Distributed event-triggered control strategies for multi-agent formation stabilization and tracking

Christophe Viel, Sylvain Bertrand, Michel Kieffer, Hélène Piet-Lahanier

Distributed event-triggered control strategies for multi-agent formation stabilization and tracking

Christophe Viel [a] Sylvain Bertrand [a] Michel Kieffer [b] Hélène Piet-Lahanier [a]

[a] *ONERA - The French Aerospace Lab, F-91123 Palaiseau (e-mail: firstname.lastname@onera.fr).*

[b] *L2S, Univ Paris-Sud, CNRS, CentraleSupelec, F-91190 Gif-sur-Yvette (e-mail: firstname.lastname@lss.supelec.fr)*

Abstract

This paper addresses the problem of formation control and tracking of some reference trajectory by an Euler-Lagrange multi-agent systems. The reference trajectory is only known by a subset of agents. This work is inspired by recent results by Yang *et al.* and adopts an event-triggered control strategy to reduce the number of communications between agents. For that purpose, to evaluate its control input, each agent maintains estimators of the states of its neighbour agents, as well as an estimate of its reference trajectory. Communication is triggered when the discrepancy between the actual state of an agent and the estimate of this state as evaluated by neighboring agents reaches some threshold. Communications are also triggered when the reference trajectory estimate is degraded. The impact of additive state perturbations on the formation control is studied. A condition for the convergence of the multi-agent system to a stable formation is studied. The time interval between two consecutive communications by the same agent is shown to be strictly positive. Simulations show the effectiveness of the proposed approach.

Key words: Communication constraints, event-triggered control, formation stabilization, multi-agent system (MAS).

1 Introduction

Distributed cooperative control of a multi-agent system (MAS) usually requires significant exchange of information between agents. In early contributions, see, *e.g.*, Olfati-Saber et al. (2007); Wei (2008), communication was considered permanent. Recently, more practical approaches have been proposed. For example, in Wen et al. (2012a,b, 2013), communication is intermittent, alternating phases of permanent communication and of absence of communication. Alternatively, communication may only occur at discrete time instants, either periodically as in Garcia et al. (2014), or triggered by some event, as in Dimarogonas et al. (2012); Fan et al. (2013); Zhang et al. (2015); Viel et al. (2016).

This paper proposes a strategy to reduce the number of communications for displacement-based formation control while following a desired reference trajectory, only known by a subset of agents. Agent dynamics are described by Euler-Lagrange models and include perturbations. This work extends results presented in Yang et al. (2015) by introducing an event-triggered strategy, and results of Liu et al. (2015); Sun et al. (2015); Tang et al. (2011) by addressing systems with more complex dynamics than a simple integrator.

To evaluate its control input in a distributed way, each agent estimates the state of its neighbors and as well as its reference trajectory. In absence of permanent communication, the quality of the state and reference trajectory estimates is difficult to evaluate. To address this issue, each agent maintains also an estimate of its own state using only the information it has shared with its neighbors. Information is communicated by the considered agent with its neighbors as soon as the discrepancy between its actual state and its own state estimate reaches some threshold. Communication is also used to maintain the quality of the estimate of the reference trajectory of each agent. The main difficulty consists in determining the communication triggering condition (CTC) that will ensure the completion of the task assigned to the MAS while reducing the number of communications between agents.

This paper is organized as follows. Some assumptions are introduced in Section 2 and the formation parametrization is described in Section 3. As the problem considered here is to drive a formation of agents along a desired reference trajectory, the designed distributed control law consists of two parts. The first part (see Section 3) drives the agents to some target formation and maintains the formation, despite the presence of perturbations. It is based on estimates of the states of the agents described in Section 5.3. The second part (see Section 4) is dedicated to the tracking of the desired trajectory. Communication instants are chosen locally by each agent using an event-triggered approach introduced in Section 6. A simulation example is considered in Section 7 to illustrate the reduction of the communications obtained by the proposed approach. Finally, conclusions are drawn in Section 8.

2 Notations and hypotheses

Consider a MAS consisting of a network of N agents whose topology is described by an undirected connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$. $\mathcal{N}$ is the set of nodes and $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ the set of edges of the network. The set of neighbours of Agent i is $\mathcal{N}_i = \{j \in \mathcal{N} | (i,j) \in \mathcal{E},\ i \neq j\}$. N_i is the cardinal number of $\mathcal{N}_i$. For some vector $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$, we define $|x| = [|x_1|, |x_2|, \ldots, |x_n|]^T$ where $|x_i|$ is the absolute value of the i-th component of x. Similarly, $x \geq 0$ indicates that each component x_i of x is non negative, *i.e.*, $x_i \geq 0\ \forall i \in \{1 \ldots n\}$.

Let $q_i \in \mathbb{R}^n$ be the vector of *coordinates* of Agent i in some global fixed reference frame $\mathcal{R}$ and let $q = \left[q_1^T, q_2^T, \ldots, q_N^T\right]^T \in \mathbb{R}^{N.n}$ be the *configuration* of the MAS. The dynamics of each agent is described by the Euler-Lagrange model

$$M_i(q_i)\,\ddot{q}_i + C_i(q_i,\,\dot{q}_i)\,\dot{q}_i + G = \tau_i + d_i, \qquad (1)$$

where $\tau_i \in \mathbb{R}^n$ is some control input, $M_i(q_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n\times n}$ is the matrix of the Coriolis and centripetal term, G accounts for gravitational acceleration supposed to be known and constant, and d_i is a time-varying state perturbation satisfying $\forall t\ \|d_i(t)\| \leqslant D_{\max}$. The state vector of Agent i is $x_i^T = \left[q_i^T, \dot{q}_i^T\right]$. The convergence proof of the control strategy developed in this paper requires considering the following assumptions on the dynamics. Assumptions A1-A3 have been already considered, *e.g.*, in Makkar et al. (2007); Mei et al. (2011).

A1) $M_i(q_i)$ is symmetric positive and there exists $k_M > 0$ satisfying $\forall x,\ x^T M_i(q_i)\,x \leq k_M x^T x$.

A2) $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric or negative definite and there exists $k_C > 0$ satisfying $\forall x$, $x^T C_i(q_i, \dot{q}_i)\,x \leq k_C \|\dot{q}_i\|\, x^T x$.

A3) The left-hand side of (1) is linearly parametrized as

$$M_i(q_i)\,x_1 + C_i(q_i,\,\dot{q}_i)\,x_2 = Y_i(q_i,\,\dot{q}_i,\,x_1,\,x_2)\,\theta_i \quad (2)$$

for all vectors $x_1,\, x_2 \in \mathbb{R}^n$, where $Y_i(q_i,\,\dot{q}_i,\,x_1,\,x_2)$ is a regressor matrix with known structure and θ_i is a vector of unknown constant parameters associated with the i-th agent.

A4) For each $i = 1, \ldots, N$, θ_i is such that $\theta_{\min,i} < \theta_i < \theta_{\max,i}$, with known $\theta_{\min,i}$ and $\theta_{\max,i}$.

Moreover, one assumes that

A5) Each Agent i measures its state x_i without error,

A6) There are no packet losses or communication delays.

In what follows, the notations M_i and C_i are used to replace $M_i(q_i)$ and $C_i(q_i, \dot{q}_i)$.

3 Formation control problem

This section describes first the target formation parametrization. The potential energy of a MAS is introduced to quantify the discrepancy between the current and target formations. It will have to be minimized. The notion of bounded convergence is also described.

3.1 Formation parametrization

Consider the relative coordinate vector $r_{ij} = q_i - q_j$ between two agents i and j and the target relative coordinate vector r_{ij}^* for all $(i,\,j) \in \mathcal{N}$. A target formation is defined by the set $\left\{r_{ij}^*, (i,\,j) \in \mathcal{N}\right\}$. In what follows, one assumes that Agent i has only access to r_{ij}^*, with $j \in \mathcal{N}_i$. The *potential energy*

$$P(q,\,t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} k_{ij}\left\|r_{ij} - r_{ij}^*\right\|^2 \qquad (3)$$

of the formation represents the disagreement between r_{ij} and r_{ij}^*, see Yang et al. (2015). In (3), the spring coefficients $k_{ij} = k_{ji}$ can be positive or null, and $k_{ii} = 0$. The minimum number of non-zero coefficients k_{ij} to properly define a target formation is $N-1$ since $\mathcal{G}$ is connected. Then, one may choose $k_{ij} \neq 0$ iff $(i,j) \in \mathcal{E}$. As will be seen, with this choice of the spring coefficients, each agent will have to estimate only the state of its neighbours to evaluate its control input.

Definition 1 *Yang et al. (2015) The MAS asymptotically converges to the target formation with a bounded error iff there exists some $\epsilon_1 > 0$ such that*

$$\lim_{t\to\infty} P(q,\,t) \leqslant \epsilon_1. \qquad (4)$$

A control law designed to reduce the potential energy $P(q,\,t)$ leads to a bounded convergence of the MAS.

4 Time-varying formation and trajectory

4.1 Main idea and notations

In this section, the MAS has to follow some reference trajectory, only known by a subset $\mathcal{N}_{\mathrm{L}} \subset \mathcal{N}$ of $N_{\mathrm{L}} \leqslant N$ agents, named leaders. Moreover, one assumes that the target formation may be time-varying and is represented by the relative configuration matrix $r^*(t)$. Each agent i is only assumed to know $r_{ij}^*(t)$ for all $j \in \mathcal{N}_i$.

Without communication constraint, in Mei et al. (2011); Sun et al. (2009), the entire formation is driven by the leaders using some spring effect. A direct adaptation of this idea to event-triggered methods leads to a large amount of communications to update the estimates of the states of leaders by other agents.

Here each agent maintains a first estimate $\overline{\boldsymbol{q}}_i^{i*}(t) = [\overline{q}_i^{i*T}(t),\, \dot{\overline{q}}_i^{*T}(t),\, \ddot{\overline{q}}_i^{*T}(t)]^T$ of its own reference trajectory $\boldsymbol{q}_i^*(t) = [q_i^{*T}(t), \dot{q}_i^{*T}(t), \ddot{q}_i^{*T}(t)]^T$ using all information it has access to.

When a communication is triggered at time t by some Agent i, it transmits to its neighbors either its reference $\boldsymbol{q}_i^*(t)$ if $i \in \mathcal{N}_{\mathrm{L}}$, or its estimated reference $\overline{\boldsymbol{q}}_i^{i*}(t)$ if $i \notin \mathcal{N}_{\mathrm{L}}$. In both cases, the neighbors $j \in \mathcal{N}_i$ may update the estimate of their own reference trajectories $\overline{\boldsymbol{q}}_j^{j*}(t)$ using $\boldsymbol{q}_i^*(t) + \boldsymbol{r}_{ij}^*$ or $\overline{\boldsymbol{q}}_i^{i*}(t) + \boldsymbol{r}_{ij}^*$ where $\boldsymbol{r}_{ij}^* = [\, r_{ij}^{*T} \;\; \dot{r}_{ij}^{*T} \;\; \ddot{r}_{ij}^{*T}\,]^T$ (see Section 4.2). Reference trajectory estimates are thus forwarded through the network when agents trigger communications.

Each agent $i \in \mathcal{N}$ uses in its control input either the reference trajectory $\boldsymbol{q}_i^*(t)$ if $i \in \mathcal{N}_{\mathrm{L}}$ or an estimate $\overline{\boldsymbol{q}}_i^{i*}(t)$ of $\boldsymbol{q}_i^*(t)$ if $i \notin \mathcal{N}_{\mathrm{L}}$. Additionally, an estimate of the reference trajectory $\boldsymbol{q}_i^*(t)$ or $\overline{\boldsymbol{q}}_i^{i*}(t)$ used by Agent i is required by Agent j to evaluate $\widehat{q}_i^j$, its estimate of the state q_i of Agent i (see Section 5.3). The estimate of $\boldsymbol{q}_i^*(t)$ or $\overline{\boldsymbol{q}}_i^{i*}(t)$ evaluated by Agent j is denoted $\widehat{\boldsymbol{q}}_i^{i,j*}(t)$. This estimate only uses information received from Agent i and is updated only when Agent i broadcasts a message. To evaluate the quality of $\widehat{\boldsymbol{q}}_i^{i,j*}(t)$, each agent maintains a second estimate $\widehat{\boldsymbol{q}}_i^{i,i*}(t)$ of its own reference trajectory $\boldsymbol{q}_i^*(t)$ or $\overline{\boldsymbol{q}}_i^{i*}(t)$ using only information it has provided to its neighbors. Since $\widehat{\boldsymbol{q}}_i^{i,i*}(t_{i,k})$ and $\widehat{\boldsymbol{q}}_i^{i,j*}(t_{i,k})$ are evaluated using the same information broadcast by Agent i, using Assumption A6, one has for all t, $\widehat{\boldsymbol{q}}_i^{i,i*}(t) = \widehat{\boldsymbol{q}}_i^{i,j*}(t)$. A communication is triggered by Agent i when the discrepancy between $\boldsymbol{q}_i^*(t)$ or $\overline{\boldsymbol{q}}_i^{i*}(t)$ and $\widehat{\boldsymbol{q}}_i^{i,i*}(t)$, *i.e.*, between its (actual or estimated) reference trajectory and that estimated by its neighbors becomes too large.

One assumes that the evolution of the reference trajectories for all $i \in \mathcal{N}_{\mathrm{L}}$ are described by

$$\dot{\boldsymbol{q}}_i^*(t) = f(\boldsymbol{q}_i^*(t), t), \tag{5}$$

whereas the estimate of the reference trajectories by Agent $i \notin \mathcal{N}_{\mathrm{L}}$ is assumed to be described by

$$\dot{\overline{\boldsymbol{q}}}_i^{i*}(t) = \overline{f}(\overline{\boldsymbol{q}}_i^{i*}(t), t). \tag{6}$$

In what follows, the time instant at which the k-th message is sent by Agent i is denoted as $t_{i,k}$. Let $t_{i,k}^j$ be the time at which the k-th message sent by Agent i is received by Agent j. According to Assumption A6, $t_{i,k}^j = t_{i,k}$ for all $j \in \mathcal{N}_i$. Let t_k^i be the time at which Agent i received its k-th message from any other agent in the network.

To simplify description, one assumes that $\mathcal{N}_{\mathrm{L}}$ consists of a single agent $\mathcal{N}_{\mathrm{L}} = \{1\}$, so the MAS reference trajectory is $\boldsymbol{q}_1^*(t)$. Extension to multiple leaders is straightforward.

Definition 2 *The MAS reaches its tracking objective iff there exists $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that (4) is satisfied and*

$$\lim_{t\to\infty} \|q_1(t) - q_1^*(t)\| \leqslant \varepsilon_2, \tag{7}$$

i.e., *iff the reference agent asymptotically converges to the reference trajectory, and the MAS asymptotically converges to the target formation with bounded errors.*

4.2 Estimation of the reference trajectory

The aim of this section is determine when an Agent j has to update the estimate of its own reference trajectory $\overline{\boldsymbol{q}}_j^{j*}(t)$ using $\boldsymbol{q}_1^*(t) + \boldsymbol{r}_{1j}^*$ or $\overline{\boldsymbol{q}}_i^{i*}(t) + \boldsymbol{r}_{ij}^*$ when a message has been received from Agent i. The update is only performed when the estimate becomes more accurate. This is always the case when $\boldsymbol{q}_1^*(t)$ is received from the leader. When $\overline{\boldsymbol{q}}_i^{i*}(t)$ is received, the update is performed only when $\overline{\boldsymbol{q}}_i^{i*}(t)$ has been updated from $\boldsymbol{q}_1^*(t)$ more recently than $\overline{\boldsymbol{q}}_j^{j*}(t)$.

For that purpose, at time t, let $t^{i*}(t)$ be the time of the most recent information about $\boldsymbol{q}_1^*$ available by Agent i. The leader knows $\boldsymbol{q}_1^*$ and thus, $\overline{\boldsymbol{q}}_1^{1*}(t) = \boldsymbol{q}_1^*(t)$ and $t^{1*}(t) = t$ for all t. If Agent i receives a message at time $t = t_{j,k}^i$ from Agent j, it compares $t^{i*}(t)$ with $t^{j*}(t)$. If $t^{i*}(t) < t^{j*}(t)$, Agent i uses the information provided by Agent j to update its estimate of $\boldsymbol{q}_i^*$ as $\bar{\boldsymbol{q}}_i^{i*}(t_{j,k}^i) = \bar{\boldsymbol{q}}_j^{j*}(t_{j,k}^i) + \boldsymbol{r}_{ji}^*$ and $t^{i*}(t_{j,k}^i) = t^{j*}(t_{j,k}^i)$.

If $t_{j,k}^i$ is the time instant of the last message received by Agent i, the evolution of $\bar{\boldsymbol{q}}_i^{i*}(t)$ for $t \geqslant t_{j,k}^i$ is then described by (6) with $\bar{\boldsymbol{q}}_i^{i*}(t_{j,k}^i)$ known.

5 Distributed control approach

A distributed control law is designed to achieve bounded convergence of the MAS. Consider the trajectory error $\varepsilon_i = q_i - q_i^*$, $\bar{\varepsilon}_j^i = q_i - \overline{q}_j^{i*}$ and $\widehat{\varepsilon}_i^j = \widehat{q}_i^j - \widehat{q}_i^{i,j*}$ where $\widehat{q}_i^{i,j*}$ is the estimation of $\overline{q}_i^{i*}$ performed by Agent j described in Section 4.1. To describe the evolution of $P(q, t)$ and ε_i, one introduces

$$g_i = \frac{\partial P(q, t)}{\partial q_i} + k_0\varepsilon_i = \sum_{j\in\mathcal{N}_i} k_{ij}\left(r_{ij} - r_{ij}^*\right) + k_0\bar{\varepsilon}_i^i \tag{8}$$

$$\dot{g}_i = \sum_{j\in\mathcal{N}_i} k_{ij}\left(\dot{r}_{ij} - \dot{r}_{ij}^*\right) + k_0\dot{\bar{\varepsilon}}_i^i \tag{9}$$

$$s_i = \dot{q}_i - \dot{\overline{q}}_i^{i*} + k_p g_i \tag{10}$$

where g_i and $\dot{g}_i$ characterize the evolution of the discrepancy between the current and target formations, $k_0 \geq 0$ and $k_p \geq 0$ are scalar design parameters. The parameter k_0 adjusts the trade-off between the trajectory tracking error and the potential energy of the formation. When no reference trajectory is considered, $k_0 = 0$.

5.1 Control design

In a distributed context with limited communications between agents, agents cannot have permanent access to q. Thus, for all $j \in \mathcal{N}_i$, one introduces the estimate $\widehat{q}_j^i$ of q_j performed by Agent i to replace the missing information in the control law. The evaluation of $\widehat{q}_j^i$ is described in Section 5.3.

Using $\widehat{q}_j^i$, $j \in \mathcal{N}_i$, Agent i can estimate (8) and (10) as

$$\overline{g}_i = \sum_{j\in\mathcal{N}_i} k_{ij}\left(\overline{r}_{ij} - r_{ij}^*\right) + k_0\overline{\varepsilon}_i^i \quad (11)$$

$$\overline{s}_i = \dot{q}_i - \dot{\overline{q}}_i^{i*} + k_p\overline{g}_i \quad (12)$$

with $\overline{r}_{ij} = q_i - \widehat{q}_j^i$ and $\dot{\overline{r}}_{ij} = \dot{q}_i - \dot{\widehat{q}}_j^i$. Using $\overline{g}_i$ and $\overline{s}_i$, Agent i can evaluate the following adaptive distributed control input to be used in (1)

$$\tau_i = -k_s\overline{s}_i - k_g\overline{g}_i + G - Y_i\left(q_i,\, \dot{q}_i,\, \dot{\overline{p}}_i,\, \overline{p}_i\right)\overline{\theta}_i \quad (13)$$

$$\dot{\overline{\theta}}_i = \Gamma_i Y_i\left(q_i,\, \dot{q}_i,\, \dot{\overline{p}}_i,\, \overline{p}_i\right)^T\overline{s}_i \quad (14)$$

where $\overline{p}_i = k_p\overline{g}_i - \dot{\overline{q}}_i^{i*}$ and $\dot{\overline{p}}_i = k_p\dot{\overline{g}}_i - \ddot{\overline{q}}_i^{i*}$.

5.2 Communication protocol

When a communication is triggered at $t_{i,k}$ by Agent i, it transmits a message containing $t_{i,k}$, $q_i\left(t_{i,k}\right)$, $\dot{q}_i\left(t_{i,k}\right)$, $\overline{\boldsymbol{q}}_i^{i*}(t_{i,k})$, t^{i*} and $\overline{\theta}_i\left(t_{i,k}\right)$. Upon reception of this message, the neighbors of Agent i update their estimate of the state of Agent i and of the reference trajectory using this information as described in Sections 4.2.

5.3 Estimator dynamics

To evaluate its control law, Agent i maintains estimates $\widehat{q}_j^i$ of q_j for its neighbours $j \in \mathcal{N}_i$, such that

$$\widehat{x}_j^i\left(t_{j,k}^i\right) = x_j\left(t_{j,k}^i\right) \quad (15)$$

and $\forall t \in \left[t_{j,k}^i,\, t_{j,k+1}^i\right[$,

$$\widehat{M}_j^i\left(\widehat{q}_j^i\right)\ddot{\widehat{q}}_j^i + \widehat{C}_j^i\left(\widehat{q}_j^i,\, \dot{\widehat{q}}_j^i\right)\dot{\widehat{q}}_j^i + G = \widehat{\tau}_j^i. \quad (16)$$

where $\widehat{x}_j^{iT} = [\widehat{q}_j^{iT}, \dot{\widehat{q}}_j^{iT}]$, $\widehat{M}_j^i(\widehat{q}_j^i)$, and $\widehat{C}_j^i(\widehat{q}_j^i,\, \dot{\widehat{q}}_j^i)$ are estimates of M_j and C_j evaluated from $Y_j(\widehat{q}_j^i,\, \dot{\widehat{q}}_j^i,\, x_1,\, x_2)$, and $\overline{\theta}_j(t_{j,k}^i)$ using $\forall(x_1, x_2) \in \mathbb{R}^2$

$$\widehat{M}_j^i\left(\widehat{q}_j^i\right)x_1 + \widehat{C}_j^i\left(\widehat{q}_j^i,\, \dot{\widehat{q}}_j^i\right)x_2 = Y_j\left(\widehat{q}_j^i,\, \dot{\widehat{q}}_j^i,\, x_1,\, x_2\right)\overline{\theta}_j\left(t_{j,k}^i\right).$$

The estimator (16) managed by Agent i requires an estimate $\widehat{\tau}_j^i$ of τ_j evaluated by Agent j. This estimate is evaluated by Agent i as follows

$$\begin{aligned}\widehat{\tau}_j^i =& -k_s\left(\dot{\widehat{\varepsilon}}_j^i + k_pk_0\widehat{\varepsilon}_j^i\right) - k_gk_0\widehat{\varepsilon}_j^i + G \\ & - Y_j\left(\widehat{q}_j^i,\, \dot{\widehat{q}}_j^i,\, \dot{\widehat{m}}_j^i,\, \widehat{m}_j^i\right)\widehat{\theta}_j^i \quad (17)\\ \dot{\widehat{\theta}}_j^i =& \Gamma_jY_j\left(\widehat{q}_j^i,\, \dot{\widehat{q}}_j^i,\, \dot{\widehat{m}}_j^i,\, \widehat{m}_j^i\right)^T\left(\dot{\widehat{\varepsilon}}_j^i + k_pk_0\widehat{\varepsilon}_j^i\right) \quad (18)\\ \widehat{\theta}_j^i\left(t_{j,k}^i\right) =& \overline{\theta}_j\left(t_{j,k}^i\right) \quad (19)\end{aligned}$$

where $\widehat{\theta}_j^i$ is the estimate of $\overline{\theta}_j$, $\widehat{\varepsilon}_j^i = \widehat{q}_j^i - \widehat{q}_j^{j,i*}$, and $\widehat{m}_j^i = k_pk_0\widehat{\varepsilon}_j^i - \dot{\widehat{q}}_j^{j,i*}$ if $k_0 > 0$, *i.e.*, in the case of a reference trajectory to be tracked and $\widehat{m}_j^i = 0$ else. Note that if $k_0 = 0$, $\dot{q}_j^* = 0$.

The term $\widehat{\boldsymbol{q}}_i^{i,j*}$ is the estimate of $\overline{\boldsymbol{q}}_i^{i*}$ performed by Agent j, using (20). The evolution of $\widehat{\boldsymbol{q}}_j^{j,i*}$ uses (6) and is described by

$$\widehat{\boldsymbol{q}}_i^{i,j*}\left(t_{i,k}^j\right) = \overline{\boldsymbol{q}}_i^{i*}\left(t_{i,k}^j\right). \quad (20)$$

$$\dot{\widehat{\boldsymbol{q}}}_i^{i,j*}(t) = \overline{f}\left(\widehat{\boldsymbol{q}}_i^{i,j*}(t),t\right) \quad \forall t \in \left[t_{i,k}^j,\, t_{k+1}^j\right[\quad (21)$$

Note that $\widehat{\boldsymbol{q}}_i^{i,i*}$ is updated only when Agent i broadcasts a message, while $\overline{\boldsymbol{q}}_i^{i*}$ is potentially updated each time Agent i receives information from other agents.

To evaluate (16)-(19) as well as $\widehat{\boldsymbol{q}}_j^i$, Agent i only requires messages from Agent $j \in \mathcal{N}_i$.

Assumption A6 and the structure of the estimator (16)-(17) ensure that $\widehat{\boldsymbol{q}}_i^i(t) = \widehat{\boldsymbol{q}}_i^j(t)$ for all $i \in \mathcal{N}$ and $j \in \mathcal{N}_i$. This simplifies the convergence and stability analysis detailed in Viel et al. (2017b).

6 Event-triggered communications

Due to the presence of state perturbations, the non-permanent communication, and the mismatch between θ_i, $\overline{\theta}_i$, and $\widehat{\theta}_i^j$, there is usually a discrepancy between q_i and its estimate $\widehat{q}_i^j$ by Agent j denoted as

$$e_i^j = \widehat{q}_i^j - q_i,\; j \in \mathcal{N}_i, \quad (22)$$

which is used to trigger communications. Agent i can estimate e_i^j by running an estimator of its own state using only information transmitted to its neighbours. This is useful to detect when the discrepancy between $\widehat{q}_i^j$ and q_i is large.

Theorem 3 introduces a CTC used to trigger communications to ensure a bounded asymptotic convergence of the MAS to the reference trajectory. Each agent is assumed to know the initial value of the state of its neighbours. This condition can be satisfied by triggering a communication at time $t = 0$.

Let $k_{\min} = \min\left(k_{ij} \neq 0\right)$, $k_{\max} = \max\left(k_{ij}\right)$, $\alpha_i = \sum_{j=1}^N k_{ij}$, $\alpha_{\min} = \min\alpha_i$, and $\alpha_{\mathrm{M}} = \max\alpha_i$. Define also for $\overline{\theta}_i \in \mathbb{R}^p$, $\Delta\theta_i = \overline{\theta}_i - \theta_i$, $\overline{\theta}_i = \left[\overline{\theta}_{i,1}, \ldots, \overline{\theta}_{i,p}\right]^T$, and, using Assumption A4,

$$\Delta\theta_{i,\max} = \begin{bmatrix}\max\left\{\left|\overline{\theta}_{i,1} - \theta_{\min,i,1}\right|, \left|\overline{\theta}_{i,1} - \theta_{\max,i,1}\right|\right\}\\ \vdots \\ \max\left\{\left|\overline{\theta}_{i,p} - \theta_{\min,i,p}\right|, \left|\overline{\theta}_{i,p} - \theta_{\max,i,p}\right|\right\}\end{bmatrix}. \quad (23)$$

Theorem 3 *Consider a MAS with agent dynamics given by* (1) *and the control law* (13). *Consider some design parameters* $\eta \geq 0$, $\eta_2 > 0$, $0 < b_i < \frac{k_s}{k_s k_p + k_g}$,

$$c_3 = \frac{3}{4} \frac{\min\left\{\frac{1}{2}, k_1, k_p, k_0, 2\left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right)\right\}}{\max\{1, k_M\}}$$

and $k_1 = k_s - (1 + k_p(k_M + 1))$. *In absence of communication delays, the system (1) is input-to-state practically stable (ISpS),see Jiang et al. (1996), and the agents can be driven to some target formation such that*

$$\lim_{t\to\infty}\left(\sum_{i\in\mathcal{N}\setminus\mathcal{N}_L} k_0 \left\|\bar{\varepsilon}_i^i\right\|^2 + \sum_{i\in\mathcal{N}_L} k_0 \left\|\varepsilon_i\right\|^2 + \frac{1}{2}P(q,t)\right) \leq \xi$$

with

$$\xi = \frac{N}{k_g c_3}\left[D_{\max}^2 + \eta + c_3 \Delta_{\max}\right] \tag{24}$$

where $\Delta_{\max} = \max_{i=1:N}\left(\sup_{t>0}\left(\Delta\theta_i^T \Gamma_i^{-1} \Delta\theta_i\right)\right)$, *if the communications are triggered when one of the following conditions is satisfied*

$$\|\dot{q}_i\| \geq \left\|\dot{\widehat{q}}_i^i\right\| + \eta_2 \tag{25}$$

$$k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta \leq \alpha_M^2\left(k_e e_i^{iT} e_i^i + k_p k_M \dot{e}_i^{iT} \dot{e}_i^i\right)$$
$$+ \alpha_M k_C^2 k_p \left\|e_i^i\right\|^2 \sum_{j=1}^N k_{ji}\left[\left\|\dot{\widehat{q}}_j^i\right\| + \eta_2\right]^2 \tag{26}$$
$$+ k_p \left\|e_i^i\right\|\left[\alpha_M^2 k_y\left(1 + \left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2\right)\right.$$
$$\left. + \frac{\left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2}{k_y\left(1 + \left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2\right)}\right] \tag{27}$$
$$+ \frac{k_g}{k_3}\sum_{j=1}^N k_{ij}\left\|\dot{\bar{q}}_i^{i*} - \dot{\tilde{q}}_i^{j,i*}\right\|^2 + 4\alpha_M k_g k_m \sum_{j=1}^N k_{ij}\left\|\bar{q}_i^{i*} - \tilde{q}_i^{j,i*}\right\|^2$$
$$+ k_g b_i \left\|\dot{q}_i - \dot{\bar{q}}_i^{i*}\right\|^2, \tag{28}$$

where $k_3 = \min\left\{\frac{1}{2}k_0, 2\left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right)k_0, 1\right\}$, $k_m = \min\{k_1, k_p\}$, $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$, $Y_i = Y_i\left(q_i, \dot{q}_i, \dot{\bar{p}}_i, \bar{p}_i\right)$, *and* $k_y > 0$ *a design parameter. Moreover, consecutive communication triggering time instants satisfy* $t_{i,k+1} > t_{i,k}$. □

In Theorem 3, $\tilde{q}_i^{j,i*}$ denotes the last estimate of the reference trajectory shared between Agents i and j, such that

$$\tilde{q}_i^{j,i*} = \begin{cases} \widehat{q}_i^{i,i*} & \text{if } t_{i,k_i} \geq t_{j,k_j} \\ \widehat{q}_i^{j,i*} & \text{if } t_{i,k_i} < t_{j,k_j}. \end{cases}$$

The proof of Theorem 3 is given in Viel et al. (2017b). When Agent i broadcasts a message, $\widehat{x}_i^i$ is updated using x_i as shown in (15), thus e_i^i and $\dot{e}_i^i$ are reset. Similarly, $\widehat{x}_i^{i,i*}$ is updated using $\tilde{x}_i^{i*}$ as shown in (20) and consequently $\widehat{q}_i^{i,i*}$ and $\dot{\widehat{q}}_i^{i,i*}$ are reset to $\bar{q}_i^{i*}$ and $\dot{\bar{q}}_i^{i*}$. This ensures that the CTC is no more satisfied immediately after Agent i has broadcast its message, avoiding continuous communication. The proof of $t_{i,k+1} - t_{i,k} > 0$ is provided in Viel et al. (2017b).

The CTCs proposed in Theorem 3 are analyzed assuming that the estimators of the state and reference trajectory of the agents and the communication protocol are such that $\forall i, j \in \mathcal{N} \times \mathcal{N}$,

$$\widehat{x}_i^i(t) = \widehat{x}_i^j(t) \tag{29}$$
$$\widehat{x}_i^i(t_{i,k}) = x_i^i(t_{i,k}) \tag{30}$$
$$\widehat{\boldsymbol{q}}_1^{i,j*}(t) = \widehat{\boldsymbol{q}}_1^{i,i*}(t) \tag{31}$$
$$\widehat{\boldsymbol{q}}_1^{i,j*}(t_{i,k}) = \widehat{\boldsymbol{q}}_1^{i,i*}(t_{i,k}) \tag{32}$$

These properties are actually satisfied if the communication protocol described in Section 5.3 and the state estimator (16) and reference trajectory estimator (20) are employed. Theorem 3 is valid independently of the way the estimate $\widehat{x}_i^i$ of x_i is evaluated provided that (29) and (32) are satisfied.

From (3) and (28), one sees that η can be used to adjust the trade-off between the bound ξ on the formation and tracking errors and the amount of triggered communications. If $\eta = 0$, there is no perturbation and θ_i is perfectly known, the system converges asymptotically.

The left term in (26) depends on the potential energy of the formation, which measures the discrepancy of the MAS with its target formation. When this term is large, larger estimation errors may be tolerated than when the potential energy is low, since the MAS requires more estimation accuracy to reach its formation.

The right term in (26) mainly depends on e_i^i and $\dot{e}_i^i$, the error of Agent i state estimate. When the discrepancy between the estimate $\widehat{x}_i^i$ of its own state x_i is large, the estimates $\widehat{x}_i^j$, $j \in \mathcal{N}_i$ of x_i are also of poor quality. A message has to be sent by Agent i to update $\widehat{x}_i^j$, $j \in \mathcal{N}_i$. To reduce the number of triggered communications, one has to keep e_i^i and $\dot{e}_i^i$ as small as possible. This may be achieved by more sophisticated estimators, as proposed in Viel et al. (2017a).

The term (27) is the error of Agent i dynamic parameters estimation. The discrepancy between the actual values of M_i and C_i and of their estimates $\widehat{M}_i^i$ and $\widehat{C}_i^i$ determines the accuracy of $\bar{\theta}_i$, that of $\Delta\theta_{i,\max}$, and the estimation errors. Even in absence of state perturbations, due to the linear parametrization, it is likely that $\widehat{M}_i^i \neq M_i$, $\widehat{C}_i^i \neq C_i$ and $\Delta\theta_{i,\max} > 0$, which leads to the satisfaction of the CTCs at some time instants. Thus, the CTC (27) is more frequently satisfied when the model of the agent dynamics is not accurate, requiring thus subsequent increase of the number of updates of the estimate of the states of agents.

The discrepancy between the estimate of the reference trajectory made by Agent i and by its neighbors is evaluated via (28). The estimates have to remain close to the reference trajectory known by the leaders. The reference trajectory estimation process differs from the state estimation process. In the state estimation process, when Agent j receives a message from Agent i, Agent j updates its estimate $\widehat{x}_i^j$ using x_i. In the reference trajectory estimation, Agent j updates its reference trajectory estimate $\overline{\boldsymbol{q}}_j^{j*}$ using $\overline{\boldsymbol{q}}_i^{i*}$ only when the information provided by Agent i is more recent than that already known by Agent j. The terms $\tilde{q}_i^{j,i*}$ and $\dot{\tilde{q}}_i^{j,i*}$ are used to keep track of the last estimate of the reference trajectory shared between Agents i and j and avoid sending too many useless messages.

The CTC (25) is related to the discrepancy between $\dot{q}_i$ and $\dot{\widehat{q}}_i^i$. The norm of the actual value $\dot{q}_i$ has to remain lower than that of the estimate $\dot{\widehat{q}}_i^i$ evaluated by neighboring agents to avoid that the discrepancy increases faster than that could be predicted by the other agents. Satisfaction of CTC (25) is obtained for small value of η_2 whereas large value of η_2 leads to (28) being satisfied more frequently. A value of η_2 that corresponds to a trade-off between the two CTCs (25) and (28) has thus to be found.

The choice of the parameters α_{M}, k_g, k_p and b_i also determines the number of messages broadcast. Choosing the spring coefficients k_{ij} such that $\alpha_i = \sum_{j=1}^{N} k_{ij}$ is small leads to a reduction in the number of communications triggered resulting from the satisfaction of (28), at the cost of a less precise formation.

7 Simulation results

The proposed approach is evaluated considering $N = 6$ agents and two different models of their dynamics.

7.1 Models of the agent dynamics and estimator

7.1.1 Double integrator with Coriolis term (DI)

The first model is such that $q_i = [x_i, y_i]^T \in \mathbb{R}^2$, $M_i = I_2$, $C_i(\dot{q}_i) = 0.1 \|\dot{q}_i\| I_2$, and $G = 0_{2\times 1}$. The vectors $\overline{\theta}_i(0) = \widehat{\theta}_i^j(0)$, $i = 1, \ldots, N$ are obtained using (2). To better observe the trade-off between the potential energy of the formation and the communication requirements, a first less accurate estimator of x_j made by Agent i is evaluated as

$$\widehat{x}_j^i(t) = x_j\left(t_{j,k}^i\right) \quad \forall t \in [t_{i,k}, t_{i,k+1}[\tag{33}$$

The parameters of the control law (13) and the CTC (28) are $k_M = \|M_i\| = 1$, $k_C = \|C_i\| = 0.1$, $k_p = 1$, $k_g = 15$, $k_s = 1 + k_p(k_M + 1)$, $b_i = \frac{1}{k_g}$, and $k_0 = 2$.

7.1.2 Surface ship (SS)

The second model considers surface ships with coordinate vectors $q_i = [\, x_i \;\, y_i \;\, \psi_i \,]^T \in \mathbb{R}^3$, $i = 1 \ldots N$, in a local earth-fixed frame. For Agent i, (x_i, y_i) represents its position and ψ_i its heading angle. The agent dynamics are assumed identical for all agents and are taken from Kyrkjeb et al. (2007). They are expressed in the body frame as

$$M_{\mathrm{b},i}\dot{\mathrm{v}}_i + C_{\mathrm{b},i}(\mathrm{v}_i)\,\mathrm{v}_i + D_{\mathrm{b},i}\mathrm{v}_i = \tau_{\mathrm{b},i} + d_{\mathrm{b},i}, \tag{34}$$

where v_i is the velocity vector in the body frame. The values of $M_{\mathrm{b},i}$, $D_{\mathrm{b},i}$, and $C_{\mathrm{b},i}(\mathrm{v}_i)$ are taken from Kyrkjeb et al. (2007). At $t = 0$, each Agent i has access to estimates $\widehat{M}_{\mathrm{b},i}^i$ of $M_{\mathrm{b},i}$, $\widehat{C}_{\mathrm{b},i}^i$ of $C_{\mathrm{b},i}$, and $\widehat{D}_{\mathrm{b},i}^i$ of $D_{\mathrm{b},i}$ described as

$$\begin{aligned}
\widehat{M}_{\mathrm{b},i}^i &= \left(1_{3\times 3} + 0.1\Xi_i^{\mathrm{M}}\right) \odot M_{\mathrm{b},i} \\
\widehat{C}_{\mathrm{b},i}^i &= \left(1_{3\times 3} + 0.1\Xi_i^{\mathrm{C}}\right) \odot C_{\mathrm{b},i} \\
\widehat{D}_{\mathrm{b},i}^i &= \left(1_{3\times 3} + 0.1\Xi_i^{\mathrm{D}}\right) \odot D_{\mathrm{b},i},
\end{aligned}$$

where $1_{3\times 3}$ is the 3×3 matrix of ones, Ξ_i^{M}, Ξ_i^{C}, and Ξ_i^{D} are matrices whose components are independent uniform random variables with values in $[-1, 1]$, and $\odot$ is the Hadamard product. These estimates are transmitted at $t = 0$ to neighbouring agents. As a consequence, the estimates of $M_{\mathrm{b},i}$ and $C_{\mathrm{b},i}$ made by all agents at $t = 0$ are all identical.

The model (34) may be expressed as (1) with $G = 0$ using an appropriate change of variables detailed in Kyrkjeb et al. (2007). The vectors $\overline{\theta}_i(0) = \widehat{\theta}_i^j(0)$, $i = 1, \ldots, N$ are obtained using (2). The estimator described in Section 5.3 is employed.

The parameters of (13) and (28) are $k_M = \|M_i\| = 33.8$, $k_C = \|C_v(1_N)\| = 43.96$, $k_p = 6$, $k_g = 20$, $k_s = 1 + k_p(k_M + 1)$, $b_i = \frac{1}{k_g}$, and $k_0 = 1.5$.

7.1.3 Parameters

The initial value are $q(0) = [x(0)^T, y(0)^T]^T$, $\dot{q}(0) = 0_{2N\times 1}$ for the DI and $q(0) = [x(0)^T, y(0)^T, \psi(0)^T]^T$, $\dot{q}(0) = 0_{3N\times 1}$ for the SS, where

$$\begin{aligned}
x(0) &= [-0.35, 4.59, 4.72, 0.64, 3.53, -1.26] \\
y(0) &= [-1.11, -4.59, 2.42, 1.36, 1.56, 3.36]
\end{aligned}$$

and $\psi(0) = 0_N$. An hexagonal target formation is considered with $r^*(0) = [\, r_{(1)}^*(0)^T \;\, r_{(2)}^*(0)^T \,]^T$ for DI and $r^*(0) = [\, r_{(1)}^*(0)^T \;\, r_{(2)}^*(0)^T \;\, r_{(3)}^*(0)^T \,]^T$ for SS where

$$\begin{aligned}
r_{(1)}^*(0) &= [0, 2, 3, 2, 0, -1] \\
r_{(2)}^*(0) &= \left[0, 0, \sqrt{3}, 2\sqrt{3}, 2\sqrt{3}, \sqrt{3}\right] \\
r_{(3)}^*(0) &= 0_N
\end{aligned}$$

Each agent communicates with $N/2 = 3$ other agents. From Yang et al. (2015), one obtains $k_{ij} = 0 \;\forall j$, except $k_{i,(i+1)} = k_{i,(i-1)} = 0.185$ and $k_{i,(i+3)} = 0.0926$. One has $\alpha_i = \sum_{j=1}^{N} k_{ij} = 0.463$, for all $i = 1, \ldots, N$ and $\alpha_{\mathrm{M}} = 0.463$.

The simulation duration is $t = T$ with $T = 4\,$s, taken sufficiently large to have a steady-state behavior, with an integration step size $\Delta t = 0.01$ s. Since time has been discretized, the minimum delay between the transmission of two messages by the same agent is set to Δt. The perturbation $d_i(t)$ is assumed constant over each interval $[k\Delta t, (k+1)\Delta t[$. The components of $d_i(t)$ are independent realizations of zero-mean uniformly distributed noise $U\left(-D_{\max}/\sqrt{3}, D_{\max}/\sqrt{3}\right)$ and are thus such that $\|d_i(t)\| \leq D_{\max}$. Let N_{m} be the total number of messages transmitted during a simulation. The performance of the proposed approach is evaluated with $R_{\mathrm{com}} = 100 N_{\mathrm{m}}/\overline{N}_{\mathrm{m}}$, where $\overline{N}_{\mathrm{m}} = NT/\Delta t \geq N_{\mathrm{m}}$.
The tracking target trajectory speed of the first agent is $\dot{q}_1^*(t) = 4\left[\sin(0.4t), \cos(0.4t), 0.1t\right]^T$, the other agents having to remain in formation. Agent 1 is taken as the leader, *i.e.* $\mathcal{N}_L = \{1\}$. The estimation model $\dot{\overline{\boldsymbol{q}}}_i^*(t) = \overline{f}\left(\overline{\boldsymbol{q}}_i^*(t)\right)$ is taken as a double integrator initialized at each t_k^i by $\overline{q}_i^{i*}$ so that $\overline{q}_i^{i*}(t) = \ddot{\overline{q}}_i^{i*}\left(t_k^i\right)\frac{\left(t-t_k^i\right)^2}{2} + \dot{\overline{q}}_i^{i*}\left(t_k^i\right)\left(t-t_k^i\right) + \overline{q}_i^{i*}\left(t_k^i\right)$.

7.2 Tracking control with DI

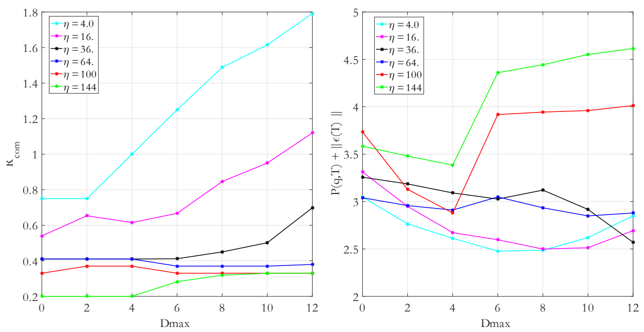


Fig. 1. Evolution of R_{com} and $P(q,t) + \|\epsilon\|$ for different values of $D_{\max}$ and η, with $\eta_2 = 7.5$. The DI model DI as well as the constant estimator (33) are considered.

Figure 1 shows the evolution of R_{com} and of $P(q,t) + \|\epsilon\|$ at $t = T$ for different values of $D_{\max} \in \{0, 2, 4, \ldots, 12\}$, $\eta \in \{4, 16, 36, 64, 100, 144\}$, and $\eta_2 = 7.5$.
In Figure 1 (a), one observes that R_{com} decreases with η and increases with $D_{\max}$, as expected observing the CTC (28). In Figure 2 (b), one observes that when η increases, $P(q,t) + \|\epsilon\|$ also increases. The evolution with $D_{\max}$ is more complex to explain, since $D_{\max}$ impacts both sides of the CTC (28). When $D_{\max}$ increases, the threshold for the CTC to be satisfied increases, but due to the noise, the CTC is also more likely to be satisfied. For all considered values of η, the increase of $D_{\max}$ is well compensated by the increase of R_{com} leading to small variations of $P(q,t) + \|\epsilon\|$.

7.3 Tracking with surface ship model

The simulation duration is $T = 5\ s$. Figure 2 shows the evolution of R_{com} and of $P(q,t) + \|\epsilon\|$ at $t = T$, for different values of $D_{\max} \in \{0, 100, 200, 400, 600, 700\}$ and $\eta \in \{10^2, 200^2, 400^2, 600^2, 700^2\}$ with $\eta_2 = 7.5$.

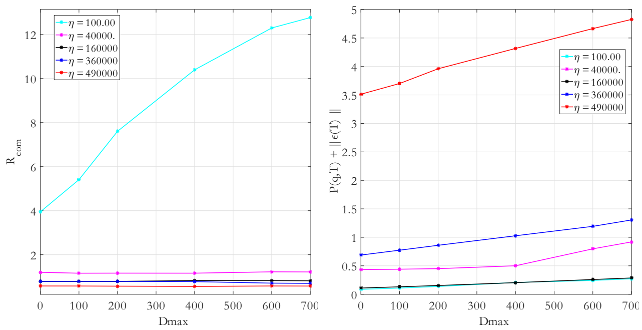


Fig. 2. Evolution of R_{com}, $P(q,t)$ and ε_0 for different values of $D_{\max}$ and η, with $\eta_2 = 7.5$. The SS model (34) and the accurate estimator (16) are considered.

In Figure 2 left, one observes again that R_{com} decreases when η increases, and increases with $D_{\max}$. Figure 2 right shows that larger values of $P(q,t) + \|\epsilon\|$ are obtained for large values of η since less communications are triggered. Moreover, as previously observed, whatever the value of η, $P(q,t) + \|\epsilon\|$ increases only slightly with $D_{\max}$ due to the increased amount of communications which compensates increasing perturbation levels.

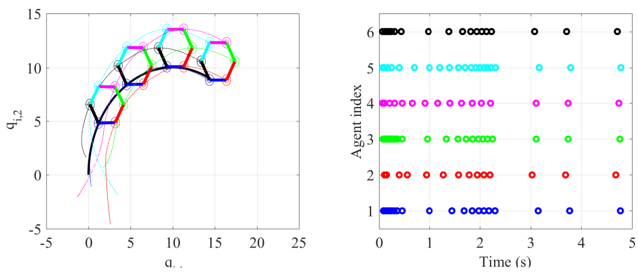
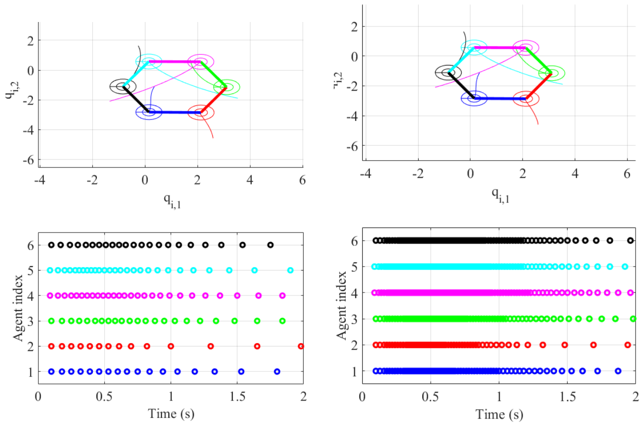


Fig. 3. Hexagonal formation and tracking problem with $D_{\max} = 20$, $\eta = 50$, and $\eta_2 = 7.5$. Circles represents agents (left figure) and communication events (right figure). $R_{\mathrm{com}} = 2.43\%$, $P(q,T) = 0.001$ and $\|\varepsilon_0\| = 0.1$. $T = 5\,$s.

8 Conclusion

This paper presents a distributed event-triggered control strategy to drive a MAS to some possibly time-varying target formation. Perturbed Euler-Lagrange dynamics are considered. The event-triggered approach requires that each agent maintains an estimate of the state of its neighbours, to be able to evaluate its control law, without requiring a permanent communication between agents. Each agent has also to estimate its own state using information it has transmitted to the other agents. The discrepancy between its actual state value and its estimate is used to trigger communications to other agents, so that they can update their estimates. Convergence properties and influence of state perturbations on the amount of required communications have been studied. Tracking of time-varying formations has also been considered. The time interval between consecutive communications has been shown to be strictly positive in Viel et al. (2017b).



(a) Estimator (16). (b) Estimator (33).

Fig. 4. Hexagonal formation with $D_{\max} = 20$, $\eta = 20$ and $\eta_2 = 7.5$. Agents are represented by circles. In (a), $R_{\text{com}} = 10.75\%$ and $P(q, T) = 0.001$. In (b) $R_{\text{com}} = 40.25\%$ and $P(q, T) = 0.001$. $T = 2$ s.

Simulations have shown the effectiveness of the proposed method in presence of state perturbations when their level remains moderate. In future work, the considered problem will be extended to communication delay and packet losses.

Acknowledgments

This work has been partly supported by the Direction Generale de l'Armement (DGA) and by ICODE.

References

Dimarogonas, D.V., Frazzoli, E., and Johansson, K.H. (2012). Distributed event-triggered control for multi-agent systems. *IEEE Trans. Aut. Contr.*, 57(5), 1291–1297.

Fan, Y., Feng, G., and Wang, Y. (2013). Distributed event-triggered control of multi-agent systems with combinational measurements. *Automatica*, 49(2), 671–675.

Garcia, E., Cao, Y., Wang, X., and Casbeer, D.W. (2014). Cooperative control with general linear dynamics and limited communication: Periodic updates. In *Proc. ACC*, 3195–3200.

Jiang, Z.P., Mareels, I.M., and Wang, Y. (1996). A Lyapunov formulation of the nonlinear small-gain theorem for interconnected ISS systems. *Automatica*, 32(8), 1211–1215.

Kyrkjeb, E., Pettersen, K., Wondergem, M., and Nijmeijer, H. (2007). Output synchronization control of ship replenishment operations: Theory and experiments. *Control Engineering Practice*, 15(6), 741–755.

Liu, Q., Sun, Z., Qin, J., and Yu, C. (2015). Distance-based formation shape stabilisation via event-triggered control. In *Proc. CCC*, 6948–6953.

Makkar, C., Hu, G., Sawyer, W.G., and Dixon, W. (2007). Lyapunov-based tracking control in the presence of uncertain nonlinear parameterizable friction. *IEEE Trans. Aut. Contr.*, 52(10), 1988–1994.

Mei, J., Ren, W., and Ma, G. (2011). Distributed coordinated tracking with a dynamic leader for multiple euler-lagrange systems. *IEEE Trans. Aut. Contr.*, 56(6), 1415–1421.

Olfati-Saber, R., Fax, A.J., and Murray, R.M. (2007). Consensus and cooperation in networked multi-agent systems. *Proc. of the IEEE*, 95(1), 215–233.

Sun, D., Wang, C., Shang, W., and Feng, G. (2009). A synchronization approach to trajectory tracking of multiple mobile robots while maintaining time-varying formations. *IEEE Trans. Robotics*, 25(5), 1074–1086.

Sun, Z., Liu, Q., Yu, C., and Anderson, B. (2015). Generalized controllers for rigid formation stabilization with application to event-based controller design. In *Proc. ECC*, 217–222.

Tang, T., Liu, Z., and Chen, Z. (2011). Event-triggered formation control of multi-agent systems. In *Proc. CCC*, 4783–4786.

Viel, C., Bertrand, S., Kieffer, M., and Piet-Lahanier, H. (2017a). New state estimators and communication protocol for distributed event-triggered consensus of linear multi-agent systems with bounded perturbations. *IET Contr. Th.& Appl.*, 11(11),1736–1748.

Viel, C., Bertrand, S., Piet-Lahanier, H., and Kieffer, M. (2016). New state estimator for decentralized event-triggered consensus for multi-agent systems. In *Proc. IFAC ICONS 2016*, 49(5), 365–370.

Viel, C., Bertrand, S., Piet-Lahanier, H., and Kieffer, M. (2017b). Distributed event-triggered control for multi-agent formation stabilization and tracking. *arXiv preprint arXiv:1709.06652*.

Wei, R. (2008). Consensus tracking under directed interaction topologies: Algorithms and experiments. In *Proc. ACC*, 742–747.

Wen, G., Duan, Z., Li, Z., and Chen, G. (2012a). Flocking of multi-agent dynamical systems with intermittent nonlinear velocity measurements. *Int. Jnl Robust and Nonl. Contr.*, 22(16), 1790–1805.

Wen, G., Duan, Z., Yu, W., and Chen, G. (2012b). Consensus in multi-agent systems with communication constraints. *Int. Jnl Robust and Nonl. Contr.*, 22(2), 170–182.

Wen, G., Duan, Z., Yu, W., and Chen, G. (2013). Consensus of second-order multi-agent systems with delayed nonlinear dynamics and intermittent communications. *Int. Jnl Contr.*, 86(2), 322–331.

Yang, Q., Cao, M., Fan, H., Chen, J., and Huang, J. (2015). Distributed formation stabilization for mobile agents using virtual tensegrity structures. In *Proc. CCC*, 447–452.

Zhang, H., Yang, R., Yan, H., and Chen, Q. (2015). Distributed event-triggered control for consensus of multi-agent systems. *Jnl Franklin Institute*, 352(9), 3476–3488.