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## A BILATERAL CONTROL SCHEME OF A HAPTIC-VIRTUAL SYSTEM USING PROPORTIONAL-DELAYED CONTROLLERS

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### ABSTRACT

This paper presents an appropriate method to compute the stability analysis for a bilateral virtual-haptic system with the purpose of maintaining a kinematic correspondence between the haptic and the virtual device. Furthermore, the stability analysis is developed in two senses, independent-delay stability and a general analysis considering a fixed delay. At last, we present experimental results evaluated in a experimental bench using the Phantom Omni haptic device and the Malab-Simulink toolkit Phansim.

### INTRODUCTION

Lately, the development of virtual systems have become a very popular tool in multiple domains, to name a few in prototyping (see, for instance [1]), training for different devices and task assistance/supervision (see, for instance [2,3]). The gap between the design of virtual and teleoperation systems is quite close, in the ideal case both scenarios must have a perfect degree of telep-

resence on the virtual or remote environment, respectively and a fully sense feedback to the operator. As mentioned by [4,5] a system which can have such performance is nothing else than a perfectly transparent system, therefore, in order to create an efficient and comfortable workspace a teleoperation or a virtual system must be perfectly transparent. There are two conditions in order to achieve a perfect transparency, these are: *impedance matching* and *kinematic correspondence*, this paper focuses on the *kinematic correspondence* problem between a haptic and a virtual device.

One of the most used linear controllers in robotics are the so called P-D (Proportional-Derivative) controllers, but even when several techniques has been developed for a better performance there are still some scenarios in which the noise provoked by non-desired high frequency signals can be a problem in some experimental tests. In order to achieve a kinematic correspondence between the haptic and the virtual device this paper studies this problem through the control scheme shown in [6] using a P- $\delta$  (Proportional-Delayed) controller previously proposed by [7] in-

stead of using a P-D controller.

The main result presented in this paper concerns the stability analysis in two different senses, delay-independent stability and a more general stability analysis considering a fixed delay in the controller. We propose a straightforward method to compute the delay independent stability using a geometric approach, and a general stability analysis via  $\mathcal{D}$ -Partition curves, the *Boundary Crossing Theorem* and the *Implicit Function Theorem*. The main goal is to find all the controllers regions in the parameters-space such that the closed loop system satisfies asymptotic stability. The approach proposed in the sequel requires three simple steps:

1. Constructing the *stability crossing boundaries* in the parameters-space.
2. Computing the crossing direction when by an appropriate choice of parameters we are moving from one region to another crossing the same boundary.
3. Explicit localization of the stability regions in the parameters-space.

In the procedure above, the first step define a partition of the parameters-space defined by the controllers parameters in several regions, each region having a constant number of unstable roots for all the controllers gains inside the boundaries of the region, such a method is nothing else than the  $\mathcal{D}$ -Decomposition method introduced by Neimark in 1949 ([8]) and largely discussed in the literature (see, for instance, [9, 10]), here by a *stability crossing boundary* (curve) we understand the set of parameters for which the characteristic equation of the closed-loop system has at least one root on the imaginary axis. In step two it can be seen if a region has more or less unstable roots compared with its neighboring regions by an argument based on the *implicit function theorem*. Finally, concluding with step three this procedure allows detecting the regions in which the controllers guarantee the stability of the closed-loop system. This paper presents experimental results on the haptic device Phantom Omni shown in Fig.1 using a test bench developed in the Malab-Simulink toolkit Phansim (see, for instance, [11]).



FIGURE 1: PHANTOM OMNI HAPTIC DEVICE.

## PRELIMINARY RESULTS

It is well known that the dynamics of a haptic device as the one shown in Fig. 1 can be modeled by a Lagrangian formulation, as follows:

$$M\ddot{\theta} + C\dot{\theta} + B = \lambda, \quad (1)$$

where  $M$  is the inertia matrix,  $C$  is the coriolis matrix,  $B$  is a vector associated to the effect of gravity,  $\lambda$  is the torque input vector and  $\theta$  is the angular position vector, also the obtained model is a non-linear coupled model. Now, heavily influenced by the work made in [6] a few assumptions can be taken into account in order to describe the dynamics of the system as a decoupled time-invariant linear model, this is formed by the three mechanical admittances of each joint in the following form:

$$P(s) := \frac{\Theta(s)}{\Lambda(s)} = \frac{1}{s(ms+b)} \quad (2)$$

where each mechanical admittance  $P(s)$  is nothing else than the transfer function from each torque input  $\Lambda(s)$  to its respectively angular position  $\Theta(s)$  considering the behavior of each of the three joints of the system in a general form.

The bilateral control scheme proposed in this article is shown in Fig. 2 where  $\Lambda_h$  and  $\Lambda_e$  are the exogenous torques related to the human operator and the virtual environment, respectively,  $P_H$  and  $P_V$  are the mechanical admittances of the haptic and the virtual device, respectively; furthermore a similar notation is used for the controllers  $C_H$  and  $C_V$  and the angular positions  $\Theta_H$  and  $\Theta_V$ . This scheme is a variation of the one presented in [6] without considering the time delays due to the signal processing and instead of using a P-D controller a P- $\delta$  controller is proposed and discussed in detail later.

From Fig. 2, after some algebraic manipulations the equations

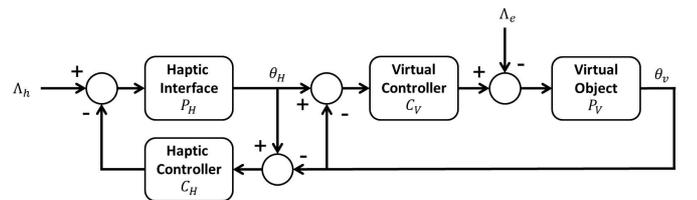


FIGURE 2: BILATERAL CONTROL SCHEME.

describing the system response can be written as follows:

$$\begin{bmatrix} 1 + P_H(s)C_H(s) & -P_H(s)C_H(s) \\ -P_V(s)C_V(s) & 1 + P_V(s)C_V(s) \end{bmatrix} \begin{bmatrix} \Theta_H(s) \\ \Theta_V(s) \end{bmatrix} = \begin{bmatrix} P_H(s)\Lambda_h \\ -P_V(s)\Lambda_e \end{bmatrix}.$$

This article studies the ideal case in which  $P(s) := P_H(s) = P_V(s)$  and we try to find a solution setting  $C(s) := C_H(s) = C_V(s)$ , then,

the characteristic equation of the closed loop system can be written as:

$$2P(s)C(s) + 1 = 0 \quad (3)$$

## MAIN RESULTS - STABILITY ANALYSIS

One of the main goals in this article is to find a solution for the kinematic correspondence problem discussed above using the P- $\delta$  (Proportional-Delayed) controller as:

$$C(s) = K_p + K_\delta e^{-\tau s}, \quad (4)$$

where  $K_p$  and  $K_\delta$  are scalar gains and  $\tau$  is a fixed value. From now on, without losing of generality we can say that the analysis presented in this paper can be used in any of the decoupled time-invariant systems of each joint (2), that is the reason why all the results are expressed in a general form. The characteristic equation of the system (3) can be rewritten as:

$$\Delta(s) := ms^2 + bs + 2K_p + 2K_\delta e^{-\tau s} = 0, \quad (5)$$

which has the structure of a quasi-polynomial and in which  $m$  and  $b$  are the parameters of any of the systems (2) presented in a general form as discussed before.

### Delay-Independent Stability

**Proposition 1.** *Let  $m, b \in \mathbb{R}^+$ , then, the bilateral haptic-virtual system is asymptotically stable independent of the delay value  $\tau$  if the controller gains satisfy the following conditions:*

$$K_\delta < \sqrt{\frac{1}{2} \frac{b^2}{m} K_p - \frac{1}{16} \frac{b^4}{m^2}}, \quad (6)$$

$$K_\delta > -K_p \quad (7)$$

*Proof.* Consider the characteristic equation (5), after substituting  $s = i\omega$ , the following is obtained:

$$(2K_p - m\omega^2 + 2K_\delta \cos(\tau\omega)) + i(b\omega - 2K_\delta \sin(\tau\omega)) = 0.$$

Then, the equation (5) has at least one root on the imaginary axis if and only if there exists  $\omega \in \mathbb{R} \cup \{0\}_+$  such that the real and imaginary part of the above equation are simultaneously equal to zero:

$$2K_p - m\omega^2 + 2K_\delta \cos(\tau\omega) = 0, \quad (8)$$

$$b\omega - 2K_\delta \sin(\tau\omega) = 0. \quad (9)$$

Equations (8) and (9) leads directly to the following expressions:

$$\cos(\tau\omega) = \frac{m\omega^2 - 2K_p}{2K_\delta},$$

$$\sin(\tau\omega) = \frac{b\omega}{2K_\delta}.$$

Using the fact that  $\sin^2(\tau\omega) + \cos^2(\tau\omega) = 1$  provides the following equation:

$$m^2\omega^4 + (b^2 - 4K_p m)\omega^2 + 4(K_p^2 - K_\delta^2) = 0.$$

It is clear that there is no real solution of  $\omega$  if the following condition holds:

$$(b^2 - 4K_p m)^2 - 16(K_p^2 - K_\delta^2)m^2 < 0,$$

which is nothing else than the condition (6). Now, using the fact that the characteristic equation (5) is a retarded type quasi-polynomial in which there is no change of degree if  $m \neq 0$  and  $b \neq 0$  implies that the movement of the roots from a semi-plane to another happens only through the imaginary axis. Therefore, the condition (6) implies that each root of the characteristic equation (5) remains in an specific semi-plane of the complex plane independently of the delay value  $\tau$ .

Finally, analyzing the delay free scenario, substituting  $\tau = 0$  in the characteristic equation (5) leads to a simple second-order polynomial, in which is easy to see that all of its roots are located in the left half plane if  $m, b \in \mathbb{R}^+$  and condition (7) holds.

### $\mathcal{D}$ -partition Curves

**Remark 1.** *It is well known that a continuous variation of the coefficients of a quasi-polynomial in which there is no change of degree implies a continuous variation of the roots of the quasi-polynomial on the complex plane (see, for instance [12, 13]).*

Let  $\Gamma$  denote the set of all  $K = [K_p, K_\delta]^T \in \mathbb{R}^2$  forming the  $K_p - K_\delta$  parameters-space such that (5) has at least one root on the imaginary axis, also let denote  $\Omega$  the frequency set of all real numbers  $\omega$ . Any  $K \in \Gamma$  is known as a *crossing point* and each subset of  $\Gamma$  which is continuous in  $\mathbb{R}^2$  is known as a  *$\mathcal{D}$ -partition curve* or a *stability crossing curve*.

**Remark 2.** *If  $\omega$  is a real number,  $K \in \mathbb{R}^2$  and  $\tau$  is a fixed value such that  $\tau \in \mathbb{R}_+$  then*

$$\Delta(-i\omega) = \overline{\Delta(i\omega)}.$$

Therefore, in order to find all the stability crossing curves we only need to consider positive  $\omega$ .

Bearing in mind the discussion above, the following result describes the set of all crossing points:

**Proposition 2.** Let  $\tau \in \mathbb{R}_+$  be a fixed value. Then, all the crossing points  $K \in \Gamma$  are given by the following equations:

$$K_p(\omega) = \frac{1}{2}(m\omega^2 - b\omega \cot(\tau\omega)), \quad (10)$$

$$K_\delta(\omega) = \frac{1}{2}b\omega \csc(\tau\omega), \quad (11)$$

*Proof.* Consider the characteristic equation (5), then, it is clear that all the crossing points  $K \in \Gamma$  are given by the pairs  $K \in \mathbb{R}^2$  solving (5) for  $s = i\omega$ , it is easy to see that such solutions can be obtained by the solution of the following system of equations:

$$\Re(\Delta(i\omega)) = 0, \quad (12)$$

$$\Im(\Delta(i\omega)) = 0. \quad (13)$$

This system of equations is solved for  $K_p$  and  $K_\delta$  easily in a straightforward way by simply algebraic manipulations, and its solutions are nothing else than the equations (10) and (11), respectively.

**Remark 3.** There exist some particular case in which a solution  $K(\omega)$  is not well defined for  $\omega = 0$ . In this case the stability crossing curve related to  $s = 0$  can be found through (5) as:

$$\Delta(0) = 2K_p + 2K_\delta = 0, \quad (14)$$

or which is the same all the points on the line:

$$K_\delta = -K_p. \quad (15)$$

Given all the stability crossing curves in the  $K_p - K_\delta$  plane, each  $\omega$  for any solution  $K(\omega) = [K_p(\omega)K_\delta(\omega)]$  and considering the boundary crossing theorem (for further details, see, for instance [14]), it is clear that the  $K_p - K_\delta$  plane is partitioned by the stability curves in stable and unstable regions with a finite number of unstable roots.

### Crossing Directions

**Proposition 3.** A root or a pair of roots of equation (5) moves from the left half complex plane (LHP) to the right half complex plane (RHP) as  $K$  crosses a stability crossing curve with  $\omega = 0$  or  $\omega \neq 0$ , respectively in the increasing direction of  $K_x$  for  $x \in \{p, \delta\}$  if:

$$C_d = \tau K_\delta \cos(\eta_x \tau \omega) + (1 - \eta_x)m\omega \sin(\tau\omega) - \frac{b}{2} > 0 \quad (16)$$

where the indicative function  $\eta$  is defined as:

$$\eta_x = \begin{cases} 1 & \text{if } x = p \\ 0 & \text{if } x = \delta \end{cases} \quad (17)$$

Furthermore, the crossing is from the RHP to the LHP if the inequality (16) is reversed.

*Proof.* The proof follows straightforwardly from the fact that the derivative of the implicit function  $s(K_x)$  along  $K_x$  is given by

$$\frac{ds}{dK_x} = -\frac{\frac{\partial \Delta}{\partial K_x}}{\frac{\partial \Delta}{\partial s}}.$$

Thus, the real part of the previous derivative evaluated on a stability crossing point for  $x = p$  and  $x = \delta$  are computed as:

$$\Re \left\{ \frac{ds}{dK_p} \right\} = \frac{\tau K_\delta \cos(\tau\omega) - \frac{b}{2}}{(\frac{b}{2} - \tau K_\delta \cos(\tau\omega))^2 + (m\omega + \tau K_\delta \sin(\tau\omega))^2},$$

$$\Re \left\{ \frac{ds}{dK_\delta} \right\} = \frac{\tau K_\delta + m\omega \sin(\tau\omega) - \frac{b}{2}}{(\frac{b}{2} - \tau K_\delta \cos(\tau\omega))^2 + (m\omega + \tau K_\delta \sin(\tau\omega))^2},$$

respectively. It is clear to see that the sign of both equations can be described by the inequality (16) which is arranged in a practical structure by the indicative function  $\eta$  previously defined. On the other hand, in order to prove the unique existence of a pair of roots crossing from plane to plane as  $K$  crosses a stability crossing curve, we need to prove that the characteristic equation (5) do not have any multiple root in the form  $s = i\omega$  for  $\omega \in \Omega$ . Covering a more general scenario the characteristic equation have at least a pair of roots with multiplicity greater than one if the equations (5) and the following equation:

$$\frac{\partial \Delta}{\partial s} = 2ms + b - 2\tau K_\delta e^{-\tau s} = 0, \quad (18)$$

has a simultaneously solution for  $s = i\omega$  with some  $\omega \in \mathbb{R}$ . After some algebraic manipulations a solution for both equations (5) and (18) can be obtained by the solution of the equation below:

$$\tau m s^2 + (\tau b + 2m)s + (2\tau K_p + b) = 0, \quad (19)$$

it is clear to see that this equation has a solution in the form  $s = i\omega$  with some  $\omega \in \mathbb{R}$  if and only if:

$$\tau b + 2m \neq 0 \quad (20)$$

this situation is not possible for the solution of (19), therefore (5) do not have any multiple root in the form  $s = i\omega$  with  $\omega \in \Omega$ , finally this proofs the arguments discussed above.

**Remark 4.** The stability of the regions mentioned above can be determined as follows. In order to know the stability of an initial region, it is convenient to observe the number  $N$  of roots in the RHP for  $K_\delta = 0$  for some  $K_p \in \mathbb{R}$ , this particular case transforms the characteristic equation (5) in a second order polynomial. Now, given  $N$  for an initial region, an analysis using the proposition above on some neighboring stability crossing curves can be used to obtain the number  $N$  of unstable roots on these neighboring regions, then, an iterative process is used to characterize the qualitative behavior of the system response as the controller gains traverse the  $K_p - K_\delta$  parameters-space.

## EXPERIMENTAL RESULTS

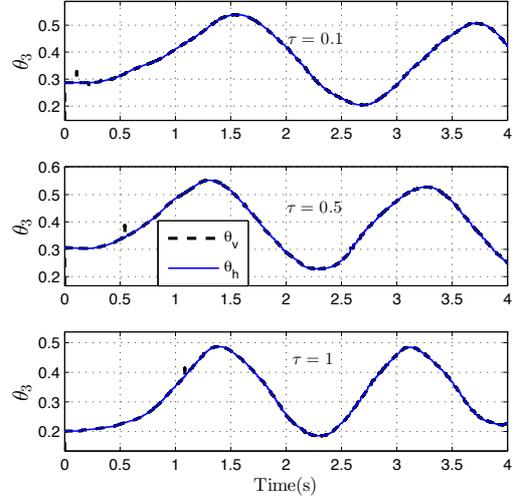
All the results presented below were obtained using the Malab-Simulink toolkit Phansim (see for instance [11]) and the Phantom Omni Haptic Device shown in Fig. 1. The angular position is normalized with respect to the mechanical stops of each joint and the parameters of the linear invariant decoupled models of each joint (2) shown in Tab.1 are computed by the least squares algorithm.

**TABLE 1: PARAMETERS OF THE SYSTEM.**

Joint	$m$	$b$
1	0.0131	0.0941
2	0.0307	0.1719
3	$4.28 \times 10^{-5}$	0.1066

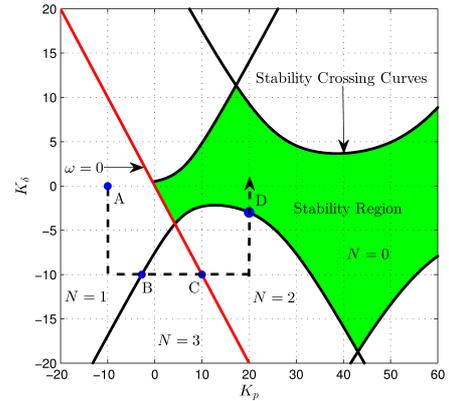
The first test presented is related to the independent-delay stability in the system formed by the joint three, the controller gains are computed using Proposition 1 in which the controller gains chosen are  $K_p = 70$  and  $K_\delta = 30$ , the results are shown in Fig. 3 where it can be observed a stable response and almost a perfect tracking between the haptic and the virtual device under different  $\tau$  values.

Now, considering a fixed  $\tau = 0.1$  the *stability crossing curves* shown in Figs. 4, 5 and 6 are obtained as mentioned in Proposition 2 and Remark 3. In order to describe the process of finding the stability regions it will be taken the system formed by the joint one as an illustrative example. As mentioned in Remark 4 the number of unstable roots ( $N = 1$ ) in an initial region is easily computed from the polynomial obtained in the point  $A$  with  $K_p = -10$  and  $K_\delta = 0$ , then using Proposition 3 the *crossing points*  $B$  and  $C$  are evaluated in(16) in the increasing direction of  $K_p$ , in the same way the *crossing point*  $D$  is evaluated in the increasing direction of  $K_\delta$  obtaining the results shown in Tab. 2. This results reflects how (5) gains two unstable roots as  $K$  crosses for  $B$ , loses one root as the crossing is for  $C$  and finally loses the two remaining roots as the crossing is for  $D$  finding a



**FIGURE 3: SYSTEM RESPONSE UNDER DIFFERENT TIME DELAYS FOR JOINT THREE.**

stability region in the  $K_p - K_\delta$  parameters-space.

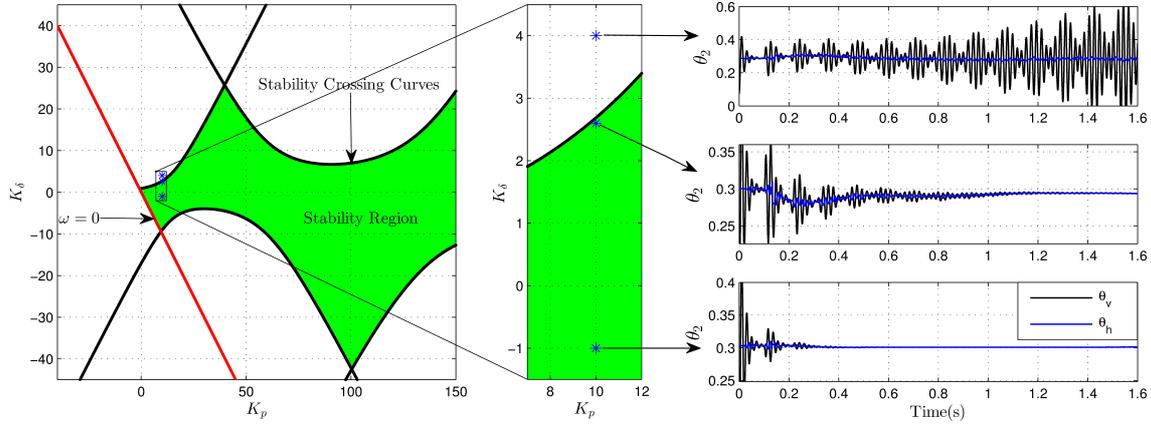


**FIGURE 4: CROSSING DIRECTIONS ANALYSIS FOR THE JOINT ONE.**

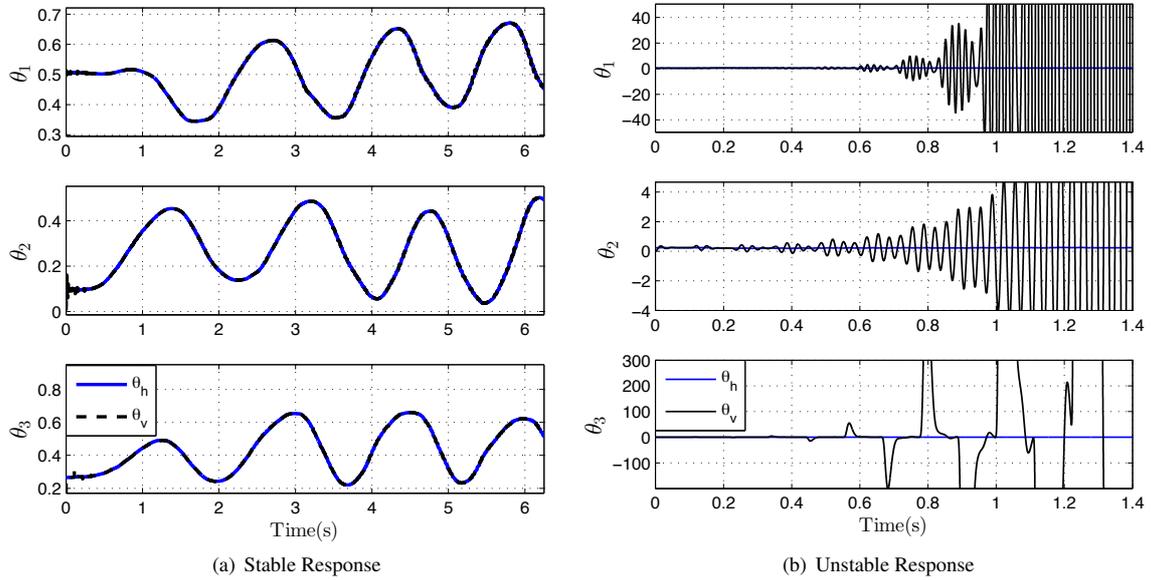
**TABLE 2: CROSSING DIRECTIONS.**

Point	$K_p$	$K_\delta$	$\omega$	$C_d$	Sign
$B$	-2.7619	-10	$\pm 32.974$	0.9409	+
$C$	10	-10	0	-1.0471	-
$D$	20	-3	$\pm 52.95$	-0.9032	-

As a second test, in order to observe the qualitative behaviour of the system response formed by the joint two, the



**FIGURE 6: STABILITY REGIONS VS EXPERIMENTAL BEHAVIOR FOR THE JOINT TWO**



**FIGURE 7: TOTAL SYSTEM RESPONSE**

system is subjected with a fixed  $K_p = 10$  to a change of the gain  $K_\delta$  moving through the stability region, a stable point near a *stability crossing curve* and directly to an unstable region with  $K_\delta = -1$ ,  $K_\delta = 2.6$  and  $K_\delta = 4$ , respectively, this results are shown in Fig. 6. This clearly shows how the system is having an oscillatory behaviour in greater magnitude as  $K$  tends to an unstable region, this is provoked by the continuous moving of the roots of (5) as  $K$  approaches to an *stability crossing curve*.

Finally, we test the complete system choosing the controller gains either in stability regions and unstable regions as shown in Tab.3. The system response in both cases is shown in Fig.7, from Fig.7(a) it can be observed that choosing the controller gains from the stability region leads to a stable response with a good tracking between the haptic and the virtual device, and on the

other hand from Fig.7(b) choosing the controller gains in a unstable region produces an unstable system response and therefore no tracking between both devices is achieved.

**TABLE 3: CONTROLLER GAINS.**

(a) STABLE			(b) UNSTABLE		
Joint	$K_p$	$K_\delta$	Joint	$K_p$	$K_\delta$
1	16	-1	1	6	8
2	14	2	2	5	4
3	10	5	3	2	8

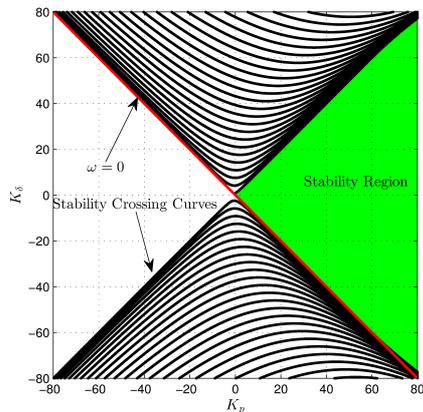


FIGURE 5: STABILITY REGIONS FOR THE JOINT THREE

### CONCLUDING REMARKS

The bilateral control scheme using the P- $\delta$  controller presented in this paper shows a good performance in order to have a kinematic correspondence between the haptic and virtual device. Furthermore, both methods related to the development of the stability analysis are derived in a straightforward way, this makes both methods easily to automate.

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