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***Abstract*—We introduce a new methodology for modeling and analyzing downlink cellular networks, where the Base Stations (BSs) constitute a motion-invariant Point Process (PP) that exhibits some degree of interactions among the points, e.g., spatial inhibition (repulsion) or spatial aggregation (clustering). The proposed approach is based on the theory of Inhomogeneous Poisson PPs (I-PPPs) and is referred to as Inhomogeneous Double Thinning (IDT) approach. The proposed approach consists of approximating a motion-invariant PP with an equivalent PP that is made of the superposition of two conditionally independent I-PPPs. The inhomogeneities are mathematically modeled through two distance-dependent thinning functions and a tractable expression of the coverage probability is obtained.**

***Index Terms*—Cellular Networks, Stochastic Geometry.**

I. Introduction

The theory of Poisson Point Processes (PPPs) has been extensively employed for modeling emerging cellular network architectures [1], [2]- [4]. Modeling cellular networks by using PPPs has the inherent advantage of analytical tractability. Practical cellular network deployments, however, are likely to exhibit some degree of interactions among the locations of the Base Stations (BSs) [5]. Therefore, several other spatial models have been proposed for overcoming the complete spatial randomness property of PPPs [6]- [9].

In this paper, we propose a new approach for modeling cellular networks, which is based on the theory of Inhomogeneous PPPs (I-PPPs) and is referred to as Inhomogeneous Double Thinning (IDT) approach. In particular: i) we introduce two distance-dependent intensity functions to create the inhomogeneities based on the spatial inhibition properties empirically observed in practical cellular networks, ii) we devise a method for approximating the network panorama of the typical user of a generic motion-invariant PP with the network panorama of a probe user located at the origin of the approximating I-PPP, and iii) we introduce a new tractable analytical expression of the coverage probability of cellular networks.

II. The IDT Approach

A. Cellular Networks Modeling Using I-PPPs

The BSs constitute the points of two conditionally independent isotropic I-PPPs, denoted by $\Phi_{\rm BS}^{(F)}$ and $\Phi_{\rm BS}^{(K)}$, with intensity measures $\Lambda_{\Phi_{\rm BS}^{(F)}}(\cdot)$ and $\Lambda_{\Phi_{\rm BS}^{(K)}}(\cdot)$ [4], respectively. Since I-PPPs are non-stationary, we are interested in computing the coverage probability of a probe user that is located at the origin. The BS serving the probe user is assumed to belong to $\Phi_{\rm BS}^{(F)}$ and the interfering BSs are assumed to belong to $\Phi_{\rm BS}^{(K)}$.

The aim of the proposed IDT approach is to approximate a generic network model based on a motion-invariant PP, $\Psi_{\rm BS}$, by appropriately choosing the inhomogeneities of the two conditionally independent I-PPPs $\Phi_{\rm BS}^{(F)}$ and $\Phi_{\rm BS}^{(K)}$. The approximation is devised in a such a way that the coverage probability, $\rm P_{cov}$, of the typical user under the network model corresponding to $\Psi_{\rm BS}$ is closely approximated with the coverage probability, $\widetilde{\rm P}_{\rm cov}^{(\mathbf{o})}$, of the probe user located at the origin under the network model corresponding to $\Phi_{\rm BS}^{(F)}$ and $\Phi_{\rm BS}^{(K)}$, i.e., $\widetilde{\rm P}_{\rm cov}^{(\mathbf{o})} \approx \rm P_{cov}$. More precisely, the spatial inhomogeneities of $\Phi_{\rm BS}^{(F)}$ and $\Phi_{\rm BS}^{(K)}$ are parameterized by using the triplet of parameters $(\rm a_F, b_F, c_F)$ and $(\rm a_K, b_K, c_K)$, respectively, which are obtained from empirical data [5]- [9].

B. Tractable Framework of the Coverage Probability

The following theorem provides a tractable expression of $\widetilde{\rm P}_{\rm cov}^{(\mathbf{o})}$. Two case studies are considered: i) the network is infinitely large and ii) the network has a finite size whose radius is $\rm R_A$. The second case study is useful to compare the analytical framework against estimates obtained by using empirical data, especially for small values of the path-loss exponent. The following notation is used: $\rm T$ is the decoding threshold, $\rm P_{tx}$ is the transmit power, $\sigma_{\rm N}^2$ is the noise power, κ and γ are the path-loss constant and the path-loss exponent, respectively, $\lambda_{\rm BS}$ is the BS density, $\mathbb{1}(\cdot)$ is the indicator function, ${}_2F_1(\cdot,\cdot,\cdot,\cdot)$ is the Gauss hypergeometric function.

Theorem 1: $\widetilde{\rm P}_{\rm cov}^{(\mathbf{o})}$ can be formulated as follows:

$$\begin{aligned}\widetilde{\rm P}_{\rm cov}^{(\mathbf{o})} &= \int_0^{\kappa(d_{\rm F})^\gamma} \exp\left(-\xi {\rm T}\sigma_{\rm N}^2/{\rm P_{tx}}\right)\exp\left(-\mathcal{I}(\xi)\right)\mathcal{U}_{\rm IN}(\xi)\,d\xi \\ &+ \int_{\kappa(d_{\rm F})^\gamma}^{\Theta} \exp\left(-\xi {\rm T}\sigma_{\rm N}^2/{\rm P_{tx}}\right)\exp\left(-\mathcal{I}(\xi)\right)\mathcal{U}_{\rm OUT}(\xi)\,d\xi\end{aligned} \tag{1}$$

where $d_{\rm F} = \frac{c_{\rm F}-b_{\rm F}}{a_{\rm F}}$, $d_{\rm K} = \frac{c_{\rm K}-b_{\rm K}}{a_{\rm K}}$, $\Theta \to \infty$ and $\mathcal{I}(\xi) = \mathcal{I}_\infty(\xi)$ for infinite-size networks, $\Theta \to \kappa {\rm R}_{\rm A}^\gamma$ and $\mathcal{I}(\xi) = \mathcal{I}_{\rm R_A}(\xi)$ for finite-size networks, and $\mathcal{I}_\infty(\cdot)$, $\mathcal{I}_{\rm R_A}(\cdot)$, $\mathcal{U}_{\rm IN}(\cdot)$, $\mathcal{U}_{\rm OUT}(\cdot)$ are defined in (2).

III. Numerical Results

In this section, we illustrate numerical results that substantiate the applicability of the IDT approach for modeling and

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$$
\begin{aligned}
&\mathcal{U}_{\mathrm{IN}}(\xi) = 2\pi\lambda_{\mathrm{BS}} \left(\frac{\mathrm{a_F}}{\gamma\xi} \left(\frac{\xi}{\kappa}\right)^{3/\gamma} + \frac{\mathrm{b_F}}{\gamma\xi} \left(\frac{\xi}{\kappa}\right)^{2/\gamma} \right) \exp\left(-2\pi\lambda_{\mathrm{BS}} \left(\frac{\mathrm{a_F}}{3} \left(\frac{\xi}{\kappa}\right)^{3/\gamma} + \frac{\mathrm{b_F}}{2} \left(\frac{\xi}{\kappa}\right)^{2/\gamma} \right) \right) \\
&\mathcal{U}_{\mathrm{OUT}}(\xi) = 2\pi\lambda_{\mathrm{BS}} \frac{\mathrm{c_F}}{\gamma\xi} \left(\frac{\xi}{\kappa}\right)^{2/\gamma} \exp\left(-2\pi\lambda_{\mathrm{BS}} \left(\frac{\mathrm{c_F}}{2} \left(\frac{\xi}{\kappa}\right)^{2/\gamma} - \frac{(\mathrm{c_F}-\mathrm{b_F})^3}{6\mathrm{a_F^2}} \right) \right) \\
&I_1(\xi) = \frac{\mathrm{a_K d_K^3}}{3} {}_2F_1\left(1, \frac{3}{\gamma}, 1+\frac{3}{\gamma}, -\frac{\kappa}{\mathrm{T}\xi}\mathrm{d_K^\gamma}\right) \mathbb{1}\left(\xi \leqslant \kappa \mathrm{d_K^\gamma}\right) \;,\; \mathcal{I}_2(\xi) = \frac{\mathrm{b_K d_K^2}}{2} {}_2F_1\left(1, \frac{2}{\gamma}, 1+\frac{2}{\gamma}, -\frac{\kappa}{\mathrm{T}\xi}\mathrm{d_K^\gamma}\right) \mathbb{1}\left(\xi \leqslant \kappa \mathrm{d_K^\gamma}\right) \\
&\mathcal{I}_3(\xi) = -\frac{\mathrm{a_K}}{3}\left(\frac{\xi}{\kappa}\right)^{3/\gamma} {}_2F_1\left(1, \frac{3}{\gamma}, 1+\frac{3}{\gamma}, -\frac{1}{\mathrm{T}}\right) \mathbb{1}\left(\xi \leqslant \kappa \mathrm{d_K^\gamma}\right) \;,\; \mathcal{I}_4(\xi) = -\frac{\mathrm{b_K}}{2}\left(\frac{\xi}{\kappa}\right)^{2/\gamma} {}_2F_1\left(1, \frac{2}{\gamma}, 1+\frac{2}{\gamma}, -\frac{1}{\mathrm{T}}\right) \mathbb{1}\left(\xi \leqslant \kappa \mathrm{d_K^\gamma}\right) \\
&\mathcal{I}_5(\xi) = -\frac{\mathrm{c_K d_K^2}}{2}\left(1 - {}_2F_1\left(1, -\frac{2}{\gamma}, 1-\frac{2}{\gamma}, -\frac{\mathrm{T}\xi}{\kappa}\mathrm{d_K^{-\gamma}}\right)\right) \mathbb{1}\left(\xi \leqslant \kappa \mathrm{d_K^\gamma}\right) \;,\; \mathcal{I}_6(\xi) = -\frac{\mathrm{c_K}}{2}\left(\frac{\xi}{\kappa}\right)^{2/\gamma}\left(1 - {}_2F_1\left(1, -\frac{2}{\gamma}, 1-\frac{2}{\gamma}, -\mathrm{T}\right)\right) \mathbb{1}\left(\xi \geqslant \kappa \mathrm{d_K^\gamma}\right) \\
&\mathcal{I}_7(\xi) = \frac{\mathrm{c_K}}{2}\mathrm{R_A^2} {}_2F_1\left(1, \frac{2}{\gamma}, 1+\frac{2}{\gamma}, -\frac{\kappa}{\mathrm{T}\xi}\mathrm{R_A^\gamma}\right) \mathbb{1}\left(\xi \leqslant \kappa \mathrm{d_K^\gamma}\right) \;,\; \mathcal{I}_8(\xi) = -\frac{\mathrm{c_K d_K^2}}{2} {}_2F_1\left(1, \frac{2}{\gamma}, 1+\frac{2}{\gamma}, -\frac{\kappa}{\mathrm{T}\xi}\mathrm{d_K^\gamma}\right) \mathbb{1}\left(\xi \leqslant \kappa \mathrm{d_K^\gamma}\right) \\
&\mathcal{I}_9(\xi) = \frac{\mathrm{c_K}}{2}\mathrm{R_A^2} {}_2F_1\left(1, \frac{2}{\gamma}, 1+\frac{2}{\gamma}, -\frac{\kappa}{\mathrm{T}\xi}\mathrm{R_A^\gamma}\right) \mathbb{1}\left(\xi \geqslant \kappa \mathrm{d_K^\gamma}\right) \;,\; \mathcal{I}_{10}(\xi) = -\frac{\mathrm{c_K}}{2}\left(\frac{\xi}{\kappa}\right)^{2/\gamma} {}_2F_1\left(1, \frac{2}{\gamma}, 1+\frac{2}{\gamma}, -\frac{1}{\mathrm{T}}\right) \mathbb{1}\left(\xi \geqslant \kappa \mathrm{d_K^\gamma}\right) \\
&\mathcal{I}_\infty(\xi) = 2\pi\lambda_{\mathrm{BS}} \sum_{k=1}^{6} \mathcal{I}_k(\xi) \;,\; \mathcal{I}_{\mathrm{R_A}}(\xi) = 2\pi\lambda_{\mathrm{BS}} \left(\sum_{k=1}^{4} \mathcal{I}_k(\xi) + \sum_{k=7}^{10} \mathcal{I}_k(\xi) \right)
\end{aligned} \tag{2}
$$

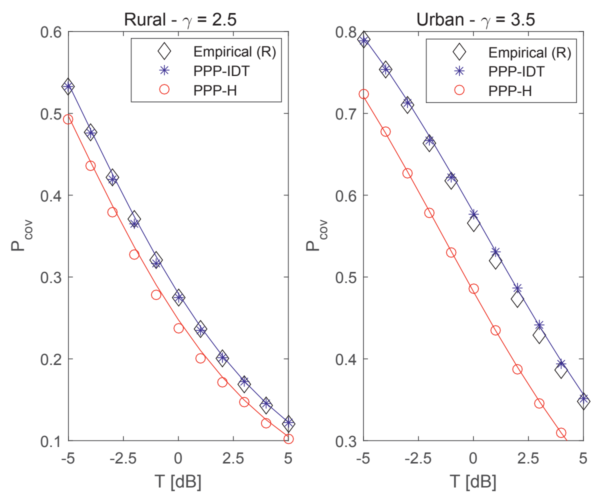

Fig. 1: Coverage probability of a GPP spatial model [5]. Markers: simulations. Solid lines: Framework (*Theorem 1*).

analyzing practical cellular network deployments. The numerical results are depicted in Figs. 1. In the figure, three curves are shown: i) the curve labelled "Empirical (R)" corresponds to a cellular network whose BSs are distributed according to a Ginibre Point Process (GPP) [7], ii) the curve labelled "PPP-IDT" is obtained by using the IDT approach in *Theorem 1*, and iii) the curve labelled "PPP-H" corresponds to the benchmark cellular network deployment where the BSs are distributed according to a homogeneous PPP of density λ_{BS}. We evince that the IDT approach is accurate, tractable and capable of reproducing the spatial interactions of a widely used non-PPP spatial model, such as the GPP model.

IV. Conclusion

In this paper, we have introduced a new tractable approach for modeling and analyzing cellular networks where the locations of the BSs exhibit some degree of spatial interactions. The proposed IDT approach has been shown to be analytically tractable and accurate with the aid of Monte Carlo simulations. It can be used for analyzing and optimizing several emerging transmission technologies, e.g., [10] and [11].

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