely. The resonator feed-
back can be implemented using position, velocity or ac-
celeration measurements, depending on the type of sensor
selected for a particular vibration control application at
hand. In this paper, the acceleration feedback is consid-
ered as it allows closing the feedback directly from the
accelerometer sensors, which are easy to apply and of
relatively low cost.

Modifications of the resonator concept include a single-
mass dual frequency absorber Olgac (1996), relative po-
sition feedback absorber in Olgac and Hosek (1997), a
torsional vibrational absorber Hosek et al. (1997), to name
a few. An auto-tuning algorithm enhancing robustness
against uncertainties in the absorber parameters was pre-
presented in Hosek and Olgac (2002). An algorithm for multi-
degree of freedom (MDOF) mechanical structures with
multiple delayed resonators is described in Jalili and Olgac
(1999), where multiple absorbers are used to suppress
different harmonics. Stability of the vibration absorber
using acceleration and displacement feedback was derived
in Alujević et al. (2012) utilizing the Nyquist criterion. In
Rivaz and Rohling (2007) a delay free PI alternative of the
resonator has been proposed for the acceleration feedback.
However, due to risky noise integration phenomenon, the
feedback needs to be supplemented with high-pass filters,
which makes the overall feedback more complex compared
to the time delay case.

Recently, a complete dynamics analysis of a DR with
acceleration feedback was performed in Vyhlidal et al.
(2014) revealing unfavorable neutral character of spectral
properties of the DR, which is also transferred to the
overall set-up. In order to mitigate this undesirable effect,
an alternative distributed delay resonator (DDR) was
proposed and analyzed in Pilbauer et al. (2016) resulting
in retarded spectral properties, which are more convenient.
The second benefit of this novel resonator scheme is that
the distributed delay provides filtering of the measured
noise. By applying the method of Cluster Treatment of
Characteristic Roots Olgac and Sipahi (2005), it was shown
in Vyhlidal et al. (2014) and Pilbauer et al. (2016) that
for both the DR and DDR, their operable frequency
range is limited. From below, it is limited by the stability
boundary of the DR, while the delay implementation
aspects limit the range from the above - due to the
exponential decay of the delay length with respect to
growing frequency. A methodology for further extension
of the operable frequency range was recently proposed in [Kučera et al. (2017)]. It is based on extending the feedback by a delay free factor virtually modifying the mass of the absorber and thus its natural frequency. Besides, implementation aspects of the resulting, so-called extended delay resonators have been discussed in [Kučera et al. (2017)].

As it results from the frequency domain analysis performed in application sections of [Pilbauer et al. (2016); Kučera et al. (2017)], the entire vibration suppression takes place only if the resonant frequency adjusted by the resonator feedback is equivalent with the true excitation frequency. The performance of the resonator decays considerably even for very small differences in these frequencies. The main objective of this paper is to propose a robust alternative to the classical delayed resonator, which will widen the frequency stop-band of the resonator. Note that in this aspect, the paper is analogous to the recent work [Pilbauer et al. (2017)], where the optimization approach is applied directly on shaping the frequency characteristic of the DR. In this paper, we propose a more straightforward and easy to apply approach, which is based on double root assignment.

A fundamental feature of the resonator concept is that the parameters of the primary structure do not play a role in the resonator tuning. The purpose of the design is only to suppress the vibrations, while stability aspects of the overall set-up are left aside. As discussed in [Vyhídal et al. (2014); Pilbauer et al. (2016)], in fact, the resonator may decrease the stability margin or even destabilize the overall set-up. Thus, the second objective of the presented work is to handle this stability aspect by including a stabilizing controller, similarly as it was done in [Penuži et al. (2017)].

The paper is organized as follows. In Section 2, the concept of vibration absorber and functioning of the overall set-up including a stabilizing controller is outlined. Section 3 then provides the main result with introducing the robust delayed resonator concept and providing two analytic rules for its parametrizations. Next, design of the additional finite order controller is addressed together with its parametrization by minimizing the spectral abscissa. The validation of the proposed concept is done in section 4, where it is also compared with the classical delayed resonator concept. In section 5, brief conclusions are given.

2. PRELIMINARIES AND PROBLEM STATEMENT

The dynamic model of the absorber, depicted in Fig. 1, with the physical parameters \( m_a, c_a, k_a \) denoting the mass, the damping and the stiffness coefficients, can be described by

\[
m_a \ddot{x}_a (\vartheta) + c_a \dot{x}_a (\vartheta) + k_a x_a (\vartheta) = u_a (\vartheta)
\]

with \( x_a \) being the displacement of the absorber and \( u_a \) a (scaled) external input. The damping ratio and the natural frequency of the absorber are given by \( \zeta = \frac{c_a}{2 \sqrt{m_a k_a}} \) and \( \bar{\Omega} = \sqrt{\frac{k_a}{m_a}} \), respectively. The objective of the absorber is to suppress vibrations of the primary structure being excited by a harmonic force \( f(t) \) at the frequency \( \bar{\Omega} \).

The absorber equation can be turned to

\[
\ddot{x}_a (\vartheta) + 2 \zeta \bar{\Omega} \dot{x}_a (\vartheta) + \bar{\Omega}^2 x_a (\vartheta) = u_a (\vartheta).
\]

The absorber time delay can be expressed as

\[
\bar{x}_a (t) + 2 \zeta \bar{\Omega} \dot{x}_a (t) + \bar{\Omega}^2 x_a (t) = u_a (t),
\]

with the only parameter \( \zeta \), assuming \( u_a = \frac{1}{m_a \bar{\Omega}} u_a \). Note that scaled excitation frequency is given by \( \omega_f = \frac{\bar{\Omega}}{\bar{\Omega}} \).

Considering the scaled parameters with respect to \( m_a \) and \( \overline{\Omega} \), the overall set-up in Fig. 1 is given by

\[
\ddot{x}_a (t) + 2 \zeta \bar{\Omega} \dot{x}_a (t) + x_a (t) - 2 \bar{\Omega} \dot{x}_a (t) = u_a (t),
\]

\[
m_p \ddot{x}_p (t) + (2 \zeta \bar{\Omega}) \dot{x}_a (t) + (1 + k_p) x_p (t) - 2 \bar{\Omega} \dot{x}_a (t) - x_a (t) = -u_a (t) + u_p (t) + f(t),
\]

where \( x_p (t) \) is the position, \( u_p (t) \) is the control input for positioning the primary structure and \( m_p, c_p, k_p \) are the scaled mass, the damping and the stiffness parameters of the primary structure.

The absorber input \( u_a \) is to be used for enhancing the absorber parameters so that the vibrations at the frequency \( \omega_f \) are entirely suppressed. Due to problem generalization purpose, we consider the feedback in the form

\[
U_a (s) = P (s) X_a (s),
\]

where \( P(s) \) is the feedback transfer function.

Compared to the classical delayed resonator concept, where (6) is the only active feedback at the set-up, we assume that also the primary structure is equipped with an active feedback

\[
U_p (s) = K (s) X_p (s),
\]

with controller \( K(s) \), which can be used for platform positioning or just for improving the overall dynamical properties in vibration suppression.

For the overall set-up (4)-(7), the transfer function between the excitation force \( f \) and the position of the platform with feedback (6) is given by

\[
G_{x_p,f}(s) = \frac{X_p(s)}{F(s)} = \frac{R(s) - P(s)}{(R(s) - P(s))(V(s) - K(s)) + (P(s) - Q(s))Q(s)},
\]

with

\[
R(s) = s^2 + 2 \zeta s + 1
\]
being the characteristic function of the absorber (1), and
\[ V(s) = m_p s^2 + (2ζ + c_p) s + (1 + k_p), \]
\[ Q(s) = 2ζ s + 1. \]  
(10)

If the transfer function \( P(s) \) is parameterized so that the characteristic equation of the resonator composed of the absorber (1) and the feedback (6), given by
\[ M(s) = R(s) - P(s) = 0, \]  
(11) 
has a root couple \( s_{1,2} = ±jω_f \), composing a pole couple of the resonator, then
\[ G_{x_p,f}(jω_f) = 0, \]  
(12) 
indicating that no vibrations at the given frequency \( ω = ω_f \) are transferred in this \( f \) to \( x_p \) channel and the vibrations are ideally suppressed.

Note however, that including the resonator feedback affects the dynamical properties of the entire system, which is given by the roots of the characteristic equation \( (R(s) - P(s)) (V(s) - K(s)) + (P(s) - Q(s)) Q(s) = 0 \)  
(13) 
Assuming presence of the delay terms in \( P(s) \), the equation (13) has infinitely many roots. For the stability implications, all of them need to be located safely in the left half of the complex plane.

3. ROBUST RESONATOR FEEDBACK

The resonator feedback is considered in the following form
\[ u(θ) = -h_\bar{a}(θ) + \int_0^\θ \bar{p}(θ) x_a(θ - θ)dθ, \]  
(14) 
where \( \bar{p}(θ) \) is the weighting function (impulse response) corresponding to the resonator transfer function \( P(s) \), which is to be parameterized to \( P(s) \) in the dimensionless form. Motivated by Kučera et al. (2017), the term with \( h \) is included in (14) to modify virtually the mass of the absorber so that the natural frequency of the absorber \( Ω = \sqrt{\frac{k_a}{m_a + τ}} \) is identical with the excitation frequency \( ω_f \), i.e. \( ω_f = 1 \), which results to
\[ h(ω_f) = \frac{k_a}{ω_f^2} - m_a. \]  
(15) 
Note that if \( h \neq 0 \), for the design purposes, the absorber (1) as well as the primary structure needs to be scaled with respect to \( m+h \), i.e. with respect to the virtually modified mass.

Coupling the absorber (1) with the feedback (14), and scaling the time with respect to \( Ω = ω_f \), the delayed resonator is given in the form
\[ \ddot{x}_a(t) + 2ζ \dot{x}_a(t) + x_a(t) = \int_0^t \bar{p}(θ) x_a(t - θ)dθ, \]  
(16) 
where \( p(θ) \) is the scaled impulse response \( \bar{p}(θ) \).

The transfer function is considered in the form
\[ P(s) = \mathcal{L}\{p(t)\} = \frac{X_a(s)}{U_a(s)} = \sum_{i=0}^{2} a_i e^{-s_\tau_i} s, \]  
(17) 
and the term \( s^2 \) corresponds to the acceleration measurements \( \ddot{x}_a \). When considering the measured output as \( y_a = \ddot{x}_a \), the transfer function turns to the mentioned integral form
\[ \frac{Y_a(s)}{U_a(s)} = \sum_{i=0}^{2} a_i e^{-s_\tau_i} s. \]

As will be shown later, selection of this specific form of the transfer function (17) is advantageous due to easy parametrization of the resonator. However, on the other hand, it should be noted that from the practical point of view, the integral character of the transfer function can be risky when the measurements in the feedback path are noisy (Kučera et al. 2017). As has been shown in Rivaz and Rohling (2007), this problem can be solved by supplying the feedback path by a high-pass filter. Another option is to impose the finite impulse response of \( P(s) \), which can be achieved by satisfying the condition
\[ \sum_{i=0}^{2} a_i = 0. \]  
(18) 

3.1 DR parametrization by assigning a double root

The objective in the robust resonator design is widening the frequency stop-band of \( [G_{x_p,f}(jω)] \), where \( G_{x_p,f}(s) \) is given by (8), in the vicinity of \( ω_f = 1 \). This can be done by assigning a double root \( s_{1,2} = j \) to the characteristic equation of the delayed resonator (11), which appears in the numerator of (8). To perform the double root assignment, the root needs to be assigned not only to (11) given by
\[ M(s) = s^2 + \left( 2ζ - \sum_{i=0}^{2} a_i e^{-s_\tau_i} \right) s + 1 = 0, \]  
(19) 
but also to its first derivative
\[ \frac{dM(s)}{ds} = 2s + 2ζ - \sum_{i=0}^{2} a_i e^{-s_\tau_i} (1 - τ_i s) = 0. \]  
(20) 
This will naturally lead to widening the range where \( |M(jω)| \) is close to zero in the vicinity of \( ω_f = 1 \), i.e. to its locally parabolic shape. This property is transferred to \( G_{x_p,f}(jω) \) so that the poles of \( G_{x_p,f}(s) \) are not located close to the zeros at \( j \), which will be imposed by the stability requirement and will be targeted in the subsequent step.

Assigning the complex root \( s_{1,2} = j \) to (20) and splitting the equation to real and imaginary parts, we obtain
\[ \Re\{M(s)\}_{z=j} = -\sum_{i=0}^{2} a_i \sin τ_i = 0, \]  
(21) 
\[ \Im\{M(s)\}_{z=j} = 2ζ - \sum_{i=0}^{2} a_i \cos τ_i = 0. \]  
(22) 
Analogously, assigning the root to (20), we obtain the following two equations
\[ \Re\left( \frac{dM(s)}{ds}\right)_{z=j} = 2ζ - \sum_{i=0}^{2} a_i (\cos τ_i - τ_i \sin τ_i) = 0, \]  
(23) 
\[ \Im\left( \frac{dM(s)}{ds}\right)_{z=j} = 2 + \sum_{i=0}^{2} a_i (\sin τ_i + τ_i \cos τ_i) = 0. \]  
(24) 
Taking into account the equality (22), the equation (23) reduces to
Lemma 1: Consider the preselected delay values \( \tau_0 = 0, \tau_1 = \pi \) and \( \tau_2 = 2\pi \), then a universal parametrization of the resonator feedback (17) is given by

\[
\begin{align*}
\dot{x}(t) &= A x(t) + B_1 u_a(t) + B_2 u_p(t) + B_3 f(t), \\
y_a(t) &= C_a x(t), \\
y_p(t) &= C_f x(t),
\end{align*}
\]

where the output \( y_a \) is the measured acceleration \( y_a = \ddot{x}_a \), and matrices \( A, B_1, B_2, B_3 \) correspond to a linear model of a mechanical system (4)–(5) translated into state space representation with a state vector \( x = [\ddot{x}_a \ \dot{x}_a \ \dot{x}_p \ \dot{x}_v]^T \). Matrix \( C_a = [0 \ 1 \ 0 \ 0] \) defines the measured acceleration of the absorber, and \( C_f = [0 \ 0 \ 1 \ 0] \) determines the control system output \( y_p = \dot{x}_v \), i.e. the position of the primary structure.

Concerning the resonator feedback implementation, which in Laplace form is given as

\[
U_a(s) = \sum_{i=0}^{N} a_i e^{-s\tau_i} Y_a(s),
\]

it can be turned to

\[
\dot{u}_a(t) = \sum_{i=0}^{N} a_i y_a(t - \tau_i).
\]

It should be mentioned that this transformation results in additional dynamics, characterized by the introduction of two eigenvalues at zero [Pflabau et al. 2015].

In order to stabilize and optimize the spectrum of the resulting infinite dimensional system, following the methodology proposed in [Michiels 2011] we consider a fixed order dynamic feedback controller in the form

\[
K : \begin{cases}
\dot{x}_K(t) = A_K x_K(t) + B_K y_p(t), \\
u_p(t) = C_K x_K(t) + D_K y_p(t),
\end{cases}
\]

with \( k \) denoting order of the controller.
The system (30) and the feedback (32) create together System of Delay Differential Algebraic Equations (DDAEs)

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + B_1u_a(t) + B_2u_p(t) + B_3f(t), \\
\dot{u}_a(t) &= \sum_{i=0}^{2} a_i y_a(t - \tau_i), \\
y_a(t) &= C_a Ax(t) + C_1 B_1 u_a(t) + C_2 B_2 u_p(t) + CB_3 f(t), \\
y_p(t) &= C_3 r(t)
\end{align*}
\]

which can be rewritten in a compact form of DDAEs as

\[
E\dot{\xi}(t) = A_0 \xi(t) + \sum_{i=1}^{2} A_i \xi(t - \tau_i) + B_0 f(t),
\]

where \(\xi = [x \ u_a \ y_a \ y_p]^T\) is a new state vector.

For the resulting system, it is required that the spectrum of (35) lies safely in the left half of the complex plain. Defining the spectral abscissa of the system as

\[
\alpha(K) := \sup(\Re(s) : s \in \Sigma),
\]

with

\[
\Sigma := \left\{ s \in C : \det\left(s E - A_0 - \sum_{i=1}^{2} A_i e^{-s\tau_i}\right) = 0 \right\},
\]

the requirement on the stability is given as \(\alpha < 0\). Note that if \(\alpha\) is very close to the imaginary axis, next to the undesirably small stability margin, it may result in undesirably long transients due to the low damping or slow modes brought by the rightmost roots. Therefore, it will be required so that the spectral abscissa is located sufficiently far to the left of the imaginary axis. This motivates a controller design based on minimizing the spectral abscissa over the free parameters, the elements of matrices \((A_K, B_K, C_K, D_K)\). For this task we use the algorithm and software described in [Michiels and Niculescu (2014)]

It is important to note that the function \(K \mapsto \alpha(K)\) is in general not everywhere differentiable, even not everywhere Lipschitz continuous, even though the dependence of the system matrices of the closed-loop system on the controller parameters is smooth. To tackle these challenges, the core optimization tasks in the method of [Michiels (2011)] are done using the software HANSO (Hybrid Algorithm for Nonsmooth Optimization) [Overton (2009)].

4. CASE STUDY EXAMPLE

Let’s consider coupled absorber with the platform as shown in Fig. 1 with following parameters \(u_a = 0.223\) kg, \(c_a = 1.273\) kgs s\(^{-1}\), \(k_a = 350\) Nm\(^{-1}\) on the absorber’s side and \(m_p = 1.52\) kg, \(c_p = 10.11\) kgs s\(^{-1}\), \(k_p = 1960\) Nm\(^{-1}\) on the primary structure. These parameters were also considered in [Kučera et al. (2017)].

To describe this set-up by (4)-(5) we use normalized parameters. Absorber has natural frequency \(\Omega = 39.627\) s\(^{-1}\) and damping ratio \(\zeta = 0.072\). For simplicity, we assume that the excitation frequency \(\Omega_f = \Omega\), which implies \(h = 0\) in (14). Then, the primary structure has normalized weight \(m_p = 6.816\), damping \(c_p = 1.144\) and stiffness \(k_p = 5.6\). System is excited by the external harmonic force with amplitude \(\Delta f = 10\) N, which implies the normalized amplitude \(\Delta f = 0.029\).

For parametrization of the robust resonator, we apply Lemma 1. In Fig. 3 the spectrum of the double root delayed resonator (drDR) (16)-(17) as well as the spectrum of its coupling with the primary structure (34) is shown. As can be seen, the spectrum of the interconnected system is unstable, as two root couples are located to the right of the stability boundary. Thus, the controller (33) of the order \(k = 3\) is tuned by minimizing the spectral abscissa, as described above. After spectral abscissa optimization, defining the controller matrices

\[
A_K = \begin{bmatrix} -1.0897 & 0.0147 & 0.4740 \\ 1.4290 & 0.8037 & -3.0751 \\ 0.2966 & 0.6138 & -0.8099 \end{bmatrix}, \quad B_K = \begin{bmatrix} 1.3048 \\ 0.5531 \\ 0.9221 \end{bmatrix},
\]

\[
C_K = \begin{bmatrix} 6.6243 & -0.1113 & -0.3087 \end{bmatrix}, \quad D_K = \begin{bmatrix} -4.2371 \end{bmatrix},
\]

the drDR with the primary structure is stable (as shown in Fig. 3).

In the next step, the performance validation is done via simulations. Next to the nominal excitation frequency \(\omega = 1\) case, two cases with 3% mismatch between the design and true excitation frequency \((\omega = 0.97\) and \(\omega = 1.03\)) are considered to study the robustness. For comparison results by the classical delayed resonator with lumped delay feedback

\[
u_a(t) = g_{DR} v_a(t - \tau_{CDR}),
\]

are shown, with gain \(g_{DR} = 0.1441\) and time delay \(\tau_{CDR} = 1.5708\), parametrized by assigning a single root \(s_1 = j\) to the delayed resonator [Olgac (1994), Olgac (1995)]. From the results shown in Fig. 4 it can be seen that for nominal frequency \(\omega = 1\) both DRs performs well. However, for the cases with the 3% frequency mismatch, the drDR performs considerably better with approximately ten times smaller residual vibration amplitudes.
Finally, the enhancement of the robustness is studied via the frequency responses. Next to the magnitude $|G_{f,x_p}(j\omega)|$ of the transfer function (8) between the excitation force $f$ and the position of the primary $x_p$, also the normalized transmissibility function

$$T(\omega) = \frac{|R(j\omega) - P(j\omega)|}{Q(j\omega)}$$

in the vicinity of $\omega = 1$ is studied. Notice that this measure, which was introduced in [Pilbauer et al. 2017], is fully determined by the absorber parameters in $R(s)$ and $Q(s)$ defined by (9) and (10) and by the parameters of active feedback $P(s)$. The denominator $Q(s)$ normalizes the function (40) in order to achieve $\lim_{\omega \to 0} T(\omega) = 1$. Note that by coupling equation (4) with the resonator feedback, the transmissibility can be interpreted as $T(\omega) = 1/\left|\frac{G_{f,x_p}(j\omega)}{X_a(j\omega)}\right|$, where $G_{f,x_p}(s) = \frac{X_a(s)}{X_p(s)}$.

If the cDR (39) is applied, then $T(\omega)$ goes to zero for the nominal frequency $\omega = 1$ as shown in Fig. 5. For the frequencies in the vicinity of $\omega = 1$, this function tends to rapidly increase and therefore, if there is an uncertainty in the excitation frequency, the quality of the vibration suppression is likely to decay. As can be seen in Fig. 5, drDR transmissibility has parabolic shape in the vicinity of $\omega = 1$ with considerably smaller increase in its neighborhood, compared to the cDR. This broadening of the effective frequency stop-band is transformed to $[G_{f,x_p}(j\omega)]$ shown in the Fig. 6. Another positive aspect is considerably lower $H_\infty$ norm (maximum of the frequency response) compared to the case with cDR, caused by applying the platform position controller $K$, which is not coupled for the cDR case.

5. CONCLUSION

A new type of delayed resonator with a double root assigned at nominal frequency was presented as the main result. For the parametrization of the proposed resonator feedback with time delays, two fully analytic formulas have been proposed. The overall design procedure gains from the fact that it was proposed for the dimensionless models, normalized with respect to the absorber mass and natural frequency. An important aspect is also application of delay free feedback part in the resonator feedback (14), by which the resonant frequency of the absorber is unified with the excitation frequency. This step makes the successive parametrization much easier, as already revealed in [Kucera et al. 2017]. It was also demonstrated that application of the additional controller of the platform position is beneficial from both the stability and robustness points of view.

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REFERENCES


