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# 1 Hybrid Probabilistic-Possibilistic Treatment of Uncertainty in 2 Building Energy Models: A Case Study of Sizing Peak 3 Cooling Loads

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## 18 **Abstract**

19 Optimal sizing of peak loads has proven to be an important factor affecting the overall energy consumption of  
20 HVAC systems. Uncertainty quantification of peak loads enables optimal configuration of the system by opting  
21 for a suitable size factor. However, the representation of uncertainty in HVAC sizing has been limited to  
22 probabilistic analysis and scenario-based cases, which may limit and bias the results. This study provides a  
23 framework for uncertainty representation in building energy modeling, due to both random factors and imprecise  
24 knowledge. The framework is shown by a numerical case study of sizing cooling loads, in which uncertain climatic  
25 data is represented by probability distributions and human-driven activities are described by possibility  
26 distributions. Cooling loads obtained from the hybrid probabilistic-possibilistic propagation of uncertainty are  
27 compared to those obtained by pure probabilistic and pure possibilistic approaches. Results indicate that a pure  
28 possibilistic representation may not provide detailed information on the peak cooling loads, whereas a pure  
29 probabilistic approach may underestimate the effect of uncertain human behavior. The proposed hybrid

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30 representation and propagation of uncertainty in this paper can overcome these issues by proper handling of both  
31 random and limited data.

## 32 **1. Introduction**

33 Energy efficient building design merits special attention as the construction sector holds the largest share of energy  
34 consumption in most countries (Birol 2010). The magnitude of energy consumption by the building sector has  
35 resulted in governmental concerns that has led to implementing global and national regulations for promoting  
36 energy efficiency in buildings (Guillén-Lambea, Rodríguez-Soria, and Marín 2016, Allouhi et al. 2015). To  
37 comply with these regulations, new buildings are designed with special attention to both indoor comfort and  
38 energy efficiency, while existing buildings undergo retrofits at envelope and systems levels. In either case, this  
39 practice is associated with careful (re)design of the Heating Ventilation and Air-Conditioning (HVAC) systems.  
40 Indeed, HVAC design is very sensitive to the implementation of optimal temperature and humidity control, which  
41 may account for up to 60% of the total electric energy consumption of a building (Pérez-Lombard, Ortiz, and Pout  
42 2008, Zhao et al. 2013, Vakiloroya et al. 2014). Studies show that cooling loads dominate the majority of HVAC  
43 energy consumption in office buildings (Wan Mohd Nazi et al. 2017) and optimal configuration of chillers can  
44 result in substantial energy saving (Salari and Askarzadeh 2015).

45 The first necessary step for optimal design of HVAC system (that eventually results in the optimal configuration  
46 of chillers/boilers) is to quantify the peak load on the heating/cooling system, which is commonly known as the  
47 sizing process. Sizing the cooling system is frequently conducted according to the ASHRAE (Mitchell and Braun  
48 2013) procedure that estimates peak loads by means of a nominal day (Design-Day) representing the hottest  
49 climatic conditions throughout a year. The calculation procedure commonly known as the Radiant Time Series  
50 method is a simplified approximation of the Heat Balance method. This procedure is among the most conventional  
51 approaches for estimating peak cooling loads as it is reliable, easy to interpret, computationally inexpensive and  
52 is accompanied by climatic design conditions for thousands of locations around the world (Schuetter, DeBaillie,  
53 and Ahl 2014). However, the ASHRAE procedure is based on deterministic and conservative assumptions, which  
54 overlook the uncertainty in environmental and occupant-related variables. It is argued that this approach results  
55 in overestimating the peak loads (Djunaedy et al. 2011), since a common practice is to apply size factors to the  
56 calculated cooling loads to reduce the risk of an undersized system. Inadequate size factors can cause the HVAC  
57 system to rarely reach the intended load and result in inefficient energy performance (Yik et al. 1999).

58 Optimal characterization of the system by accounting for uncertainty in input quantities can be a reliable  
59 alternative to experimental-based application of safety factors. Various studies have considered uncertain  
60 quantities in the problem of sizing HVAC systems and calculating peak cooling loads (Sun, Huang, and Huang  
61 2015, Cheng, Wang, and Yan 2016, Shen and Sun 2016, Gang, Wang, Xiao, et al. 2015, Burhenne et al. 2013,  
62 Yıldız and Arsan 2011, Cheng et al. 2015, Lee and Schiavon 2014, Cheng et al. 2017). This has allowed more  
63 accurate cooling load best estimates (Mui and Wong 2007), optimal cost-energy design (Rasouli et al. 2013) and  
64 ideal configuration of the cooling systems (Gang, Wang, Shan, et al. 2015). In (Domínguez-Muñoz, Cejudo-  
65 López, and Carrillo-Andrés 2010), the authors propose a method for calculating peak loads based on stochastic  
66 simulation, showing that calculating peak cooling loads while considering uncertainty can reduce the risk of  
67 oversizing the HVAC system. In (Sun et al. 2014), a new framework for sizing the HVAC systems considering  
68 uncertainty is introduced, combining actual weather data and random sampling of other uncertain variables to  
69 obtain the peak loads.

70 In the context of building energy modelling dissimilar levels of information are available for different uncertain  
71 input quantities, which should be handled with their respective appropriate representations (Wang et al. 2016,  
72 Corotis 2015). Considering for example, the uncertainty associated with occupancy, in some cases, measured  
73 historical data are available and occupancy patterns are represented through Markov Models (Page et al. 2008,  
74 Richardson, Thomson, and Infield 2008, Wang, Yan, and Jiang 2011, Tahmasebi and Mahdavi 2015) or clustered  
75 into a number of scenarios (D'Oca and Hong 2015, Miller, Nagy, and Schlueter 2015). Whereas in other cases,  
76 limited historical data on the peak number of occupants were treated by probability density functions (Eisenhower  
77 et al. 2012, Azar and Amodi 2016, Kim 2016). As a result, representing occupant-related uncertainty in building  
78 energy simulation calls for a change of perspective towards a *fit-for-purpose* treatment (Gaetani, Hoes, and Hensen  
79 2016). This challenge is specifically important in peak load calculations, as it can result in (under)oversizing the  
80 HVAC system.

81 In the present work, we distinguish between two types of uncertain quantities: (1) those affected by stochastic  
82 uncertainty, such as climatic parameters, whose randomness is due to their inherent variability, and (2) those  
83 affected by epistemic uncertainty, such as internal gains, whose uncertainty is due to lack of knowledge and  
84 information (Dubois and Prade 2009).

85 Stochastic uncertainty is typically represented by probability distributions whose parameters are estimated using  
86 experimental (Oberkampff et al. 2002). For example, large amount of data collected from weather stations (e.g.

87 temperature, relative humidity and wind speed) are available for estimating the parameters of the probability  
88 distributions representing the stochastic uncertainty of the climatic quantities. Specifically, in this study the  
89 probability distributions representing uncertainties in the urban microclimatic are based on a large dataset of  
90 hourly climatic data, collected from a weather station with close proximity to the studied building.

91 With respect to the epistemic uncertainty, in some cases, laboratory experiments are performed to quantify the  
92 uncertainty of the physical properties of building components (e.g. thermal conductivity, solar heat gain  
93 coefficient, moisture content, specific heat and mass). The repetition of experiments allows for a reliable  
94 information representation, properly characterizing the quantity variability. In other cases where very scarce  
95 information is available (e.g. internal gains of an unoccupied building which may be still in the design phase or  
96 evacuated for restoration) one may resort to the elicitation of expert knowledge to represent uncertainty. Expert  
97 elicitation is often of ambiguous quality in nature, and, therefore, may be difficult to describe through probability  
98 distributions. Let us, for example, consider the case, in which we are aware of the minimum and maximum values  
99 of an uncertain quantity. Since this information does not imply that the probability of occurrence of all  
100 intermediate values is the same, the use of a uniform probability distribution is questionable (Klir 1994).  
101 Similarly, the knowledge of the minimum, maximum and most probable values of an uncertain quantity, does not  
102 allow the use of a triangular probability distribution. To our opinion, the literature of building energy modelling  
103 have frequently (and inappropriately) represented the uncertainty on epistemic quantities, for which scarce  
104 knowledge is available through probability distributions (Tian 2013, D'Oca, Hong, and Langevin 2018).

105 Possibilistic representation of scarce information is an alternative to the conventional probabilistic quantification  
106 of uncertainty (Parsons 1996). This type of representation is particularly helpful in quantifying the uncertainty  
107 associated with incomplete knowledge, where opting for probability distributions may distort the actual  
108 information. In the practice of building energy modelling, a framework for handling both probabilistic and  
109 possibilistic representations of uncertainty is necessary.

110 In this study, we describe different representations of uncertainties involved in the problem of sizing HVAC loads,  
111 in support of a successive optimal design of the HVAC system. To handle both probabilistic and possibilistic  
112 uncertainty representations, we resort to a hybrid uncertainty propagation method (Guyonnet et al. 2003). A  
113 homogeneous post-processing approach is introduced to process the outputs obtained by the hybrid uncertainty  
114 propagation. To highlight the effectiveness of the hybrid method, fully probabilistic and fully possibilistic  
115 treatments of the uncertainties are presented in a comparative numerical case study.

116 The main original contributions of this study include:

- 117 - Introducing a possibilistic representation of occupant-related uncertainty in building energy modelling.
- 118 - Introducing the hybrid uncertainty treatment method for joint propagation of uncertainties represented
- 119 by probability distributions (i.e. climatic data) and possibility distributions (i.e. internal gains).
- 120 - Contrasting the advantages and drawbacks of pure probabilistic and pure possibilistic treatments of
- 121 uncertainty, compared to the introduced hybrid method.

122 The paper is structured as follows: Section 2 provides a detailed description of the possibilistic representation of  
123 scarce knowledge and introduces the hybrid method for uncertainty propagation. Section 3 applies the hybrid  
124 method to a case study of sizing cooling loads for an office building, and demonstrates the results of the presented  
125 method in comparison with pure probabilistic and pure possibilistic representations. Section 4 draws the  
126 conclusions and provides suggestions for future work.

## 127 **2. Possibilistic representation of uncertainty**

128 Uncertainty can be categorized into two classes, i.e. aleatory and epistemic. Aleatory uncertainty deals with  
129 randomness due to inherent variability in the system behavior (e.g. outdoor temperature fluctuation), while  
130 epistemic uncertainty is derived from lack of knowledge on the process or system (e.g. the state of an HVAC  
131 system) (Zio 2013). For example, lack of accessible information on the value of a quantity, which enters as a  
132 parameter of the system or process model, can result in epistemic uncertainty (e.g. due to difficulties in collecting  
133 accurate measurements or the lack of time for data collection).

134 Although one may argue that probability theory is sufficient for handling both aleatory and epistemic uncertainty  
135 (Lindley 1987, Zadeh 2008), recent studies have challenged the probabilistic framework, highlighting its  
136 limitations in representing incomplete knowledge (Dubois and Prade 2001, Cobb and Shenoy 2003, Haenni and  
137 Lehmann 2003). Studies have reasoned that a fully probabilistic approach can distort the actual scarce knowledge  
138 and impact the calculations obtained from the model (Dubois, Prade, and Smets 1996, Kohlas and Monney 2013).

139 It has been shown that misrepresenting epistemic uncertainty - as a result of incomplete knowledge - can lead to  
140 faulty intuitions on the system's reliability (Chen et al. 2016, Zhang et al. 2017), and therefore, imprecise  
141 probabilistic frameworks have been introduced to properly handle both classes of uncertainty (Rocchetta and  
142 Patelli 2016). Take for example the uncertainty affecting the quantification of internal gains in a building, which  
143 is commonly represented by means of uniform, triangular or Gaussian probability density functions (Eisenhower

144 et al. 2012, Hopfe and Hensen 2011, Heo, Choudhary, and Augenbroe 2012, Azar and Amoodi 2016, Kim 2016).  
 145 Adopting a uniform probability distribution for occupant density does not correctly characterize the uncertain  
 146 parameter, as we are not in complete ignorance of the number of occupants. On the other hand, assigning triangular  
 147 or Gaussian distributions to occupant density – based on linguistic propositions of the number of occupants – will  
 148 misrepresent the scarce information, as we do not know the frequency of occurrence, but rather, a range within  
 149 which occupant density may vary (Cooper, Ferson, and Ginzburg 1996, Baudrit, Dubois, and Perrot 2008). In this  
 150 study, we seek a reliable alternative for probabilistic treatment of epistemic uncertainty in building energy  
 151 modelling, namely, a framework that can faithfully represent the imperfect knowledge on occupant behavior  
 152 without distorting the information.

153 This goes under the framework of possibility theory for representing epistemic uncertainty (Zadeh 1999), where  
 154 a possibility distribution value  $\pi(x) \in [0,1]$  is allocated to each real value  $x$  in the range  $X$ . Expressing  
 155  $\pi(x) = 0$ , indicates that the value  $x$  is considered impossible, whereas  $\pi(x) = 1$  implies that at least one  
 156 interpretation of the value  $x$  is completely possible. Take for example the number of occupants in a bank at 10:00  
 157 a.m. of weekdays, where 10 employees work full-time and 7 to 10 visitors are anticipated. In this example,  
 158 observing less than 10 occupants is unexpected and surprising i.e.  $\pi(x) = 0$ , while encountering 17 occupants  
 159 is considered normal and the routine state of affairs i.e.  $\pi(x) = 1$ . Any number of occupants between 10 and  
 160 17, as well as 17 to 20, is characterized with a degree of certainty i.e.  $0 < \pi(x) < 1$ . According to the theory  
 161 of possibility, the likelihood of an event  $A$  is described by two limiting measures, the possibility  $\Pi$  and the  
 162 necessity  $N$ , defined as (Dubois et al. 2000):

$$163 \quad \Pi(A) = \sup_{x \in A} \pi(x) \quad (\text{Eq.1})$$

$$164 \quad N(A) = 1 - \Pi(\bar{A}) = \inf_{x \notin A} (1 - \pi(x)). \quad (\text{Eq.2})$$

165 Let  $\mathcal{P}(\pi)$  be a family of probability distributions such that for any event  $A$ , the probability measure of that event  
 166  $P(A)$  is within the assigned necessity and possibility limits, i.e.  $N(A) \leq P(A) \leq \Pi(A)$ ; then,

$$167 \quad N(A) = \inf P(A) \quad \Pi(A) = \sup P(A) \quad (\text{Eq.3})$$

168 where the **infimum** and **supremum** probabilities represent the largest lower bound and the least upper bound of  
169 all probability measures in  $\mathcal{P}$ . This representation of uncertainty is particularly helpful when the available data is  
170 scarce or only the upper and lower bounds can be defined (e.g. uniform, triangular probability distributions). It is  
171 possible to transform a possibility distribution into a family of probability distributions (Figure 1). For this, a  
172 possibility distribution can be seen as a nested set of confidence intervals (Dubois and Prade 1992), which are the  
173  $\alpha$ -cuts of the distribution i.e.  $[\underline{x}_\alpha, \bar{x}_\alpha] = \{x, \pi(x) \geq \alpha\}$ . In this case, the necessity measure  $N([\underline{x}_\alpha, \bar{x}_\alpha])$   
174 gives the degree of certainty contained in the  $\alpha$ -cuts  $[\underline{x}_\alpha, \bar{x}_\alpha]$ . Then, each interval is represented with a range of  
175 probability measures, such that  $P(X \in [\underline{x}_\alpha, \bar{x}_\alpha]) \geq 1 - \alpha$  and  $P(X \notin [\underline{x}_\alpha, \bar{x}_\alpha]) \approx \alpha$ .

176 **FIGURE 1. TRANSFORMATION OF POSSIBILITY DISTRIBUTION TO BELIEF FUNCTION.**

177

### 178 2.1. Hybrid probabilistic-possibilistic uncertainty propagation

179 Uncertainty propagation is the process of numerically propagating the uncertainty associated to input quantities  
180 of the model to the outputs of that model. In this section, we describe how randomness (represented using  
181 probability distributions) and imprecision (represented using possibility distributions) can be jointly propagated  
182 (Guyonnet et al. 2003). Let us consider a model  $Z = f(X_1, X_2, \dots, X_k, X_{k+1}, \dots, X_n)$ , in which the output is  
183 a function of  $n$  uncertain quantities  $X_i, i = 1, 2, \dots, n$ . For ease of illustration, we consider that the first  $k$   
184 quantities are aleatory with uncertainty represented by the probability distributions  $p_{X_i}(x), i = 1, 2, \dots, k$ ,  
185 whereas the remaining  $n - k$  quantities are epistemic with uncertainty represented by the possibility  
186 distributions  $\pi^{X_i}(x), i = k + 1, k + 2, \dots, n$ . The procedure of propagating both types of uncertainty  
187 consists of two nested loops (Baraldi and Zio 2008): Monte Carlo sampling from the probabilistic distributions  
188 (outer loop) and approximation of the possibilistic distributions through  $\alpha$ -cuts (inner loop). The following steps  
189 are to be performed:

- 190 1. A  $k$  dimensional vector of random realizations  $(x_1, \dots, x_k)$  is generated by Monte Carlo sampling  
191 from the uncertain (probabilistic) quantities  $(X_1, \dots, X_k)$ .



- 192 2.  $\alpha$  is set to zero and the related  $\alpha$ -cuts of all possibility distributions  $(\pi^{X_{k+1}}, \dots, \pi^{X_n})$  are found.
- 193 The possibility distributions are intervals of possible values of the possibilistic quantities
- 194  $(X_{k+1}, \dots, X_n)$ .
- 195 3. The supremum and infimum values  $[\bar{f}_\alpha, \underline{f}_\alpha]$  of  $f(x_1, \dots, x_k, X_{k+1}, \dots, X_n)$  are calculated, where
- 196  $(x_1, \dots, x_k)$  is the vector of Monte Carlo-sampled probabilistic quantities from step 1, and
- 197  $(X_{k+1}, \dots, X_n)$  are the values of the possibilistic quantities obtained from step 2.
- 198 4. A small increment  $(\Delta\alpha)$  is added to the value of  $\alpha$  (e.g.  $\Delta\alpha = 0.05$ ) and the new  $\alpha$ -cuts are found.
- 199 5. Steps 3 and 4 are repeated while  $\alpha \leq 1$ .
- 200 6. Steps 1 to 5 are repeated until the desired number of Monte Carlo samples ( $m$ ) are generated.

201 The outcomes of this procedure are  $m$  random realizations of  $n$  ( $n = \left(\frac{1}{\Delta\alpha}\right)$ ) possibility measures i.e.

202  $(\pi_1, \dots, \pi_m)$ . It is worth noting that the number of realizations ( $m \cdot n$ ) should come from a tradeoff between

203 computational cost and desired accuracy in the uncertainty description. Choosing a large value for  $\Delta\alpha$  will fail to

204 adequately describe the possibilistic representations, while selecting a small value can result in a considerable

205 increase in computation time. Similarly, a small value of  $m$  could fail to appropriately define the probabilistic

206 representations, while a large value could lead to a large computational time.

207 Notice that a two nested loops uncertainty propagation scheme has been adopted in the context of building energy

208 modelling (de Wilde and Tian 2009), but in a different case in which all uncertainties are treated probabilistically.

## 209 2.2. Post-processing analysis of uncertain model outputs

210 The outcomes of the hybrid procedure i.e. the  $m$  possibility distributions  $(\pi_1, \dots, \pi_m)$  can be combined by

211 using different methods, such as separate affectation of probability and possibility descriptions (Dubois, Fargier,

212 and Fortin 2005), fuzzy prediction interval method (Guyonnet et al. 2003) and homogenous post-processing

213 (Baudrit, Dubois, and Guyonnet 2006). The homogeneous post-processing method provides a tradeoff between

214 the other two techniques and is based on the use of evidence theory, which is based on the allocation of basic

215 probability assignments denoted by  $v(E_i)$  to subsets  $(E_i = 1, 2, \dots, K)$  of the uncertain quantity domain, with

216  $\sum_{i=1}^K v_i = 1$  (Shafer 1976). From the basic probability assignments  $v(E_i)$ , it is possible to obtain the belief  
 217 ( $Bel$ ) and plausibility ( $Pl$ ) measures:

$$218 \quad Bel(A) = \sum_{E, E \subseteq A} v(E_i) \quad (\text{Eq.4})$$

$$219 \quad Pl(A) = \sum_{E, E \cap A \neq \emptyset} v(E_i) = 1 - Bel(\bar{A}). \quad (\text{Eq.5})$$

220  $Bel(A)$  measures the degree of belief that  $A$  will occur, while  $Pl(A)$  measures the extent to which  $A$  evidence  
 221 does not support the negation of  $A$  (Mauris et al. 2000). Notice that the possibility theory is a special case of the  
 222 evidence theory through which we can interpret belief and plausibility measures as probability bounds similar to  
 223 what is done in the possibility theory for the interpretation of the necessity and possibility measures (Eq. 1 - Eq.2).  
 224 Therefore, the probability that  $A$  lies within the belief and plausibility interval satisfies:

$$225 \quad \forall P \in \mathcal{P}, Bel(A) \leq P(A) \leq Pl(A). \quad (\text{Eq.6})$$

226 Since the basic probability assignments,  $v$ , of evidence theory generalizes both probability and possibility  
 227 distributions, evidence theory provides a common framework for the hybrid modelling of uncertainty using both  
 228 probability and possibility distributions. The homogeneous post processing method used in this work for  
 229 combining the  $m$  possibility distributions  $\pi_i, i = 1, 2, \dots, m$ , is based on the aggregation of the basic belief  
 230 assignments  $v_i, i = 1, 2, \dots, m$ , which can be obtained from the possibility distribution  $\pi_i$  (Masson and Denœux  
 231 2006), into a joint basic belief assignment. In (Baudrit, Dubois, and Guyonnet 2006) it is shown that this  
 232 aggregation step can be performed by using the evidence theory laws and allows directly computing, for any  
 233 subset  $A$ , the corresponding belief and plausibility measures:

$$234 \quad Pl(A) = \frac{1}{m} \sum_{i=1}^m \Pi_i(A) \quad (\text{Eq.7})$$

$$235 \quad Bel(A) = \frac{1}{m} \sum_{i=1}^m N_i(A) \quad (\text{Eq.8})$$

236 where  $N_i(A)$  and  $\Pi_i(A)$  are the necessity and possibility measures derived from the possibility distributions  
 237  $(\pi_1, \dots, \pi_m)$ , and  $m$  is the number of Monte Carlo realizations.

### 238 3. A case study of cooling load design for an office building

239 The effectiveness of the hybrid propagation method is illustrated by evaluating the peak cooling loads of a 30'000  
240 m<sup>2</sup> office building (Figure 2) located in the center of Milano (Italy), which is undergoing a Core and Shell retrofit.  
241 The renovations cover a wide range of properties i.e. internal architectural layout, building's envelope (walls,  
242 windows, and roofs), and both electrical and mechanical systems. The logic behind choosing the current building  
243 as the case study is that both aleatory and epistemic classes of uncertainty are available due to the in situ conditions  
244 of the building. Aleatory uncertainty is unavoidable due to the randomness in climatic data. Epistemic uncertainty  
245 is inevitable as the building is evacuated for renovation purposes, and therefore, the only information on occupant  
246 behavior is accessible through experts' knowledge on occupancy patterns. The building serves as the headquarters  
247 of a bank, consisting of six stories and two courtyards, as well as a ten story tower. To keep the simulation time  
248 below 30 seconds per simulation (i.e. the maximum time available to run the numerous simulations needed for  
249 performing the following analysis), each story is modelled as single thermal zone, **except for the ground and fifth**  
250 **floors, which account for 3 and 2 thermal zones, respectively, giving rise to a 13 thermal zone model, because, to**  
251 **some extent, they have non-continuous (detached) thermal boundaries.** Non-occupied spaces (i.e. toilets,  
252 staircases, and hallways are modelled as non-conditioned zones (Figure 3). Server rooms and **Uninterrupted Power**  
253 **Supply** rooms are also modelled as non-conditioned zones, as cooling for these spaces will be provided by separate  
254 Variable Refrigerant Flow (VRF) units. The surface to volume ratio of the building is 0.39, with a window to wall  
255 ratio of approximately 42 %. The properties of the renovated envelope components, are displayed in Table 1.  
256 Schedules of occupancy, lighting system, electric equipment and the HVAC system operation for the Design-Day  
257 calculation are provided in Table 2. **It should be noted that keeping the number of thermal zones low (equal to 13)**  
258 **may lead to an underestimation of the peak cooling load, introducing a systematic error into the results obtained**  
259 **with the application of the uncertainty propagation methods that will follow. As a matter of fact, reliance on a**  
260 **detailed model (with 143 zones) and sizing peak cooling loads based on (already mentioned) overconservative**  
261 **ASHRAE method (with 0.4% summer design condition) returns a peak cooling load equal to 615.3 kW, which is**  
262 **3.2% larger than the 595.9 kW peak cooling load that is obtained using the simplified model (with 13 zones)**  
263 **adopted in this study.**

264 **FIGURE 2. THE CASE-STUDY BUILDING AS MODELLED IN DESIGNBUILDER SOFTWARE.**

265 **FIGURE 3. ZONING SCHEMA OF A SAMPLE FLOOR. THE SHADED ZONES ARE CONSIDERED AS NON-**  
266 **CONDITIONED SPACES.**

267 **TABLE 1. ENVELOPE PROPERTIES OF THE CASE STUDY BUILDING (AFTER RETROFIT).**

268 **TABLE 2. PRESUMED PROPERTIES OF INTERNAL GAINS AND HVAC SYSTEMS. \*THE METABOLIC**  
269 **RATE IS CONSIDERED TO BE 120 W/PERSON FOR LIGHT OFFICE WORK.**

270 This latter cooling load (595.9 kW) follows the conventional Design-Day method as described in the ASHRAE  
271 Handbook of Fundamentals (HoF) (Handbook 2009), and is used as the baseline for comparison. In this method,  
272 a 24 hour temperature profile (representing the hottest day of the year) is generated based on the ASHRAE's  
273 "Fraction of Daily Temperature Range" Table. ASHRAE argues that the daily temperature variation is driven by  
274 the heat from the sun, therefore, the table assigns a series of weights (i.e. 24 weights corresponding to each hour  
275 of the day) to a single dry bulb temperature. This process generates a sinusoidal temperature profile that reaches  
276 minimum at early morning and hits peak in the afternoon. In ASHRAE's Design-Day method, a single dry bulb  
277 temperature value is chosen from the "Climatic Design Conditions" Table, which is published for each climate  
278 by ASHRAE. In this study however, ASHRAE's deterministic Design-Day dry bulb temperature is replaced with  
279 randomly sampled temperatures, whose probability of occurrence is obtained from a dataset gathered at a nearby  
280 weather station. Then, daily temperature profiles are created by applying ASHRAE's "Fraction of Daily  
281 Temperature Range" weights to the sampled dry bulb temperatures. A limitation of the method proposed in this  
282 study is that the weights used for generating daily temperature profiles are deterministic. Since the temperature  
283 transition from hour  $t$  to hour  $t + 1$  in different days will always follow the same predefined pattern, the  
284 generated sinusoidal daily temperature profiles is an oversimplification of the reality, as it disregards the inevitable  
285 noise in the actual daily temperature fluctuation. Methods for simulating realistic time-series climatic profiles  
286 have been proposed in (Li and Zio 2012, Sansavini et al. 2014, Rocchetta, Li, and Zio 2015, Naseri et al. 2016),  
287 and can be considered for expanding the current study. The incident solar radiation on each surface is calculated  
288 based on the "Clear-Sky Solar Radiation" method in an hourly interval. ASHRAE's "Clear-Sky Solar Radiation"  
289 is the sum of beam, diffuse and ground reflected solar radiation received on each surface. The HoF design day  
290 sizing procedure provides climatic properties for calculating peak cooling loads, based on three possible choices  
291 of 0.4%, 1% and 2% design conditions. Each design condition is based on the maximum number of hours in a  
292 year, during which the HVAC system may not be able to fully maintain the desired internal conditions. The dry  
293 bulb temperature for 0.4% design condition can be obtained from the 0.4<sup>th</sup> percentile of the inverse cumulative  
294 distribution function of all temperatures that are collected during a 25 year span and scaled to one year. In other

295 words, the 0.4% design condition tends to guarantee adequate cooling power for all conditions, except the most  
296 extremely hot 35 hours of a year (one year is considered to be 8760 hours).

297 The building is modelled using the DesignBuilder software (Tindale 2005), which performs sizing calculations  
298 according to the ASHRAE HoF procedure with the EnergyPlus calculation engine (Trčka and Hensen 2010,  
299 Crawley et al. 2001). The JEPlus simulation manager is utilized to facilitate the rapid initialization of the  
300 EnergyPlus software (Zhang 2009, Zhang and Korolija 2010, Zhang 2012). The same quantities selected in  
301 (Domínguez-Muñoz, Cejudo-López, and Carrillo-Andrés 2010, Sun et al. 2014) through a sensitivity analysis of  
302 the variables affecting the peak cooling loads have been considered as affected by uncertainty. Although the  
303 analysis in (Domínguez-Muñoz, Cejudo-López, and Carrillo-Andrés 2010, Sun et al. 2014) refers to different  
304 buildings and cities, the results of the sensitivity analysis are applicable to the current case study, given the  
305 similarity between the two buildings in terms of comparable characteristics, recurrent internal gain profiles and  
306 climatic design properties (Table 3). Note that the identification of the quantities whose uncertainty should be  
307 treated within the analysis is typically a complex process, which in a general case should be based on the  
308 application of sensitivity and uncertainty analysis techniques. Readers interested in this topic may refer to (Das et  
309 al. 2014) in the context of probabilistic uncertainty representation, (Zahiri, Tavakkoli-Moghaddam, and Pishvae  
310 2014) in the context of possibilistic uncertainty representation, and (Singh and Markeset 2014) in the context of  
311 hybrid probabilistic-possibilistic uncertainty treatment. The climatic variables with uncertain quantities consist of  
312 outdoor dry bulb temperature and relative humidity, while the internal gains are occupant-related variables, with  
313 uncertain quantities concerning the number of occupants (occupant density), lighting power and appliance power.

314 **TABLE 3. ASHRAE CLIMATIC DESIGN PROPERTIES FOR MILANO COMPARED TO ATLANTA AND**  
315 **MALAGA**

316 Concerning the former type of variables, HoF provides the required climatic information for sizing HVAC loads  
317 from databases of weather stations. For the current case study, the HoF resorts to Linate or Malpensa weather  
318 stations, where Linate is often more preferable considering its proximity to Milano city center, where the studied  
319 building is located. However, it is argued that weather stations located in open flat areas outside the city (e.g.  
320 airports), cannot adequately characterize the Urban Heat Island (UHI) intensity of the city center (Thevenard  
321 2010, Paolini et al. 2016, 2017). To address UHI intensity in the cooling load calculations, in this work we use  
322 on-site climatic data from Brera weather station, which is located in the center of Milano city (Lombardia 2006).  
323 The extracted climatic data consist of dry bulb temperature and relative humidity. This data is used to calculate  
324 the design dry bulb temperature, the Mean Coincident Wet Bulb temperature (MCWB), the Mean Coincident Dry

325 Bulb temperature Range (MDBR) for 0.4%, 1% and 2% design conditions. Three “*Control Samples*” are proposed  
326 that correspond to 0.4%, 1% and 2% design conditions of Linate airport weather station. Similarly, 3 “*Case*  
327 *Studies*” are considered for Brera weather station corresponding to 0.4%, 1% and 2% design conditions (Table  
328 4). It is found that for 0.4% design condition, the dry bulb temperature in city center is 0.9°C higher than in  
329 Linate, while the MCWB and the MDBR in the city center are 1.9°C and 1.5°C lower, respectively. The  
330 obtained climatic differences between Linate airport and Brera weather station are in agreement with the UHI  
331 effects reported in literature (Thevenard and Cornick 2013).

332 The latter category of variables with uncertain quantities, namely occupant density, lighting and appliance power,  
333 are parameters that are difficult to quantify, as accurate measurements of human activities require a great amount  
334 of time and effort (Wang et al. 2016). In many cases, little to no information is available on occupant behavior  
335 and, therefore, designers often rely on nominal values provided by ASHRAE or occasionally resort to personal  
336 experience. Meanwhile, the reliability of ASHRAE recommended occupancy profiles for office buildings has also  
337 been subject to debate (Duarte, Van Den Wymelenberg, and Rieger 2013). Therefore, in this work, we sought  
338 experts’ opinions on internal gains during the peak cooling load period. Experts suggested the presence of 1200  
339 people during peak cooling loads, while recommending 150 kW and 180 kW for lighting and appliance power,  
340 respectively. The occupant density (0.04 person/m<sup>2</sup>) suggested by the experts is slightly lower than the ASHRAE’s  
341 nominal value for office spaces density (0.05 person/m<sup>2</sup>). The presumed lighting power density (5 W/m<sup>2</sup>) is  
342 notably lower than ASHRAE’s recommendations for office spaces (10.5 W/m<sup>2</sup>). This is due to the fact that the  
343 newly designed lighting configuration benefits from a fully dimmable LED system (2.2 - 9.6 W/m<sup>2</sup>). Since we  
344 are accounting for a clear sky in the ASHRAE Design-Day calculations and have access to adequate daylight,  
345 experts have set the lighting power density to almost half of the maximum available power.

346 **TABLE 4. COMPARISON OF 0.4%, 1% AND 2% DESIGN CONDITIONS FOR LINATE AIRPORT AND**  
347 **BRERA CITY CENTER.**

348

349 Considering that the process of sizing cooling loads for 0.4%, 1% and 2% design conditions follows a similar  
350 pattern, explaining each design condition individually is redundant. Therefore, in the following sections, the  
351 uncertainty quantification process is fully described for 0.4% design conditions, although it has been performed  
352 and analyzed for all three design conditions.

353 **FIGURE 4. REPRESENTATION OF CLIMATIC VARIABLES WITH UNCERTAIN QUANTITIES: DRY BULB**  
354 **TEMPERATURE (TOP) AND MCWB TEMPERATURE (BOTTOM) THROUGH PROBABILISTIC**

355 REPRESENTATIONS (BLUE-SOLID) AND THEIR TRANSFORMATION INTO POSSIBILISTIC MEASURES  
356 (RED-DASHED).

357 **FIGURE 5. REPRESENTATION OF HUMAN-DOMINATED VARIABLES WITH UNCERTAIN QUANTITIES:**  
358 **OCCUPANT DENSITY (TOP), LIGHTING POWER (MIDDLE) AND APPLIANCE POWER (BOTTOM)**  
359 **THROUGH PROBABILISTIC REPRESENTATIONS (BLUE-SOLID) AND THEIR TRANSFORMATION INTO**  
360 **POSSIBILISTIC MEASURES (RED-DASHED).**

361

### 362 3.1. Probabilistic representation

363 The most common approach for quantifying uncertainty in peak cooling load calculations is through a pure  
364 probabilistic approach. In this method, all measures (temperature, relative humidity, occupant density, lighting  
365 power and appliance power) are represented by means of a probability distribution.

366 Alongside UHI effects, the return period of extreme climatic conditions is also associated with uncertainty (Huang  
367 2014). The collected weather data from Brera weather station cover a 17-year span from 1998 to 2015. Since this  
368 period may not be adequate to account for a reliable return period of 25 years, we incorporated a 0.5°C standard  
369 deviation to the calculated dry bulb temperature of Brera station (Figure 4 - Top). This variation is recommended  
370 for 0.4% cooling design conditions at locations with climatic characteristics comparable to Milano (standard  
371 deviations of 0.4°C and 0.3°C have been suggested for 1% and 2% design conditions, respectively) (Thevenard  
372 2010). The MCWB is calculated from the dry bulb temperature and relative humidity, and adopts a “Burr Type  
373 XII” probability distribution function (Figure 4 - Bottom) (Handbook 2009).

374 The probabilistic approach for characterizing scarce knowledge on occupant behavior is commonly characterized  
375 by probability density functions, either normal (Hopfe and Hensen 2011) or triangular (Heo, Choudhary, and  
376 Augenbroe 2012). Although, the experts recommended the presence of 1200 occupants, they did expect this value  
377 to vary between 1000 and 1250. Similarly, the peak lighting and appliance power are expected to have  $\pm 15$  kW  
378 and  $\pm 30$  kW variation respectively. Here, we adopt triangular probability distributions to characterize the  
379 uncertainties concerning internal gains (Figure 5).

380 A total of 1000 random samples have been generated by Latin hypercube sampling (Helton and Davis 2003) ( $V^i =$   
381  $x_1^i, x_2^i, x_3^i, x_4^i, x_5^i, i = 1, \dots, 1000$ ), and fed to JEPlus tool for sizing calculations. The output is a set of 1000  
382 random cooling loads.

### 383 3.2. Possibilistic representation

384 A pure possibilistic approach is also studied. It is worth noting that this approach is not common in sizing HVAC  
385 loads. However, (Ruparathna, Hewage, and Sadiq 2017) performed a life cycle assessment of building energy

386 retrofit by resorting to the theory of possibility. Furthermore, epistemic uncertainty has been implemented in  
387 HVAC control techniques by means of fuzzy parameters (Sunitha and Behera 2016, Keshtkar et al. 2015).

388 It is also important to note that using a possibilistic representation of an uncertain quantity takes a more  
389 conservative attitude. A possibility measure of 1 is a weaker statement compared to a probability of 1, as the  
390 possibilistic unit value states that the occurrence of an event is possible, expected and not surprising, whereas a  
391 probability of 1 states that the event is certain. For comparison purposes, we perform probability-possibility  
392 transformation to describe a probabilistic representation in possibility theory terms. Detailed descriptions on  
393 probability-possibility (possibility-probability) transformations can be found in (Dubois, Prade, and Sandri 1993,  
394 Dubois et al. 2004, Dubois, Prade, and Smets 2008, Flage et al. 2013). In this study, we adopt the Variable  
395 Transformation method described in (Mouchaweh et al. 2006), as it is easy to implement and provides a reasonable  
396 approximation of both representations. Figure 4 and 5 illustrate the possibilistic transformations of climatic  
397 variables and internal gains, respectively.

398 According to the theory of possibility, distributions may be transformed to a set of  $\alpha$ -cuts (see section 2). The  
399 increment  $\Delta\alpha$  is set to 0.001 and, therefore, 1001 values corresponding to lower bounds of each quantity, and  
400 another 1001 values as the upper limiting bounds are found. Eventually 2002 inputs are generated for all lower  
401 and upper bounds, where each input vector consists of 5 variables i.e.  $\underline{V}^i = \underline{x}_1^i, \underline{x}_2^i, \underline{x}_3^i, \underline{x}_4^i, \underline{x}_5^i$  and  $\overline{V}^i =$   
402  $\overline{x}_1^i, \overline{x}_2^i, \overline{x}_3^i, \overline{x}_4^i, \overline{x}_5^i$ ,  $i = 1, \dots, 1001$ . The inputs are fed into JEPlus tool and two sets of cooling loads are  
403 obtained. The outputs are presented through the possibility  $\Pi$  and necessity  $N$  measures.

### 404 3.3. Hybrid probabilistic – possibilistic representation

405 In this section, random (aleatory) variability is considered for climatic features i.e. dry bulb temperature and  
406 MCWB, which are described by probability distributions (see section 3.1). Since the building was under retrofit,  
407 not much information regarding human-dominated quantities can be obtained. In fact, resorting to experts'  
408 knowledge is the only way to obtain information on an evacuated building without any historical data on occupant  
409 behavior. This situation may be encountered in many cooling design scenarios, and therefore, dealing with scarce  
410 data merits careful consideration from two different points of view. First, the variability of occupant density and  
411 lighting/appliance power are prone to have dependencies between them. Therefore, representing each of these  
412 variables by a separate probability density function will neglect their dependencies and distort the original  
413 knowledge. Second, gathering information on human behavior can be associated with high imprecision, where



414 the information may be categorized into an epistemic class. For this reason, the scarce data on occupant-related  
415 variables has been represented by possibilistic representations (see Section 3.2).

416 The vector of the input quantities which will be considered uncertain is formed by 5 features. Dry bulb temperature  
417 and MCWB uncertainty is described by probability distributions and the two quantities are indicated by  $X_1$  and  
418  $X_2$ , respectively, whereas occupant density, lighting power and appliance power uncertainty is described by  
419 possibility distributions and they are indicated by  $\pi^{X_3}$ ,  $\pi^{X_4}$ , and  $\pi^{X_5}$  respectively. A set of  $m = 1000$  random  
420 samples  $(x_1^i, x_2^i)$ ,  $i = 1, \dots, m$  are generated from their respective probability distributions  $p_{x_1}(x)$  and  
421  $p_{x_2}(x)$ . For the remaining 3 features (possibilistic quantities), the value of  $\Delta\alpha$  is set to 0.01 and therefore 101  
422  $\alpha$ -cuts are defined. The corresponding upper and lower boundaries of each  $\alpha$ -cut are identified from the  
423 possibility distributions of occupant density ( $\pi^{X_3}$ ), lighting power ( $\pi^{X_4}$ ) and appliance power ( $\pi^{X_5}$ ). One vector  
424 containing a temperature value (denoted  $x_1^1$ ) and a MCWB value (denoted  $x_2^1$ ) is selected from the matrix of  $m$   
425 random samples. This vector is concatenated with the lower limiting bounds obtained from the first  $\alpha$ -cut of the  
426 possibility distributions  $\pi^{X_3}(x)$ ,  $\pi^{X_4}(x)$  and  $\pi^{X_5}(x)$ , generating the vector  $\underline{V}_1^1 = x_1^1, x_2^1, \underline{x}_3^1, \underline{x}_4^1, \underline{x}_5^1$ .  
427 Inserting the vector  $\underline{V}_1^1$  in the EnergyPlus \*.idf script and executing the sizing calculation produces a single value,  
428 namely, the infimum bound of all plausible peak cooling loads that correspond to dry-bulb temperature  $x_1^1$ ,  
429 MCWB  $x_2^1$ , and  $\alpha$ -cut one (occupant density  $\underline{x}_3^1$ , lighting power  $\underline{x}_4^1$  and appliances power  $\underline{x}_5^1$ ). Similarly, we  
430 concatenate the same vector of random values ( $x_1^1, x_2^1$ ) with the upper limiting bounds of the first  $\alpha$ -cut of  
431 possibility distributions  $\pi^{X_3}(x)$ ,  $\pi^{X_4}(x)$  and  $\pi^{X_5}(x)$ , generating vector  $\bar{V}_1^1 = x_1^1, x_2^1, \bar{x}_3^1, \bar{x}_4^1, \bar{x}_5^1$ . Once  
432 more, values of dry-bulb temperature, MCWB, occupant density, lighting power and appliance power is replaced  
433 in the EnergyPlus \*.idf script with  $\bar{V}_1^1$  and the sizing calculation is executed. The output of the second simulation  
434 returns the upper bound of all plausible peak cooling loads that correspond to dry-bulb temperature  $x_1^1$ , MCWB  
435  $x_2^1$  and  $\alpha$ -cut one (occupant density  $\bar{x}_3^1$ , lighting power  $\bar{x}_4^1$  and appliances power  $\bar{x}_5^1$ ). At this point we have  
436 obtained the two (upper and lower limiting) values, within which lies all plausible peak cooling loads  
437 corresponding to random vector  $i=1$  ( $x_1^i, x_2^i$ ) and  $\alpha$ -cut  $j=1$  ( $x_3^j, x_4^j, x_5^j$ ). This procedure, based on the use of  
438 the extrema of the  $\alpha$ -cuts of the epistemic quantities for the computation of the  $\alpha$ -cut extrema of the

439 corresponding peak cooling loads is derived from the intuition that the system's response to epistemic uncertain  
440 quantities  $(x_3^j, x_4^j, x_5^j)$  is monotonic, arguing that larger occupant density, lighting power and appliance power  
441 will result in larger internal heat gains, and therefore, larger cooling loads.

442 The process of obtaining the limiting bounds is repeated until the randomly generated vector  $i=1$   $(x_1^i, x_2^i)$  is  
443 concatenated with all  $n=101$  lower bounds and  $n=101$  upper bounds obtained from the  $\alpha$ -cuts of the possibility  
444 distributions  $\pi^{X_3}$ ,  $\pi^{X_4}$  and  $\pi^{X_5}$ . As a result, we obtain  $\underline{V}_1^j = x_1^1, x_2^1, x_3^j, x_4^j, x_5^j, j = 1, \dots, n$  and  $\overline{V}_1^j =$   
445  $x_1^1, x_2^1, x_3^j, x_4^j, x_5^j, j = 1, \dots, n$ , through which 202 sizing calculations are executed. At this stage, we have  
446 defined all plausible ranges of peak cooling loads that correspond to temperature  $x_1^1$ , MCWB  $x_2^1$  and all plausible  
447 internal gains. The same procedure is repeated  $m=1000$  times for different random vectors of dry bulb  
448 temperature and MCWB  $(x_1^i, x_2^i), i = 1, \dots, m$ , and their corresponding  $n=101$   $\alpha$ -cuts  $(x_3^j, x_4^j, x_5^j), j =$   
449  $1, \dots, n$ , producing  $m * n$  peak cooling loads for each limiting (upper/lower) bound. To facilitate this process  
450 through JEPlus, a batch of 1000 iterations are executed, each containing two sets of 101 simulations (1000\*2\*101  
451 jobs). The described procedure is presented in the form of a flowchart in Figure 6. The flowchart consists of two  
452 loops, where the outer loop is responsible for generating random vectors and the inner loop handles the  
453 possibilistic variables. The two loops provide the  $n=101$   $\alpha$ -cuts of  $m=1000$  possibility distributions  $(\underline{V}_i^j, \overline{V}_i^j),$   
454  $i = 1, \dots, m; j = 1, \dots, n$ , which are aggregated by using the homogenous post-processing technique (see  
455 Section 2.2). This produces the the two limiting cumulative distributions,  $Pl$  and  $Bel$ , of the cooling load reported  
456 in Figure 7. In practice, the lower cumulative distribution,  $Bel$ , of the output is obtained by computing the average  
457 of the  $m=1000$  necessity measures and the upper cumulative distribution,  $Pl$ , by computing the average of the  
458  $m=1000$  possibility distributions according to Eqs. 7 and 8.

459 **FIGURE 6. FLOWCHART OF HYBRID PROBABILISTIC-POSSIBILISTIC UNCERTAINTY PROPAGATION**  
460 **DERIVED FROM (BARALDI AND ZIO 2008).**

461 **FIGURE 7. LIMITING PROBABILITY BOUNDS DERIVED FROM THE OUTPUTS OF THE HYBRID**  
462 **METHOD BY USING HOMOGENEOUS POST PROCESSING.**

463

464 **3.4. Results and comparison**

465 In the post-processing stage, the outputs of all three methods (probabilistic, possibilistic and hybrid) are presented  
466 as cumulative distributions. Contrary to the probabilistic approach which returns a single percentile for each  
467 cooling load, the possibilistic and hybrid methods provide a range of percentiles. The range in the possibilistic  
468 approach is enclosed by the possibility ( $\Pi$ ) and necessity ( $N$ ) measures, whereas the hybrid method returns the  
469 boundary of the range through plausibility ( $Pl$ ) and belief ( $Bel$ ) functions. Figure 9 displays a comparison  
470 between the cumulative distributions of the hybrid probabilistic-possibilistic uncertainty propagation and the pure  
471 probabilistic and pure possibilistic methods for 0.4% design condition of the case study. It is observed that the  
472 outputs of the pure probabilistic representation (green continuous line denoted “MC”) are contained between the  
473 plausibility function (blue dashed line denoted “ $Pl$ ”) and the belief functions (blue dotted line denoted “ $Bel$ ”) of  
474 the hybrid method. Also, the  $Pl$  and  $Bel$  functions extracted from the hybrid method are within the possibility  
475 function (red dashed line denoted “ $\Pi$ ”) and necessity function (red dotted line denoted “ $N$ ”) of the pure  
476 possibilistic representation. The effects of treating all uncertain variables probabilistically, versus treating climatic  
477 data probabilistically and internal gains possibilistically, can be seen by the distance between the  $Pl$  and MC  
478 distributions, as well as the distance between MC and  $Bel$ . This distance represents our incomplete knowledge  
479 on internal gains and, therefore, appears as a range with limiting bounds. Similarly, the effects of representing all  
480 data possibilistically, versus treating climatic data probabilistically and internal gains possibilistically, are shown  
481 by the distance between  $\Pi$  and  $Pl$ , as well as the distance between  $Bel$  and  $N$ .

482 **FIGURE 8. MAPPING OUTPUTS FROM PURE PROBABILISTIC (MC), HYBRID (Pl, Bel) AND PURE**  
483 **POSSIBILISTIC ( $\Pi$ , N) UNCERTAINTY TREATMENTS FOR 0.4% DESIGN CONDITION.**

484  
485 From the computational point of view, the hybrid method is considerably more demanding than the pure  
486 probabilistic and possibilistic approaches. Since simulations are only conducted for the sizing process, each  
487 simulation lasts roughly 35 seconds on an Intel® Core™ i7-3610QM @ 2.30- 3.10 GHz processor with 8 logical  
488 cores and enabled multi-processing, executing mini-batches of 8 parallel jobs at a time. The overall simulation  
489 time for the pure probabilistic and the pure possibilistic methods are 1.2 and 2.5 hours respectively (35 seconds  
490 per simulation where 1000 and 2002 simulations are run for the pure probabilistic and possibilistic approach

491 respectively). Execution of the hybrid method requires 250 hours (35 seconds per simulations for 202'000 total  
492 runs).

493 To compare the outputs of each representation, a quantitative assessment is provided (Table 5). Each value  
494 selected from the x axis (denoted “Peak cooling load”), will cross the  $\Pi$ ,  $Pl$ ,  $MC$ ,  $Bel$  and  $N$  distributions in  
495 five different points along the y axis (denoted “ecdf”). Take Control sample A from Table 4 with a peak cooling  
496 load of 595.9 kW. This value intercepts the  $MC$  distribution at the 51<sup>st</sup> percentile, or in other words, would suffice  
497 to cover 51% of uncertain events (Figure 10). Mapping Control Sample A on the  $Pl$  and  $Bel$  measures of the  
498 hybrid method reveals that 595.9 kW would be able to cover between 23% and 73% of uncertain events. The  
499 observed difference between the pure probabilistic approach and the hybrid method is the result of forcing a  
500 probability density function onto human-dominated uncertain measures (i.e. internal gains). Mapping Control  
501 Sample A on the  $\Pi$  and  $N$  distributions will correspond to the 0<sup>th</sup> and 97<sup>th</sup> percentiles, implying that 595.9 kW  
502 would be able to cover anywhere between 0% and 97% of uncertain events. This, basically means that the pure  
503 possibilistic approach provides no information regarding the performance of Control Sample A. As expected, the  
504 pure probabilistic approach returns a crisp output and is the most sensitive of the three methods, whereas the pure  
505 possibilistic approach is the most cautious of the three approaches.

506 **FIGURE 9. MAPPING “CONTROL SAMPLE A” AND “CASE STUDY A” ON THE CUMULATIVE**  
507 **DISTRIBUTIONS OF UNCERTAIN PEAK COOLING LOADS FOR THE 0.4% DESIGN CONDITION. TOP:**  
508 **PURE PROBABILISTIC. MIDDLE: HYBRID PROBABILISTIC-POSSIBILISTIC. BOTTOM: PURE**  
509 **POSSIBILISTIC.**

510 **TABLE 5. ADEQUACY ASSESSMENT OF PEAK COOLING LOADS FOR LINATE AND BRERA STATIONS**  
511 **BASED ON PROBABILISTIC (PROB.), POSSIBILISTIC (POSS.) AND HYBRID UNCERTAINTY**  
512 **TREATMENTS.**

513 In practice, HVAC designers often make up for the uncertainty by applying sizing factors to the estimated peak  
514 cooling load. However, based on the building type and designer’s experience the magnitude of the safety margin  
515 may greatly vary. Previous studies argued that the application of uncertainty quantification is a reliable alternative  
516 to the experimental-based application of safety factors, consequently, lowering the risk of oversizing the system  
517 (Domínguez-Muñoz, Cejudo-López, and Carrillo-Andrés 2010, Sun et al. 2014). In the case study, we are looking  
518 for a value which can cover 90% of uncertain events for the 0.4% design condition. Therefore, we seek the cooling  
519 load corresponding to the 90<sup>th</sup> percentile on the cumulative distributions. Also, we seek the suitable size factor  
520 that enables Control Sample A (595.9 kW) to cover all uncertain events except the worst 10%. The pure  
521 probabilistic approach reaches the 90<sup>th</sup> percentile at 607.2 kW and, therefore, a size factor of 1.019 is assigned.

522 To achieve the same level of confidence on the outputs of the hybrid method we intercept the 90<sup>th</sup> percentile on  
523 the *Bel* distribution, which returns a peak cooling load of 612.3 kW and a 1.028 size factor. The desired  
524 confidence from the pure possibilistic method is obtained by intercepting the 90<sup>th</sup> percentile at the *N* distribution,  
525 returning a peak cooling load of 619.6 kW and a 1.04 size factor. It is worth noting that these results are effected  
526 by the systematic error introduced by the simplification of the building's thermal zones (i.e. 3.2% for the 0.4%  
527 summer design condition). Therefore, designers should take caution when simplifying the thermal zoning, to  
528 obtain a reasonable tradeoff between the overhead systematic error due to building model simplifications and  
529 computational cost of simulations, to be run within the uncertainty propagation method proposed, that are shown  
530 to be methodologically suitable to address these problems of epistemic and aleatory uncertainty propagation.

531 Improper representation of internal gains through probability distributions neglects the lack of accurate  
532 measurements, and the inherent epistemic nature of uncertainty associated with the peak lighting and equipment  
533 power, as well as the maximum number of occupants. In each random generation, the pure probabilistic  
534 approach forces a single probability of occurrence on each internal gain component. Therefore, the pure  
535 probabilistic approach is the most risky among the three uncertainty representation and propagation methods. It  
536 is observed that the output of the pure possibilistic approach is the most conservative of the three methods.  
537 Consequently, a possibilistic representation of climatic variables is prone to overestimate the peak cooling load  
538 and is not recommended for sizing cooling loads under uncertainty. The hybrid method on the other hand,  
539 provides a somewhat conservative range of confidence that is less risky compared to the pure probabilistic  
540 approach and less conservative than the pure possibilistic approach. Therefore, the outputs of the hybrid method  
541 are less likely to undersize the system with respect to the pure probabilistic approach and to oversize it in  
542 comparison with the pure possibilistic approach. Since the pure probabilistic approach returns a crisp output for  
543 every percentile, it can be a suitable starting point for estimating the size factor. In the meantime, the belief and  
544 plausibility measures obtained from the hybrid method can work as the support of the pure probabilistic method,  
545 by quantifying the magnitude of confidence due to incomplete knowledge on internal gains.

546 **FIGURE 10. ASSIGNING SIZE FACTORS FOR DIFFERENT UNCERTAINTY REPRESENTATIONS. TOP:**  
547 **0.4% DESIGN CONDITION, MIDDLE: 1% DESIGN CONDITION, BOTTOM: 2% DESIGN CONDITION.**

548

549 Figure 11 displays the results of all three uncertainty quantification methods i.e. probabilistic (denoted  
550 “Probability Theory), possibilistic (denoted “Possibility Theory”) and hybrid (denoted “Evidence Theory”) for  
551 0.4%, 1% and 2% cooling design conditions. It is observed that the pure probabilistic method and the hybrid

552 method demonstrate close performances, specifically in high percentiles. This phenomenon is generally evident  
553 after the 90<sup>th</sup> percentile, regardless of the type of design condition (0.4%, 0.1% or 2%). It also indicates that in  
554 our case study, a probabilistic representation of internal gains will result in a slightly undersized system. Therefore,  
555 we suggest the application of pure probabilistic uncertainty treatment for sizing cooling loads, only in buildings  
556 with deterministic occupancy profiles. The slopes of the necessity ( $N$ ) and possibility ( $II$ ) measures reveal the  
557 effect of treating climatic variables (dry bulb, MCWB) as scarce knowledge and is mainly evident at 2% design  
558 conditions. The fixed distance between the  $Pl$  and  $Bel$  measures indicate that the magnitude of effect of  
559 uncertainties in human-dominated variables (i.e. internal gains) is constant at all climatic conditions. This is  
560 related to the fact that in our case study, all design conditions are mainly dominated by climatic variables rather  
561 than internal gains.

562 The probabilistic treatment of uncertainty provides a crisp output for each percentile, which may deem risky as it  
563 overlooks the lack of knowledge on activities dominated by human-behavior. The hybrid approach on the other  
564 hand provides a range of values (cooling loads) which are equally plausible. This means that even though the real  
565 value of the peak cooling load is unknown, one may assume that encountering any value outside the provided  
566 range is unlikely (implausible). To contrast how the provided range may assist designers in decision support, a  
567 comparative assessment of resorting to each method is provided. Let us assume that the designer tends to select a  
568 single chiller by using ASHRAE's 1% summer design conditions, while seeking a reliability of 99%. In this case,  
569 choosing the 99<sup>th</sup> percentile from the pure probabilistic framework returns a peak cooling load of 609 kW. In this  
570 case, designers often opt for the closet option that is larger than the estimated load, i.e. a chiller with a reference  
571 cooling capacity of 650 kW. The hybrid method returns two values for the 99<sup>th</sup> percentile which define the range  
572 of most plausible occurrences, i.e. encountering any peak cooling load between 594 kW and 610 kW is equally  
573 plausible. By resorting to the 99<sup>th</sup> percentile of the  $Pl$  measure (594 kW), one can opt for a chiller with a reference  
574 cooling power of 600 kW. It is also possible to rely on the 99<sup>th</sup> percentile of the  $Bel$  measure (610 kW) and select  
575 a chiller with a 650 kW reference cooling capacity, which will be identical to the outcome of a purely probabilistic-  
576 based decision. Therefore, it can be inferred that forcing unavailable information on occupant density, lighting  
577 power and appliance power in the form of probability distributions, results in overestimating the peak cooling  
578 load and eventually eliminating some design choices. It is observed that opting different frameworks for handling  
579 uncertainty can provide dissimilar options, and therefore adds to the degrees of freedom provided to the designer  
580 for decision making. This level of versatility will explicitly come in handy when sequencing multiple chillers for

581 achieving optimal operational COP. Meanwhile, gathering further information on the aforementioned epistemic  
582 uncertain quantities can prevent overestimation of peak cooling loads and provide more reliable cooling capacity.

#### 583 4. Conclusion

584 The existing literature on uncertainty assessment of HVAC systems assumes random variability for all uncertain  
585 parameters. This approach can be challenged in situations of scarce and incomplete knowledge. In this paper, we  
586 address this concern by characterizing aleatory uncertainties with probability and epistemic uncertainty with  
587 possibility representations. The study offers a hybrid uncertainty propagation method so that both aleatory and  
588 epistemic classes of uncertainty are properly introduced to the model. The proposed method is able to propagate  
589 the uncertainty through the model with minimal information loss. Results are presented in the form of plausibility  
590 and belief functions.

591 A numerical case study is provided to compare the effects of adopting the hybrid uncertainty treatment to the pure  
592 probabilistic and possibilistic representations. In this study, climatic data (i.e. temperature and humidity) are  
593 characterized with probability density functions while human-dominated events (i.e. occupant density, appliance  
594 power and lighting power) are represented by possibility distributions. The introduced hybrid treatment of  
595 uncertainty can be useful in support of the optimal design of chillers' configuration (balancing operating expenses  
596 and capital expenditure). The hybrid treatment of uncertainty is particularly important from the HVAC system  
597 design point of view, when designers have limited access to complete information regarding building  
598 characteristics. The hybrid approach may be well fitting for buildings with unpredictable occupancy patterns (e.g.  
599 hospitals), where quantifying the exact number of occupants is extremely difficult to quantify. Therefore, opting  
600 for a hybrid probabilistic-possibilistic framework for Bayesian calibration of building energy models is a potential  
601 for future studies (Pedroni et al. 2015). The hybrid treatment of uncertainty could also be evaluated when dealing  
602 with control regimes of autonomous building components (shading and lighting systems), where incomplete  
603 knowledge over occupant behavior and a system's state may have dependencies. In other situations, i.e. buildings  
604 with more predictable occupancy patterns (e.g. schools), results may be less sensitive to the hybrid approach,  
605 when compared to pure probabilistic representations.

606 In this study, cooling loads are estimated by means of dry bulb and MCWB temperature. However, resorting to  
607 dew point temperature, humidity ratio and mean coincident dry bulb temperature (for dehumidification purposes)  
608 is not expected to alter the methodological scheme followed for the analysis. Also, it is advised to perform  
609 simplifications on the thermal zoning with cautious, and seek for a tradeoff that does not oversimplify the model

610 while maintaining a reasonable simulation time. As a last remark, scarce information (such as occupant density)  
611 can occasionally contain more information than the studied example, although not enough to fit a probability  
612 density function. In such cases, the application of Chebyshev's inequality can provide a suitable fit for all family  
613 of probability distributions and, therefore, is a potential for expanding the provided framework.

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