Recent advances in HARQ communications

— Tutorial to be presented at ICT 2019, Hanoi —

Presented by Pierre Duhamel (CNRS/CentraleSupelec/L2S, France)
Co-authors: Leszek Szczecinski (INRS, Canada), Philippe Ciblat (Telecom ParisTech, France) and Francesca Bassi (CNRS/CentraleSupelec, France)

April, 2019

With many contributions from

Faton Maliqi, Alaa Khreis, Mohamed Jabi

This work was partly supported by the Labex Digicosme PhD scholarship from Université Paris-Saclay
Context: (Short) description of a simplified wireless commutation scenario

Transmitter (TX):

Traditional presentation:
- Adaptive Modulation and Coding: adapts the amount of information transmitted to the "quality" of the channel
  - obviously requires the transmitter to know the channel parameters
  - and to have a performance model for the considered channel
- Transmitter does not know if the transmission failed
Example: AMC with QAM modulation

In actual situations: there exists a target error rate...
Another example: AMC with QAM modulation in 802.11n
Drawbacks

- Not many degrees of freedom in the design of AMC
- Would require full knowledge of the instantaneous channel parameters
- When used with average channel conditions, lack of adaptivity (true propagation conditions, noise level ....)

Note also that practical implementations require anyway a feedback channel:
The receivers estimates the "quality" of the channel (usually the SNR), and sends it back to the transmitter, which is then transmitting with the most appropriate Modulation and Coding Scheme (MCS)
Part 1 : The general picture

However, this is a pure ”Physical Layer” point of view, and there could be many problems in the interactions between the various ingredients of a wireless communication network...

Therefore, we spend some time in giving an overview of the aspects that are strongly interconnected... (in order to propose the smartest HARQ... )
Motivation

- Rapid varying radio channel
  - Time-variant: coherence time (Doppler spread)
  - Frequency-selective: coherence bandwidth (delay spread)
  - Interference
- Exploit the channel variation *prior to* transmission
  - Link adaptation: Set transmission parameters to handle radio channel variation
  - Channel-dependent scheduling: Efficient resource sharing among users
- Handle the channel variation *after* transmission
  - Hybrid ARQ: Retransmission request of erroneously received data packets
Link adaptation (1)

Power control:
- Dynamically adjust the transmit power to compensate for the varying radio channel condition
- Maintain a certain SNR at the receiver
- Constant data rate regardless of the channel variation

![Power Control](image-url)
Rate control:
- Packet-data traffic: not a strong desire for constant rate (as high rate as possible)
- Dynamically adjust the data rate to compensate for the varying radio channel condition
- Full constant transmit power (desirable in multiuser systems)
Link adaptation (3)

- Rate control
  - Adaptive Modulation and Coding (AMC) scheme
  - ”Good” channel condition: Bandwidth limited (High-order modulation + high-rate coding)
  - ”Poor” channel condition: Power limited (Low-order modulation + low-rate coding)

- In HSDPA link adaptation
  - QPSK for noisy channels and 16 QAM for clearer channels
  - 14Mbps, on clear channels using 16-QAM and close to 1/1 coding rate.
  - 2.4 Mbps, on noisy channels using QPSK and 1/3 coding rate (14 Mbps x 1/2 x 1/3)
  - This adaptation is performed up to 500 times per second
Link adaptation (4)

- Power control: constant rate
  - Desired for voice/video (Short-term rate variation not an issue with constant average data rate)
  - Inefficient use of transmit power

- Rate control: constant (max) transmit power
  - Adaptive data rate
  - Efficient use of transmit power
  - Desired in multiuser systems to reduce variations in interference power

[Chung & Goldsmith, 2001] Little spectral efficiency is lost when the power or rate is constrained to be constant, with optimal adaptation.
Scheduling

- The allocation of the shared resources among the users at each time instant
  - Whom?
  - How?
- Joint function with link adaptation
- Channel dependent
- Downlink scheduling $\Rightarrow$ Centralized resource
- Uplink scheduling $\Rightarrow$ Distributed resource

Two examples below of extreme choices for Downlink scheduling, and a more reasonable one (we do not consider uplink in this context description...
Downlink Scheduling (1)

- Channel-dependent scheduling
  - Max-C/I (Max rate) scheduler
    - Schedule at the fading peaks
  - Independently varying radio links
    - Multiuser diversity gain
  - High system throughput but not fair

\[ k = \arg \max_i R_i \]

Exploit fading rather than combat

Starve the poor channel user

By S. Parkvall
• **Round-robin** scheduling
  - Regardless of channel conditions
  - Fair? … same amount of the radio resources
  - Unfair! … service quality (more resources needed for poor channel)
  - Simple but poor performance
Downlink Scheduling (3)

- Two-fold requirement
  - Take advantage of the fast channel variations
  - Ensure the same average user throughput
- Proportional-fair scheduler
  - Proportion between the instantaneous data rate and the average data rate during a certain period
  - High throughput and fairness

\[ k = \arg \max_i \frac{R_i}{R} \]

Schedule on fading peaks, regardless of the absolute quality

By S. Parkvall
• LTE
  – channel-dependent scheduling in time and frequency domains

By S. Parkvall
Requirements on Channel state information

In what follows, we implicitly work with Block fading channels: even if an average situation is safe, very bad channels may occur...

- CSI: Needed at TX for link adaptation and channel-dependent scheduling
- Downlink
  - Pilot signal? e.g., Correlation channel estimator
  - Measured channel conditions reported to BS → Outdated if high mobility
  - Channel prediction: Additional complexity and constraint
  - Link adaptation based on ”long-term” average channel
How to adapt to channel’s variation? : from AMC to ARQ

Summary: advanced packet radio wireless networks such as HSDPA, channel-dependent scheduling may be used to take advantage of favourable channel conditions to increase the throughput and system spectral efficiency ... (wireless communications are a very "liberal" situation: efficient channels / users should be used as much as possible)

- Since AMC is working with average (non instantaneous) performance,
- Idea: trial and error
  - First send a packet of symbols
  - if correctly received (ACK), ↗
  - if residual errors (NACK), ↘ and send again a packet containing "same" information...

- This requires feedback channel: information on the instantaneous channel, and the success of the transmission.

.... and do not forget that there is delay in the feedback: processing time, transmission time, framing time, etc...
Part 2: Classical ARQ/HARQ situations

- ARQ
- HARQ
- HARQ taxonomy:
  - Type I and II
  - Chase Combining, Incremental Redundancy

And we first assume that everything is instantaneous
ARQ (Automatic ReQuest) overview: the ingredients

- **Forward Error Correction (FEC)**
  - Add redundancy for error correction

- **Automatic Repeat Request (ARQ)**
  - Compatible with TCP behavior for packet data
  - Error-detecting code by Cyclic Redundancy Check (CRC)
  - CRC used as a check sum to detect errors (Division of polynomials in Galois field GF(2)...remainder...)
  - No error? Positive acknowledgement (ACK)
  - Error? Negative acknowledgement (NAK)

- **Hybrid ARQ**
  - Combination of FEC and ARQ
  - FEC: correct a subset of errors
  - ARQ: if still error detected
Let $S = [s_0, \cdots, s_{N-1}]$ be a packet composed by $N$ uncoded symbols.
Towards Hybrid ARQ (HARQ): Type-I HARQ

Remark
Retransmission does not contradict forward error coding (FEC)

Type-I HARQ: packet $S$ is composed by coded symbols $s_n$
- first packet is more protected
- there is less retransmission
- transmission delay is reduced

- Efficiency is upper-bounded by the code rate

Drawbacks
- Each received packet is treated independently
- Mis-decoded packet is thrown in the trash
Type-II HARQ

Memory at RX side is considered ⇒ Type-II HARQ

Main examples:
- *Chase Combining (CC)*
- *Incremental Redundancy (IR)*
Examples: CC-HARQ and IR-HARQ

**CC**

\[
Y_1 = S_1 + N_1 \\
Y_2 = S_1 + N_2
\]

then detection on

\[
Y = (Y_1 + Y_2)/2
\]

SNR-Gain equal to 3dB

**IR**

\[
Y_1 = S_1(1) + N_1 \\
Y_2 = S_1(2) + N_2
\]

then detection on

\[
Y = [Y_1, Y_2]
\]

Coding gain
Hybrid ARQ (Automatic Repeat reQuest)

$m_k$ \rightarrow \mathbf{p}_k(1) \rightarrow \mathbf{y}_t = h(t)\mathbf{x}_t + \mathbf{w}_t \rightarrow \mathbf{m}_k$

$m_{k+1}$ \rightarrow \mathbf{p}_{k+1}(1) \rightarrow \mathbf{y}_{t+1} \rightarrow \mathbf{m}_{k+1}

$h(t)$: Rayleigh flat fading channel

$\mathbf{p}_k(\ell)$: $\ell$-th packet of message $m_k$, $\ell \in \{1, \cdots, C\}$

$\mathbf{p}_k(1) = \mathbf{p}_k(2)$ for CC-HARQ (Chase Combining) $\rightarrow$ diversity gain

$\mathbf{p}_k(1) \neq \mathbf{p}_k(2)$ for IR-HARQ (Incremental Redundancy) $\rightarrow$ diversity + coding gain
Part 3: Performance metrics

- **Packet Error Rate (PER):**
  \[ \text{PER} = \text{Prob(} \text{information packet is not decoded} \text{)} \]

- **Efficiency (Throughput/Goodput/etc):**
  \[ \eta = \frac{\text{information bits received without error}}{\text{transmitted bits}} \]

- **(Mean) delay:**
  \[ d = \# \text{ transmitted packets when information packet is received} \]

- **Jitter:**
  \[ \sigma_d = \text{delay standard deviation} \]

**Quality of Service (QoS)**

- Data: PER and efficiency
- Voice on IP: delay
- Video Streaming: efficiency and jitter
HARQ in its context: which tools would allow for some improvement?

4.1 Power adaptation
4.2 Bandwidth adaptation
4.3 Rate (reward) adaptation
4.4 Layered coded HARQ
4.5 Non-orthogonal HARQ; reducing the delay and improving the throughput (Pierre)
Back to Basics: Canonical HARQ (fixed rate, Rayleigh)

Figure below explicits the subcodewords, and the reward (\# bits, normalized by \# symbols)

```
Transmitter    Receiver

m2          m1

\begin{align*}
\text{x3} & \text{x2} & \text{x1} \\
\text{snr}_1 & \text{snr}_2 \\
\text{NACK}_1 & \text{ACK}_2
\end{align*}
```

Reward

- R = 0
- R = R

“Canonical” model, to be questioned below

- Constant power
- Constant bandwidth
- Binary reward R ∈ \{0, R\}
Renewal-Reward Theorem

variable bandwidth (multiple rounds) + variable reward (final NACK)

Throughput

\[ \eta_K \triangleq \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} R(t) = \frac{\mathbb{E}[R]}{\mathbb{E}[D]} \]

\[ = \frac{R(1 - f_1) + R(f_1 - f_2) + \ldots + R(f_{K-1} - f_K)}{(1 - f_1) + 2(f_1 - f_2) + \ldots + (K-1)(f_{K-2} - f_{K-1}) + Kf_{K-1}} = \frac{R(1 - f_K)}{1 + \sum_{k=1}^{K-1} f_k}, \quad (1) \]

Sequence of / decoding errors

\[ f_l \triangleq \Pr \{ \text{NACK}_l \} \]
Example: 16-QAM, Rayleigh fading, $R = 3.75, K \in \{2, 4\}$

Turbo-codes, fixed rate, varying $T = 2, 4$

Incremental redundancy

- Shannon bounds predict well the performance of practical codes; throughput grows with $K$
- Gains appear in “low” throughout, i.e., for $\eta_K < R$
- No/negligible gains for “high” throughput $\eta_K \approx R$ (obvious!)
Example: .... adjusting the rate $R \in \{0.25, 0.5, \ldots, 7.75\}$

- Throughput can be improved adjusting $R$, but
- No significant gains even for $\eta_K \approx R$ even when for $K = \infty$
- Theoretical result: $\lim_{K \to \infty} \eta_K = \bar{C}$, but only if $R \to \infty$ (not practical!)
Power adaptation

The receiver sends additional feedback (about the instantaneous SINR)

- The transmitter varies the power of the subcodeword in each round
- The sub-codewords have the same length
- Adaptation: power varies according to the extra feedback (precalculated function)
- Allocation: power varies according to the index of the round (precalculated scalar)
Length adaptation

- The transmitter varies the bandwidth (e.g., length) $N_s,k$ in each round.
- $\ell_k \triangleq N_{s,k}/N_{s,1}$ is the normalized bandwidth; $\ell_1 = 1$.
- Transmission with constant power, $P_k = 1 \ \forall k$.
- Adaptation: bandwidth varies according to the extra feedback (precalculated function).
- Allocation: bandwidth varies according to the index of the round (precalculated scalar).
Degrees of freedom in the design of HARQ

Throughput

Variable power HARQ

\[ \eta_{K}^{VP} = \frac{R(1 - f_{K})}{1 + \sum_{k=1}^{K-1} f_{k}} \]  

(2)

constraint (in allocation):

\[ \bar{P} = \frac{\sum_{k=1}^{K} P_{k}(f_{k-1} - f_{k})}{1 + \sum_{k=1}^{K-1} f_{k}} \]  

(3)

Variable length HARQ

\[ \eta_{K}^{VL} = \frac{R(1 - f_{K})}{1 + \bar{\ell}} \]  

(4)

where (for allocation)

\[ \bar{\ell} = \sum_{k=2}^{K} \ell_{k} f_{k-1} \]

In both cases, the reward (rate) does not change, only the expression of the constraint...
VL vs. VP example: “Shannon codes”, Rayleigh, $R = 4$

After optimization (using dynamic programming)

- Adaptive power does not help throughput (but can decrease packet loss)
- Adaptive bandwidth yields significant gains in terms of throughput
Variable bandwidth HARQ

Now let us do the converse w.r.t. eq. 4: fix bandwidth, make full use of it, and check what happens on the reward.

System level considerations

- Manage “empty” space within the block via
  - frequency allocation (4G) or
  - use of many packets within a single block
- Potential issues: increased signaling overhead and optimization problem.
Reward/rate adaptation

Manipulating the term in the numerator of the throughput expression

- **Fixed reward**
  \[ \eta_K = \frac{R(1 - f_1) + R(f_1 - f_2) + \ldots + R(f_{K-1} - f_K)}{1 + \sum_{k=1}^{K-1} f_k} \]

- **Variable reward**
  \[ \eta_K = \frac{R(1 - f_1) + R_2^\Sigma (f_1 - f_2) + \ldots + R_K^\Sigma (f_{K-1} - f_K)}{1 + \sum_{k=1}^{K-1} f_k} \]

  **Notation:** \( R_2^\Sigma \): accumulated reward with 2 transmissions

  **Interpretation:** multi-packet transmission per round
  - Proposition 1: Time-sharing (TS)
  - Proposition 2: Cross-packet coding (XP)
Time Sharing HARQ

- Time sharing: the sub-codwords of two packets are transmitted in non-overlapping manner.
- $p$ the portion of time/bandwidth allocated to different packets in the same block (depends on the outdated snr)

\[ m_2 - m_1 \]

\[ \text{snr}_1 \]

\[ \text{snr}_2 \]

\[ \text{Reward} \]

\[ \text{NACK}_1, p \]

\[ \text{ACK}_1, \text{ACK}_2, p \]
Cross-packet coding HARQ

- $R_1$ is used in the first round, i.e., $m_1 \in \{0, 1\}^{R_1 N_s}$.
- $m_2 = [m_1, m_2] \in \{0, 1\}^{(R_1 + R_2)N_s}$ is encoded using a conventional code.
Example: XP vs TS; 16QAM, Rayleigh fading

Cross-packet (XP) coding is the winner but...
XP-HARQ: encoding, decoding and reward/rate adaptation

\[ \Phi_i: \text{encoder for packet } i; \]
\[ m_{i+1} \text{ is jointly encoded with } m_i \implies \text{encoder and decoder become increasingly complex} \]
Cross-packet coding: practical issues

Issues with encoding

- Growing size of inputs $m_{[k]} = [m_1, \ldots, m_k] \in \{0, 1\}^{N_s R_k^\Sigma}$
- Sub-optimality of the encoder design, due to the growing rate, e.g., $R_k$ exceed constellation size, concatenation of codes, etc.

Issues with decoding

- Joint decoding (on multiply-concatenated codes); possible but non-standard
- Multidimensional-multiparametric PER curves (surfaces) are hard to measure, store, and use (for adaptation)

$$\Pr \{\text{NACK}_k\} = \text{PER}(\text{snr}_1, \ldots, \text{snr}_k; R_1, \ldots, R_k)$$
Layer-coded HARQ (L-HARQ)

Practical implementation of XP-HARQ
- now: same encoder $\Phi$ (same # of bits at the inputs)
- Multipacket encoding $\rightarrow$ puncturing (with rate $\rho$) and binary packet mixing $+$ Off-the-shelf (optimized) encoder
- Practically: transmit part of $m_1$ (punctured) mixed with part of $m_2$ (punctured)
- Joint decoding $\rightarrow$ conventional decoding $+$ backtrack decoding (using priors)

Higher rate, and mix at binary level $\Rightarrow$ simpler encoders and decoders
“Artificial” inclusion of systematic bits: if $\hat{m}_2$ is OK, $m_2$ is recovered, which provides the corresponding contribution to $m_3$
Example: 16-QAM, Turbo code, Rayleigh-fading, $R = 3.75$

- Gain for high throughput region (this is what we wanted!)
- Loss for low throughput (error propagation; should be combated with rate adaptation)
What if feedback not instantaneous?

Management for $T$:
- Stop-and-Wait
- Parallel Stop-and-Wait/Selective Repeat
Management for $T$

STOP-AND-WAIT

$T = 4$

\[
\begin{array}{c}
S_1 \\
\text{empty} \\
S_1
\end{array}
\]

NACK$_1$

PARALLEL/SELECTIVE-AND-REPEAT

\[
\begin{array}{cccccccc}
S_1 & S_2 & S_3 & S_4 & S_1 & S_5 & S_3 & S_6 \\
\text{NACK}_1 & \text{ACK}_2 & \text{NACK}_3 & \text{ACK}_4
\end{array}
\]

Why $T \neq 1$?

- Decoding processing time at RX
- Framing: traffic for return channel
- Propagation time

Example: $T = 8$ in LTE
Non orthogonal HARQ; reducing the delay and improving the throughput

Another way of building multi-layer HARQ, with corresponding protocol.

- State of the Art \( T = 1 \)
- Application to \( T \neq 1 \)
State of the Art \((T = 1)\)

Sending the superposition of two streams instead of one!

\[ \mathbf{y} = \mathbf{x}_1 + \mathbf{x}_2 + \mathbf{w} \]

But superposition does not increase the capacity

\[ R = R_1 + R_2 < \log_2(1 + P_1 + P_2) = \log_2(1 + P) \]

with \(P\) the transmit power.

However a way to be closer to the capacity, especially with retransmission (since ACK/NACK provides information)
Main Idea [Steiner06]:

- Frame 1: send two messages under superposition coding (SC), i.e., two layers with short power constraints $P$
- Frame 2: if one layer not decoded, send it again with full power $P$
- Frame 3: start with two new messages

Two contexts:

- Channel constant over each retransmission
- Channel time-varying at each retransmission

Additional works:

- Practical implementation of [Steiner06] with $P_1 = 0.8P$ [Assimi2009]
- CSI at the TX for relevant actions (SC or not with Markov Decision Process) [Jabi2015]
- At TCP level: flooding the TCP packet with hierarchical superposition coding [Zhang2009]
Application to $T \neq 1$

Idea To reduce the delay, send in advance (before receiving any ACK/NACK) redundant packets in superposition to standard parallel HARQ with low power (for minimizing the disturbance):

\[
\begin{align*}
S_k(\ell), & \quad \text{if no superposition,} \\
\sqrt{\alpha} S_k(\ell) + \sqrt{1 - \alpha} S_{k'}(\ell'), & \quad \text{if superposition.}
\end{align*}
\]

with $k, k'$ the messages.

We have two layers:

- The first one is standard parallel HARQ
- The second one corresponds to superposed packets chosen as:
  1. $S_{k'}(\ell')$ is not superposed if $m_{k'}$ is in timeout or previously ACKed
  2. Superposed packet is the unsent packet of the lowest index $\ell'$ of the most recent message $m_{k'}$, with $k' \neq k$
  3. If the transmitter already sent all the packets, superposed packet is with the lowest index $\ell'$ not previously sent in the second layer.
  4. No packet is superposed to a packet of the first layer that has $\ell = L$. 
Hybrid ARQ (Automatic Repeat reQuest)

\[ y_t = h(t)x_t + w_t \]

- **Channel**
  - \( m_k \) (Tx)
  - \( p_k(1) \)
  - \( p_k(2) \)
  - \( y_t \)
  - \( y_{t+1} \)
  - \( m_k \) (Rx)
  - \( h(t) \)
  - NACK
  - ACK

- **Tx**
  - \( m_k \)
  - \( p_k(1) \)
  - \( p_k(2) \)

- **Rx**
  - \( y_t \)
  - \( y_{t+1} \)
  - \( m_k \)

\( h(t) \): Rayleigh flat fading channel

- \( p_k(\ell) \): \( \ell \)-th packet of message \( m_k \), \( \ell \in \{1, \cdots, C\} \)

- \( p_k(1) = p_k(2) \) for CC-HARQ (Chase Combining) \( \rightarrow \) diversity gain

- \( p_k(1) \neq p_k(2) \) for IR-HARQ (Incremental Redundancy) \( \rightarrow \) diversity + coding gain
HARQ with feedback delay \( (T = 3) \)

**Why \( T \neq 1? \) \( (T = 8 \text{ in LTE}) \)**

- Decoding (processing) time at the receiver
- Framing: traffic for return channel
- Propagation time
Non-orthogonal transmission

Idea
- Superpose (re)transmitted packets to increase the throughput [Shamai08, Assimi09, Szczecinski14]

Objectives
- Low latency
- High reliability
- Large throughput

Why non-orthogonal transmission?
- Non-orthogonal transmission exploits the potential of MAC
- Other strategies usually require CSI at the transmitter [Kasper17]
  - time-sharing
  - rate adaptation

MAC: Multiple Access Channel - CSI: Channel State Information
Degrees of freedom in the design of HARQ

Multi-layer HARQ with feedback delay

General idea

Send additional redundant packets using two layers
Before receiving the ACK/NACK feedback
Superposed to parallel HARQ
With low power

Layer 1: parallel HARQ VERY important
Layer 2: superposed packets

\[
\sqrt{\alpha} p_k(\ell) + \sqrt{1 - \alpha} p_{k'}(\ell')
\]
without superposition
with superposition

<table>
<thead>
<tr>
<th>time-slot</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tx</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 1</td>
<td>(p_1(1))</td>
<td>(p_2(1))</td>
<td>(p_3(1))</td>
<td>(p_1(2))</td>
<td>(p_4(1))</td>
<td>(p_3(2))</td>
</tr>
<tr>
<td>Layer 2</td>
<td>(p_1(2))</td>
<td>(p_2(2))</td>
<td>(p_3(2))</td>
<td>(p_3(3))</td>
<td>(p_4(2))</td>
<td></td>
</tr>
<tr>
<td><strong>Channel</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Rx</strong></td>
<td>(y_1)</td>
<td>(y_2)</td>
<td>(y_3)</td>
<td>(y_4)</td>
<td>(y_5)</td>
<td>(y_6)</td>
</tr>
</tbody>
</table>

Proposed protocol, \(T = 3\)
Transmitter

How do we choose the superposed redundant packets?

- Superpose packets of the most recent messages
  → Low latency

- Superpose unsent redundant packets
  → Transmit diversity
  → High reliability

Proposed protocol, $T = 3$

Low latency + High reliability → Large throughput
Decoding

Let $\mathcal{M}$ be the set of messages that the receiver is attempting to decode at time-slot $t$.

- If the receiver successfully decodes the subset $\mathcal{D} \subseteq \mathcal{M}$ and none of the messages in $\mathcal{M} \setminus \mathcal{D}$, we say that the decoder operates in the rate region $\mathcal{R}_D$.
- The set $\mathcal{D}$, along with the rules of the transmit protocol, allows to obtain $\mathcal{F}_t$ the set of ACK/NACK.
- In order to characterize the decoding outcome, we
  1. evaluate the rate region $\mathcal{R}_D$ for every possible $\mathcal{D} \subseteq \mathcal{M}$, by checking the corresponding rate inequalities
  2. determine, on the basis of the available observations, the operating rate region $\mathcal{R}_D$ of the receiver.
Degrees of freedom in the design of HARQ

Multi-layer HARQ with feedback delay

Performance with capacity-achieving codes

Received signals

\[ y_1 = h(1)p_1(1) + w_1 \]

\[ y_2 = h(2)\sqrt{\alpha}p_2(1) + h(2)\sqrt{1 - \alpha}p_1(2) + w_2 \]

Rate regions at \( t = 2 \) [ElGamal12]

\[ g(t) = |h(t)|^2 \]
Setup for numerical evaluation

Distance between the transmitter and the receiver
\[ d = 15u \text{ where } u \text{ is a unit of distance} \]

- **Variance**: \[ \sigma^2 = \left( \frac{c}{d^2} \right)^2 \] where \( c \) is a constant, fixed as \( c = 400u^2 \)
- **HARQ protocol**: IR-HARQ with \( C = 4 \) and \( R = 0.8 \)
- **Feedback delay**: \( T = 3 \) time-slots
- **Transmit energy**: \( E_s \) per symbol
Throughput using capacity-achieving codes

$1dB$ to $2.5dB$ gain at moderate SNR, Much more for high SNR

10% throughput gain at $0dB$
Message Error Rate using capacity-achieving codes

Additional diversity gain due to multi-layer transmission
Latency using capacity-achieving codes

Parallel IR-HARQ

Proposed protocol

More packets are served with small delays (< 4 time-slots)
Numerical optimization of $\alpha$

$E_s/N_0 = 0dB$

$\alpha = 0.7$ provides the best performance at $0dB$

$\alpha$ can be numerically optimized for each SNR
Proposed protocol in comparison to 3GPP LTE

Throughput using $C = 4$, $T = 8$ and capacity-achieving codes

![Graph showing throughput vs $E_s/N_0$ dB]

**LTE:** Long-Term Evolution  
**3GPP:** 3rd Generation Partnership Project [TS 36.213] [TS 36.321]
Part 5: HARQ and AMC; Friends or Foes?

5.1 Model again; the source of errors
5.2 HARQ on top of AMC; problems and remedies
5.3 Connecting L-HARQ with AMC

Previously: the rate was fixed, but now, we take into account the fact that (average) CSI knowledge allows AMC: what happens when combined with HARQ?
Model: AMC+HARQ, saturated buffer

Transmitter

\[ y_k = \sqrt{s\text{snr}_k} x_k + z_k, \quad k = 1, \ldots, K \]

Receiver

- Only PHY throughput counts: LLC-level ARQ removes all residual errors from PHY
- Modulation and coding set (MCS) is decided by the receiver (using measured CSI)
- Measured CSI (snr) is delayed with respect to the actual CSI (s\text{nr})
Decoding Errors due to the delayed CSI (Doppler)

2 different SNR’s perceived at the transmitter: the ”average” on which AMC is chosen, and the instantaneous (but outdated) one coming from receiver...

Assumptions

- Propagation time is (often) negligible
- Processing time is non-negligible for decoding, CSI acquisition, encoding
- $\tau_{tot} f_D \gg 1$ (snr$'_1$ and snr$'_2$ are independent)
- $\tau_{tot} f_D > 0$ (snr$_2$ and snr$'_2$ are correlated)
- $T f_D \approx 0$ (no channel variation when receiving)
Example of PER curves; 16-QAM; Doppler $f_D \tau = 0.05$

- Theoretical and practical curves are similar: Turbo-C (solid) and PerfectC (dashed)
- In practice: fix decoding threshold, $\text{PER}_{th}$ and select MCS using $\text{SNR}$
  - For example: $\text{PER}_{th} = 0.1$, $R([8.5\text{dB}, 10.5\text{dB}]) = 1.5$, $R([10.5\text{dB}, 13\text{dB}]) = 2.25$, etc.

Problem: curves should be ”observed” (measured) for each possible receiver: decoding time has an impact...
Reminder: How much we gain with HARQ (no AMC)

- Throughput improved in low SNR
- No gain for high nominal rate, i.e., in high SNR
Throughput degradation in high SNR due to HARQ

Source of the problem: i) first round rate $R_1 = R(snr_1)$; ii) after NACK, second round’s reward is only $R_1$; iii) in AMC the reward might be $R_2 = R(snr_2) > R_1$

Patching: if $snr_2 > snr_1$, abandon HARQ and use AMC (packet dropping)
AMC+L-HARQ: decoding example, $k = 3$

\[ \Phi \text{'s are controlled by AMC, } \Phi^b \text{'s controlled by HARQ} \]

- NACK$_1$, NACK$_2$, ACK$_3$, $\rightarrow$ $\hat{m}_{[3]}$ and $\hat{m}^\prime_{[2]}$ are error-free
- Backtrack decoding: using $\hat{m}^\prime_{[2]}$, decoder #2 produces $\hat{m}^b_{[2]}$ and $\hat{m}^\prime_{[1]}$, which are error-free
- Backtrack decoding: using $\hat{m}^\prime_{[1]}$, decoder #1 produces $\hat{m}^b_{[1]}$ which is error-free
AMC+L-HARQ: Decoupled control

- AMC round $k$: Channel encoder $\Phi_k$ (MCS) adapts to fresh CSI (measured at round $k$)
- HARQ round $k$: Puncturer $\Phi^b_{k-1}$ adapts to old CSI (from the round $k-1$)
- Joint optimization of rates not needed
- Knowledge of the channel model not needed for optimization (of the rates)
Numerical example: Turbo C, 16QAM, Rayleigh, $\tau f_D = 0.05$ and $R \in \{1.5, 2.25, 3, 3.75\}$

- The adaptation does not depend on the channel model (emphasized again)
- HARQ improves with number of rounds (that’s what we wanted!)
- Gains $\sim 3$dB for high rates,
Part 6: Extensions and wrap up

Content:

- cooperative communications
- conclusions on theoretical and practical issues
Interaction between Relaying and HARQ:

- Both techniques applied solely will bring improvement;
- What improvement will bring if these two techniques are applied together?
- What is the best way of combining them?
Reference literature

- Combination of these two techniques in literature:
  - Energy efficiency is studied in [Stanojev, 2009], and from the perspective of information theory is studied in [Falavarjani, 2010];
  - The interaction is mostly studied via deterministic protocols [Krikidis, 2007]; We focus on both: deterministic and probabilistic protocols;
  - The Relay is mostly considered in Decode-and-Forward (DCF) mode; We focus more on the Demodulate-and-Forward (DMF) mode.

- For theoretical analysis we focus on Finite State Markov Chain (FSMC).
System model

- Example scenario:
  - Source-Relay-Destination network;
  - ARQ mechanism (stop-and-wait policy);
  - All the nodes listen to control messages (ACK/NACK) issued by D.

- Relay mode:
  - Decode-and-Forward (DCF) - Relay always forward the correct copy.
  - Demodulate-and-Forward (DMF) - demodulation errors of R are taken into account when evaluating likelihood function at the decoder:

\[
p(y_{RD,n}|c_{n,i}) = p(y_{RD,n}|D_R = 0, c_{n,i}) \cdot p(D_R = 0|c_{n,i}) + p(y_{RD,n}|D_R = 1, c_{n,i}) \cdot p(D_R = 1|c_{n,i})
\]
The deterministic protocol, DMF mode

- The example protocol:

- Finite State Machine (FSM):
  - Systematic way for analyzing protocols;
  - FSM enters a state in each time-slot;
  - The state determines the action that is going to be taken during the time-slot;
  - The outcome of the action determines the transition to the next state.
Monte Carlo simulation for evaluation of:

- \( \pi_{[1,0]} \) - probability of NACK on the channel S-D;
- \( \pi_{[0,1]} \) - probability of NACK on the channel R-D;
- \( \pi_{[A,B]} \) - prob. of NACK combining A cop. from S and B cop. from R.
Probability transition matrices, DMF

\[
P_I = \begin{pmatrix}
1 - \pi_{[1,0]} & 0 & \pi_{[1,0]} & 0 & \cdots & 0 & \cdots \\
1 - \pi_{[1,0]} & 0 & \pi_{[1,0]} & 0 & \cdots & 0 & \cdots \\
1 - \pi_{[0,1]} & 0 & 0 & \pi_{[0,1]} & \cdots & 0 & \cdots \\
& \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \\
1 - \pi_{[1,0]} & 0 & 0 & 0 & \cdots & \pi_{[1,0]} & \cdots \\
& \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \\
1 - \pi_{[0,1]} & \pi_{[0,1]} & 0 & 0 & \cdots & 0 & \cdots \\
\end{pmatrix}
\]

\[
P_{II} = \begin{pmatrix}
1 - \pi_{[1,0]} & 0 & \pi_{[1,0]} & 0 & \cdots & 0 & \cdots \\
1 - \pi_{[1,0]} & 0 & \pi_{[1,0]} & 0 & \cdots & 0 & \cdots \\
1 - \pi_{[1,1]} & 0 & 0 & \pi_{[1,1]} & \cdots & 0 & \cdots \\
& \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \\
1 - \pi_{[1,N_R]} & 0 & 0 & 0 & \cdots & \pi_{[1,N_R]} & \cdots \\
& \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \\
1 - \pi_{[N_S, N_S N_R]} & \pi_{[N_S, N_S N_R]} & 0 & 0 & \cdots & 0 & \cdots \\
\end{pmatrix}
\]
Performance evaluation using FSMC

- **Performance metrics:**
  - **PDU error rate (PER)** - the proportion of PDUs that were transmitted but never ACK-ed by D;
  - **$\overline{T}$** - average number of transmissions per PDU;
  - **Goodput (G)** - the number of successfully delivered information PDU’s per unit of time.

- **Performance analysis using FSMC representation:**
  - We evaluate the steady state vector $p$ from matrix $P_1$ or $P_{II}$;
  - We obtain the steady state probabilities of the initial states $p_0$ and $p_1$;
  - The performance metrics can be obtained as:

$$
PER = \frac{p_1}{p_0 + p_1}, \quad \overline{T} = \frac{1}{p_0 + p_1}
$$

$$
G = R_c \cdot \frac{1 - PER}{\overline{T}} \left[ \frac{PDUs}{tu} \right] = R_c \cdot p_0 \left[ \frac{PDUs}{tu} \right]
$$
Accurate performance evaluation…. but can become computationally expensive

- As the protocol gets more sophisticated, the FSMC analysis becomes more complex:
  - Increasing the number of nodes or the number of transmissions, the number of states increases very quickly;
  - Switching the Relay from DMF mode to DCF mode, the number of states increases also quickly.

- Resulting number of nodes can quickly become much larger than 100, hence:
  - can we reduce the size of the FSMC while keeping PER, $\overline{T}$ and $G$, untouched ? (equivalent to keep State 0 and State 1 untouched);
  - Since each state is associated with an action, it is more straightforward to aggregate states with the same actions.
State aggregation on the FSMC

- Let us consider the following example:

Let $I$ be a new state resulting from the aggregation of the set of states $\mathcal{I}$, then the steady state probability of being in state $I$ is:

$$z_I = \sum_{i \in \mathcal{I}} p_i.$$

- The transition probabilities between the aggregated states can be evaluated as:

$$Z_{IJ} = \frac{\sum_{i \in \mathcal{I}} p_i \left( \sum_{j \in \mathcal{J}} P_{ij} \right)}{\sum_{i \in \mathcal{I}} p_i}.$$
State aggregation: simplified FSMC, DMF

- The simplified transition matrix contains only four states:

\[
Z = \begin{bmatrix}
1 - \pi_{[1,0]} & 0 & \pi_{[1,0]} & 0 \\
1 - \pi_{[1,0]} & 0 & \pi_{[1,0]} & 0 \\
1 - \pi_{[RF]} & \gamma \cdot \beta \pi_{[RF]} & (1 - \gamma) \pi_{[RF]} & \gamma (1 - \beta) \pi_{[RF]} \\
1 - \pi_{[SF]} & 0 & \pi_{[SF]} & 0
\end{bmatrix}
\]

where parameters \( \pi_{[RF]}, \pi_{[SF]}, \gamma \) and \( \beta \) link the original transition matrix with the simplified one, and can be obtained from the state aggregation procedure;

- The idea of state aggregation can be extended similarly to the case of DCF mode.
Protocol associated with the simplified FSMC

- Aggregation of states:
  - The actions remain the same;
  - Some transitions now will become probabilistic;
  - If we define:
    \( \gamma \) - the probability that R is not allowed to retransmit one more time after it failed previously;
    \( \beta \) - the probability that S is not allowed to retransmit one more time after R failed and is not allowed to retransmit anymore.
  - We can associate the simplified transition matrix \( Z \) with a FSM and a protocol.
Definition of the probabilistic protocol:

- The protocol starts either from State 0 or from State 1;
- If NACK from D: the first retransmission comes from R;

If R is retransmitting, the next action is chosen by realization of two random parameters $V_S$ and $V_R$:

- R retransmits with probability $(1 - \gamma)$;
- S retransmits with probability $(\gamma(1 - \beta))$;
- Neither S or R are allowed to retransmit, with probability $\gamma \cdot \beta$. The PDU is lost.
Comparison with a referent deterministic protocol:

- Comparison in PER and $\overline{T}$;

![Graph 1: PER vs. $E_b/N_0$ for different protocols]

![Graph 2: Average number of transmissions per PDU vs. $E_b/N_0$ for different protocols]
In summary

- HARQ is "yet another" way of adapting the communication protocol to the actual channel values, therefore ...
  - the compatibility with other ingredients of the protocol has to be checked
  - and some adaptation has to be implemented
- but these adaptations also open new possibilities, with improved performance... or not!
- Clearly, non orthogonal superposition instead of orthogonal retransmission has a great potential of improvement...
References (1)

pp. 31-33

pp. 34-37

pp. 38-42
References (2)

pp. 45-47

pp. 55-65

pp. 67-73

pp. 77-87