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Abstract

This paper studies the ergodic sum rate performance of regularized zero-forcing (RZF) precoding in the downlink (DL) of a multi-user multiple-input single-output (MISO) system, where the channel between the base station (BS) and each user is modeled by the double scattering model. This non-Gaussian channel model is a function of both the antenna correlation and the structure of scattering in the propagation environment. This paper first makes the preliminary contribution of deriving the minimum-mean-square-error (MMSE) channel estimate for this model. The system model accounts for channel estimation errors and per-group channel correlation matrices for the users. The analysis assumes that the number of BS antennas, the number of single-antenna users and the number of scatterers in each group grow large while their ratio remains bounded. Under this setting, this paper derives deterministic equivalents of the signal-to-interference plus noise ratio (SINR) and the sum rate which are almost surely tight in the large system limit. The derived approximations are expressed in a closed-form for the special case of multi-keyhole channel. We show that the performance of a massive MIMO system does not scale linearly with the number of antennas under uncorrelated channel conditions and is actually limited by the number of scatterers in the propagation environment. Simulation results confirm the close match provided by the asymptotic analysis for moderate system dimensions.

I. Introduction

Massive multiple-input multiple-output (MIMO) is widely considered as a promising technology for next generation wireless communication systems due to its potential to improve both

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energy efficiency and spectral efficiency [1]–[3]. However, most works on this subject share the underlying assumption of rich scattering conditions and, thus, work with full rank Rayleigh or Rician fading channel matrices. Although the use of full rank channel matrices facilitates the derivation of closed-form capacity bounds and approximations, these models do not capture the characteristics of realistic propagation environments, where the presence of spatial correlation and poor scattering conditions significantly deteriorates the system performance [4]–[6].

Many works have already considered correlated Rayleigh fading channel models to study the impact of antenna correlation on the performance of massive MIMO systems [3], [7]–[9]. One popular correlation-based channel model utilized in several works is the Kronecker model [10], [11]- the rank of which is determined by the spatial correlation in the transmit (Tx) and receive (Rx) arrays. However, low rank channels have been observed in MIMO systems that have low antenna correlation at both ends of the transmission link [12]. Gesbert *et al* showed in [4] that the MIMO capacity is governed by both the spatial correlation at the communication ends and the structure of scattering in the propagation environment. Motivated by this, the authors devised a "double-scattering channel model" which utilized the geometry of the propagation environment to model spatial correlation, rank deficiency and limited scattering. A special case of the double-scattering model is the keyhole channel [5], [13], which exhibits null correlation between the entries of the channel matrix but only a single degree of freedom.

A. Related Literature

The main literature related to the theoretical analysis of the double-scattering channel model is represented by [14]–[21]. The authors in [14] studied its diversity order and showed that a MIMO system with t Tx and r Rx antennas and s scatterers achieves a diversity of order $trs/\max(t, r, s)$. The authors in [15] investigated the channel hardening property and showed that unlike Rayleigh fading channels, keyhole channels do not harden, resulting in a degradation of the spectral efficiency. The authors in [16] analyzed the ergodic MIMO capacity taking into account the presence of spatial fading correlation, double scattering, and keyhole effects in the propagation environment and showed that the use of multiple antennas in keyhole channels only offers diversity gains, but no spatial multiplexing gain. The MIMO multiple access channel (MAC) with double-scattering fading was studied in [19], where the authors obtained closed-form upper-bounds on the sum-capacity and proved that signals sent along the eigenvectors of the Tx correlation matrix maximize capacity.

A few papers have analyzed the double scattering model in the asymptotic regime targeting massive MIMO settings [12], [21]. The authors in [12] studied this model without Tx and Rx correlation using tools from free probability theory and derived implicit expressions of the asymptotic mutual information and signal-to-interference-plus-noise ratio (SINR) of the minimum-mean-square-error (MMSE) detector. The authors in [21] studied a MIMO multiple access system with double-scattering channels and derived almost surely tight deterministic approximations of the mutual information and the SINR of the MMSE detector in the asymptotic regime. However, none of these papers have considered practical massive MIMO systems with linear signal processing schemes and channel estimation. The authors are only aware of [20] which describes the behavior of the double scattering model with channel estimation and linear signal detection using analytical and numerical examples instead of theoretical analysis.

B. Main Contributions

The focus of this work is on the downlink (DL) of a single-cell multi-user multiple-input single-output (MISO) system. We consider double scattering fading between the BS and the users in its most general form. The users are further divided into G groups, such that the users in the same group experience similar propagation conditions and are therefore characterized by common correlation matrices. We use a realistic system model where the BS obtains channel state information (CSI) from uplink pilot transmissions and applies MMSE estimation technique. Under the assumption that the number of BS antennas, scatterers and users grow large, we derive asymptotically tight deterministic approximations of the SINR and the ergodic sum rate with regularized zero-forcing (RZF) precoding. These approximations can be easily computed and are shown to be accurate for realistic system dimensions through simulations. The deterministic equivalents are expressed in a closed-form for the multi-keyhole channel under perfect CSI assumption and some important insights into the impact of different system parameters on the sum rate performance are drawn. Our results show that massive MIMO techniques are only useful in rich scattering environments.

C. Outline and Notation

The rest of the paper is organized as follows. Section II presents the transmission model and introduces the double scattering channel model along with its MMSE estimate. In section III, the asymptotically tight deterministic equivalents of the SINR and user rates with RZF precoding are provided. A few case studies for the Rayleigh product channel are also considered. Simulation results are provided in Section IV and section V concludes the paper. All technical proofs are presented in the appendices.

The following notation is used throughout this work. Boldface lower-case and upper-case characters denote vectors and matrices respectively. The superscript $(\cdot)^H$ represents the conjugate transpose, $\mathbb{E}[\cdot]$ represents the expectation and $\log(\cdot)$ represents the logarithm. The operator tr $(\mathbf{X})$ denotes the trace of a matrix $\mathbf{X}$. The spectral norm of a matrix $\mathbf{X}$ is denoted by $||\mathbf{X}||$. The $N \times N$ identity matrix is denoted by $\mathbf{I}_N$ and the $N \times N$ diagonal matrix of entries $\{x_n\}$ is denoted by $\mathbf{X} = \text{diag}(x_1, x_2, \dots, x_N)$. A random vector $\mathbf{x} \sim \mathcal{CN}(\mathbf{m}, \mathbf{\Phi})$ is complex Gaussian distributed with mean vector $\mathbf{m}$ and covariance matrix $\mathbf{\Phi}$. The notation $\xrightarrow{a.s.}$ denotes almost sure convergence.

II. System Model

Consider a single-cell multi-user MISO system where a BS equipped with N antennas serves K single-antenna users. We assume transmissions over flat-fading double scattering channels under a time-division duplexing (TDD) protocol. The K users are divided into G groups of K_g, $g = 1, \dots, G$, users such that the users in the same group experience similar propagation conditions. In this section, we outline the transmission model, discuss the double scattering channel model and introduce its MMSE estimate.

A. Transmission Model

Under narrow-band transmission, the signal $y_{k,g}$ received by user k in group g is given as,

$$y_{k,g} = \mathbf{h}_{k,g}^H \mathbf{x} + n_{k,g}, \quad k = 1, \dots, K_g, \;\; g = 1, \dots, G, \tag{1}$$

where $\mathbf{h}_{k,g}^H \in \mathbb{C}^{1 \times N}$ is the channel vector from the BS to user k in group g, $\mathbf{x} \in \mathbb{C}^{N \times 1}$ is the Tx signal vector and $n_{k,g} \sim \mathcal{CN}(0, \sigma^2)$ is the receiver noise.

The Tx signal vector $\mathbf{x}$ is given as,

$$\mathbf{x} = \sum_{g=1}^{G} \sum_{k=1}^{K_g} \sqrt{p_{k,g}} \mathbf{g}_{k,g} s_{k,g}, \tag{2}$$

where $\mathbf{g}_{k,g} \in \mathbb{C}^{N \times 1}$ is the precoding vector for user k in group g, and $p_{k,g} \geq 0$ and $s_{k,g} \sim \mathcal{CN}(0, 1)$ are the signal power and the data symbol for user k in group g respectively. The precoding vectors satisfy the average total power constraint as,

$$\mathbb{E}[||\mathbf{x}||^2] = \mathbb{E}[\text{tr }(\mathbf{P}\mathbf{G}^H\mathbf{G})] \leq \bar{P}, \tag{3}$$

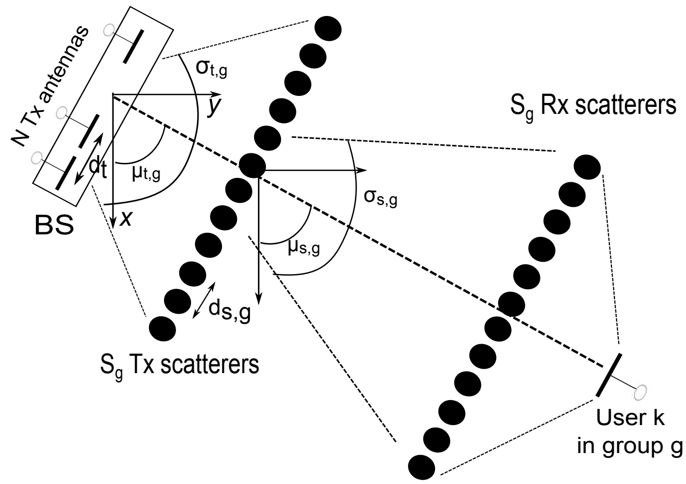

Fig. 1: Geometric model of the double scattering channel between the BS and the user k in group g.

where $\bar{P} > 0$ is the average total Tx power, $\mathbf{P} = \text{diag}(p_{1,1}, p_{2,1}, \ldots, p_{K_1,1}, p_{1,2}, \ldots, p_{K_G-1,G}, p_{K_G,G}) \in \mathbb{R}^{K\times K}$ and $\mathbf{G} = [\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_G] \in \mathbb{C}^{N\times K}$ is the precoding matrix, where $\mathbf{G}_g = [\mathbf{g}_{1,g}, \ldots, \mathbf{g}_{K_g,g}] \in \mathbb{C}^{N\times K_g}$.

B. Double-Scattering Channel Model

A main contribution of this paper is to apply the double scattering channel model proposed in [4] to a multi-user MISO system. This model provides non-Gaussian channels between the BS and the users with ranks that are determined by both the spatial correlation between the antennas at the BS and the structure of scattering in the propagation environment. The expression of the channel vector $\mathbf{h}_{k,g}$ described by the double scattering model in [4] is given as,

$$\mathbf{h}_{k,g} = \sqrt{S_g}\left(\frac{1}{\sqrt{S_g}}\mathbf{R}_{BS_g}^{1/2}\mathbf{W}_g\bar{\mathbf{S}}_g^{1/2}\right)\tilde{\mathbf{w}}_{k,g}, \tag{4}$$

where S_g is the number of scatterers at the Tx and the Rx sides in group g, $\mathbf{R}_{BS_g} \in \mathbb{C}^{N\times N}$ is the correlation matrix between the BS antennas and the S_g Tx scatterers in group g, $\bar{\mathbf{S}}_g \in \mathbb{C}^{S_g\times S_g}$ is the correlation matrix between the Tx and Rx scatterers in group g, $\mathbf{W}_g \sim$ i.i.d. $\mathcal{CN}(0,1) \in \mathbb{C}^{N\times S_g}$ describes the small-scale fading between the BS and the scattering cluster at the Tx side in group g and $\tilde{\mathbf{w}}_{k,g} \sim$ i.i.d. $\mathcal{CN}(0, \frac{1}{S_g}) \in \mathbb{C}^{S_g\times 1}$ describes the small-scale fading between the user k in group g and the scattering cluster at the Rx side. Since the distributions of $\mathbf{W}_g$ and $\tilde{\mathbf{w}}_{k,g}$ are unitarily invariant, we can assume $\bar{\mathbf{S}}_g$ to be diagonal, i.e. $\bar{\mathbf{S}}_g = \text{diag}(\bar{s}_{1,g}, \bar{s}_{2,g}, \ldots, \bar{s}_{S_g,g})$ without any loss of generality for the statistics of the received signal.

Schematic of the double scatting channel from the BS to user k in group g has been illustrated in Fig. 1, where $\sigma_{t,g}$ and $\sigma_{s,g}$ represent the angular spread of the radiated signal from the BS array and the Tx scatterers respectively in group g, $\mu_{t,g}$ and $\mu_{s,g}$ determine the mean angle of departure (AoD) of the radiated signal from the BS array and the Tx scatterers respectively in group g, where $\mu_{t,g} = \mu_{s,g}$, and d_t and $d_{s,g}$ determine the spacing between the adjacent antennas at the BS and between the adjacent scatterers in group g respectively.

C. Channel Estimation

During a dedicated uplink training phase, the users transmit mutually orthogonal pilot sequences that allow the BS to compute the MMSE estimates $\hat{\mathbf{h}}_{k,g}$ of the channel vectors $\mathbf{h}_{k,g}$. After correlating the received training signal with the pilot sequence of user k in group g, the BS estimates the channel vector $\mathbf{h}_{k,g}$ based on the received observation, $\mathbf{y}_{k,g}^{tr} \in \mathbb{C}^{N\times 1}$, given as,

$$\mathbf{y}_{k,g}^{tr} = \mathbf{h}_{k,g} + \frac{1}{\sqrt{\rho_{tr}}}\mathbf{n}_{k,g}^{tr}, \tag{5}$$

where $\mathbf{n}_{k,g}^{tr} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ and $\rho_{tr} > 0$ is the effective training SNR, assumed to be given here.

Lemma 1: The MMSE estimate $\hat{\mathbf{h}}_{k,g}$ of the double scattering channel vector $\mathbf{h}_{k,g}$ in (4) is given as,

$$\hat{\mathbf{h}}_{k,g} = d_g \mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{y}_{k,g}^{tr}, \tag{6}$$

where $d_g = \frac{1}{S_g}(\text{tr}\ \bar{\mathbf{S}}_g)$ and $\mathbf{Q}_g = \left(d_g\mathbf{R}_{BS_g} + \frac{1}{\rho_{tr}}\mathbf{I}_N\right)^{-1}$.

The proof of *Lemma 1* is postponed to Appendix A.

We stress that the estimate in Lemma 1 has been derived for a non-Gaussian channel. In fact, MMSE estimator is very general and is not specific for Gaussian channels, as commonly used in most of the massive MIMO literature. Under the orthogonality property of the MMSE estimate, we can decompose the channel vector $\mathbf{h}_{k,g}$ as $\mathbf{h}_{k,g} = \hat{\mathbf{h}}_{k,g} + \tilde{\mathbf{h}}_{k,g}$, where $\tilde{\mathbf{h}}_{k,g}$ is the uncorrelated channel estimation error. Note that $\hat{\mathbf{h}}_{k,g}$ and $\tilde{\mathbf{h}}_{k,g}$ although uncorrelated are not generally independent due to the non-Gaussian nature of the channel. However, we will show that $\mathbb{E}[\tilde{\mathbf{h}}_{k,g}^H\mathbf{A}\hat{\mathbf{h}}_{k,g}] = 0$, where $\mathbf{A}$ is a $N \times N$ matrix with a bounded spectral norm, implying the following lemma.

Lemma 2: Let $\mathbf{A}$ be a $N \times N$ matrix independent of $\mathbf{h}_{k,g}$ and $\hat{\mathbf{h}}_{k,g}$, with a bounded spectral norm; that is, there exists a $C_A < \infty$ such that $||\mathbf{A}|| \leq C_A$. Then,

$$\frac{1}{N}\tilde{\mathbf{h}}_{k,g}^H\mathbf{A}\hat{\mathbf{h}}_{k,g} \xrightarrow[N\to\infty]{a.s.} 0. \tag{7}$$

The proof is postponed to Appendix A.

D. Achievable Rates

Since the users do not have any channel estimate, we provide an ergodic achievable rate based on the techniques utilized in [3], [22]. To this end, we decompose $y_{k,g}$ as,

$$\begin{aligned} y_{k,g} &= \sqrt{p_{k,g}}\mathbb{E}[\mathbf{h}_{k,g}^H \mathbf{g}_{k,g}]s_{k,g} + \sqrt{p_{k,g}}(\mathbf{h}_{k,g}^H \mathbf{g}_{k,g} - \mathbb{E}[\mathbf{h}_{k,g}^H \mathbf{g}_{k,g}])s_{k,g} \\ &+ \sum_{(k',g')\neq(k,g)} \sqrt{p_{k',g'}}\mathbf{h}_{k,g}^H \mathbf{g}_{k',g'} s_{k',g'} + n_{k,g}, \end{aligned} \tag{8}$$

and assume that the average effective channels $\mathbb{E}[\mathbf{h}_{k,g}^H \mathbf{g}_{k,g}]$ can be perfectly learned at the users. Then the SINR $\gamma_{k,g}$ of user k in group g is defined as,

$$\gamma_{k,g} = \frac{p_{k,g}|\mathbb{E}[\mathbf{h}_{k,g}^H \mathbf{g}_{k,g}]|^2}{p_{k,g}\text{var}[\mathbf{h}_{k,g}^H \mathbf{g}_{k,g}] + \sum_{(k',g')\neq(k,g)} p_{k',g'}\mathbb{E}[|\mathbf{h}_{k,g}^H \mathbf{g}_{k',g'}|^2] + \sigma^2}. \tag{9}$$

The ergodic achievable downlink rate $R_{k,g}$ of user k in group g is given as,

$$R_{k,g} = \log(1 + \gamma_{k,g}), \tag{10}$$

while the ergodic achievable sum rate is given as,

$$R_{sum} = \sum_{g=1}^{G}\sum_{k=1}^{K_g} R_{k,g}. \tag{11}$$

This paper considers RZF precoding, which is a state-of-the-art heuristic precoding scheme with a simple closed-form expression given as [3], [7],

$$\mathbf{G} = \zeta(\hat{\mathbf{H}}^H\hat{\mathbf{H}} + K\alpha\mathbf{I}_N)^{-1}\hat{\mathbf{H}}^H, \tag{12}$$

where $\hat{\mathbf{H}}$ is the compound channel defined as $[\hat{\mathbf{H}}_1^H, \hat{\mathbf{H}}_2^H, \dots, \hat{\mathbf{H}}_G^H]^H \in \mathbb{C}^{K\times N}$, where $\hat{\mathbf{H}}_g = [\hat{\mathbf{h}}_{1,g}, \hat{\mathbf{h}}_{2,g}, \dots, \hat{\mathbf{h}}_{K_g,g}]^H \in \mathbb{C}^{K_g\times N}$, α is the regularization parameter and ζ is a normalization factor to ensure that the power constraint in (3) is satisfied and is obtained as,

$$\zeta^2 = \frac{\bar{P}}{\mathbb{E}[\text{tr}\mathbf{P}\hat{\mathbf{H}}(\hat{\mathbf{H}}^H\hat{\mathbf{H}} + K\alpha\mathbf{I}_N)^{-2}\hat{\mathbf{H}}^H]} = \frac{\bar{P}}{\Theta}, \tag{13}$$

where $\Theta = \mathbb{E}[\text{tr}\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{V}}^2\hat{\mathbf{H}}^H]$, where $\hat{\mathbf{V}} = (\hat{\mathbf{H}}^H\hat{\mathbf{H}} + K\alpha\mathbf{I}_N)^{-1}$. The SINR in (9) is now defined as,

$$\gamma_{k,g\text{RZF}} = \frac{p_{k,g}|\mathbb{E}[\mathbf{h}_{k,g}^H\hat{\mathbf{V}}\hat{\mathbf{h}}_{k,g}]|^2}{\mathbb{E}[\mathbf{h}_{k,g}^H\hat{\mathbf{V}}\hat{\mathbf{H}}_{[k,g]}^H\mathbf{P}_{[k,g]}\hat{\mathbf{H}}_{[k,g]}\hat{\mathbf{V}}\mathbf{h}_{k,g}] + p_{k,g}\text{var}\ [\mathbf{h}_{k,g}^H\hat{\mathbf{V}}\hat{\mathbf{h}}_{k,g}] + \frac{\Theta}{\rho}}, \tag{14}$$

where $\rho = \frac{\bar{P}}{\sigma^2}$, $\mathbf{h}_{k,g}$ is given by (4), $\hat{\mathbf{h}}_{k,g}$ is given by (6) and $\hat{\mathbf{H}}_{[k,g]} = [\hat{\mathbf{H}}_1^H, \dots, \hat{\mathbf{H}}_{g-1}^H, \hat{\mathbf{h}}_{1,g}, \dots, \hat{\mathbf{h}}_{k-1,g}, \hat{\mathbf{h}}_{k+1,g}, \dots, \hat{\mathbf{h}}_{K_g,g}, \hat{\mathbf{H}}_{g+1}^H, \dots, \hat{\mathbf{H}}_G^H]^H \in \mathbb{C}^{K-1\times N}$.

III. Main Results

As the ergodic rate with RZF precoding is very difficult to derive for finite system dimensions, we consider the large system limit, where N, K, S grow infinitely large while keeping finite ratios. Under this setting and the double scattering channel model in (4) and its estimate in (6), this section presents the deterministic equivalents of the SINR and sum-rate with RZF precoding.

A. Preliminaries

The analysis in this work makes the following three assumptions.

A-1. For all g, N, S_g, K_g and K tend to infinity such that,

$$0 < \lim\inf \frac{S_g}{N} \leq \lim\sup \frac{S_g}{N} < \infty, \tag{15}$$

$$0 < \lim\inf \frac{K_g}{N} \leq \lim\sup \frac{K_g}{N} < \infty, \tag{16}$$

$$0 < \lim\inf \frac{K_g}{K} \leq \lim\sup \frac{K_g}{K} < \infty. \tag{17}$$

In the sequel, the notation $N \to \infty$ denotes Assumption **A-1**.

A-2. For all g, $\lim\sup_N ||\mathbf{R}_{BS_g}|| < \infty$ and $\lim\sup_{S_g} ||\bar{\mathbf{S}}_g|| < \infty$.

A-3. The random matrix $\frac{1}{K}\hat{\mathbf{H}}^H\hat{\mathbf{H}}$ has uniformly bounded spectral norm with probability 1, i.e. $\lim\sup_N ||\frac{1}{K}\hat{\mathbf{H}}^H\hat{\mathbf{H}}|| < \infty$, with probability 1.

The derivations in this work rely extensively on the Fubini Theorem. The complete statement of this theorem can be found in [23] and its application to the study of a double-scattering MIMO multiple access channel can be found in [21], [24]. In plain words this theorem implies that if a deterministic equivalent g_n exists for a function f_n of a random series $(\mathbf{H}''_n)_{n\geq 1}$ and a deterministic series $(\mathbf{H}'_n)_{n\geq 1}$ of matrices and if it can be proved that this deterministic equivalent holds true for almost every such $(\mathbf{H}'_n)_{n\geq 1}$ generated by a space Ω, then the latter is also a deterministic equivalent of the random series $(\mathbf{H}'_n, \mathbf{H}''_n)_{n\geq 1}$.

Using this mathematical idea, the derivation of the deterministic equivalents in this work interprets the double-scattering channel model in (4) as,

$$\mathbf{h}_{k,g} = \sqrt{S_g}\mathbf{Z}_g\tilde{\mathbf{w}}_{k,g}, \tag{18}$$

where,

$$\mathbf{Z}_g = \frac{1}{\sqrt{S_g}}\mathbf{R}_{BS_g}^{1/2}\mathbf{W}_g\bar{\mathbf{S}}_g^{1/2}. \tag{19}$$

Note that (18) is essentially a Rayleigh correlated channel model [7] but with a random Tx correlation matrix $\mathbf{Z}_g$, which itself is a Kronecker model. The idea is to first assume $\mathbf{Z}_g$ to be deterministic and use RMT results for the Kronecker model to derive the deterministic equivalent of the SINR. Under this setting, the estimate of the double scattering model in (6) can be interpreted as,

$$\hat{\mathbf{h}}_{k,g} = \mathbf{\Phi}_g^{1/2}\bar{\mathbf{q}}_{k,g}, \tag{20}$$

where $\bar{\mathbf{q}}_{k,g}$ has entries distributed as $\sim \mathcal{CN}(0,1)$ and $\mathbf{\Phi}_g$ is the covariance matrix of the channel estimate given as,

$$\mathbf{\Phi}_g = d_g^2 \mathbf{R}_{BS_g}\mathbf{Q}_g\left(\mathbf{Z}_g\mathbf{Z}_g^H + \frac{1}{\rho_{tr}}\mathbf{I}_N\right)\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H. \tag{21}$$

Once we obtain the deterministic equivalent of the SINR in terms of certain fixed point equations that depend on the matrices $\mathbf{Z}_g$ using RMT results for the Kronecker model from [25], we extend the analysis by allowing $\mathbf{Z}_g$ to be random based on the Fubini theorem. In this second step, we derive the deterministic equivalents of the fixed point equations under the actual random $\mathbf{Z}_g$s.

Our first theorem will introduce a set of $3G$ implicit equations which uniquely determines the quantities $(m_g, \bar{m}_g, \delta_g)$, $1 \leq g \leq G$. These will be needed later to provide the deterministic equivalents of $\gamma_{k,g}$ and corresponding rates.

Theorem 1: Consider the resolvent matrix $\hat{\mathbf{C}}^{-1}(\alpha) = \left(\frac{1}{K}\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \alpha\mathbf{I}_N\right)^{-1}$, where the columns of $\hat{\mathbf{H}}^H$ are distributed according to (6). Then the following system of $3G$ implicit equations in $m_g, \bar{m}_g$ and δ_g, $1 \leq g \leq G$,

$$\begin{aligned} m_g(\alpha) &= \frac{1}{K}d_g^2\left(\sum_{j=1}^{S_g}\frac{\bar{s}_{g,j}\delta_g(\bar{\mathbf{D}}_g,\alpha)}{1+\frac{K_g}{K}d_g^2\bar{s}_{g,j}\bar{m}_g(\alpha)\delta_g(\bar{\mathbf{D}}_g,\alpha)} + \frac{S_g}{\rho_{tr}}\delta_g(\tilde{\mathbf{D}}_g,\alpha)\right), \\ \bar{m}_g(\alpha) &= \frac{1}{1+m_g(\alpha)}, \\ \delta_g(\mathbf{R}_g,\alpha) &= \frac{1}{S_g}\mathrm{tr}\ \mathbf{R}_g\bar{\mathbf{T}}(\alpha), \end{aligned} \tag{22}$$

where,

$$\bar{\mathbf{T}}(\alpha) = \left(\sum_{i=1}^{G}\frac{\bar{\mathbf{D}}_i}{S_i}\left(\sum_{j=1}^{S_i}\frac{\frac{K_i}{K}d_i^2\bar{s}_{i,j}\bar{m}_i(\alpha)}{1+\frac{K_i}{K}d_i^2\bar{s}_{i,j}\bar{m}_i(\alpha)\delta_i(\bar{\mathbf{D}}_i,\alpha)}\right) + \frac{K_i}{K}d_i^2\frac{\bar{m}_i(\alpha)}{\rho_{tr}}\tilde{\mathbf{D}}_i + \alpha\mathbf{I}_N\right)^{-1}, \tag{23}$$

has a unique solution satisfying $m_g, \bar{m}_g,\ \delta_g > 0$ for all g and $\alpha > 0$, where $\mathbf{R}_g$ is an arbitrary matrix with a uniformly bounded spectral norm, $\bar{\mathbf{D}}_g = \mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{R}_{BS_g}\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H$ and $\tilde{\mathbf{D}}_g =$

$\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H$. Let $\mathbf{U}$ be any deterministic matrix with a uniformly bounded spectral norm. Under assumptions **A-1**, **A-2**, **A-3** and for $\alpha > 0$, we have

$$\frac{1}{K}\text{tr}(\mathbf{U}\hat{\mathbf{C}}^{-1}(\alpha)) - \frac{1}{K}\text{tr}(\mathbf{U}\bar{\mathbf{T}}(\alpha)) \xrightarrow[N\to\infty]{a.s.} 0. \tag{24}$$

Proof: The proof of Theorem 1 is postponed to Appendix B.

B. Asymptotic Analysis

This section provides the major contributions of this work as it copes with the channel model in (4) and its estimate in (6) and provides the large system SINR analysis of RZF precoding.

To begin with, we first present the deterministic equivalent of the quantity $\frac{1}{K}\text{tr}\ \mathbf{\Phi}_g\hat{\mathbf{C}}^{-1}(\alpha)$, which will repeatedly appear in our analysis. The result is presented in the following lemma.

Lemma 3: Under the setting of assumptions **A-1**, **A-2** and **A-3** and for $\alpha > 0$,

$$\frac{1}{K}\text{tr}\ \mathbf{\Phi}_g\hat{\mathbf{C}}^{-1}(\alpha) - m_g(\alpha) \xrightarrow[N\to\infty]{a.s.} 0, \tag{25}$$

where $m_g(\alpha)$ is obtained as the unique solution of (22).

Proof: Utilizing the result in (74) for $\mathbf{U} = \mathbf{\Phi}_g$ under the assumption of deterministic $\mathbf{Z}_g$s yields,

$$\frac{1}{K}\text{tr}\ \mathbf{\Phi}_g\hat{\mathbf{C}}^{-1}(\alpha) - e_g(\alpha) \xrightarrow[N\to\infty]{a.s.} 0. \tag{26}$$

The result can be extended to random $\mathbf{Z}_g$s using Fubini Theorem and the deterministic equivalent of e_g is obtained as m_g as shown in Appendix B.

The next lemma introduces the deterministic equivalent of an important quantity that forms the mathematical basis of the subsequent large system analysis of the SINR in (14).

Lemma 4: Define $\boldsymbol{\chi}(\alpha) = [\chi_1(\alpha), \chi_2(\alpha), \ldots, \chi_G(\alpha)]^T$, where $\chi_g(\alpha) = \frac{1}{K^2}\text{tr}\ \mathbf{\Phi}_g\hat{\mathbf{C}}^{-1}(\alpha)\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}(\alpha)$, where $\mathbf{\Phi}_g$ is given by (21). Under the setting of assumptions **A-1**, **A-2** and **A-3** and for $\alpha > 0$,

$$\boldsymbol{\chi}(\alpha) - \boldsymbol{\chi}^o(\alpha) \xrightarrow[N\to\infty]{a.s.} 0, \tag{27}$$

where,

$$\boldsymbol{\chi}^o(\alpha) = (\mathbf{I}_N - \bar{\mathbf{J}}(\alpha))^{-1}\bar{\mathbf{v}}(\alpha), \tag{28}$$

where,

$$[\bar{\mathbf{J}}(\alpha)]_{g,i} = d_g^2\frac{K_i}{K}\frac{1}{(1+m_i(\alpha))^2}\left(\bar{\beta}_{g,i}(\mathbf{R}_{BS_g}\mathbf{Q}_g,\alpha) + \frac{1}{\rho_{tr}}\tilde{\beta}_i(\mathbf{R}_{BS_g}\mathbf{Q}_g,\alpha)\right), \tag{29}$$

$$\bar{\mathbf{v}}_g(\alpha) = d_g^2\frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{p_{l,i}}{(1+m_i(\alpha))^2}\left(\bar{\beta}_{g,i}(\mathbf{R}_{BS_g}\mathbf{Q}_g,\alpha) + \frac{1}{\rho_{tr}}\tilde{\beta}_i(\mathbf{R}_{BS_g}\mathbf{Q}_g,\alpha)\right), \tag{30}$$

where,

$$\bar{\beta}_{g,i}(\mathbf{A},\alpha) = \bar{\beta}^1_{g,i}(\mathbf{A},\alpha) + \bar{\beta}^2_{g,i}(\mathbf{A},\alpha), \tag{31}$$

where,

$$\bar{\beta}^1_{g,i}(\mathbf{A},\alpha) = \begin{cases} \frac{d_i^2}{S_i K} \sum_{j=1}^{S_i} \frac{\bar{s}_{i,j}}{(1+\frac{K_i}{K} d_i^2 \bar{m}_i(\alpha)\bar{s}_{i,j}\delta_i(\bar{\mathbf{D}}_i,\alpha))^2} \sum_{n=1}^{S_g} \frac{\bar{s}_{g,n} M'_g(\mathbf{A}\mathbf{R}_{BS_g}\mathbf{A}^H, \bar{\mathbf{D}}_i, \alpha)}{(1+\frac{K_g}{K} d_g^2 \bar{m}_g(\alpha)\bar{s}_{g,n}\delta_g(\bar{\mathbf{D}}_g,\alpha))^2}, & \text{if } g \neq i, \\ \frac{d_i^2}{K} \sum_{j=1}^{S_i} \frac{\bar{s}_{i,j}}{(1+\frac{K_i}{K} d_i^2 \bar{m}_i(\alpha)\bar{s}_{i,j}\delta_i(\bar{\mathbf{D}}_i,\alpha))^2} \left(\sum_{\substack{n=1 \\ n\neq j}}^{S_i} \frac{\frac{1}{S_i}\bar{s}_{i,n} M'_i(\mathbf{A}\mathbf{R}_{BS_i}\mathbf{A}^H, \bar{\mathbf{D}}_i, \alpha)}{(1+\frac{K_i}{K} d_i^2 \bar{m}_i(\alpha)\bar{s}_{i,n}\delta_i(\bar{\mathbf{D}}_i,\alpha))^2} + \bar{s}_{i,j}\delta_i(\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{R}_{BS_i}\mathbf{A}^H,\alpha)\delta_i(\mathbf{A}\mathbf{R}_{BS_i}\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H,\alpha) \right), & \text{if } g = i, \end{cases}$$

$$\bar{\beta}^2_{g,i}(\mathbf{A},\alpha) = \frac{d_i^2}{K\rho_{tr}} \sum_{j=1}^{S_g} \frac{\bar{s}_{g,j} M'_g(\mathbf{A}\mathbf{R}_{BS_g}\mathbf{A}^H, \tilde{\mathbf{D}}_i, \alpha)}{(1+\frac{K_g}{K} d_g^2 \bar{m}_g(\alpha)\bar{s}_{g,j}\delta_g(\bar{\mathbf{D}}_g,\alpha))^2}, \tag{32}$$

and,

$$\tilde{\beta}_i(\mathbf{A},\alpha) = \frac{d_i^2}{K} \left(\sum_{j=1}^{S_i} \frac{\bar{s}_{i,j} M'_i(\bar{\mathbf{D}}_i, \mathbf{A}\mathbf{A}^H, \alpha)}{(1+\frac{K_i}{K} d_i^2 \bar{m}_i(\alpha)\bar{s}_{i,j}\delta_i(\bar{\mathbf{D}}_i,\alpha))^2} + \frac{S_i}{\rho_{tr}} M'_i(\tilde{\mathbf{D}}_i, \mathbf{A}\mathbf{A}^H, \alpha) \right), \tag{33}$$

where,

$$M'_g(\mathbf{R}_g, \mathbf{L}) = \frac{1}{S_g} \text{tr } \mathbf{R}_g \left[\bar{\mathbf{T}} \left(\sum_{z=1}^{G} \frac{\bar{\mathbf{D}}_z \bar{m}_z^2}{S_z} \left(\frac{K_z}{K}\right)^2 d_z^4 \text{tr } (\bar{\mathbf{S}}_z \mathbf{W}_z^2 \bar{\mathbf{S}}_z) M'_z(\bar{\mathbf{D}}_z, \mathbf{L}) + \mathbf{L} \right) \bar{\mathbf{T}} \right], \tag{34}$$

where $\mathbf{R}_g$ and $\mathbf{L}$ are arbitrary matrices with uniformly bounded spectral norm,

$$\mathbf{W}_i = \left(\mathbf{I}_{S_i} + \frac{K_i}{K} d_i^2 \bar{m}_i \delta_i(\bar{\mathbf{D}}_i,\alpha)\bar{\mathbf{S}}_i \right)^{-1}, \tag{35}$$

and $\mathbf{M}'(\bar{\mathbf{D}}, \mathbf{L}) = [M'_1(\bar{\mathbf{D}}_1, \mathbf{L}), M'_2(\bar{\mathbf{D}}_2, \mathbf{L}), \ldots, M'_G(\bar{\mathbf{D}}_G, \mathbf{L})]^T$, which can be expressed as a system of linear equations as follows,

$$\mathbf{M}'(\bar{\mathbf{D}}, \mathbf{L}) = (\mathbf{I}_N - \mathbf{J}(\bar{\mathbf{D}}))^{-1} \mathbf{v}(\bar{\mathbf{D}}, \mathbf{L}), \tag{36}$$

$$[\mathbf{J}(\bar{\mathbf{D}})]_{g,i} = \frac{1}{S_g} \text{tr } (\bar{\mathbf{D}}_g \bar{\mathbf{T}} \bar{\mathbf{D}}_i \bar{\mathbf{T}}) \left(\frac{\bar{m}_i^2}{S_i} \left(\frac{K_i}{K}\right)^2 d_i^4 \text{tr } (\bar{\mathbf{S}}_i \mathbf{W}_i^2 \mathbf{S}_i) \right), \tag{37}$$

$$[\mathbf{v}(\bar{\mathbf{D}}, \mathbf{L})]_g = \frac{1}{S_g} \text{tr } (\bar{\mathbf{D}}_g \bar{\mathbf{T}} \mathbf{L} \bar{\mathbf{T}}), \tag{38}$$

for $g, i = 1, \ldots, G$.

Proof: The proof of *Lemma 4* is provided in Appendix C.

Based on the results in *Theorem 1*, *Lemma 3* and *Lemma 4*, the deterministic equivalent of the SINR in (14) can be derived and is presented in the next theorem.

Theorem 2: Under the setting of assumptions **A-1**, **A-2** and **A-3** and for $\alpha > 0$, the downlink SINR of user k in group g defined in (14) converges almost surely as,

$$\gamma_{k,g\text{RZF}} - \gamma^o_{k,g\text{RZF}} \xrightarrow[N\to\infty]{a.s.} 0, \tag{39}$$

where,

$$\gamma_{k,g\text{RZF}}^{o} = \frac{p_{k,g}(m_g(\alpha))^2}{\Upsilon_{k,g}^{o}(1+m_g(\alpha))^2 + \frac{\xi^{o}(\mathbf{I}_N,\alpha)(1+m_g(\alpha))^2}{\rho}}, \tag{40}$$

where,

$$\Upsilon_{k,g}^{o} = \kappa_g^{o}(\mathbf{I}_N,\alpha) - \chi_g^{o}(\alpha) + \left(\frac{1}{1+m_g(\alpha)}\right)^2 \chi_g^{o}(\alpha), \tag{41}$$

where,

$$\kappa_g^{o}(\mathbf{I}_N,\alpha) = \frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{1}{(1+m_i(\alpha))^2}\bar{\beta}_{g,i}(\mathbf{I}_N,\alpha)(p_{l,i}+\chi_i^{o}(\alpha)), \tag{42}$$

$$\xi^{o}(\mathbf{I}_N,\alpha) = \frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{1}{(1+m_i(\alpha))^2}\tilde{\beta}_{i}(\mathbf{I}_N,\alpha)(p_{l,i}+\chi_i^{o}(\alpha)). \tag{43}$$

Proof: The proof of *Theorem 2* is given in Appendix E.

Corollary 1: Assume that **A-1**, **A-2** and **A-3** hold true and $\alpha > 0$. Then the individual downlink rates $R_{k,g}$ of users converge as,

$$R_{k,g} - R_{k,g}^{o} \xrightarrow[N\to\infty]{a.s.} 0, \tag{44}$$

where,

$$R_{k,g}^{o} = \log(1+\gamma_{k,g\text{RZF}}^{o}), \tag{45}$$

where $\gamma_{k,g\text{RZF}}^{o}$ is given by (40).

Proof: The proof follows from the application of the continuous mapping theorem [23] to the logarithm function and the almost sure convergence of $\gamma_{k,g\text{RZF}}$ in (39).

An approximation of the average system sum rate can be obtained by replacing $R_{k,g}$ in (11) with its asymptotic approximation as,

$$R_{sum}^{o} = \sum_{g=1}^{G}\sum_{k=1}^{K_g}\log(1+\gamma_{k,g\text{RZF}}^{o}). \tag{46}$$

The asymptotic expressions provided in Theorem 2 and Corollary 1 will be shown to be tight by the means of simulations in Section IV, even for finite system dimensions. This means they can be used for evaluating the performance of practical systems without relying on time-consuming Monte-Carlo simulations. Despite being useful for this purpose, the expressions are quite involved and do not provide direct insights into the interplay between different system parameters. We therefore focus on the perfect CSI case and obtain some simplifications under different operating conditions.

C. Rayleigh Product Channel

A special case of the double-scattering channel is the Rayleigh product channel which does not exhibit any form of correlation between the Tx and Rx antennas or the scatterers [18]. This model is popularly known as the multi-keyhole channel. We study this model for perfect CSI and find that Theorem 2 can be given in a closed-form as shown in the next corollary.

Corollary 2: For $G=1$, let $S_1=S$, $K_1=K$, and assume $\bar{\mathbf{S}}_1=\mathbf{I}_S$ and $\mathbf{R}_{BS_1}=\mathbf{I}_N$. Then under perfect CSI, $\gamma^o_{k\text{RZF}}$ can be given in a closed-form as,

$$\gamma^o_{k\text{RZF}}=\frac{\frac{p_k}{P/K}(1-\bar{m})^2(1-\bar{m}^2a)}{\bar{m}^4a+\frac{\bar{m}^2b}{\rho}}, \tag{47}$$

where,

$$a=\frac{S(N-K+K\bar{m})^2}{K(S\alpha-K\bar{m}+N\bar{m}+K\bar{m}^2)^2}+\frac{S^3\alpha^2(S-1)(N-K+K\bar{m})}{K(S\alpha-K\bar{m}+N\bar{m}+K\bar{m}^2)^3(S\alpha-2K\bar{m}+N\bar{m}+S\bar{m}+2K\bar{m}^2)}, \tag{48}$$

$$b=\frac{S^2(N-K+K\bar{m})}{K(S\alpha-K\bar{m}+N\bar{m}+K\bar{m}^2)(S\alpha-2K\bar{m}+N\bar{m}+S\bar{m}+2K\bar{m}^2)}, \tag{49}$$

and $\bar{m}$ is given as the unique root to,

$$\bar{m}^3-\left(2-\frac{S}{K}-\frac{N}{K}\right)\bar{m}^2+\left(1-\frac{S}{K}-\frac{N}{K}+\frac{SN}{K^2}+\frac{S\alpha}{K}\right)\bar{m}-\frac{S\alpha}{K}=0, \tag{50}$$

such that $\bar{m}\in\left(1-\frac{S}{K},1\right)$.

Proof: The proof is postponed to Appendix F.

Corollary 3: Under the setting of Corollary 2, let $\frac{S}{N},\frac{S}{K}\to\infty$. Then $\gamma^o_{k\text{RZF}}$ defined in Theorem 2 can be given in a closed-form as,

$$\gamma^o_{k\text{RZF}}=\frac{\frac{p_k}{P/K}(\frac{1}{\bar{m}}-1)(1+\frac{\alpha}{\bar{m}^2})}{1+\frac{1}{\rho\bar{m}^2}}, \tag{51}$$

where,

$$\bar{m}=\frac{1-\frac{N}{K}-\alpha+\sqrt{(\alpha+\frac{N}{K}-1)^2+4\alpha}}{2}. \tag{52}$$

Note that as $\frac{S}{N},\frac{S}{K}\to\infty$, $\mathbf{h}$ behaves as a Rayleigh fading channel, whose SINR is given as (51). A similar result has also been obtained in Corollary 2 of [7], where the authors derive the deterministic equivalent of the SINR under RZF precoding, with the channel between each user and the BS modeled as Rayleigh correlated.

Corollary 4: Under the setting of Corollary 2, let $\frac{N}{S}\to\infty$ and $\frac{N}{K}\to\infty$ with $K>S$. Then $\gamma^o_{k\text{RZF}}$ defined in Theorem 2 can be given in a closed-form as,

$$\gamma^o_{k\text{RZF}}=\frac{\frac{p_k}{P/K}S}{K-S}. \tag{53}$$

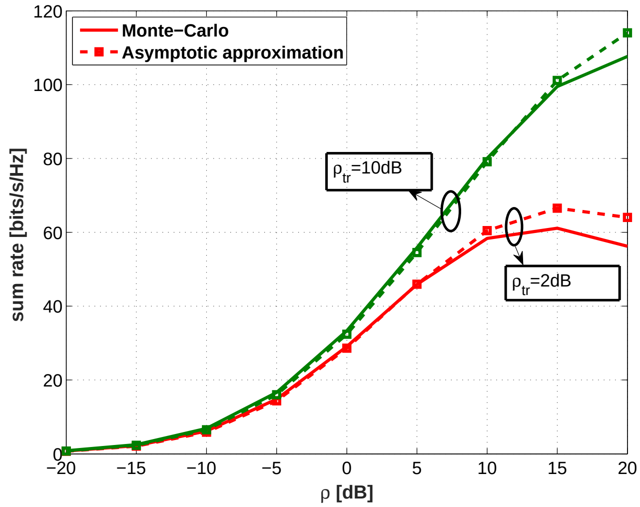

Fig. 2: Sum rate versus SNR with $\alpha = 1/\rho$.

It is commonly believed that the capacity of massive MIMO systems scales linearly as $min(N, K)$ under i.i.d. channel gains and perfect CSI. However, *Corollary 4* refutes this as it implies that the performance of a massive MIMO system is actually limited by the number of scatterers in the environment. For a low number of scatterers, deploying more antennas will yield no benefit. Massive MIMO techniques are therefore only useful in very rich scattering environments. This result will be verified using simulations as well.

IV. Simulation Results

Under the double scattering model, the correlation matrices $\mathbf{R}_{BS_g}$ and $\bar{\mathbf{S}}_g$ are given as $\mathbf{R}_{BS_g} = \mathbf{G}(\mu_{t,g}, \sigma_{t,g}, d_t, S_g)$ and $\bar{\mathbf{S}}_g = \mathbf{G}(\mu_{s,g}, \sigma_{s,g}, d_{s,g}, S_g)$, where $\mathbf{G}(\mu, \phi, d, n)$ is defined as [20], [21],

$$[\mathbf{G}(\mu, \sigma, d, n)]_{k,l} = \frac{1}{n} \sum_{j=\frac{1-n}{2}}^{\frac{n-1}{2}} \exp\left(- i2\pi d(k-l)\cos\left(\frac{\pi}{2} + \frac{j\sigma}{n-1} + \mu\right)\right). \tag{54}$$

All parameters have already been defined in Fig. 1.

The parameters are set as $G = 4$, $K = 80$, $N = 80$, S_g=$\{80, 90, 84, 94\}$, $\mu_{t,g} = \mu_{s,g}$=$\{-\pi/3, -\pi/9, \pi/9, \pi/3\}$, $\sigma_{t,g}$=$\{\pi/5, \pi/6, \pi/5, \pi/7\}$ and $\sigma_{s,g}$=$\{\pi/6, \pi/6, \pi/6, \pi/6\}$. Also, $d_t = 0.5$ and $d_{s,g} = 2$ for all g. We assume an equal number of users in each group, i.e. $K_g = K/G$, with uniform power allocation, $\mathbf{P} = \mathbf{I}_K$. Fig. 2 compares the downlink system sum-rate $R_{sum} = \sum_{g=1}^{G}\sum_{k=1}^{K_g} \log(1 + \gamma_{k,g_{\rm RZF}})$ obtained using 2000 Monte-Carlo realizations of the SINR in (14) to the deterministic approximation provided in (46), where $\gamma^o_{k,g\rm RZF}$ is given by (40). It can be

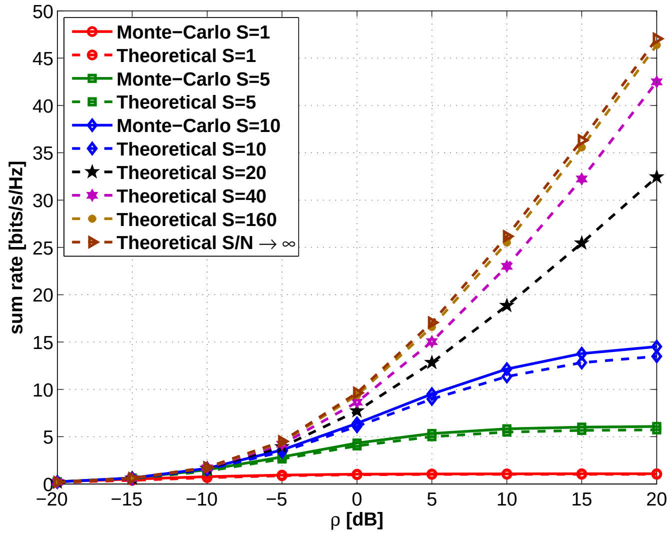

Fig. 3: Sum rate versus SNR for a multi-keyhole channel with $\alpha = 1/\rho$.

seen that the asymptotic result derived in this paper yields a good approximation for moderate system dimensions. The sum-rate is decreasing at high SNR values for $\rho_{tr} = 2\text{dB}$, because the regularization parameter α does not account for ρ_{tr} and thus the matrix $\hat{\mathbf{H}}^H\hat{\mathbf{H}} + K\alpha\mathbf{I}_N$ in the RZF precoder becomes ill-conditioned as the quality of the estimate deteriorates. Furthermore, note that the mismatch starts to increase for high SNR values due to the slower convergence of $\gamma_{k,g\text{RZF}}$ to its deterministic approximation as well documented in RMT literature [7], [26]. Therefore, higher system dimensions are needed for a better approximation at high SNR values.

Fig. 3 studies the effect of the number of scatterers on the system sum rate for a single group with multi-keyhole channel, i.e. $G = 1$, $\mathbf{R}_{BS} = \mathbf{I}_N$, $\bar{\mathbf{S}} = \mathbf{I}_S$, with $N = 20$, $K = 20$ and perfect CSI. The downlink sum-rate in (46) plotted using the closed-form expression of $\gamma^o_{k\text{RZF}}$ in *Corollary 2* is close to the Monte-Carlo result even for very low number of scatterers. The spatial multiplexing gains are seen to increase linearly with S. However, for $S > N$, the gains start to decrease since the degrees of freedom are limited by the number of antennas at the BS. The limiting sum rate as $S/N, S/K \to \infty$ is also plotted using the SINR in (51). As the number of scatterers increases, the performance approaches to that of a Rayleigh fading channel.

Finally, we study the downlink performance of the double scattering channel as the number of BS antennas increases for a single group with $S = 20$, $K = 40$ and perfect CSI. The results are shown in Fig. 4 for both the correlated and uncorrelated cases. It can be seen that as the number of antennas increases, the performance of both cases saturates to a limiting sum-rate as

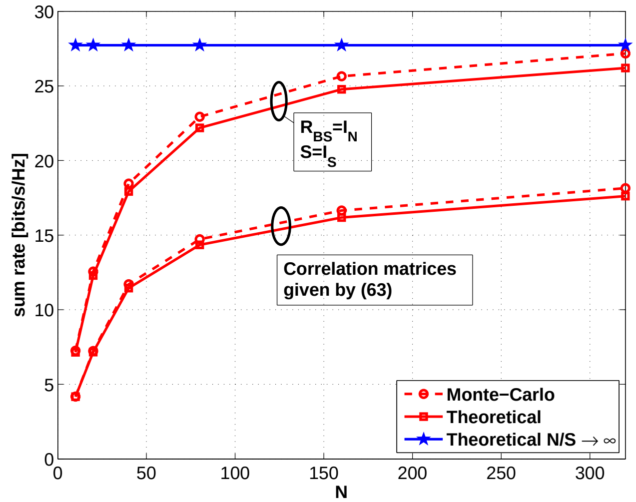

Fig. 4: Sum rate versus N with $\rho = 5\text{dB}$ and $\alpha = 1/\rho$.

$N/S, N/K \to \infty$. For the case where $\mathbf{R}_{BS} = \mathbf{I}_N$ and $\mathbf{S} = \mathbf{I}_S$, this limiting sum rate is given by substituting (53) in (46), which is also plotted in blue on the figure. This result is in contrast to the common belief that the performance of a massive MIMO system scales linearly with the number of BS antennas if the antennas are uncorrelated and perfect CSI is available. In fact, the performance is limited by the number of scatterers in the environment and the benefits of massive MIMO can not be realized in poor scattering conditions.

V. Conclusion

In this paper, we studied a multi-user MISO system with double-scattering fading channels that are more realistic than the commonly used Gaussian channel models. The system performance of this non-Gaussian channel is extremely difficult to study for finite system dimensions so we focused on the massive MISO setting. We first derived the MMSE estimate for this channel model. Then under a user-grouping setting with per-group channel correlation matrices and the assumption that the number of BS antennas, scatterers and users grow infinitely large, we derived almost surely tight deterministic approximations of the SINR and the sum rate with RZF precoding. Simulation results showed a close match between the asymptotic and the Monte-Carlo simulated sum rate for relatively small system dimensions and provided insights into the performance of multi-keyhole channels. The results highlighted that massive MIMO performance is actually limited by the number of scatterers in the environment.

APPENDIX A
MMSE ESTIMATE

A. Proof of Lemma 1

The MMSE estimate $\hat{\mathbf{h}}_{k,g}$ of $\mathbf{h}_{k,g}$ is computed as $\hat{\mathbf{h}}_{k,g} = \mathbf{F}_g \mathbf{y}_{k,g}^{tr}$ where $\mathbf{F}_g$ is obtained as the solution of $\min_{\mathbf{F}_g} \mathbb{E}[||\mathbf{h}_{k,g} - \mathbf{F}_g \mathbf{y}_{k,g}^{tr}||^2]$ resulting in,

$$\mathbf{F}_g = \mathbf{C}_{\mathbf{h}_{k,g}\mathbf{y}_{k,g}^{tr}} \mathbf{C}^{-1}_{\mathbf{y}_{k,g}^{tr}\mathbf{y}_{k,g}^{tr}}, \tag{55}$$

where,

$$\mathbf{C}_{\mathbf{h}_{k,g}\mathbf{y}_{k,g}^{tr}} = \mathbb{E}[\mathbf{h}_{k,g}\mathbf{y}_{k,g}^{tr^H}] = \mathbb{E}[\mathbf{h}_{k,g}\mathbf{h}_{k,g}^H], \tag{56}$$

$$= \frac{1}{S_g}\mathbf{R}_{BS_g}^{1/2}\mathbb{E}[\mathbf{W}_g\bar{\mathbf{S}}_g\mathbf{W}_g^H]\mathbf{R}_{BS_g}^{1/2^H}, \tag{57}$$

$$= \frac{1}{S_g}\mathbf{R}_{BS_g}^{1/2}(\mathrm{tr}\ \bar{\mathbf{S}}_g)\mathbf{I}_N\mathbf{R}_{BS_g}^{1/2^H} = \frac{1}{S_g}(\mathrm{tr}\ \bar{\mathbf{S}}_g)\mathbf{R}_{BS_g}, \tag{58}$$

and,

$$\mathbf{C}_{\mathbf{y}_{k,g}^{tr}\mathbf{y}_{k,g}^{tr}} = \mathbb{E}[\mathbf{y}_{k,g}^{tr}\mathbf{y}_{k,g}^{tr^H}] = \mathbb{E}\left[\mathbf{h}_{k,g}\mathbf{h}_{k,g}^H + \frac{1}{\rho_{tr}}\mathbf{n}_{k,g}^{tr}\mathbf{n}_{k,g}^{tr^H}\right], \tag{59}$$

$$= \frac{1}{S_g}(\mathrm{tr}\ \bar{\mathbf{S}}_g)\mathbf{R}_{BS_g} + \frac{1}{\rho_{tr}}\mathbf{I}_N. \tag{60}$$

Therefore,

$$\hat{\mathbf{h}}_{k,g} = \frac{1}{S_g}(\mathrm{tr}\ \bar{\mathbf{S}}_g)\mathbf{R}_{BS_g}\mathbf{Q}_g\left(\mathbf{h}_{k,g} + \frac{1}{\sqrt{\rho_{tr}}}\mathbf{n}_{k,g}^{tr}\right). \tag{61}$$

B. Proof of Lemma 2

We deal with $\mathbb{E}[\mathbf{h}_{k,g}^H\mathbf{A}\hat{\mathbf{h}}_{k,g}]$ and $\mathbb{E}[\hat{\mathbf{h}}_{k,g}^H\mathbf{A}\hat{\mathbf{h}}_{k,g}]$ separately.

$$\mathbb{E}[\mathbf{h}_{k,g}^H\mathbf{A}\hat{\mathbf{h}}_{k,g}] = d_g\mathbb{E}[\mathbf{h}_{k,g}^H\mathbf{A}\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{h}_{k,g}] + \frac{d_g}{\sqrt{\rho_{tr}}}\mathbb{E}[\mathbf{h}_{k,g}^H\mathbf{A}\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{n}_{k,g}^{tr}], \tag{62}$$

$$= T_1 + T_2. \tag{63}$$

Due to the independence of $\mathbf{n}_{k,g}^{tr}$ and $\mathbf{h}_{k,g}$ we can show that $T_2 = 0$. Now,

$$T_1 = \frac{d_g}{S_g}\mathbb{E}[\tilde{\mathbf{w}}_{k,g}^H\bar{\mathbf{S}}_g^{1/2}\mathbb{E}[\mathbf{W}_g^H\mathbf{R}_{BS_g}^{1/2}\mathbf{A}\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{R}_{BS_g}^{1/2}\mathbf{W}_g|\tilde{\mathbf{w}}_{k,g}]\bar{\mathbf{S}}_g^{1/2}\tilde{\mathbf{w}}_{k,g}], \tag{64}$$

$$= \frac{d_g}{S_g}\mathbb{E}[\tilde{\mathbf{w}}_{k,g}^H\bar{\mathbf{S}}_g\tilde{\mathbf{w}}_{k,g}]\mathrm{tr}(\mathbf{R}_{BS_g}\mathbf{A}\mathbf{R}_{BS_g}\mathbf{Q}_g), \tag{65}$$

$$= \frac{d_g}{S_g}\mathrm{tr}(\bar{\mathbf{S}}_g)\mathrm{tr}(\mathbf{R}_{BS_g}\mathbf{A}\mathbf{R}_{BS_g}\mathbf{Q}_g). \tag{66}$$

Similarly we deal with $\mathbb{E}[\hat{\mathbf{h}}_{k,g}^H \mathbf{A}\hat{\mathbf{h}}_{k,g}]$ as,

$$\mathbb{E}[\hat{\mathbf{h}}_{k,g}^H \mathbf{A}\hat{\mathbf{h}}_{k,g}] = d_g^2 \mathbb{E}[\mathbf{h}_{k,g}^H \mathbf{Q}_g^H \mathbf{R}_{BS_g}^H \mathbf{A}\mathbf{R}_{BS_g}\mathbf{Q}_g \mathbf{h}_{k,g}] + \frac{d_g^2}{\rho_{tr}} \mathbb{E}[\mathbf{n}_{k,g}^{tr^H} \mathbf{Q}_g^H \mathbf{R}_{BS_g}^H \mathbf{A}\mathbf{R}_{BS_g}\mathbf{Q}_g \mathbf{n}_{k,g}^{tr}], \tag{67}$$

$$= \frac{d_g^2}{S_g} \text{tr } (\bar{\mathbf{S}}_g)\text{tr}(\mathbf{R}_{BS_g}\mathbf{Q}_g^H \mathbf{R}_{BS_g}^H \mathbf{A}\mathbf{R}_{BS_g}\mathbf{Q}_g) + \frac{d_g^2}{\rho_{tr}} \text{tr}(\mathbf{Q}_g^H \mathbf{R}_{BS_g}^H \mathbf{A}\mathbf{R}_{BS_g}\mathbf{Q}_g), \tag{68}$$

$$= \frac{d_g}{S_g} \text{tr}(\bar{\mathbf{S}}_g)\text{tr}\left(\left(d_g \mathbf{R}_{BS_g} + \frac{1}{\rho_{tr}}\mathbf{I}_N\right) \mathbf{Q}_g^H \mathbf{R}_{BS_g}^H \mathbf{A}\mathbf{R}_{BS_g}\mathbf{Q}_g\right), \tag{69}$$

$$= \frac{d_g}{S_g} \text{tr}(\bar{\mathbf{S}}_g)\text{tr}(\mathbf{R}_{BS_g}\mathbf{A}\mathbf{R}_{BS_g}\mathbf{Q}_g). \tag{70}$$

As a consequence $\mathbb{E}[(\mathbf{h}_{k,g} - \hat{\mathbf{h}}_{k,g})^H \mathbf{A}\hat{\mathbf{h}}_{k,g}] = 0$. Using standard tools from RMT, we can show that the $\mathbb{E}\left[\left|\frac{1}{N}(\mathbf{h}_{k,g} - \hat{\mathbf{h}}_{k,g})^H \mathbf{A}\hat{\mathbf{h}}_{k,g}\right|^4\right]$ is bounded and converges to zero, implying,

$$\frac{1}{N}(\mathbf{h}_{k,g} - \hat{\mathbf{h}}_{k,g})^H \mathbf{A}\hat{\mathbf{h}}_{k,g} \xrightarrow[N\to\infty]{a.s.} 0. \tag{71}$$

Appendix B
Proof of Theorem 1

As a starting point we assume $\mathbf{Z}_g$ to be deterministic such that lim $\text{sup}_N ||\mathbf{Z}_g \mathbf{Z}_g^H|| < \infty$, which allows us to use the expression of $\hat{\mathbf{h}}_{k,g}$ in (20) to have,

$$\hat{\mathbf{C}} = \frac{1}{K}\sum_{g=1}^{G}\sum_{k=1}^{K_g} \mathbf{\Phi}_g^{1/2} \bar{\mathbf{q}}_{k,g} \bar{\mathbf{q}}_{k,g}^H \mathbf{\Phi}_g^{1/2^H} + \alpha \mathbf{I}_N, \tag{72}$$

$$= \sum_{g=1}^{G} \tilde{\mathbf{\Phi}}_g^{1/2} \bar{\mathbf{Q}}_g \mathbf{I}_{K_g} \bar{\mathbf{Q}}_g^H \tilde{\mathbf{\Phi}}_g^{1/2^H} + \alpha \mathbf{I}_N, \tag{73}$$

where $\bar{\mathbf{Q}}_g \sim \mathcal{CN}(0, \frac{1}{K_g}) \in \mathbb{C}^{N\times K_g}$ and $\tilde{\mathbf{\Phi}}_g = \frac{K_g}{K}\mathbf{\Phi}_g$, where $\mathbf{\Phi}_g$ is given by (21).

We now rely on the observation that $\tilde{\mathbf{\Phi}}_g^{1/2} \bar{\mathbf{Q}}_g \mathbf{I}_{K_g} \bar{\mathbf{Q}}_g^H \tilde{\mathbf{\Phi}}_g^{1/2^H}$ can be considered as a double scattering channel model with deterministic correlation matrices $\tilde{\mathbf{\Phi}}_g^{1/2}$. Under this setting, the deterministic equivalent of $\frac{1}{K}\text{tr } \mathbf{U}\hat{\mathbf{C}}^{-1}$ can be obtained using Theorem 3 in Appendix G as,

$$\frac{1}{K}\text{tr } \mathbf{U}\hat{\mathbf{C}}^{-1} - \frac{1}{K}\text{tr } \mathbf{U}\left(\sum_{i=1}^{G} \bar{e}_i \tilde{\mathbf{\Phi}}_i + \alpha \mathbf{I}_N\right)^{-1} \xrightarrow[N\to\infty]{a.s.} 0, \tag{74}$$

where $(e_g, \bar{e}_g)$ are given as a unique solution to the following set of implicit equations,

$$\bar{e}_g = \frac{1}{1+e_g}, \tag{75}$$

$$e_g = \frac{1}{K_g}\text{tr } \tilde{\mathbf{\Phi}}_g \left(\sum_{i=1}^{G} \bar{e}_i \tilde{\mathbf{\Phi}}_i + \alpha \mathbf{I}_N\right)^{-1}. \tag{76}$$

Now for the actual double-scattering channel model, $\mathbf{Z}_g$s are random and modeled as $\frac{1}{\sqrt{S_g}}\mathbf{R}_{BS_g}^{1/2}\mathbf{W}_g\bar{\mathbf{S}}_g^{1/2}$. Using the Fubini Theorem [24] we can extend the result in (74) to random $\mathbf{Z}_g$s. This does not affect the expression of $\bar{e}_g$, but e_g are now random quantities and we need to find a deterministic equivalent, denoted as m_g for them, such that, $e_g - m_g \xrightarrow[N\to\infty]{a.s.} 0$.

To do this, we first define quantities $e_{g,i,j}$ and $\bar{e}_{g,i,j}$ for $i = 1, \dots, G$, $j = 1, \dots, S_g$, which are given as unique solution to following set of fixed point equations.

$$\bar{e}_{g,i,j} = \frac{1}{1 + e_{g,i,j}}, \tag{77}$$

$$e_{g,i,j} = \frac{1}{K_g}\text{tr } \tilde{\mathbf{\Phi}}_{g,i,j}(\sum_{l=1}^{G} \bar{e}_{l,i,j}\tilde{\mathbf{\Phi}}_{l,i,j} + \alpha\mathbf{I}_N)^{-1}. \tag{78}$$

where,

$$\tilde{\mathbf{\Phi}}_{g,i,j} = \frac{K_g}{K} d_g^2 \mathbf{R}_{BS_g}\mathbf{Q}_g\left(\mathbf{Z}_{g,i,j}\mathbf{Z}_{g,i,j}^H + \frac{1}{\rho_{tr}}\mathbf{I}_N\right)\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H, \tag{79}$$

where,

$$\mathbf{Z}_{g,i,j} = \begin{cases} \mathbf{Z}_g, & \text{if } i \neq g, \\ [\mathbf{z}_{g,1}, \dots, \mathbf{z}_{g,j-1}, \mathbf{z}_{g,j+1}, \dots, \mathbf{z}_{g,S_g}], & \text{if } i = g. \end{cases} \tag{80}$$

It can be shown following the techniques used in Appendix E of [24] that for all g, i, j,

$$e_g - e_{g,i,j} \xrightarrow[K\to\infty]{a.s.} 0, \tag{81}$$

$$\bar{e}_g - \bar{e}_{g,i,j} \xrightarrow[K\to\infty]{a.s.} 0. \tag{82}$$

Now using the expression of $\mathbf{\Phi}_g$ from (21) in e_g we have,

$$e_g = \frac{d_g^2}{K}\text{tr } \mathbf{R}_{BS_g}\mathbf{Q}_g\left(\mathbf{Z}_g\mathbf{Z}_g^H + \frac{1}{\rho_{tr}}\mathbf{I}_N\right)\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H\left(\sum_{i=1}^{G}\bar{e}_i\tilde{\mathbf{\Phi}}_i + \alpha\mathbf{I}_N\right)^{-1}, \tag{83}$$

$$= \frac{d_g^2}{K}\sum_{j=1}^{S_g}\mathbf{z}_{g,j}^H\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H\left(\sum_{i=1}^{G}\bar{e}_i\tilde{\mathbf{\Phi}}_i + \alpha\mathbf{I}_N\right)^{-1}\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{z}_{g,j} + \frac{d_g^2}{K}\frac{1}{\rho_{tr}}\text{tr } \tilde{\mathbf{D}}_g\left(\sum_{i=1}^{G}\bar{e}_i\tilde{\mathbf{\Phi}}_i + \alpha\mathbf{I}_N\right)^{-1},$$

$$= T_1 + T_2, \tag{84}$$

where $\tilde{\mathbf{D}}_g = \mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H$. First we treat T_1 using Lemma 11 in Appendix G to remove the dependence of $\left(\sum_{i=1}^{G}\bar{e}_i\tilde{\mathbf{\Phi}}_i + \alpha\mathbf{I}_N\right)^{-1}$ on the vector $\mathbf{z}_{g,j}$ as,

$$T_1 = \frac{d_g^2}{K}\sum_{j=1}^{S_g}\frac{\mathbf{z}_{g,j}^H\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H\left(\sum_{i=1}^{G}\frac{K_i}{K}d_i^2\bar{e}_{i,g,j}\mathbf{R}_{BS_i}\mathbf{Q}_i\left(\mathbf{Z}_i\mathbf{Z}_i^H + \frac{1}{\rho_{tr}}\mathbf{I}_N\right)\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H - \frac{K_g}{K}d_g^2\bar{e}_{g,g,j}\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{z}_{g,j}\mathbf{z}_{g,j}^H\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H + \alpha\mathbf{I}_N\right)^{-1}\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{z}_{g,j}}{1 + \bar{e}_{g,g,j}\frac{K_g}{K}d_g^2\mathbf{z}_{g,j}^H\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H\left(\sum_{i=1}^{G}\frac{K_i}{K}d_i^2\bar{e}_{i,g,j}\mathbf{R}_{BS_i}\mathbf{Q}_i\left(\mathbf{Z}_i\mathbf{Z}_i^H + \frac{1}{\rho_{tr}}\mathbf{I}_N\right)\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H - \frac{K_g}{K}d_g^2\bar{e}_{g,g,j}\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{z}_{g,j}\mathbf{z}_{g,j}^H\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H + \alpha\mathbf{I}_N\right)^{-1}\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{z}_{g,j}}.$$

Then from Lemma 7 and Lemma 9 in Appendix G we have,

$$T_1 - \frac{d_g^2}{K}\sum_{j=1}^{S_g}\frac{\frac{\bar{\mathbf{s}}_{g,j}}{S_g}\mathrm{tr}\ \bar{\mathbf{D}}_g\left(\sum_{i=1}^{G}\bar{e}_i\tilde{\mathbf{\Phi}}_i+\alpha\mathbf{I}_N\right)^{-1}}{1+\bar{e}_g\frac{K_g}{K}d_g^2\frac{\bar{\mathbf{s}}_{g,j}}{S_g}\mathrm{tr}\ \bar{\mathbf{D}}_g\left(\sum_{i=1}^{G}\bar{e}_i\tilde{\mathbf{\Phi}}_i+\alpha\mathbf{I}_N\right)^{-1}} \xrightarrow[N\to\infty]{a.s.} 0,. \tag{85}$$

where $\bar{\mathbf{D}}_g = \mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{R}_{BS_g}\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H$.

Notice that $\tilde{\mathbf{\Phi}}_i$ is still a function of $\mathbf{Z}_i$ which is random. To remove the dependence of T_1 on $\mathbf{Z}_i$, we need the deterministic equivalent of $\frac{1}{S_g}\mathrm{tr}\ \bar{\mathbf{D}}_g\left(\sum_{i=1}^{G}\bar{e}_i\tilde{\mathbf{\Phi}}_i+\alpha\mathbf{I}_N\right)^{-1}$. Using the expression of $\tilde{\mathbf{\Phi}}_i$ in (21) and Theorem 3 in Appendix G we have the following convergence,

$$\frac{1}{S_g}\mathrm{tr}\ \mathbf{R}_g\left(\sum_{i=1}^{G}\bar{e}_i\tilde{\mathbf{\Phi}}_i+\alpha\mathbf{I}_N\right)^{-1} - \frac{1}{S_g}\mathrm{tr}\ \mathbf{R}_g(\sum_{i=1}^{G}\bar{f}_i\bar{\mathbf{D}}_i+\frac{K_i}{K}d_i^2\frac{\bar{e}_i}{\rho_{tr}}\tilde{\mathbf{D}}_i+\alpha\mathbf{I}_N)^{-1} \xrightarrow[N\to\infty]{a.s.} 0, \tag{86}$$

where $\mathbf{R}_g$ is any deterministic matrix that satisfies lim $\sup_N||\mathbf{R}_g|| < \infty$ and,

$$\bar{f}_g = \frac{1}{S_g}\sum_{j=1}^{S_g}\frac{\frac{K_g}{K}d_g^2\bar{e}_g\bar{s}_{g,j}}{1+\frac{K_g}{K}d_g^2 f_g(\bar{\mathbf{D}}_g)\bar{e}_g\bar{s}_{g,j}}, \tag{87}$$

$$f_g(\mathbf{R}_g) = \frac{1}{S_g}\mathrm{tr}\ \mathbf{R}_g(\sum_{i=1}^{G}\bar{f}_i\bar{\mathbf{D}}_i+\frac{K_i}{K}d_i^2\frac{\bar{e}_i}{\rho_{tr}}\tilde{\mathbf{D}}_i+\alpha\mathbf{I}_N)^{-1}, \tag{88}$$

such that $(f_g(\mathbf{R}_g), \bar{f}_g) \geq 0$. Substituting $\bar{f}_g$ in $f_g(\mathbf{R}_g)$, we have,

$$f_g(\mathbf{R}_g) = \frac{1}{S_g}\mathrm{tr}\ \mathbf{R}_g\left(\sum_{i=1}^{G}\frac{\bar{\mathbf{D}}_i}{S_i}\left(\sum_{j=1}^{S_i}\frac{\frac{K_i}{K}d_i^2\bar{e}_i\bar{s}_{i,j}}{1+\frac{K_i}{K}d_i^2 f_i(\bar{\mathbf{D}}_i)\bar{e}_i\bar{s}_{i,j}}\right)+\frac{K_i}{K}d_i^2\frac{\bar{e}_i}{\rho_{tr}}\tilde{\mathbf{D}}_i+\alpha\mathbf{I}_N\right)^{-1}. \tag{89}$$

Consequently,

$$T_1 - \frac{1}{K}d_g^2\sum_{j=1}^{S_g}\frac{\bar{s}_{g,j}f_g(\bar{\mathbf{D}}_g)}{1+\frac{K_g}{K}\bar{e}_g d_g^2\bar{s}_{g,j}f_g(\bar{\mathbf{D}}_g)} \xrightarrow[N\to\infty]{a.s.} 0. \tag{90}$$

Similarly T_2 can be derived as,

$$T_2 - \frac{S_g}{K}d_g^2\frac{1}{\rho_{tr}}f_g(\tilde{\mathbf{D}}_g) \xrightarrow[N\to\infty]{a.s.} 0. \tag{91}$$

Plugging T_1 and T_2 into the expression of e_g in (84) yields,

$$e_g = \frac{1}{K}d_g^2\left(\sum_{j=1}^{S_g}\frac{\bar{s}_{g,j}f_g(\bar{\mathbf{D}}_g)}{1+\frac{K_g}{K}d_g^2\bar{s}_{g,j}\bar{e}_g f_g(\bar{\mathbf{D}}_g)}+\frac{S_g}{\rho_{tr}}f_g(\tilde{\mathbf{D}}_g)\right)+\epsilon_g, \tag{92}$$

where $\epsilon_g \xrightarrow[N\to\infty]{a.s.} 0$. Consider the deterministic counterpart of $(e_g(\alpha), \bar{e}_g(\alpha), f_g(\mathbf{R}_g,\alpha))$ as,

$$\begin{aligned} m_g(\alpha) &= \frac{1}{K}d_g^2\left(\sum_{j=1}^{S_g}\frac{\bar{s}_{g,j}\delta_g(\bar{\mathbf{D}}_g,\alpha)}{1+\frac{K_g}{K}d_g^2\bar{s}_{g,j}\bar{m}_g(\alpha)\delta_g(\bar{\mathbf{D}}_g,\alpha)}+\frac{S_g}{\rho_{tr}}\delta_g(\tilde{\mathbf{D}}_g,\alpha)\right), \\ \bar{m}_g(\alpha) &= \frac{1}{1+m_g(\alpha)}, \\ \delta_g(\mathbf{R}_g,\alpha) &= \frac{1}{S_g}\mathrm{tr}\ \mathbf{R}_g\bar{\mathbf{T}}(\alpha), \end{aligned} \tag{93}$$

where,

$$\bar{\mathbf{T}}(\alpha)=\left(\sum_{i=1}^{G}\frac{\bar{\mathbf{D}}_i}{S_i}\left(\sum_{j=1}^{S_i}\frac{\frac{K_i}{K}d_i^2\bar{s}_{i,j}\bar{m}_i(\alpha)}{1+\frac{K_i}{K}d_i^2\bar{s}_{i,j}\bar{m}_i(\alpha)\delta_i(\bar{\mathbf{D}}_i,\alpha)}\right)+\frac{K_i}{K}d_i^2\frac{\bar{m}_i(\alpha)}{\rho_{tr}}\tilde{\mathbf{D}}_i+\alpha\mathbf{I}_N\right)^{-1}. \tag{94}$$

Define $\Upsilon_1=\max_g|e_g(\alpha)-m_g(\alpha)|$, $\Upsilon_2=\max{}_g|\bar{e}_g(\alpha)-\bar{m}_g(\alpha)|$, $\Upsilon_3=\max_g|f_g(\mathbf{R}_g,\alpha)-\delta_g(\mathbf{R}_g,\alpha)|$ and $\epsilon=\max_g|\epsilon_g|$. It can be shown that,

$$\Upsilon_1,\Upsilon_2,\Upsilon_3\xrightarrow[N\to\infty]{a.s.}0, \tag{95}$$

for α sufficiently large and $\epsilon\xrightarrow[N\to\infty]{a.s.}0$. The result can be extended to all α by Vitali convergence theorem [27].

Now combining (74) with the result in (86) for $\mathbf{R}_g=\mathbf{U}$ will yield,

$$\frac{1}{K}\mathbf{U}\hat{\mathbf{C}}^{-1}(\alpha)-\frac{1}{K}\mathbf{U}\bar{\mathbf{T}}(\alpha)\xrightarrow[N\to\infty]{a.s.}0, \tag{96}$$

where $\bar{\mathbf{T}}(\alpha)$ is given by (94).

This completes the proof of Theorem 1. The uniqueness of the solution $(m_g,\bar{m}_g,\delta_g)$ can be proved by showing that the G-variate function in (93) is a standard interference function [see Definition 2 and Theorem 8 in [28]]. The proof of uniqueness will follow similar steps as done for Theorem 2 in [28] and has been omitted.

APPENDIX C
PROOF OF LEMMA 4

We are interested in finding the deterministic equivalent of:

$$\chi_g=\frac{1}{K^2}\text{tr }\mathbf{\Phi}_g\hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}. \tag{97}$$

First we need to control the variance of χ_g and prove that var (χ_g) converges to zero. This can be done using standard tools from RMT (see [8]) and will imply that,

$$\chi_g-\chi_g^o\xrightarrow[N\to\infty]{a.s.}0, \tag{98}$$

where $\chi_g^o=\mathbb{E}[\chi_g]$. This allows us to focus directly on $\mathbb{E}[\chi_g]$.

Using the expression of $\mathbf{\Phi}_g$ in (21) we have,

$$\chi_g^o=\frac{1}{K^2}d_g^2\mathbb{E}\left[\text{tr }\mathbf{R}_{BS_g}\mathbf{Q}_g\left(\mathbf{Z}_g\mathbf{Z}_g^H+\frac{1}{\rho_{tr}}\mathbf{I}_N\right)\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H\hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}\right], \tag{99}$$

$$=d_g^2\left(\kappa_g^o(\mathbf{R}_{BS_g}\mathbf{Q}_g)+\frac{1}{\rho_{tr}}\xi^o(\mathbf{R}_{BS_g}\mathbf{Q}_g)\right), \tag{100}$$

where,

$$\kappa_g(\mathbf{R}_{BS_g}\mathbf{Q}_g) = \frac{1}{K^2}\text{tr }\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H\hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}, \tag{101}$$

$$\xi(\mathbf{R}_{BS_g}\mathbf{Q}_g) = \frac{1}{K^2}\text{tr }\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H\hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}, \tag{102}$$

and κ_g^o and ξ^o denote their respective expectations. We therefore start the proof by deriving the deterministic equivalents of $\kappa_g(\mathbf{A})$ and $\xi(\mathbf{A})$, where $\mathbf{A}$ is a deterministic matrix of uniformly bounded spectral norm. Plugging these results in (100) will complete the proof of Lemma 4.

A. *Deterministic equivalent of $\kappa_g(\boldsymbol{A})$:*

The aim of this section is to derive a deterministic equivalent for the random quantity,

$$\kappa_g(\mathbf{A}) = \frac{1}{K^2}\text{tr }\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}. \tag{103}$$

We can again control the variance of κ_g and show that it goes to zero implying that,

$$\kappa_g(\mathbf{A}) - \kappa_g^o(\mathbf{A}) \xrightarrow[N\to\infty]{a.s.} 0, \tag{104}$$

where $\kappa_g^o(\mathbf{A}) = \mathbb{E}[\kappa_g(\mathbf{A})]$ This allows us to focus directly on $\mathbb{E}[\kappa_g(\mathbf{A})]$. Using the result from last section in (74) that $\frac{1}{K}\text{tr }\hat{\mathbf{C}}^{-1} - \frac{1}{K}\text{tr }\mathbf{T} \xrightarrow[N\to\infty]{a.s.} 0$, where $\mathbf{T} = (\sum_{i=1}^{G}\frac{\tilde{\mathbf{\Phi}}_i}{1+m_i} + \alpha\mathbf{I}_N)^{-1}$ and the resolvent identity given as,

$$\hat{\mathbf{C}}^{-1} - \mathbf{T} = \mathbf{T}(\mathbf{T}^{-1} - \hat{\mathbf{C}})\hat{\mathbf{C}}^{-1} = \mathbf{T}\left(\sum_{i=1}^{G}\frac{\tilde{\mathbf{\Phi}}_i}{1+m_i} - \frac{1}{K}\hat{\mathbf{H}}_i^H\hat{\mathbf{H}}_i\right)\hat{\mathbf{C}}^{-1}, \tag{105}$$

we decompose κ_g as,

$$\begin{aligned}\kappa_g(\mathbf{A}) &= \frac{1}{K^2}\text{tr }\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1} + \frac{1}{K^2}\sum_{i=1}^{G}\text{tr }\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\frac{\tilde{\mathbf{\Phi}}_i}{1+m_i}\hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1} \\ &- \frac{1}{K^3}\sum_{i=1}^{G}\text{tr }\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{H}}_i^H\hat{\mathbf{H}}_i\hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1} \qquad (106) \\ &= Z_1 + Z_2 + Z_3. \qquad (107)\end{aligned}$$

We will only deal with the expectations of the terms Z_1 and Z_3, since Z_2 will be compensated by terms in Z_3. We begin with applying Lemma 6 from Appendix G on Z_1 as,

$$\mathbb{E}[Z_1] = \frac{1}{K^2}\sum_{i=1}^{G}\sum_{l=1}^{K_i}p_{l,i}\mathbb{E}\left[\frac{\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}}{1+\frac{1}{K}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i}}\right], \tag{108}$$

$$= \frac{1}{K^2}\sum_{i=1}^{G}\sum_{l=1}^{K_i}p_{l,i}\Bigg(\mathbb{E}\left[\frac{\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}\left(\frac{1}{K}\text{tr }\mathbf{\Phi}_i\hat{\mathbf{C}}_{[l,i]}^{-1} - \frac{1}{K}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i}\right)}{(1+\frac{1}{K}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i})(1+\frac{1}{K}\text{tr }\mathbf{\Phi}_i\hat{\mathbf{C}}_{[l,i]}^{-1})}\right] + \mathbb{E}\left[\frac{\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}}{1+\frac{1}{K}\text{tr }\mathbf{\Phi}_i\hat{\mathbf{C}}_{[l,i]}^{-1}}\right]\Bigg). \tag{109}$$

Assuming $\mathbf{Z}_g$ to be deterministic and using the fact that lim $\sup_N ||\mathbf{Z}_g\mathbf{Z}_g^H|| < \infty$ almost surely (proved in [24]), we can use the expression of $\hat{\mathbf{h}}_{k,g}$ in (20) and show with the help of Lemma 7 from Appendix G that the first term on the right side of the above equation is negligible. Now using the Lemma 9 and Lemma 3 on the denominator of the second term will yield,

$$\mathbb{E}[Z_1] = \frac{1}{K^2}\sum_{i=1}^{G}\sum_{l=1}^{K_i} p_{l,i} \frac{\mathbb{E}\left[\hat{\mathbf{h}}_{l,i}^H \hat{\mathbf{C}}_{[l,i]}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}\right]}{1+m_i} + o(1), \tag{110}$$

Using Lemma 7 with the expression of $\hat{\mathbf{h}}_{k,g}$ in (20) we have,

$$\mathbb{E}[Z_1] = \frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i} \frac{p_{l,i}}{K_i} \frac{\mathbb{E}\left[\text{tr }\tilde{\mathbf{\Phi}}_i \hat{\mathbf{C}}_{[l,i]}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\right]}{1+m_i} + o(1). \tag{111}$$

Now using Lemma 9 and Theorem 3 from Appendix G we have,

$$\mathbb{E}[Z_1] = \frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i} \frac{p_{l,i}}{K_i} \frac{\mathbb{E}\left[\text{tr }\tilde{\mathbf{\Phi}}_i \mathbf{T}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\right]}{1+m_i} + o(1). \tag{112}$$

Note that the analysis in the last two steps can be extended to random $\mathbf{Z}_g$s using Fubini Theorem and we need the deterministic equivalent of $\bar{\beta}_{g,i}(\mathbf{A}) = \frac{1}{K_i}\mathbb{E}[\text{tr }\tilde{\mathbf{\Phi}}_i\mathbf{T}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}]$ under the actual random $\mathbf{Z}_g$s. Using the expression of $\mathbf{\Phi}_i$ from (21) results in,

$$\bar{\beta}_{g,i}(\mathbf{A}) = \frac{d_i^2}{K}\mathbb{E}\left[\text{tr }\mathbf{R}_{BS_i}\mathbf{Q}_i\left(\mathbf{Z}_i\mathbf{Z}_i^H + \frac{1}{\rho_{tr}}\mathbf{I}_N\right)\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\mathbf{T}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\right], \tag{113}$$

$$= \mathbb{E}\left[\frac{d_i^2}{K}\sum_{j=1}^{S_i}\mathbf{z}_{i,j}^H\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\mathbf{T}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{z}_{i,j}\right] + \mathbb{E}\left[\frac{d_i^2}{K\rho_{tr}}\text{tr }\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\mathbf{T}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\right],$$

$$= T_4 + T_5. \tag{114}$$

We analyze the two terms separately. First we use Lemma 11 from Appendix G on T_4 to remove the dependence of $\mathbf{T}$ on $\mathbf{z}_{i,j}$ as follows,

$$T_4 = \begin{cases} \frac{d_i^2}{K}\sum_{j=1}^{S_i}\mathbb{E}\left[\frac{\mathbf{z}_{i,j}^H\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\tilde{\mathbf{T}}_i\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\tilde{\mathbf{T}}_i\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{z}_{i,j}}{(1+\bar{m}_i\frac{K_i}{K}d_i^2\mathbf{z}_{i,j}^H\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\tilde{\mathbf{T}}_i\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{z}_{i,j})^2}\right] & \text{if } g \neq i, \\ \frac{d_i^2}{K}\sum_{j=1}^{S_i}\mathbb{E}\left[\frac{\mathbf{z}_{i,j}^H\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\tilde{\mathbf{T}}_i\mathbf{A}(\mathbf{Z}_g\mathbf{Z}_g^H - \mathbf{z}_{i,j}\mathbf{z}_{i,j}^H)\mathbf{A}^H\tilde{\mathbf{T}}_i\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{z}_{i,j} + \mathbf{z}_{i,j}^H\mathbf{A}^H\tilde{\mathbf{T}}_i\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{z}_{i,j}\mathbf{z}_{i,j}^H\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\tilde{\mathbf{T}}_i\mathbf{A}\mathbf{z}_{i,j}}{(1+\bar{m}_i\frac{K_i}{K}d_i^2\mathbf{z}_{i,j}^H\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\tilde{\mathbf{T}}_i\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{z}_{i,j})^2}\right] & \text{if } g = i, \end{cases}$$

where $\tilde{\mathbf{T}}_i = \left(\sum_{l=1}^{G}\frac{K_l}{K}d_l^2\bar{m}_l\mathbf{R}_{BS_l}\mathbf{Q}_l\left(\mathbf{Z}_l\mathbf{Z}_l^H + \frac{1}{\rho_{tr}}\mathbf{I}_N\right)\mathbf{Q}_l^H\mathbf{R}_{BS_l}^H - \frac{K_i}{K}d_i^2\bar{m}_i\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{z}_{i,j}\mathbf{z}_{i,j}^H\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H + \alpha\mathbf{I}_N\right)^{-1}$.

Next we use Lemma 7 and Lemma 9 from Appendix G to get the following,

$$T_4 = \begin{cases} \frac{d_i^2}{K}\sum_{j=1}^{S_i}\mathbb{E}\left[\frac{\frac{\bar{s}_{i,j}}{S_i}\text{tr }\mathbf{R}_{BS_i}\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\mathbf{T}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\mathbf{R}_{BS_i}\mathbf{Q}_i}{(1+\bar{m}_i\frac{K_i}{K}d_i^2\frac{\bar{s}_{i,j}}{S_i}\text{tr }\mathbf{R}_{BS_i}\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\mathbf{T}\mathbf{R}_{BS_i}\mathbf{Q}_i)^2}\right] + o(1) \\ \frac{d_i^2}{K}\sum_{j=1}^{S_i}\mathbb{E}\left[\frac{\frac{\bar{s}_{i,j}}{S_i}\text{tr }\mathbf{R}_{BS_i}\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\mathbf{T}\mathbf{A}(\mathbf{Z}_g\mathbf{Z}_g^H - \mathbf{z}_{i,j}\mathbf{z}_{i,j}^H)\mathbf{A}^H\mathbf{T}\mathbf{R}_{BS_i}\mathbf{Q}_i + (\frac{\bar{s}_{i,j}}{S_i}\text{tr }\mathbf{R}_{BS_i}\mathbf{A}^H\mathbf{T}\mathbf{R}_{BS_i}\mathbf{Q}_i)(\frac{\bar{s}_{i,j}}{S_i}\text{tr }\mathbf{R}_{BS_i}\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\mathbf{T}\mathbf{A})}{(1+\bar{m}_i\frac{K_i}{K}d_i^2\frac{\bar{s}_{i,j}}{S_i}\text{tr }\mathbf{R}_{BS_i}\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\mathbf{T}\mathbf{R}_{BS_i}\mathbf{Q}_i)^2}\right] + o(1) \end{cases}$$

Using the Lemma 11 to remove the dependence of $\mathbf{T}$ on $\mathbf{z}_{g,j}$ and using the deterministic equivalent of $f_i(\bar{\mathbf{D}}_i) = \frac{1}{S_i}\text{tr }\bar{\mathbf{D}}_i\mathbf{T}$ derived in Appendix B as $\delta_i(\bar{\mathbf{D}}_i)$, where $\bar{\mathbf{D}}_i = \mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{R}_{BS_i}\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H$, yields,

$$T_4 = \begin{cases} \frac{d_i^2}{K}\sum_{j=1}^{S_i}\mathbb{E}\left[\frac{\bar{s}_{i,j}}{S_i}\sum_{n=1}^{S_g}\frac{\mathbf{z}_{g,n}^H\mathbf{A}^H\tilde{\mathbf{T}}_g\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{R}_{BS_i}\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\tilde{\mathbf{T}}_g\mathbf{A}\mathbf{z}_{g,n}}{(1+\frac{K_i}{K}d_i^2\bar{m}_i\bar{s}_{i,j}\delta_i(\bar{\mathbf{D}}_i))^2(1+\bar{m}_g\frac{K_g}{K}d_g^2\mathbf{z}_{g,n}^H\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H\tilde{\mathbf{T}}_g\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{z}_{g,n})^2}\right] & \text{if } g \neq i, \\ \frac{d_i^2}{K}\sum_{j=1}^{S_i}\mathbb{E}\left[\frac{\bar{s}_{i,j}}{S_i}\sum_{\substack{n=1\\n\neq j}}^{S_g}\frac{\mathbf{z}_{g,n}^H\mathbf{A}^H\tilde{\mathbf{T}}_g\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{R}_{BS_i}\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H\tilde{\mathbf{T}}_g\mathbf{A}\mathbf{z}_{g,n}}{(1+\frac{K_i}{K}d_i^2\bar{m}_i\bar{s}_{i,j}\delta_i(\bar{\mathbf{D}}_i))^2(1+\bar{m}_g\frac{K_g}{K}d_g^2\mathbf{z}_{g,n}^H\mathbf{Q}_g^H\mathbf{R}_{BS_g}^H\tilde{\mathbf{T}}_g\mathbf{R}_{BS_g}\mathbf{Q}_g\mathbf{z}_{g,n})^2}\right] & \\ +\frac{d_i^2}{K}\sum_{j=1}^{S_i}\mathbb{E}\left[\frac{\bar{s}_{i,j}^2\delta_i(\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{R}_{BS_i}\mathbf{A}^H)\delta_i(\mathbf{A}\mathbf{R}_{BS_i}\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H)}{(1+\frac{K_i}{K}d_i^2\bar{m}_i\bar{s}_{i,j}\delta_i(\bar{\mathbf{D}}_i))^2}\right] & \text{if } g = i. \end{cases}$$

Finally we use Lemma 7 and 9 from Appendix G to get the following approximation for T_4,

$$T_4 = \begin{cases} \frac{d_i^2}{K}\sum_{j=1}^{S_i}\frac{\bar{s}_{i,j}}{S_i}\sum_{n=1}^{S_g}\frac{\bar{s}_{g,n}\mathbb{E}[\frac{1}{S_g}\text{tr }\mathbf{R}_{BS_g}\mathbf{A}^H\mathbf{T}\bar{\mathbf{D}}_i\mathbf{T}\mathbf{A}]}{(1+\frac{K_i}{K}d_i^2\bar{m}_i\bar{s}_{i,j}\delta_i(\bar{\mathbf{D}}_i))^2(1+\frac{K_g}{K}d_g^2\bar{m}_g\bar{s}_{g,n}\delta_g(\bar{\mathbf{D}}_g))^2} + o(1) & \text{if } g \neq i, \\ \frac{d_i^2}{K}\sum_{j=1}^{S_i}\frac{\bar{s}_{i,j}}{S_i}\sum_{\substack{n=1\\n\neq j}}^{S_g}\frac{\bar{s}_{g,n}\mathbb{E}[\frac{1}{S_g}\text{tr }\mathbf{R}_{BS_g}\mathbf{A}^H\mathbf{T}\bar{\mathbf{D}}_i\mathbf{T}\mathbf{A}]}{(1+\frac{K_i}{K}d_i^2\bar{m}_i\bar{s}_{i,j}\delta_i(\bar{\mathbf{D}}_i))^2(1+\frac{K_g}{K}d_g^2\bar{m}_g\bar{s}_{g,n}\delta_g(\bar{\mathbf{D}}_g))^2} & \\ +\frac{d_i^2}{K}\sum_{j=1}^{S_i}\frac{\bar{s}_{i,j}^2\delta_i(\mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{R}_{BS_i}\mathbf{A}^H)\delta_i(\mathbf{A}\mathbf{R}_{BS_i}\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H)}{(1+\frac{K_i}{K}d_i^2\bar{m}_i\bar{s}_{i,j}\delta_i(\bar{\mathbf{D}}_i))^2} + o(1) & \text{if } g = i. \end{cases}$$

Similar steps would yield the following deterministic equivalent for T_5,

$$T_5 = \frac{d_i^2}{K\rho_{tr}}\sum_{j=1}^{S_g}\frac{\bar{s}_{g,j}\mathbb{E}[\frac{1}{S_g}\text{tr }\mathbf{R}_{BS_g}\mathbf{A}^H\mathbf{T}\tilde{\mathbf{D}}_i\mathbf{T}\mathbf{A}]}{(1+\frac{K_g}{K}\bar{m}_g d_g^2\bar{s}_{g,n}\delta_g(\bar{\mathbf{D}}_g))^2} + o(1). \tag{115}$$

where $\tilde{\mathbf{D}}_i = \mathbf{R}_{BS_i}\mathbf{Q}_i\mathbf{Q}_i^H\mathbf{R}_{BS_i}^H$.

In order to complete the calculation of the deterministic equivalent of $\bar{\beta}_{g,i}$, we need the deterministic equivalent of $M_g'(\mathbf{R}_g,\mathbf{L}) = \mathbb{E}[\frac{1}{S_g}\text{tr }\mathbf{R}_g\mathbf{T}\mathbf{L}\mathbf{T}]$ which is stated in the following Lemma.

Lemma 5: Define $M_g'(\mathbf{R}_g,\mathbf{L}) = \mathbb{E}[\frac{1}{S_g}\text{tr }\mathbf{R}_g\mathbf{T}\mathbf{L}\mathbf{T}]$ where $\mathbf{R}_g$ and $\mathbf{L}$, $g = 1,\dots,G$ are deterministic matrices with uniformly bounded spectral norm. Then under the setting of assumptions **A-1**, **A-2** and **A-3** and for $\alpha > 0$,

$$M_g'(\mathbf{R}_g,\mathbf{L}) = \frac{1}{S_g}\text{tr }\mathbf{R}_g\left[\bar{\mathbf{T}}\left(\sum_{z=1}^{G}\frac{\bar{\mathbf{D}}_z\bar{m}_z^2}{S_z}\left(\frac{K_z}{K}\right)^2 d_z^4\text{tr }(\bar{\mathbf{S}}_z\mathbf{W}_z^2\bar{\mathbf{S}}_z)M_z'(\bar{\mathbf{D}}_z,\mathbf{L}) + \mathbf{L}\right)\bar{\mathbf{T}}\right] + o(1), \tag{116}$$

where,

$$\mathbf{W}_i = \left(\mathbf{I}_{S_i} + \frac{K_i}{K}d_i^2\bar{m}_i\delta_i(\bar{\mathbf{D}}_i)\bar{\mathbf{S}}_i\right)^{-1}, \tag{117}$$

and $\mathbf{M}'(\bar{\mathbf{D}},\mathbf{L}) = [M_1'(\bar{\mathbf{D}}_1,\mathbf{L}), M_2'(\bar{\mathbf{D}}_2,\mathbf{L}),\dots,M_G'(\bar{\mathbf{D}}_G,\mathbf{L})]^T$, which can be expressed as a system of linear equations as follows:

$$\mathbf{M}'(\bar{\mathbf{D}},\mathbf{L}) = (\mathbf{I}_N - \mathbf{J}(\bar{\mathbf{D}}))^{-1}\mathbf{v}(\bar{\mathbf{D}},\mathbf{L}), \tag{118}$$

$$[\mathbf{J}(\bar{\mathbf{D}})]_{g,i} = \frac{1}{S_g}\text{tr }(\bar{\mathbf{D}}_g\bar{\mathbf{T}}\bar{\mathbf{D}}_i\bar{\mathbf{T}})\left(\frac{\bar{m}_i^2}{S_i}\left(\frac{K_i}{K}\right)^2 d_i^4\text{tr }(\bar{\mathbf{S}}_i\mathbf{W}_i^2\mathbf{S}_i)\right), \tag{119}$$

$$[\mathbf{v}(\bar{\mathbf{D}},\mathbf{L})]_g = \frac{1}{S_g}\text{tr }(\bar{\mathbf{D}}_g\bar{\mathbf{T}}\mathbf{L}\bar{\mathbf{T}}), \tag{120}$$

for $g, i = 1, \ldots, G$. The proof of Lemma 5 can be found in Appendix D.

Using Lemma 5, we have the deterministic equivalent of terms T_4 and T_5 and hence $\bar{\beta}_{g,i}(\mathbf{A})$. This yields the expression of $\mathbb{E}[Z_1]$ as,

$$\mathbb{E}[Z_1] = \frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i} p_{l,i}\frac{\bar{\beta}_{g,i}(\mathbf{A})}{1+m_i} + o(1). \tag{121}$$

We now study Z_3. Using Lemma 6 from Appendix G we have,

$$\begin{aligned} Z_3 &= -\frac{1}{K^3}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{\text{tr }\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}}{1+\frac{1}{K}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i}}, \qquad (122)\\ &= -\frac{1}{K^3}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{\text{tr }\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}_{[l,i]}^{-1}}{1+\frac{1}{K}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i}} + \frac{1}{K^4}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{\text{tr }\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}}{(1+\frac{1}{K}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i})^2},\\ &= Z_{31} + Z_{32}. \qquad (123) \end{aligned}$$

We sequentially deal with terms Z_{31} and Z_{32}. Note that $\frac{1}{K}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i}$ converges to m_i using Lemma 7 and Lemma 3. This yields,

$$\begin{aligned} \mathbb{E}[Z_{31}] &= -\frac{1}{K^3}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{\mathbb{E}\left[\text{tr }\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}_{[l,i]}^{-1}\right]}{1+m_i} + o(1), \qquad (124)\\ &= -\frac{1}{K^3}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{\mathbb{E}\left[\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{H}}_{[l,i]}^H\mathbf{P}_{[l,i]}\hat{\mathbf{H}}_{[l,i]}\hat{\mathbf{C}}_{[l,i]}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}\right]}{1+m_i} - \frac{1}{K^3}\sum_{i=1}^{G}\sum_{l=1}^{K_i}p_{l,i}\frac{\mathbb{E}\left[\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}\right]}{1+m_i} + o(1),\\ &= \Psi_1 + \Psi_2. \qquad (125) \end{aligned}$$

The quadratic forms in Ψ_2 can be shown to have variance $O(K^{-2})$. Assuming $\mathbf{Z}_g$ to be deterministic, we can use Lemma 10, Lemma 7, Lemma 9 and Theorem 3 from Appendix G sequentially to obtain,

$$\Psi_2 = -\frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}p_{l,i}\frac{\mathbb{E}\left[\frac{1}{K_i}\text{tr }\left(\tilde{\mathbf{\Phi}}_i\hat{\mathbf{C}}_{[l,i]}^{-1}\right)\right]\mathbb{E}[\frac{1}{K_i}\text{tr }\left(\tilde{\mathbf{\Phi}}_i\hat{\mathbf{C}}_{[l,i]}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\right)]}{1+m_i} + o(1). \tag{126}$$

Extending the analysis to random $\mathbf{Z}_g$ based on the Fubini Theorem and using Lemma 3 we have,

$$\Psi_2 = -\frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}p_{l,i}\frac{m_i\bar{\beta}_{g,i}(\mathbf{A})}{1+m_i} + o(1). \tag{127}$$

The term Ψ_1 is compensated by Z_2. To see this, observe that the first order of the term does not change if we substitute $\hat{\mathbf{H}}_{[l,i]}$ by $\hat{\mathbf{H}}$ and $\mathbf{P}_{[l,i]}$ by $\mathbf{P}$. Then using Fubini theorem, Lemma 7 and Lemma 9 from Appendix G we have,

$$\begin{aligned} \Psi_1 &= -\frac{1}{K^3}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\mathbb{E}\left[\frac{\text{tr }\mathbf{\Phi}_i\hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}}{1+m_i}\right] + o(1), \qquad (128)\\ &= -\mathbb{E}[Z_2] + o(1). \qquad (129) \end{aligned}$$

Finally, it remains to deal with Z_{32}. Substituting $\frac{1}{K}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i}$ by its deterministic equivalent from Lemma 3, we get,

$$\mathbb{E}[Z_{32}] = \frac{1}{K^4}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{\mathbb{E}\left[\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{H}}_{[l,i]}^H\mathbf{P}_{[l,i]}\hat{\mathbf{H}}_{[l,i]}\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i}\right]}{(1+m_i)^2}$$
$$+\frac{1}{K^4}\sum_{i=1}^{G}\sum_{l=1}^{K_i}p_{l,i}\frac{\mathbb{E}\left[\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\hat{\mathbf{h}}_{l,i}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i}\hat{\mathbf{h}}_{l,i}^H\hat{\mathbf{C}}_{[l,i]}^{-1}\hat{\mathbf{h}}_{l,i}\right]}{(1+m_i)^2}+o(1). \tag{130}$$

Analogously to before, $\mathbb{E}[Z_{32}]$ can be simplified as,

$$\mathbb{E}[Z_{32}] = \frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{\mathbb{E}\left[\frac{1}{K_i}\mathrm{tr}\,\left(\tilde{\mathbf{\Phi}}_i\hat{\mathbf{C}}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\right)\right]\mathbb{E}\left[\frac{1}{K^2}\mathrm{tr}\,\left(\mathbf{\Phi}_i\hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}\right)\right]}{(1+m_i)^2}$$
$$+\frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}p_{l,i}\frac{\mathbb{E}\left[\frac{1}{K_i}\mathrm{tr}\,\left(\tilde{\mathbf{\Phi}}_i\hat{\mathbf{C}}^{-1}\mathbf{A}\mathbf{Z}_g\mathbf{Z}_g^H\mathbf{A}^H\mathbf{T}\right)\right]\mathbb{E}\left[\frac{1}{K_i}\mathrm{tr}\,\left(\tilde{\mathbf{\Phi}}_i\hat{\mathbf{C}}^{-1}\right)\right]\mathbb{E}\left[\frac{1}{K_i}\mathrm{tr}\,\left(\tilde{\mathbf{\Phi}}_i\hat{\mathbf{C}}^{-1}\right)\right]}{(1+m_i)^2}+o(1), \tag{131}$$

$$=\frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{\bar{\beta}_{g,i}(\mathbf{A})\chi_i^o}{(1+m_i)^2}+\frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}p_{l,i}\frac{m_i^2\bar{\beta}_{g,i}(\mathbf{A})}{(1+m_i)^2}+o(1), \tag{132}$$

where χ_i^o is defined in (100).

Combining (121), (127) and (132), we obtain,

$$\kappa_g^o(\mathbf{A}) = \frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{\bar{\beta}_{g,i}(\mathbf{A})}{(1+m_i)^2}(p_{l,i}+\chi_i^o)+o(1). \tag{133}$$

B. Deterministic equivalents of $\xi(\boldsymbol{A})$ and χ_g:

Using similar argument for $\xi(\mathbf{A}) = \frac{1}{K^2}\mathrm{tr}\,\mathbf{A}\mathbf{A}^H\hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H\mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}$ as done for $\kappa_g(\mathbf{A})$ in (104), we have $\xi(\mathbf{A})-\xi^o(\mathbf{A}) \xrightarrow[N\to\infty]{a.s.} 0$, where $\xi^o(\mathbf{A}) = \mathbb{E}[\xi(\mathbf{A})]$.

Repeating the same steps as done for $\kappa_g^o(\mathbf{A})$, we obtain,

$$\xi^o(\mathbf{A}) = \frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{\tilde{\beta}_i(\mathbf{A})}{(1+m_i)^2}(p_{l,i}+\chi_i^o)+o(1), \tag{134}$$

where,

$$\tilde{\beta}_i(\mathbf{A}) = \frac{d_i^2}{K}\left(\sum_{j=1}^{S_i}\frac{\bar{s}_{i,j}M_i'(\bar{\mathbf{D}}_i,\mathbf{A}\mathbf{A}^H)}{(1+\frac{K_i}{K}d_i^2\bar{m}_i\bar{s}_{i,j}\delta_i(\bar{\mathbf{D}}_i))^2}+\frac{S_i}{\rho_{tr}}M_i'(\tilde{\mathbf{D}}_i,\mathbf{A}\mathbf{A}^H)\right), \tag{135}$$

where $M_i'(\mathbf{R}_i,\mathbf{L})$ has been defined in Lemma 5.

Plugging (133) and (134) into (100) yields the deterministic equivalent of χ_g^o as,

$$\chi_g^o = \frac{d_g^2}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}(p_{l,i}+\chi_i^o)\left(\frac{\bar{\beta}_{g,i}(\mathbf{R}_{BS_g}\mathbf{Q}_g)}{(1+m_i)^2}+\frac{1}{\rho_{tr}}\frac{\tilde{\beta}_i(\mathbf{R}_{BS_g}\mathbf{Q}_g)}{(1+m_i)^2}\right)+o(1). \tag{136}$$

Now $\boldsymbol{\chi}^o = [\chi_1^o, \chi_2^o, \ldots, \chi_g^o]^T$ can be expressed as a system of linear equations as,

$$\boldsymbol{\chi}^o = (\mathbf{I}_N - \bar{\mathbf{J}})^{-1}\bar{\mathbf{v}}, \tag{137}$$

where,

$$[\bar{\mathbf{J}}]_{g,i} = d_g^2\frac{K_i}{K}\frac{1}{(1+m_i)^2}\left(\bar{\beta}_{g,i}(\mathbf{R}_{BS_g}\mathbf{Q}_g)+\frac{1}{\rho_{tr}}\tilde{\beta}_i(\mathbf{R}_{BS_g}\mathbf{Q}_g)\right), \tag{138}$$

$$\bar{\mathbf{v}}_g = d_g^2\frac{1}{K}\sum_{i=1}^{G}\sum_{l=1}^{K_i}\frac{p_{l,i}}{(1+m_i)^2}\left(\bar{\beta}_{g,i}(\mathbf{R}_{BS_g}\mathbf{Q}_g)+\frac{1}{\rho_{tr}}\tilde{\beta}_i(\mathbf{R}_{BS_g}\mathbf{Q}_g)\right). \tag{139}$$

Also, plugging (136) into (133) and (134) completes the deterministic equivalents of $\kappa_g^o(\mathbf{A})$ and $\xi^o(\mathbf{A})$ respectively. These terms will later be needed in the proof of Theorem 2.

Appendix D
Proof of Lemma 5

To derive the deterministic equivalent of $M_g'(\mathbf{R}_g,\mathbf{L}) = \mathbb{E}[\frac{1}{S_g}\text{tr }\mathbf{R}_g\mathbf{T}\mathbf{L}\mathbf{T}]$, where $\mathbf{R}_g$ and $\mathbf{L}$, $g=1,\ldots,G$ are deterministic matrices, we use the results from Appendix B in (86) and (89) along with $||f_g(\mathbf{R}_g)-\delta_g(\mathbf{R}_g)|| \xrightarrow[N\to\infty]{a.s.}$ to have,

$$\frac{1}{S_g}\text{tr }\mathbf{R}_g\mathbf{T}-\delta_g(\mathbf{R}_g) \xrightarrow[N\to\infty]{a.s.}, \tag{140}$$

where $\delta_g(\mathbf{R}_g)=\frac{1}{S_g}\mathbf{R}_g\bar{\mathbf{T}}$, where $\bar{\mathbf{T}}$ is given by (94). To this end note that,

$$M_g'(\mathbf{R}_g,\mathbf{L}) = \mathbb{E}\left[\frac{d}{dl}\frac{1}{S_g}\text{tr }\mathbf{R}_g(\mathbf{T}^{-1}-l\mathbf{L})^{-1}|_{l=0}\right]. \tag{141}$$

The deterministic equivalent of $\frac{1}{S_g}\text{tr }\mathbf{R}_g(\mathbf{T}^{-1}-l\mathbf{L})^{-1}$ is obtained using Theorem 3 as,

$$\frac{1}{S_g}\text{tr }\mathbf{R}_g\left(\sum_{z=1}^{G}\bar{m}_z\tilde{\mathbf{\Phi}}_z-l\mathbf{L}+\alpha\mathbf{I}_N\right)^{-1}-\frac{1}{S_g}\text{tr }\mathbf{R}_g(\sum_{z=1}^{G}\bar{F}_z\bar{\mathbf{D}}_z+\frac{K_z}{K}d_z^2\frac{\bar{m}_z}{\rho_{tr}}\tilde{\mathbf{D}}_z-l\mathbf{L}+\alpha\mathbf{I}_N)^{-1} \xrightarrow[N\to\infty]{a.s.} 0, \tag{142}$$

where $F_g(\mathbf{R}_g,\mathbf{L}),\bar{F}_g$ are defined as the unique solution to,

$$\bar{F}_g = \frac{1}{S_g}\sum_{j=1}^{S_g}\frac{\frac{K_g}{K}d_g^2\bar{m}_g\bar{s}_{g,j}}{1+\frac{K_g}{K}d_g^2F_g(\bar{\mathbf{D}}_g,\mathbf{L})\bar{m}_g\bar{s}_{g,j}}, \tag{143}$$

$$F_g(\mathbf{R}_g,\mathbf{L}) = \frac{1}{S_g}\text{tr }\mathbf{R}_g(\sum_{z=1}^{G}\bar{F}_z\bar{\mathbf{D}}_z+\frac{K_z}{K}d_z^2\frac{\bar{m}_z}{\rho_{tr}}\tilde{\mathbf{D}}_z-l\mathbf{L}+\alpha\mathbf{I}_N)^{-1}, \tag{144}$$

such that $(F_g(\mathbf{R}_g,\mathbf{L}),\bar{F}_g)\geq 0$. Substituting $\bar{F}_g$ in $F_g(\mathbf{R}_g,\mathbf{L})$, we have,

$$F_g(\mathbf{R}_g,\mathbf{L})=\frac{1}{S_g}\mathrm{tr}\ \mathbf{R}_g\left(\sum_{z=1}^{G}\frac{1}{S_z}\sum_{j=1}^{S_z}\frac{\frac{K_z}{K}d_z^2\bar{m}_z\bar{s}_{z,j}}{1+\frac{K_z}{K}d_z^2F_z(\bar{\mathbf{D}}_z,\mathbf{L})\bar{m}_z\bar{s}_{z,j}}\bar{\mathbf{D}}_z+\frac{K_z}{K}d_z^2\frac{\bar{m}_z}{\rho_{tr}}\tilde{\mathbf{D}}_z-l\mathbf{L}+\alpha\mathbf{I}_N\right)^{-1}. \tag{145}$$

Note that $F_g(\mathbf{R}_g,\mathbf{L})$ reduces to $\delta_g(\mathbf{R}_g)$ for $l=0$.

The expression of $M_g'(\mathbf{R}_g,\mathbf{L})$ can be obtained by using (141) and (142) to get $M_g'(\mathbf{R}_g,\mathbf{L})=\frac{d}{dl}F_g(\mathbf{R}_g,\mathbf{L})|_{l=0}+o(1)$ resulting in,

$$M_g'(\mathbf{R}_g,\mathbf{L})=\frac{1}{S_g}\mathrm{tr}\ \mathbf{R}_g\left[\bar{\mathbf{T}}\left(\sum_{z=1}^{G}\frac{\bar{\mathbf{D}}_z\bar{m}_z^2}{S_z}\left(\frac{K_z}{K}\right)^2d_z^4\mathrm{tr}\ (\bar{\mathbf{S}}_z\mathbf{W}_z^2\bar{\mathbf{S}}_z)M_z'(\bar{\mathbf{D}}_z,\mathbf{L})+\mathbf{L}\right)\bar{\mathbf{T}}\right]+o(1), \tag{146}$$

where $\mathbf{W}_i=\left(\mathbf{I}_{S_i}+\frac{K_i}{K}d_i^2\bar{m}_i\delta_i(\bar{\mathbf{D}}_i)\bar{\mathbf{S}}_i\right)^{-1}$.

Note that $M_g'(\mathbf{R}_g,\mathbf{L})$ depends on the values of $M_z'(\bar{\mathbf{D}}_z,\mathbf{L})$. The latter can be expressed as a system of linear equations by solving (146) for $\mathbf{R}_g=\bar{\mathbf{D}}_g$. This system is represented by (118).

APPENDIX E
PROOF OF THEOREM 2

The deterministic equivalents of the energy term $|\mathbb{E}[\mathbf{h}_{k,g}^H\hat{\mathbf{V}}\hat{\mathbf{h}}_{k,g}]|^2$, the term Θ of the power normalization, the interference term $\mathbb{E}[\mathbf{h}_{k,g}^H\hat{\mathbf{V}}\hat{\mathbf{H}}_{[k,g]}^H\mathbf{P}_{[k,g]}\hat{\mathbf{H}}_{[k,g]}\hat{\mathbf{V}}\mathbf{h}_{k,g}]$ and the variance term var $(\mathbf{h}_{k,g}^H\hat{\mathbf{V}}\hat{\mathbf{h}}_{k,g})$ are worked out separately to yield the deterministic equivalent of the SINR.

A. *Deterministic equivalent of* $|\mathbb{E}[\boldsymbol{h}_{k,g}^H\hat{\boldsymbol{V}}\hat{\boldsymbol{h}}_{k,g}]|^2$*:*

Note that $\mathbf{h}_{k,g}^H\hat{\mathbf{V}}\hat{\mathbf{h}}_{k,g}$ can be written as $\frac{1}{K}\mathbf{h}_{k,g}^H\hat{\mathbf{C}}^{-1}\hat{\mathbf{h}}_{k,g}$, where $\hat{\mathbf{C}}=\frac{1}{K}\hat{\mathbf{H}}^H\hat{\mathbf{H}}+\alpha\mathbf{I}_N$. In order to remove the dependency of $\hat{\mathbf{C}}$ on $\hat{\mathbf{h}}_{k,g}$, we use Lemma 6 from Appendix G to get,

$$\mathbf{h}_{k,g}^H\hat{\mathbf{V}}\hat{\mathbf{h}}_{k,g}=\frac{1}{K}\mathbf{h}_{k,g}^H\hat{\mathbf{C}}_{[k,g]}^{-1}\hat{\mathbf{h}}_{k,g}-\frac{1}{K^2}\frac{\mathbf{h}_{k,g}^H\hat{\mathbf{C}}_{[k,g]}^{-1}\hat{\mathbf{h}}_{k,g}\hat{\mathbf{h}}_{k,g}^H\hat{\mathbf{C}}_{[k,g]}^{-1}\hat{\mathbf{h}}_{k,g}}{1+\frac{1}{K}\hat{\mathbf{h}}_{k,g}^H\hat{\mathbf{C}}_{[k,g]}^{-1}\hat{\mathbf{h}}_{k,g}}. \tag{147}$$

We know from Lemma 2 that $\frac{1}{K}\mathbf{h}_{k,g}^H\hat{\mathbf{C}}_{[k,g]}^{-1}\hat{\mathbf{h}}_{k,g}-\frac{1}{K}\hat{\mathbf{h}}_{k,g}^H\hat{\mathbf{C}}_{[k,g]}^{-1}\hat{\mathbf{h}}_{k,g}\xrightarrow[N\to\infty]{a.s.}0$. Therefore we are only interested in the deterministic equivalent of $\frac{1}{K}\hat{\mathbf{h}}_{k,g}^H\hat{\mathbf{C}}_{[k,g]}^{-1}\hat{\mathbf{h}}_{k,g}$ which can be treated using Fubini Theorem and Lemma 7 and Lemma 9 as $\frac{1}{K}\hat{\mathbf{h}}_{k,g}^H\hat{\mathbf{C}}_{[k,g]}^{-1}\hat{\mathbf{h}}_{k,g}-\frac{1}{K}\mathbf{\Phi}_g\hat{\mathbf{C}}^{-1}\xrightarrow[N\to\infty]{a.s.}0$. The deterministic equivalent of $\frac{1}{K}\mathbf{\Phi}_g\hat{\mathbf{C}}^{-1}$ been derived in Lemma 3 as m_g. Note that Lemma 3 not only implies almost sure convergence but also convergence in mean. Therefore by dominated convergence theorem and continuous mapping theorem we have,

$$|\mathbb{E}[\mathbf{h}_{k,g}^H \hat{\mathbf{V}} \hat{\mathbf{h}}_{k,g}]|^2 - \frac{m_g^2}{(1+m_g)^2} \xrightarrow[N\to\infty]{a.s.} 0. \tag{148}$$

B. Deterministic equivalent of Θ:

$$\Theta = \mathbb{E}[\text{tr } \mathbf{P}\hat{\mathbf{H}}^H \hat{\mathbf{V}}^2 \hat{\mathbf{H}}] = \mathbb{E}\left[\frac{1}{K^2}\text{tr } \mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}\hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H\right], \tag{149}$$

$$= \mathbb{E}\left[\frac{1}{K^2}\text{tr } \hat{\mathbf{C}}^{-1}\hat{\mathbf{H}}^H \mathbf{P}\hat{\mathbf{H}}\hat{\mathbf{C}}^{-1}\right] = \xi^o(\mathbf{I}_N), \tag{150}$$

where $\xi^o(\mathbf{A})$ was derived in the last section and is given by (134).

C. Deterministic equivalent of $\mathbb{E}[\boldsymbol{h}_{k,g}^H \hat{\boldsymbol{V}} \hat{\boldsymbol{H}}_{[k,g]}^H \boldsymbol{P}_{[k,g]} \hat{\boldsymbol{H}}_{[k,g]} \hat{\boldsymbol{V}} \boldsymbol{h}_{k,g}]$:

Denote $\mathbf{h}_{k,g}^H \hat{\mathbf{V}} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{V}} \mathbf{h}_{k,g}$ as $\Upsilon_{k,g}$. Using Lemma 6 from Appendix G we decompose $\Upsilon_{k,g}$ as,

$$\begin{aligned}
\Upsilon_{k,g} &= \frac{1}{K^2}\mathbf{h}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \mathbf{h}_{k,g} - \frac{1}{K^3}\frac{\mathbf{h}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g} \hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \mathbf{h}_{k,g}}{1 + \frac{1}{K}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g}}, \\
&- \frac{1}{K^3}\frac{\mathbf{h}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g} \hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \mathbf{h}_{k,g}}{1 + \frac{1}{K}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g}} + \frac{1}{K^4}\frac{\mathbf{h}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g} \hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g} \hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \mathbf{h}_{k,g}}{(1 + \frac{1}{K}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g})^2}, \\
&= X_1 + X_2 + X_3 + X_4.
\end{aligned} \tag{151}$$

Let us begin by treating X_1. Using $\mathbf{h}_{k,g} = \hat{\mathbf{h}}_{k,g} + \tilde{\mathbf{h}}_{k,g}$ and Lemma 2 we get,

$$X_1 - \left(\frac{1}{K^2}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g} + \frac{1}{K^2}\tilde{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \tilde{\mathbf{h}}_{k,g}\right) \xrightarrow[N\to\infty]{a.s.} 0. \tag{152}$$

Let $X_1^1 = \frac{1}{K^2}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g}$. Using the expression of $\hat{\mathbf{h}}_{k,g}$ in (20), Fubini Theorem and Lemma 7 we have,

$$X_1^1 - \frac{1}{K^2}\text{tr } \mathbf{\Phi}_g \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \xrightarrow[N\to\infty]{a.s.} 0. \tag{153}$$

To this end note that,

$$\frac{1}{K^2}\text{tr } \mathbf{\Phi}_g \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} = \chi_g + o(1). \tag{154}$$

Therefore,

$$X_1^1 - \chi_g^o \xrightarrow[N\to\infty]{a.s.} 0, \tag{155}$$

where χ_g^o was derived in the last section and is given by (136).

Now let $X_1^2 = \frac{1}{K^2}\tilde{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \tilde{\mathbf{h}}_{k,g}$. Under deterministic $\mathbf{Z}_g$, $\tilde{\mathbf{h}}_{k,g} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Z}_g\mathbf{Z}_g^H - \mathbf{\Phi}_g)$. Using this along with Lemma 7 we have,

$$X_1^2 - \frac{1}{K^2}\mathrm{tr}\ (\mathbf{Z}_g\mathbf{Z}_g^H - \mathbf{\Phi}_g)\hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \xrightarrow[N\to\infty]{a.s.} 0. \tag{156}$$

The analysis can be extended to random $\mathbf{Z}_g$s using Fubini Theorem. In fact, we know from Appendix C that,

$$\frac{1}{K^2}\mathrm{tr}\ \mathbf{Z}_g\mathbf{Z}_g^H\hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} = \kappa_g(\mathbf{I}_N) + o(1), \tag{157}$$

$$\frac{1}{K^2}\mathrm{tr}\ \mathbf{\Phi}_g\hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} = \chi_g + o(1). \tag{158}$$

Therefore,

$$X_1^2 - (\kappa_g^o(\mathbf{I}_N) - \chi_g^o) \xrightarrow[N\to\infty]{a.s.} 0, \tag{159}$$

where $\kappa_g^o(\mathbf{A})$ and χ_g^o were derived in the last section and are given by (133) and (136) respectively.

Consequently,

$$X_1 - \left(\kappa_g^o(\mathbf{I}_N) - \chi_g^o + \chi_g^o\right) \xrightarrow[N\to\infty]{a.s.} 0. \tag{160}$$

Next note that,

$$X_2 = -Y_2 \frac{\frac{1}{K}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \mathbf{h}_{k,g}}{1 + \frac{1}{K}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g}}, \tag{161}$$

where,

$$Y_2 = \frac{1}{K^2}\mathbf{h}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g}, \tag{162}$$

Using Lemma 2, we have

$$Y_2 - \frac{1}{K^2}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g} \xrightarrow[N\to\infty]{a.s.} 0. \tag{163}$$

Note that $Y_2 = X_1^1$, which has already been worked out in (155). Also using Lemma 2, we have $\frac{1}{K}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \mathbf{h}_{k,g} - \frac{1}{K}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g} \xrightarrow[N\to\infty]{a.s.} 0$. Now using Lemma 3 and (155) on X_2 yields,

$$X_2 + \frac{m_g}{1+m_g}\chi_g^o \xrightarrow[N\to\infty]{a.s.} 0. \tag{164}$$

Similar analysis yields the deterministic equivalent of X_3 as,

$$X_3 + \frac{m_g}{1+m_g}\chi_g^o \xrightarrow[N\to\infty]{a.s.} 0 \tag{165}$$

Next note that,

$$X_4 = Y_4 \frac{\frac{1}{K}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \mathbf{h}_{k,g} \frac{1}{K} \mathbf{h}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g}}{(1 + \frac{1}{K}\hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g})^2}, \tag{166}$$

where,

$$Y_4 = \frac{1}{K^2} \hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{C}}_{[k,g]}^{-1} \hat{\mathbf{h}}_{k,g}. \tag{167}$$

$Y_4 = X_1^1$, which has already been worked in (155). This yields the deterministic equivalent of X_4 as,

$$X_4 - \frac{m_g^2}{(1+m_g)^2} \chi_g^o \xrightarrow[N\to\infty]{a.s.} 0. \tag{168}$$

Combining (160), (164), (165) and (168) yields the deterministic equivalent of $\Upsilon_{k,g}$. Note that the lemmas utilized in this section not only imply almost sure convergence but also convergence in mean. Therefore,

$$\mathbb{E}[\mathbf{h}_{k,g}^H \hat{\mathbf{V}} \hat{\mathbf{H}}_{[k,g]}^H \mathbf{P}_{[k,g]} \hat{\mathbf{H}}_{[k,g]} \hat{\mathbf{V}} \mathbf{h}_{k,g}] - \left(\kappa_g^o(\mathbf{I}_N) - \chi_g^o + \frac{1}{(1+m_g)^2} \chi_g^o \right) \xrightarrow[N\to\infty]{a.s.} 0. \tag{169}$$

D. Deterministic Equivalent of var $(\boldsymbol{h}_{k,g}^H \hat{\boldsymbol{V}} \hat{\boldsymbol{h}}_{k,g})$

Define the following quantities,

$$a = \hat{\mathbf{h}}_{k,g}^H \hat{\mathbf{V}} \hat{\mathbf{h}}_{k,g}, \tag{170}$$

$$\bar{a} = \mathbb{E}[\mathbf{h}_{k,g}^H \hat{\mathbf{V}} \hat{\mathbf{h}}_{k,g}], \tag{171}$$

$$b = \mathbb{E}[\tilde{\mathbf{h}}_{k,g}^H \hat{\mathbf{V}} \hat{\mathbf{h}}_{k,g}]. \tag{172}$$

By matrix inversion lemma we have $0 \leq a, \bar{a} \leq 1$. Moreover $\mathbb{E}[b] = 0$ and $\mathbb{E}[ab] = \mathbb{E}[ab^*] = 0$. Thus,

$$\text{var }(\mathbf{h}_{k,g}^H \hat{\mathbf{V}} \hat{\mathbf{h}}_{k,g}) = \mathbb{E}[|a + b - \bar{a}|^2], \tag{173}$$

$$= \mathbb{E}[(a - \bar{a})(a + \bar{a})] + \mathbb{E}[|b|^2], \tag{174}$$

$$\leq 2\mathbb{E}[|a - \bar{a}|] + \mathbb{E}[|b|^2]. \tag{175}$$

We have already shown that $a - \frac{m_g}{1+m_g} \xrightarrow[N\to\infty]{a.s.} 0$. Since a and $\bar{a}$ are bounded, so by dominated convergence theorem we have $\mathbb{E}|a - \bar{a}| \xrightarrow[N\to\infty]{a.s.} 0$. Moreover one can show that $\mathbb{E}[|b|^2] \leq \frac{N}{K^2\alpha^2} d_g^2 ||\mathbf{R}_{BS_g}||^2 \xrightarrow[N\to\infty]{a.s.} 0$. Therefore,

$$\text{var }(\mathbf{h}_{k,g}^H \hat{\mathbf{V}} \hat{\mathbf{h}}_{k,g}) \xrightarrow[N\to\infty]{a.s.} 0. \tag{176}$$

Combining the results of the five subsections completes the proof of Theorem 2.

Appendix F
Proof of Corollary 2

Note that this corollary deals with the perfect CSI case for which Theorem 1 and Theorem 2 will be reformulated by setting $\rho_{tr} = \infty$. Under the assumptions of the corollary, the fundamental equations in Theorem 1 can be reduced to,

$$\bar{m} = \frac{1}{1+m}, \tag{177}$$

$$m = \frac{S\delta}{K(1+\bar{m}\delta)}, \tag{178}$$

$$\delta = \frac{1}{\frac{\bar{m}mK}{N\delta} + \frac{S\alpha}{N}}. \tag{179}$$

From (177), we have,

$$m = \frac{1-\bar{m}}{\bar{m}}. \tag{180}$$

Solving (178) for δ and replacing m with (180) yields,

$$\delta = \frac{1-\bar{m}}{\bar{m}\left(\frac{S}{K} - 1 + \bar{m}\right)}. \tag{181}$$

Solving (179) for δ and replacing m with (180) yields,

$$\delta = \frac{N}{S\alpha} - \frac{K}{S\alpha} + \frac{K\bar{m}}{S\alpha}. \tag{182}$$

Equating (181) and (182) and re-arranging the terms as a polynomial in $\bar{m}$ yields,

$$\bar{m}^3 - \left(2 - \frac{S}{K} - \frac{N}{K}\right)\bar{m}^2 + \left(1 - \frac{S}{K} - \frac{N}{K} + \frac{SN}{K^2} + \frac{S\alpha}{K}\right)\bar{m} - \frac{S\alpha}{K} = 0, \tag{183}$$

By Theorem 1, only one of the roots of this polynomial satisfies $(m, \bar{m}, \delta) > 0$. From (180), we have $\bar{m} < 1$. From (181) we have $\bar{m} > \left(1 - \frac{S}{K}\right)$. Hence $\bar{m} \in \left(1 - \frac{S}{K}, 1\right)$.

The SINR in Theorem 2 can be simplified similarly under the assumptions of Corollary 2 by setting $\rho_{tr} = \infty$ and expressing everything in terms of $\bar{m}$. Note that for perfect CSI, the interference term will only depend on $\kappa^o(\mathbf{I}_N)$. Also note that the expression of $M'(\mathbf{R}, \mathbf{L})$ in Lemma 5 can be obtained by expressing (145), simplified for the setting of Corollary 2, as a polynomial in $F(\mathbf{R}, \mathbf{L})$ and applying implicit function theorem with respect to l.

Appendix G
Related Results and Lemmas

Theorem 3 [[25], Corollary 1]: For $k \in \{1, \dots, K\}$, let $(n_k)_{N\geq 1} = (n_k(N))_{N\geq 1}$ be a sequence of positive integers and let $(\mathbf{R}_{k,N})_{N\geq 1}, \mathbf{R}_{k,N} \in \mathbb{C}^{N\times N}$, $(\mathbf{T}_{k,N})_{N\geq 1}, \mathbf{T}_{k,N} \in \mathbb{C}^{n_k\times n_k}$ and

$(\mathbf{D}_N)_{N\geq 1}, \mathbf{D}_N \in \mathbb{C}^{N\times N}$, be three sequences of non-negative definite Hermitian matrices, satisfying lim $\sup_N ||\mathbf{R}_{k,N}|| < \infty$, lim $\sup_N ||\mathbf{T}_{k,N}|| < \infty$ and lim $\sup_N ||\mathbf{D}_N|| < \infty$. Let $(\mathbf{X}_{k,N})_{N\geq 1}, \mathbf{X}_{k,N} \in \mathbb{C}^{N\times n_k}$ be a sequence of random matrices with i.i.d. complex Gaussian entries with zero mean and variance $1/n_k$. Denote $\mathbf{B}_N = \sum_k \mathbf{R}_{k,N}^{1/2}\mathbf{X}_{k,N}\mathbf{T}_{k,N}\mathbf{X}_{k,N}^H\mathbf{R}_{k,N}^{1/2}$. Let $c_k = n_k/N$ and assume that $0 < \liminf_N c_k \leq \limsup_N c_k < \infty$ for all k. Then,

$$\frac{1}{N}\text{tr } \mathbf{D}_N\left(\mathbf{B}_N + \frac{1}{x}\mathbf{I}_N\right) - \frac{1}{N}\text{tr } \mathbf{D}_N\left(\sum_{i=1}^{K}\bar{e}_i\mathbf{R}_{i,N} + \frac{1}{x}\mathbf{I}_N\right) \xrightarrow[N\to\infty]{a.s.} 0, \tag{184}$$

where,

$$\bar{e}_{k,N} = \frac{1}{n_k}\text{tr } \mathbf{T}_{k,N}(e_{k,N}\mathbf{T}_{k,N} + \mathbf{I}_{n_k})^{-1}, \tag{185}$$

$$e_{k,N} = \frac{1}{n_k}\text{tr } \mathbf{R}_{k,N}\left(\sum_{i=1}^{K}\bar{e}_i\mathbf{R}_{i,N} + \frac{1}{x}\mathbf{I}_N\right)^{-1}, \tag{186}$$

such that $\bar{e}_{k,N}, e_{k,N} > 0$ for all k.

Lemma 6 (Common inverses of resolvents:) Given any matrix $\mathbf{H} \in \mathbb{C}^{N\times K}$. Let $\mathbf{h}_k$ denote its k^{th} column and $\mathbf{H}_{[k]}$ denote the matrix obtained after removing the k^{th} column from $\mathbf{H}$. The resolvent matrices of $\mathbf{H}$ and $\mathbf{H}_{[k]}$ are denoted by $\mathbf{C}^{-1}(\alpha) = \left(\frac{1}{K}\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}_N\right)^{-1}$ and $\mathbf{C}_{[k]}^{-1}(\alpha) = \left(\frac{1}{K}\mathbf{H}_{[k]}\mathbf{H}_{[k]}^H + \alpha\mathbf{I}_N\right)^{-1}$. Then,

$$\mathbf{C}^{-1}(\alpha) = \mathbf{C}_{[k]}^{-1}(\alpha) - \frac{1}{K}\frac{\mathbf{C}_{[k]}^{-1}(\alpha)\mathbf{h}_k\mathbf{h}_k^H\mathbf{C}_{[k]}^{-1}(\alpha)}{1 + \frac{1}{K}\mathbf{h}_k^H\mathbf{C}_{[k]}^{-1}(\alpha)\mathbf{h}_k}, \tag{187}$$

and also,

$$\mathbf{C}^{-1}(\alpha)\mathbf{h}_k = \frac{\mathbf{C}_{[k]}^{-1}(\alpha)\mathbf{h}_k}{1 + \frac{1}{K}\mathbf{h}_k^H\mathbf{C}_{[k]}^{-1}(\alpha)\mathbf{h}_k}, \tag{188}$$

Lemma 7 (Convergence of quadratic forms): Let $x_N = [x_1, x_2, \ldots, x_N]^T$ be a $N \times 1$ vector with i.i.d. complex Gaussian random variables with unit variance. Let $\mathbf{A}_N$ be an $N \times N$ matrix independent of $\mathbf{x}_N$, whose spectral norm is bounded; that is, there exists $C_A < \infty$ such that $||\mathbf{A}|| \leq C_A$. Then, for any $p \geq 1$, there exists a constant C_p depending only on p, such that,

$$\mathbb{E}_{\mathbf{x}_N}\left[\left|\frac{1}{N}\mathbf{x}_N^H\mathbf{A}_N\mathbf{x}_N - \frac{1}{N}\text{tr }(\mathbf{A}_N)\right|^p\right] \leq \frac{C_pC_A^p}{N^{p/2}}, \tag{189}$$

where the expectation is taken over the distribution of $\mathbf{x}_N$. By choosing $p \geq 2$, we thus have,

$$\frac{1}{N}\mathbf{x}_N^H\mathbf{A}_N\mathbf{x}_N - \frac{1}{N}\text{tr }(\mathbf{A}_N) \xrightarrow[N\to\infty]{a.s.} 0. \tag{190}$$

Lemma 8: Let $\mathbf{A}_N$ be as in Lemma 6 and $\mathbf{x}_N, \mathbf{y}_N$ be random, mutually independent with complex Gaussian entries of zero mean and variance 1. Then,

$$\frac{1}{N}\mathbf{x}_N^H \mathbf{A}_N \mathbf{y}_N \xrightarrow[N\to\infty]{a.s.} 0. \tag{191}$$

Lemma 9 (Rank-one perturbation lemma): Let $\mathbf{C}^{-1}(\alpha)$ and $\mathbf{C}_{[k]}^{-1}(\alpha)$ be the resolvent matrices as defined in Lemma 5. Then for any matrix $\mathbf{A}$ we have,

$$\frac{1}{N}\text{tr}(\mathbf{A}(\mathbf{C}^{-1}(\alpha) - \mathbf{C}_{[k]}^{-1}(\alpha))) \leq ||\mathbf{A}||. \tag{192}$$

Lemma 10: Let X_N, Y_N be two scalar random variables, with variances $var(X_N) = O(N^{-2})$ and $var(Y_N) = O(N^{-2})$. Then,

$$\mathbb{E}[X_N Y_N] = \mathbb{E}[X_N]\mathbb{E}[Y_N] + o(1). \tag{193}$$

Lemma 11 (Matrix Inversion Lemma): Let $\mathbf{A}$ be $N \times N$ Hermitian invertible matrix. Then for any vector $\mathbf{x} \in \mathbb{C}^{N\times 1}$ and any scalar $\tau \in \mathbb{C}$ such that $\mathbf{A} + \tau\mathbf{x}\mathbf{x}^H$ is invertible,

$$\mathbf{x}^H(\mathbf{A} + \tau\mathbf{x}\mathbf{x}^H) = \frac{\mathbf{x}^H\mathbf{A}^{-1}}{1 + \tau\mathbf{x}^H\mathbf{A}^{-1}\mathbf{x}}. \tag{194}$$

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